Nuclear (μ^-, e^+) conversion mediated by Majorana neutrinos

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We study the lepton number violating (LNV) process of (μ^-, e^+) conversion in nuclei mediated by the exchange of light and heavy Majorana neutrinos. Nuclear structure calculations have been carried out for the case of an experimentally interesting nucleus ^{48}Ti in the framework of a renormalized proton-neutron quasiparticle random phase approximation. We demonstrate that the imaginary part of the amplitude of a light Majorana neutrino exchange mechanism gives an appreciable contribution to the (μ^-, e^+) conversion rate. This specific feature is absent in the allied case of $0\nu\beta\beta$ decay. Using the present neutrino oscillations, tritium beta decay, accelerator, and cosmological data, we derived the limits on the effective masses of light $\langle m\rangle_{\mu e}$ and heavy $\langle M_N^{-1}\rangle_{\mu e}$ neutrinos. The expected rates of nuclear (μ^-, e^+) conversion, corresponding to these limits, were found to be so small that even within a distant future the (μ^-, e^+) conversion experiments will hardly be able to detect the neutrino signal. Therefore, searches for this LNV process can only rely on the presence of certain physics beyond the trivial extension of the standard model by inclusion of massive Majorana neutrinos.

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I. INTRODUCTION

Lepton number L conservation is one of the most obscure sides of the standard model (SM) not supported by an underlying principle and following from an accidental interplay between gauge symmetry and field content. Any deviation from the SM structure may introduce L nonconservation. Over the years the possibility of lepton number nonconservation has been attracting a great deal of theoretical and experimental efforts since any positive experimental signal of lepton number violating (LNV) would point to physics beyond the SM. The simplest extension of the SM allowing LNV processes implies inclusion of massive Majorana neutrinos with the $\Delta L=2$ mass term introducing the necessary source of LNV. However, the role of neutrinos in LNV processes is more intricate. The fundamental fact [1] consists of the following: observation of any LNV process would prove that neutrinos are massive Majorana particles. This is true even if their direct contribution to this process is negligible and the dominant contribution has nothing to do with neutrinos.

Recent neutrino oscillation experiments established the presence of small nonzero neutrino masses; a fact that itself points to physics beyond the SM. However, neutrino oscillations are not sensitive to the nature of neutrinos; they could be either Majorana or Dirac particles leading to the same oscillation observables.

The principal question if neutrinos are Majorana or Dirac particles can be answered only by searching for LNV processes, which, as commented above, are intimately related to the nature of neutrinos.

Various LNV processes have been discussed in the literature in this respect (for review see [2]). In principle, they can probe Majorana neutrino contribution and provide information on the so-called effective masses $\langle m_\nu\rangle_{\alpha\beta}$ and $\langle M_N^{-1}\rangle_{\alpha\beta}$ of light and heavy Majorana neutrinos (for definition see Sec. II). These quantities under certain assumptions are related to the entries of the Majorana neutrino mass matrix $M_{\alpha\beta}^{(\nu)}$.

Among these processes there are a few LNV nuclear processes having prospects for experimental searches: neutrinoless double beta decay ($0\nu\beta\beta$), muon to positron (μ^-, e^+) conversion, and, probably, muon to antimuon (μ^-, μ^+) conversion [3,4].

Currently the most sensitive experiments intended to distinguish the Majorana nature of neutrinos are those searching for $0\nu\beta\beta$ decay [5–8]. The nuclear theory side [9–11] of this process has been significantly improved in the last decade (see also [12–14] and references therein) allowing reliable extraction of fundamental particle physics parameters from experimental data.

The (μ^-, e^+) conversion is another LNV nuclear process searched for experimentally. The important role of the muon as a test particle for new physics beyond the SM has been recognized a long time ago. When negative muons penetrate into matter they can be trapped to atomic orbits. Then the bound muon may disappear, either decaying into one electron and two neutrinos or being captured by the nucleus, i.e., due to ordinary muon capture. These two processes, conserving both total lepton number and lepton flavors, are the SM processes and have been well studied both theoretically and experimentally. The physics beyond the SM resides in yet nonobserved channels of muon capture: muon-electron (μ^-, e^-) and muon-positron (μ^-, e^+) conversions in nuclei [15–30]

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$$(A,Z) + \mu_b^- \to e^- + (A,Z)^* ,$$

$$(A,Z) + \mu_b^- \to e^+ + (A,Z-2)^* . \quad (1)$$

Apparently, the (μ^-, e^-) conversion process violates lepton flavor L_f and conserves the total lepton number L, while (μ^-, e^+) conversions violate both of them. Additional differences between the (μ^-, e^-) and (μ^-, e^+) lie on the nuclear physics side. The first process can proceed on one nucleon of the participating nucleus while the second process involves two nucleons as dictated by charge conservation [16,18]. Note also that the (μ^-, e^-) conversion amplitude is quadratic and (μ^-, e^+) amplitude linear in the light neutrino mass. Thus the second process looks more sensitive to the light neutrino masses.

The currently best experimental limit on the (μ^-, e^+) conversion branching ratio has been established at PSI [31] for the ^{48}Ti nuclear target

$$R^{(\mu e^+)}(\mathrm{Ti}) = \frac{\Gamma(\mu^- + {}^{48}\mathrm{Ti} \to e^+ + {}^{48}\mathrm{Ca})}{\Gamma(\mu^- + {}^{48}\mathrm{Ti} \to \nu_\mu + {}^{48}\mathrm{Sc})} < 4.3 \times 10^{-12}. \quad (2)$$

A significant improvement of this limit is now expected in the near-future experiments: SINDRUM II (PSI) with ^{48}Ti target [31], MECO (Brookhaven) with ^{27}Al target [32], and PRIME (Tokyo) with ^{48}Ti target [33].

In the present paper we study light and heavy Majorana neutrino exchange mechanisms of the (μ^-, e^+) conversion that are conceptually most natural and simple. One of the main motivations of this study comes from the nuclear physics side of this process. The nuclear theory of (μ^-, e^+) conversion is not yet well elaborated and may show interesting features absent in the other LNV processes, such as the $0\nu\beta\beta$ decay. For instance, as we will demonstrate, the imaginary part of the (μ^-, e^+) conversion amplitude in the case of light Majorana exchange gives an appreciable contribution to the rate of this process, a fact that has not been recognized for a long time. Studying the most simple case of (μ^-, e^+) conversion via Majorana neutrino exchange, we have in mind that this process may receive contribution from other mechanisms offered by various models beyond the SM, such as the R-parity violating supersymmetric models, the leptoquark extensions of the SM, etc. Some of these mechanisms may involve light or heavy neutrino exchange and, therefore, in the part of nuclear structure calculations, they may resemble the ordinary neutrino mechanisms. Thus, our present study can be viewed as a step toward a more general description of (μ^-, e^+) conversion including all the possible mechanisms.

Below, we develop a detailed nuclear structure theory for the light and heavy neutrino exchange mechanisms of this process on the basis of the nuclear proton-neutron renormalized quasiparticle random phase approximation (pn-QRPA) [34,35]. We calculate the nuclear matrix elements of (μ^-, e^+) conversion in ^{48}Ti, which serves as target nucleus in the SINDRUM [31] and PRIME [33] experiments.

Existing limits on neutrino masses and mixing from neutrino oscillation phenomenology and other observational data allow us to estimate the typical rate of this process, assuming the dominance of light or heavy Majorana neutrino exchange mechanisms. Extremely low values for these rates, derived in this way, leave no chance to detect a neutrino signal in the (μ^-, e^+) conversion even within a distant future and, thus, to derive information on the effective masses $\langle m_\nu \rangle_{\mu e}$ and $\langle M_N^{-1} \rangle_{\mu e}$ from this process. This conclusion, nevertheless, does not diminish the importance of experiments searching for (μ^-, e^+) conversion because its observation would be an unambiguous signal of a nontrivial physics beyond the SM.

The paper is organized as follows. In Sec. II we discuss some general issues of Majorana neutrinos for LNV processes. Section III deals with the current limits on the effective Majorana neutrino masses entering to the (μ^-, e^+) conversion amplitude. The amplitude and rate of (μ^-, e^+) conversion are derived in Sec. IV. The details of nuclear calculations for (μ^-, e^+) conversion in ^{48}Ti are given in Sec. V. In Sec. VI we discuss the possible impact of (μ^-, e^+) conversion experiments on neutrino physics and visa versa. In Sec. VII we summarize our results and conclusions.

II. MAJORANA NEUTRINOS IN LNV PROCESSES

The finite masses of neutrinos are tightly related to the problem of lepton flavor and/or number violation. The Dirac, Majorana, and Dirac-Majorana neutrino mass terms in the Lagrangian offer different neutrino mixing schemes and allow various lepton number and/or flavor violating processes [36–38].

Let us consider the generic case of neutrino field contents with the three left-handed weak doublet neutrinos $\nu'_{Li} = (\nu'_{Le}, \nu'_{L\mu}, \nu'_{L\tau})$ and n species of the SM singlet right-handed neutrinos $\nu'_{Ri} = (\nu'_{R1}, \ldots, \nu'_{Rn})$. The mass term for this set of fields can be written in a general form as

$$-\frac{1}{2}\overline{\nu'}\mathcal{M}^{(\nu)}\nu'^{c} + \mathrm{H.c.} = -\frac{1}{2}(\overline{\nu'_L}, \overline{\nu'^{c}_R})\begin{pmatrix} \mathcal{M}_L & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_R \end{pmatrix}\begin{pmatrix} \nu'^{c}_L \\ \nu'_R \end{pmatrix} + \mathrm{H.c.} = -\frac{1}{2}\sum_{i=1}^{3+n} m_i \overline{\nu^c}_i \nu_i + \mathrm{H.c.} \quad (3)$$

Here $\mathcal{M}_L$, $\mathcal{M}_R$ are 3×3 and $n\times n$ symmetric Majorana mass matrices and M_D is a $3\times n$ Dirac-type matrix. Rotating the neutrino mass matrix by the unitary transformation to the diagonal form

$$U^T \mathcal{M}^{(\nu)} U = \mathrm{diag}\{m_i\}, \quad (4)$$

we end up with $n+3$ Majorana neutrinos $\nu_i = U^*_{ki}\nu'_k$ with the masses m_i. In special cases there may appear among them pairs with masses degenerate in absolute values. Each of these pairs can be collected into a Dirac neutrino field. This situation corresponds to conservation of certain lepton numbers assigned to these Dirac fields.

The considered generic model must contain at least three observable light neutrinos while the other states may be of arbitrary mass. In particular, they may include intermediate and heavy mass states. The presence or absence of these neutrino states is a question for experimental searches.

The favored neutrino model has to accommodate modern neutrino phenomenology in a natural way, in particular, to answer the question of the smallness of neutrino masses compared to the charged lepton ones. The most prominent guiding principle in this problem is the seesaw mechanism. It suggests that the typical scale of M^D matrix elements in Eq. (3) is comparable to the masses of charged leptons, meanwhile $\mathcal{M}_R$ is associated to a large hypothetical scale of lepton number violation, such as $M_{LNV} \approx 10^{12}$ GeV. Then the diagonalization in Eq. (4) brings very light ν_k and very heavy N_k Majorana neutrinos. This mechanism can be realized in various models beyond the SM with significantly lower scales, $M_{LNV} \sim 1$ TeV, leading to the neutrino masses and mixing consistent with the observational data. A particular example is given by the class of supersymmetric models with bilinear R-parity violation (see, for instance, Ref. [39] and references therein). In these models the heavy Majorana neutrinos have moderately large masses ~ 1 TeV and even lower, giving them phenomenological significance via *a priori* nonnegligible contributions to LNV processes. In the present paper we examine the contributions of light and heavy Majorana neutrinos to (μ^-, e^+) conversion.

In general, the flavor neutrino states are the superpositions of light (ν_k) and heavy (N_k) Majorana mass eigenstates

$$\nu_l(x) = \sum_{k=\text{light}} U_{lk}\nu_k(x) + \sum_{k=\text{heavy}} U_{lk}N_k(x), \tag{5}$$

with the masses m_k and M_k, respectively. Here U is neutrino mixing matrix.

Now let us consider LNV processes with two charged (anti-)leptons $(\overline{l_\alpha})l_\alpha$, $(\overline{l_\beta})l_\beta$ in the initial or final state, or with one $(\overline{l_\alpha})l_\alpha$ in the initial and another l_β, $(\overline{l_\beta})$ in the final state. Assume that the characteristic energy scale of this process is q_0 and that light and heavy neutrino masses satisfy the conditions

$$m_k \ll q_0 \text{ for } \forall\, k, \quad \text{and } M_k \gg q_0 \text{ for } \forall\, k. \tag{6}$$

Then neutrino contribution to its amplitude $\mathcal{A}_{\alpha\beta}$ can be represented in the form (for more details see, for instance, Ref. [40])

$$\mathcal{A}_{\alpha\beta} = \langle m_\nu\rangle_{\alpha\beta} G_\nu + \langle M_N^{-1}\rangle_{\alpha\beta} G_N, \tag{7}$$

where G_ν, G_N are the corresponding structure factors and

$$\langle m_\nu\rangle_{\alpha\beta} = \sum_{k=\text{light}} U_{\alpha k}U_{\beta k} m_k, \tag{8}$$

$$\langle M_N^{-1}\rangle_{\alpha\beta} = \sum_{k=\text{heavy}} \frac{U_{\alpha k}U_{\beta k}}{M_k} \tag{9}$$

are the effective light and heavy neutrino masses, respectively.

The following comment is in order. If the mixing of heavy neutrino states to the active flavors is negligible, then the light neutrino sector can be characterized by the effective light neutrino mass matrix $\mathcal{M}^{(\nu)}$, which satisfies the relation

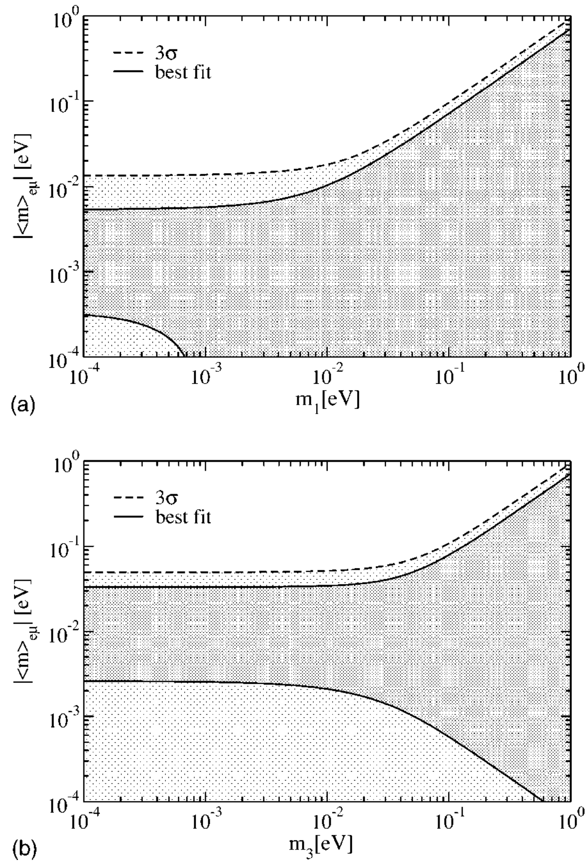

FIG. 1. Allowed regions of the effective Majorana neutrino mass $|\langle m\rangle_{\mu e}|$ for normal (left panel) and inverted (right panel) hierarchy vs the mass of lightest neutrino state: m_1 and m_3, respectively.

$$\mathcal{M}^{(\nu)}_{\alpha\beta} = \langle m_\nu\rangle_{\alpha\beta}. \tag{10}$$

If the heavy Majorana neutrino states N are appreciably mixed with the active neutrino flavors, then this equality no longer holds and LNV processes do not provide direct limits on Majorana neutrino mass matrix elements.

From the nonobservation of the LNV processes one can deduce the upper limits on the corresponding parameters $\langle m_\nu\rangle$ and $\langle M_N^{-1}\rangle$. It must be stressed that these limits have physical sense only if they satisfy the following consistency conditions:

$$|\langle m_\nu\rangle_{\alpha\beta}| \ll q_0, \quad |\langle M_N^{-1}\rangle_{\alpha\beta}|^{-1} \gg q_0, \tag{11}$$

which follow from the conditions of Eq. (6).

Currently the most stringent limits of this type stem from the $0\nu\beta\beta$ decay. Its amplitude, written in the form of Eq. (7), depends on the parameters $\langle m_\nu\rangle_{ee}$ and $\langle M^{-1}\rangle_{ee}$. Assuming that only light or heavy exchange mechanism is in operation, the following limits have been derived from the experimental data [5,13,41]:

$$|\langle m_\nu\rangle_{ee}| \leq 0.55 \text{ eV}, \quad |\langle M_N^{-1}\rangle_{ee}|^{-1} \geq 9 \times 10^7 \text{ GeV}. \tag{12}$$

Note that these limits satisfy the consistency conditions in Eq. (11) because the characteristic energy scale of $0\nu\beta\beta$ decay is of the order of $q_0 \sim 100$ MeV.

As we shall demonstrate, the current and near-future experimental searches for (μ^-, e^+) conversion are unable to reach meaningful limits on the corresponding parameters $\langle m_\nu \rangle_{\mu e}$ and $\langle M^{-1} \rangle_{\mu e}$, satisfying the consistency conditions in Eq. (11). Moreover, the limits following from the neutrino observations and cosmological data show that the sensitivities of (μ^-, e^+) conversion experiments are too far from being able to detect neutrino contributions. With the lucky exception of the $0\nu\beta\beta$ decay, this is the fate of all the experiments searching for other known LNV processes (see, for instance, [42]).

III. EFFECTIVE NEUTRINO MASS FROM NEUTRINO OBSERVATIONS

Here, we estimate the effective light $\langle m_\nu \rangle_{\mu e}$ and heavy $\langle M_N^{-1} \rangle_{\mu e}$ neutrino effective masses that determine light and heavy Majorana neutrino contributions to $(\mu^- - e^+)$ conversion according to the general formula in Eq. (7). To this end we utilize the existing neutrino oscillation, cosmological, and accelerator data, applying the methods previously used for the analysis of $\langle m_\nu \rangle_{ee}$ relevant for $0\nu\beta\beta$ decay (see, for instance, [13,14] and references therein).

Let us start with the three light neutrino scenario without heavy neutrinos. In this case we have

$$|\langle m_\nu \rangle_{\mu e}| = |U_{e1}U_{\mu 1}m_1 + U_{e2}U_{\mu 2}m_2 + U_{e3}U_{\mu 3}m_3|, \quad (13)$$

with the unitary Pontecorvo-Maki-Nakagawa-Sakata neutrino mixing matrix U. In its standard parametrization (e.g., [37]) it takes the form

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\alpha_{21}/2)} & 0 \\ 0 & 0 & e^{i(\alpha_{31}/2)} \end{pmatrix}, \quad (14)$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$. The three mixing angles vary in the range $0 \leqslant \theta_{ij} \leqslant \pi/2$. In addition, Majorana neutrino mixing matrix U contains three CP-violating phases: one Dirac δ and two Majorana phases α_{21}, α_{31}.

The global analysis of the solar, atmospheric, reactor, and accelerator neutrino oscillation data gives the following values of the neutrino mixing angles [43]:

$$\sin^2\theta_{12} = 0.30[0.23 - 0.39], \quad (15)$$

$$\sin^2\theta_{13} = 0.006[< 0.054], \quad (16)$$

$$\sin^2\theta_{23} = 0.52[0.31 - 0.72], \quad (17)$$

and the two independent mass-squared differences.[1]

$$\Delta m^2_{sol} = 6.9 \times 10^{-5}\ \mathrm{eV}^2\ [(5.4 - 9.5) \times 10^{-5}\ \mathrm{eV}^2], \quad (18)$$

$$\Delta m^2_{atm} = 2.6 \times 10^{-3}\ \mathrm{eV}^2\ [(1.4 - 3.7) \times 10^{-3}\ \mathrm{eV}^2]. \quad (19)$$

The values in the square brackets correspond to the 3σ intervals.

Using the above best values for the neutrino oscillation parameters we estimate the effective light Majorana neutrino mass $|\langle m_\nu \rangle_{\mu e}|$ for the three standard cases of neutrino mass spectrum.

(i) *Normal hierarchy*: $m_1 \ll m_2 \ll m_3$. In this case $\Delta m^2_{21} \approx \Delta m^2_{sol}$, $\Delta m^2_{32} \approx \Delta m^2_{atm}$ Therefore, one has

$$m_1 \ll \sqrt{\Delta m^2_{sol}}, \quad m_2 \simeq \sqrt{\Delta m^2_{sol}}, \quad m_3 \simeq \sqrt{\Delta m^2_{atm}}. \quad (20)$$

[1] Mass-squared difference is defined as $\Delta m^2_{ij} = m_i^2 - m_j^2$.

(ii) *Inverted hierarchy*: $m_3 \ll m_1 < m_2$. Now, $\Delta m^2_{21} \approx \Delta m^2_{sol}$, $\Delta m^2_{31} \approx -\Delta m^2_{atm}$. This results in the following estimate for neutrino masses:

$$m_3 \ll \sqrt{\Delta m^2_{atm}}, \quad m_2 \simeq \sqrt{\Delta m^2_{atm}}, \quad m_1 \simeq \sqrt{\Delta m^2_{atm}}. \quad (21)$$

Using the estimates (20) and (21) in Eq. (13) with the best-fit values for the neutrino oscillation parameters from Eqs. (15)–(19), we end up with the values of the effective light neutrino mass for normal hierarchy

$$|\langle m_\nu \rangle_{\mu e}| \simeq (0.35 - 5.3) \times 10^{-3}\ \mathrm{eV} \quad (22)$$

and for inverted hierarchy

$$|\langle m_\nu \rangle_{\mu e}| \simeq (0.3 - 3.3) \times 10^{-2}\ \mathrm{eV} \quad (23)$$

within the ranges corresponding to the variation of CP-violating phases within the intervals $0 \leqslant \delta < 2\pi$, $0 \leqslant \alpha_{12} < 2\pi$, $0 \leqslant \alpha_{23} < 2\pi$. The small terms with m_1 in Eq. (22) and m_3 in Eq. (23) were neglected. The effect of these terms is presented in Fig. 1, which shows the dependence of the allowed regions of $|\langle m_\nu \rangle_{\mu e}|$ on the mass of the lightest neutrino m_1 for the normal and m_3 for the inverted neutrino mass hierarchies.

(iii) *Quasidegenerate hierarchy*: $m_1 \simeq m_2 \simeq m_3$. This mass spectrum can be consistent with neutrino oscillation data if the characteristic neutrino mass scale is sufficiently large $m_0 \gg \sqrt{\Delta m^2_{atm}}$. In this case the effective light neutrino mass can be written as

$$|\langle m_\nu\rangle_{\mu e}| \approx m_0 \left| \sum_{k=1}^{3} U_{\mu k} U_{ek} \right|. \qquad (24)$$

In order to estimate its value one needs the values of the characteristic neutrino mass scale m_0. It can be deduced from ^{3}H experiments and cosmological data. Using the best fit values of neutrino mixing angles from Eq. (15) and adopting for the simplicity $\delta=\alpha_{12}=\alpha_{23}=0$ we obtain

$$|\langle m_\nu\rangle_{\mu e}| \lesssim 1.46 \text{ eV}, \quad m_0 < 2.05 \text{ eV} \qquad (25)$$

from the Troitsk ^{3}H experiment [44],

$$|\langle m_\nu\rangle_{\mu e}| \lesssim 1.56 \text{ eV}, \quad m_0 < 2.2 \text{ eV} \qquad (26)$$

from the Mainz ^{3}H experiment [45],

$$|\langle m_\nu\rangle_{\mu e}| \lesssim 0.16 \text{ eV}, \quad m_0 < 0.23 \text{ eV} \qquad (27)$$

from the cosmological data [46], and

$$|\langle m_\nu\rangle_{\mu e}| \sim 0.14 \text{ eV}, \quad m_0 \sim 0.2 \text{ eV} \qquad (28)$$

from the cosmological data [47]. Note that the results of the global analysis of the cosmological data in Refs. [46,47] provide significantly more stringent limits on the neutrino mass scale than those from the direct laboratory measurements of ^{3}H β-decay [44,45]. However, at the same time the cosmological limits are more model dependent than the laboratory ones.

Now, let us assume that there exist heavy neutrinos N with the masses $M_k \gg q_0 \sim m_\mu$, where $q_0 \sim m_\mu$ is the typical energy scale of $(\mu^- - e^+)$ conversion set by the muon mass m_μ. Their contribution to this process is determined by the effective mass

$$\langle M_N^{-1}\rangle_{\mu e} = \sum_{k=\text{heavy}} \frac{U_{\mu k} U_{ek}}{M_k}. \qquad (29)$$

Due to the lack of model independent information on mixing matrix elements $U_{\mu k} U_{ek}$ in the sector of heavy neutrinos it is hard to estimate this quantity. For this reason we adopt the conservative upper bound following from the existing LEP limit on the mass of heavy stable neutral lepton $M_N \geqslant 39.5$ GeV [48]. Assuming the existence of only one heavy neutrino identified with this particle, we obtain

$$|\langle M_N^{-1}\rangle_{\mu e}| \leqslant (39.5 \text{ GeV})^{-1}. \qquad (30)$$

In what follows we will use the results presented in Eqs. (22), (23), (25)–(28), and (30) for discussion of the expected rates of $(\mu^- - e^+)$ conversion induced by the Majorana neutrino exchange.

IV. NEUTRINO MEDIATED (μ^-,e^+) CONVERSION: GENERAL FORMALISM

The process of (μ^-,e^+) conversion is very similar to the $0\nu\beta\beta$ decay. Both processes violate the lepton number by two units and, therefore, take place if and only if neutrinos are Majorana particles with nonzero mass.

On the other hand, there are various important differences between (μ^-,e^+) conversion and $0\nu\beta\beta$ decay. Among them we mention the following:

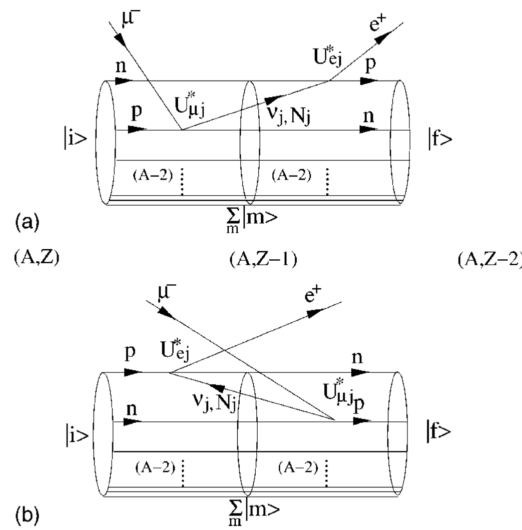

FIG. 2. Direct (a) and cross (b) Feynman diagrams of (μ^-,e^+) conversion in nuclei mediated by Majorana neutrinos.

(i) They have rather different available energies and different number of leptons in their final states. This results in a significant difference between the corresponding phase space integrals.

(ii) The emitted positron in (μ^-,e^+) conversion has large momentum, and, therefore, the long-wave approximation is not valid in contrast to $0\nu\beta\beta$ decay.

(iii) As we will show, the nuclear matrix element of (μ^-,e^+) conversion for light neutrino-exchange demonstrates a singular behavior, absent in the $0\nu\beta\beta$ decay. This feature gives rise to the large imaginary part of the (μ^-,e^+) conversion amplitude. Technically, the singularity significantly complicates the numerical calculation of the nuclear matrix elements.

(iv) In the case of the (μ^-,e^+) conversion, there is large number of nuclear final states that must be properly taken into account.

Below, we analyze the amplitude of the (μ^-,e^+) conversion in nuclei mediated by light and heavy Majorana neutrinos. The corresponding diagrams are shown in Fig. 2. We concentrate only on the nuclear transition connecting the ground states (g.s.) of the initial and final nuclei, which is favored from the experimental point of view due to the minimal background. The characteristic signature of g.s.$\rightarrow$g.s. transition is the presence of a peak in the e^+ spectrum at the energy

$$E_{e^+} = m_\mu - \varepsilon_b - (E_f - E_i), \qquad (31)$$

which allows reliable separation of signal from background. Here, m_μ, ε_b, E_i and E_f are the mass of muon, the muon atomic binding energy (for ^{48}Ti this is $\varepsilon_b=1.45$ MeV), the energies of initial and final nuclear ground states, respectively. Latter on we neglect the kinetic energy of final nucleus.

The leading order (μ^-,e^+) conversion matrix element, corresponding to the diagrams in Fig. 2, reads

$$\langle f|S^{(2)}|i\rangle = -i\left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{4E_{\mu^-}E_{e^+}}} \overline{v}(k_{e^+})(1+\gamma_5)u(k_{\mu^-}) \times \frac{m_e g_A^2}{2\pi R}\left[\eta_\nu^{\mu e}\mathcal{M}_\nu^{(\mu e^+)\Phi} + \eta_N^{\mu e}\mathcal{M}_N^{(\mu e^+)\Phi}\right] \times 2\pi\delta(E_{\mu^-}+E_i-E_f-E_{e^+}). \quad (32)$$

Here m_e and m_p are electron and proton masses, and k_{e^+} (E_{e^+}) and k_{μ^-} (E_{μ^-}) are the momentum (energy) of outgoing positron and captured muon, respectively. The conventional normalization factor involves the nuclear radius $R=1.1\ A^{1/3}$ fm. For the weak axial coupling constant g_A we adopt the value $g_A=1.254$. In the above expression we introduced for convenience the following LNV parameters:

$$\eta_\nu^{\mu e} = \frac{\langle m_\nu\rangle_{\mu e}}{m_e}, \quad \eta_N^{\mu e} = \langle M_N^{-1}\rangle_{\mu e} m_p. \quad (33)$$

The nuclear matrix elements in Eq. (32) defined as

$$\mathcal{M}_i^{(\mu e^+)\Phi} = -\frac{M_{F(i)}^{(\mu e^+)\Phi}}{g_A^2} + M_{GT(i)}^{(\mu e^+)\Phi} \text{ for } i=\nu,N \quad (34)$$

contain the Fermi $M_F^{(\mu e^+)\Phi}$ and Gamow-Teller $M_{GT}^{(\mu e^+)\Phi}$ contributions. They take the following form for *the light Majorana neutrino exchange mechanism*:

$$M_{F(\nu)}^{(\mu e^+)\Phi} = \frac{4\pi R}{(2\pi)^3}\int \frac{d\vec{q}}{2q} f_V^2(\vec{q}^2)\sum_n \left(\frac{\langle 0_i^+|\sum_l \tau_l^+ e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l}|n\rangle\langle n|\sum_m \tau_m^+ e^{i\vec{q}\cdot\vec{r}_m}\Phi(r_m)|0_f^+\rangle}{q-E_{\mu^-}+E_n-E_i+i\varepsilon_n} + \frac{\langle 0_i^+|\sum_m \tau_m^+ e^{i\vec{q}\cdot\vec{r}_m}\Phi(r_m)|n\rangle\langle n|\sum_l \tau_l^+ e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l}|0_f^+\rangle}{q+E_{e^+}+E_n-E_i+i\varepsilon_n}\right), \quad (35)$$

$$M_{GT(\nu)}^{(\mu e^+)\Phi} = \frac{4\pi R}{(2\pi)^3}\int \frac{d\vec{q}}{2q} f_A^2(\vec{q}^2)\sum_n \left(\frac{\langle 0_i^+|\sum_l \tau_l^+\vec{\sigma}_l e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l}|n\rangle\langle n|\sum_m \tau_m^+\vec{\sigma}_m e^{i\vec{q}\cdot\vec{r}_m}\Phi(r_m)|0_f^+\rangle}{q-E_{\mu^-}+E_n-E_i+i\varepsilon_n} + \frac{\langle 0_i^+|\sum_m \tau_m^+\vec{\sigma}_m e^{i\vec{q}\cdot\vec{r}_m}\Phi(r_m)|n\rangle\langle n|\sum_l \tau_l^+\vec{\sigma}_l e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l}|0_f^+\rangle}{q+E_{e^+}+E_n-E_i+i\varepsilon_n}\right), \quad (36)$$

and for *the heavy Majorana neutrino exchange mechanism*

$$M_{I(N)}^{(\mu e^+)\Phi} = \frac{4\pi R}{(2\pi)^3}\frac{2}{m_p m_e}\int d\vec{q}\langle 0_i^+|\sum_{lm}\tau_l^+\tau_m^+ h_I(\vec{q}^2)e^{-i\vec{q}\cdot(\vec{r}_l-\vec{r}_m)} \times e^{-i\vec{k}_{e^+}\cdot\vec{r}_l}\Phi(r_m)|0_f^+\rangle \quad (I=F,GT) \quad (37)$$

with

$$h_F(\vec{q}^2) = f_V^2(\vec{q}^2),$$

$$h_{GT}(\vec{q}^2) = \vec{\sigma}_l\cdot\vec{\sigma}_m f_A^2(\vec{q}^2). \quad (38)$$

We use the conventional dipole parametrization for the nucleon form factors [49]

$$f_V(\vec{q}^2) = \left(1+\frac{\vec{q}^2}{\Lambda_V^2}\right)^{-2},$$

$$f_A(\vec{q}^2) = \left(1+\frac{\vec{q}^2}{\Lambda_A^2}\right)^{-2}, \quad (39)$$

with $\Lambda_V=0.71$ GeV, $\Lambda_A=1.09$ GeV. In Eqs. (35)–(37) the factor $\Phi(r)$ is the radial part of the bound muon 1S wave function (see Appendix A). In the denominators of Eqs. (35) and (36) we introduced the widths ε_n of intermediate nuclear states.

In the calculations of nuclear matrix elements we adopt the following approximations.

(i) Taking into account slow variation of muon wave function within the nucleus, we apply the standard approximation [19]

$$|\mathcal{M}_i^{(\mu e^+)\Phi}|^2 = \langle\Phi\rangle^2|\mathcal{M}_i^{(\mu e^+)}|^2, \quad i=\nu,N. \quad (40)$$

Here $\langle\Phi\rangle^2$ is the muon average probability density and

$$|\mathcal{M}_i^{(\mu e^+)}| = |\mathcal{M}_i^{(\mu e^+)\Phi}|_{\Phi=1}. \quad (41)$$

The explicit form of $\langle\Phi\rangle^2$ is given in Appendix B.

(ii) In muon to positron conversion the typical energy of light intermediate neutrinos is about 100 MeV ($\omega\approx|q| \geqslant 1/R\sim 100$ MeV), which is much larger than the typical excitation energies of intermediate nuclear states. Therefore, to a good approximation the individual energies of these states in the energy denominators of Eqs. (35) and (36) can be neglected or replaced by some average value $\langle E_n\rangle$ to which the matrix elements are not very sensitive. Then the intermediate nuclear states can be summed up by closure. A similar situation occurs in the case of $0\nu\beta\beta$ decay [9–11].

Thus, in Eqs. (35) and (36) we complete the sum over the virtual intermediate nuclear states by closure after replacing E_n, ε_n with some average values $\langle E_n\rangle$, ε, respectively,

$$\sum_n \frac{|n\rangle\langle n|}{q-E_{\mu^-}+E_n-E_i+i\varepsilon_n} \approx \frac{1}{q-E_{\mu^-}+\langle E_n\rangle-E_i+i\varepsilon}, \tag{42}$$

$$\sum_n \frac{|n\rangle\langle n|}{q+E_{e^+}+E_n-E_i+i\varepsilon_n} \approx \frac{1}{q+E_{e^+}+\langle E_n\rangle-E_i+i\varepsilon}. \tag{43}$$

Obviously, the validity of the closure approximation is just the question of the choice of the average excitation energy which will be discussed in Sec. V.

The angular part of neutrino propagators can be integrated using the relation

$$\begin{aligned}\int e^{-i\vec{q}\cdot(\vec{r}_l-\vec{r}_m)}e^{-i\vec{k}_{e^+}\cdot\vec{r}_l}d\Omega_q &= (4\pi)^2\sum_\lambda(-1)^\lambda\sqrt{2\lambda+1}j_\lambda(k_{e^+}R_{lm})j_0(qr_{lm})j_\lambda(k_{e^+}r_{lm}/2)\\ &\times\{Y_\lambda(\Omega_{r_{lm}})\otimes Y_\lambda(\Omega_{R_{lm}})\}_{00},\end{aligned} \tag{44}$$

where j_λ is the spherical Bessel function, Y_λ is the spherical harmonic, and

$$\vec{r}_{ij}=\vec{r}_i-\vec{r}_j, \quad r_{ij}=|\vec{r}_{ij}|, \quad \vec{R}_{ij}=\frac{\vec{r}_i+\vec{r}_j}{2}, \quad R_{ij}=|\vec{R}_{ij}|. \tag{45}$$

Note that in the limit when the outgoing positron momentum $|\vec{k}_{e^+}|$ is zero the right-hand side of Eq. (44) is reduced to $4\pi j_0(qr_{lm})$.

With the above approximations and comments we can write the expressions for the nuclear matrix elements introduced in Eq. (41) in the form

$$\mathcal{M}_\nu^{(\mu e^+)} = M_{\rm dir.}^{(\mu e^+)} + M_{\rm cro.}^{(\mu e^+)},$$

$$\mathcal{M}_N^{(\mu e^+)} = -\frac{M_{{\rm F}(N)}^{(\mu e^+)}}{g_{\rm A}^2} + M_{{\rm GT}(N)}^{(\mu e^+)}. \tag{46}$$

Here the nuclear matrix element $\mathcal{M}_\nu^{(\mu e^+)}$ is decomposed into the contributions coming from direct and cross Feynman diagrams in Fig. 2. They can be written as

$$\begin{aligned}M_{\rm dir.}^{(\mu e^+)} = \langle 0_i^+|&\sum_{lm}\tau_l^+\tau_m^+4\pi\sum_\lambda(-1)^\lambda\sqrt{2\lambda+1}j_\lambda\\ &\times(k_{e^+}R_{lm})j_\lambda\left(\frac{k_{e^+}r_{lm}}{2}\right)\{Y_\lambda(\Omega_{r_{lm}})\otimes Y_\lambda(\Omega_{R_{lm}})\}_{00}\\ &\times\frac{R}{\pi}\int_0^\infty\frac{j_0(qr_{lm})}{q-E_{\mu^-}+\langle E_n\rangle-E_i+i\varepsilon}\left(\vec{\sigma}_l\cdot\vec{\sigma}_m f_{\rm A}^2(q^2)\right.\\ &\left.-\frac{f_{\rm V}^2(q^2)}{g_{\rm A}^2}\right)qdq|0_f^+\rangle,\end{aligned} \tag{47}$$

$$\begin{aligned}M_{\rm cro.}^{(\mu e^+)} = \langle 0_i^+|&\sum_{lm}\tau_l^+\tau_m^+4\pi\sum_\lambda(-1)^\lambda\sqrt{2\lambda+1}j_\lambda\\ &\times(k_{e^+}R_{lm})j_\lambda\left(\frac{k_{e^+}r_{lm}}{2}\right)\{Y_\lambda(\Omega_{r_{lm}})\otimes Y_\lambda(\Omega_{R_{lm}})\}_{00}\\ &\times\frac{R}{\pi}\int_0^\infty\frac{j_0(qr_{lm})}{q+E_{e^+}+\langle E_n\rangle-E_i+i\varepsilon}\left(\vec{\sigma}_l\cdot\vec{\sigma}_m f_{\rm A}^2(q^2)\right.\\ &\left.-\frac{f_{\rm V}^2(q^2)}{g_{\rm A}^2}\right)qdq|0_f^+\rangle.\end{aligned} \tag{48}$$

The Gamow-Teller and Fermi nuclear matrix elements of heavy Majorana neutrino exchange mechanism take the form

$$\begin{aligned}M_{I(N)}^{(\mu e^+)} = \frac{1}{m_p m_e}\langle 0_i^+|&\sum_{lm}\tau_l^+\tau_m^+4\pi\sum_\lambda(-1)^\lambda\sqrt{2\lambda+1}j_\lambda\\ &\times(k_{e^+}R_{lm})j_\lambda\left(\frac{k_{e^+}r_{lm}}{2}\right)\{Y_\lambda(\Omega_{r_{lm}})\otimes Y_\lambda(\Omega_{R_{lm}})\}_{00}\\ &\times\frac{2R}{\pi}\int_0^\infty j_0(qr_{lm})h_I(\vec{q}^2)q^2dq|0_f^+\rangle \quad (I={\rm F},{\rm GT}),\end{aligned} \tag{49}$$

with $h_I(\vec{q}^2)$ defined in Eq. (38).

It is important to note that the value of $E_r\equiv -E_{\mu^-}+\langle E_n\rangle -E_i$ is negative for the studied nuclear system $A=48$. Therefore, the contribution of direct Feynman diagram in Fig. 2(a) with the light intermediate neutrino has the pole at $q=-E_r-i\varepsilon$, as it follows from the formula (47). As a consequence, the imaginary part of the (μ^-,e^+) conversion amplitude for the case of the light neutrino exchange can be significant. This fact was noted in Ref. [22] and then in Refs. [23,24]. In Ref. [24] it was shown that the imaginary part of the amplitude dominates in the total branching ratio of the (μ^-,e^+) conversion in ^{27}Al. In Sec. V. we will demonstrate that the similar conclusion is valid for (μ^-,e^+) conversion in ^{48}Ti.

The following comment is in order. In the expressions (35)–(37) for nuclear matrix elements $\mathcal{M}_i^{(\mu e^+)\Phi}$ we neglected the contributions of the higher-order terms of nucleon current (weak magnetism, induced pseudoscalar coupling). As suggested by the analogy with $0\nu\beta\beta$ decay [50], these terms should not be essential for the light neutrino exchange mechanism meanwhile their contribution in the case of heavy Majorana neutrino exchange might be significant.

However, the detailed study of this effect is beyond the scope of this paper and will be considered elsewhere.

Now we are ready to write the expression for g.s. $\rightarrow$g.s. (μ^-,e^+) conversion rate. For simplicity we assume that only one mechanism is in operation and present the corresponding rates for light and heavy Majorana neutrino exchange mechanisms separately

$$\Gamma_i^{(\mu e^+)} = \frac{1}{\pi} E_{e^+} k_{e^+} F(Z-2,E_{e^+}) c_{\mu e} \langle \Phi \rangle^2 |\mathcal{M}_i^{(\mu e^+)}|^2 |\eta_i^{(\mu e)}|^2 \quad (i=\nu,N), \tag{50}$$

where $c_{\mu e} = 2G_F^4[(m_e m_\mu)/(4\pi m_\mu R)]^2 g_A^4$, $k_{e^+} = |\vec{k}_{e^+}|$. The relativistic Coulomb factor $F(Z,E)$ in Eq. (50) we take in the standard form [9]

$$F(Z,E) = \left[\frac{2}{\Gamma(2\gamma_1+1)}\right]^2 (2pR)^{2(\gamma_1-1)} |\Gamma(\gamma_1 - iy)|^2 e^{-\pi y}, \tag{51}$$

where $\gamma_1 = \sqrt{1-(\alpha Z)^2}$, α is the fine structure constant, and $y = \alpha Z E/p$.

To conclude this section we point out that in our analysis of (μ^-,e^+) conversion we limit ourselves by the $0^+_{\text{g.s.}} \rightarrow 0^+_{\text{g.s.}}$ transition, which represents a particular contribution to the total rate of this process. This is the most favored channel for experimental study because its signal can be reliably separated from the background as we commented above. On the other hand, in Ref. [24] it was demonstrated that $0^+_{\text{g.s.}} \rightarrow 0^+_{\text{g.s.}}$ transition constitutes about 41% of the total (μ^-,e^+) conversion rate in ^{27}Al and, therefore, neglecting the excited final states is a reasonable approximation. We expect that this conclusion holds for ^{48}Ti as well.

V. NUCLEAR MATRIX ELEMENTS

We calculate the (μ^-,e^+) conversion nuclear matrix elements within the proton-neutron renormalized quasiparticle random phase approximation (*pn*-RQRPA) [34,35,51,52]. In the present study we focus on ^{48}Ti nucleus utilized as a stopping target in the SINDRUM II [31] and PRIME [33] experiments.

Nuclear transition scheme for the studied $A=48$ nuclear system is shown in Fig. 3 Our nuclear structure calculations involve the single-particle model space both for protons and neutrons consisting of the full $0-3\hbar\omega$ shells plus $2s_{1/2}$, $0g_{7/2}$ and $0g_{9/2}$ levels. The single particle energies were obtained using the Coulomb-corrected Woods-Saxon potential. The two-body G-matrix elements were calculated from the Bonn one-boson exchange potential on the basis of the Brueckner theory. Since the considered model space is finite the pairing interactions have been adjusted to fit the empirical pairing gaps [53]. In addition, we renormalize the particle-particle and particle-hole channels of the *G*-matrix interaction of the nuclear Hamiltonian *H* by introducing the parameters g_{pp} and g_{ph}, respectively. The two-nucleon correlation effect has been taken into account in a standard way by multiplying the operators with the square of the correlation Jastrow-like function [54]. The details of our nuclear model can be found in Appendix C.

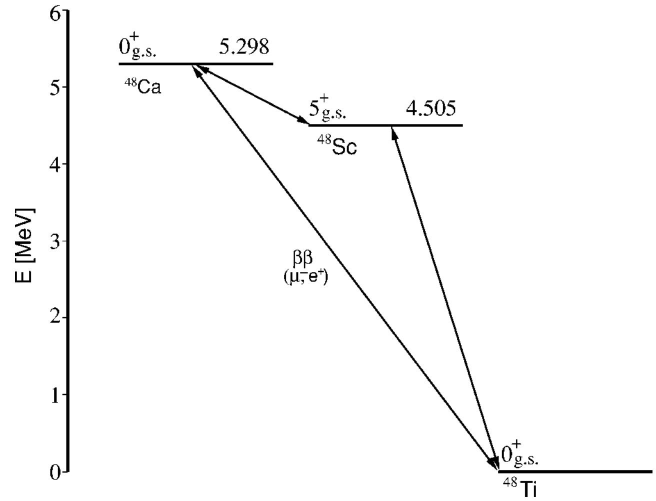

FIG. 3. Transition scheme for the $A=48$ nuclear system.

As we already commented in Sec. IV, the matrix element of the direct contribution [Fig. 2(a)] of the light neutrino exchange mechanism contains an imaginary part that stems from the pole of the integrand in Eq. (47) at $q=-E_r-i\varepsilon$. Taking into account that the widths ε of low-lying nuclear states are negligible in comparison to their energies, one can separate the imaginary and real parts of this matrix element using the well-known formula

$$\frac{1}{\alpha+i\varepsilon} = \mathcal{P}\frac{1}{\alpha} - i\pi\delta(\alpha) \tag{52}$$

valid in the limit $\varepsilon \rightarrow 0$.

In Table I we show the nuclear matrix elements of light and heavy Majorana neutrino exchange mechanisms of the (μ^-,e^+) conversion in ^{48}Ti calculated for $g_{pp}=1.0$ and $g_{pp}=0.8$, 1.0, 1.2. All of the presented results were obtained for the particular value of energy difference $\langle E_n \rangle - E_i = 10$ MeV. This choice is justified by weak dependence of the matrix elements on this parameter within the interval of its reasonable values 2 MeV$\leqslant(\langle E_n\rangle - E_i)\leqslant$15 MeV. We verified this property by the direct numerical analysis. In Fig. 4 we present the absolute value of the light neutrino exchange nuclear matrix element $|M^{(\mu e^+)}|$ as a function of the average value $\langle E_n\rangle - E_i$ for $g_{pp}=0.8$, 1.0, and 1.2. One can see that its variation within the studied range of $\langle E_n\rangle - E_i$ is about 30%. For $g_{pp}=0.8$, 1.0 ($g_{pp}=1.2$) the matrix element is an increasing (decreasing) function of $\langle E_n\rangle - E_i$. Different behavior in these two cases is related to a specific interplay between the direct $M^{(\mu e^+)}_{\text{dir.}}$ and cross $M^{(\mu e^+)}_{\text{cro.}}$ diagram terms in $M^{(\mu e^+)}$. For $g_{pp}=0.8$, 1.0 there is a mutual cancellation of the real parts of these two terms so that the imaginary part of $M^{(\mu e^+)}_{\text{dir.}}$, which is a growing function of $\langle E_n\rangle - E_i$, dominantes and determines the behavior of $M^{(\mu e^+)}$. For $g_{pp}=1.2$ the situation is opposite. The real parts, decreasing with $\langle E_n\rangle - E_i$, contribute coherently and constitute the dominant part of $M^{(\mu e^+)}$, which becomes a decreasing function of $\langle E_n\rangle - E_i$. We have also found that the nuclear matrix elements do not show an appreciable

TABLE I. Nuclear matrix elements of the light and heavy Majorana neutrino exchange mechanisms of (μ^-, e^+) conversion in ^{48}Ti [see Eqs. (46)–(49)]. The calculations have been performed within the *pn*-RQRPA without and with the inclusion of two-nucleon short-range correlations (src).

g_{pp}	$M^{(\mu e^+)}_{\text{cro.}}$	$\text{Re}(M^{(\mu e^+)}_{\text{dir.}})$	$\text{Im}(M^{(\mu e^+)}_{\text{dir.}})$	$\lvert\mathcal{M}^{(\mu e^+)}_{\nu}\rvert$	$\lvert\mathcal{M}^{(\mu e^+)}_{N}\rvert$
			Without src		
0.8	0.097	0.002	0.088	0.132	25.5
1.0	0.077	0.034	0.059	0.125	22.8
1.2	0.051	0.091	0.018	0.142	19.6
			With src		
0.8	0.049	−0.080	0.050	0.059	5.92
1.0	0.034	−0.040	0.024	0.025	5.19
1.2	0.013	0.027	−0.013	0.042	4.33
			With src, $\lvert\vec{k}_{e^+}\rvert=0$		
0.8	0.298	−0.029	0.386	0.470	31.4
1.0	0.233	0.069	0.275	0.408	27.7
1.2	0.147	0.243	0.125	0.409	23.2

variation in the physical region of the parameter g_{ph} $(0.8 \leqslant g_{ph} \leqslant 1.2)$. On the contrary, as seen from Table I they significantly depend on the renormalization parameter g_{pp} and on the two-nucleon short-range correlation. It is also worth noting that the large momentum k_{e^+} of outgoing positron is the source of strong suppression of the (μ^-, e^+) conversion matrix elements. In order to illustrate this effect, we presented in Table I the matrix elements calculated in the limit $\lvert k_{e^+}\rvert=0$ when the suppression of this type is absent. The cross check of Table I reveals the corresponding suppression factor of about ~ 10.

An important issue of our analysis is the presence of the significant imaginary part of matrix element $\mathcal{M}^{\mu e^+}_{\nu}$ corresponding to the light Majorana neutrino exchange mechanism. This fact was first noticed in Ref. [22] and then in Refs. [23,24]. In the previous studies of (μ^-, e^+) conversion [9,15,16,18] the role of imaginary part was overlooked.

In the presented detailed study we have found, that the relative contribution of the imaginary part to the rate of (μ^-, e^+) conversion in ^{48}Ti is always significant, but appreciably depends on the value of the nuclear model parameter g_{pp} and on the short-range correlations. It absolutely dominates over the real part by the factor of ~ 16 for the most conventional case when $g_{pp}=1$ and the short-range correlations are taken into account (for the motivation of this choice see, for instance, Refs. [13,14]). This conclusion is consistent with the result of Ref. [24] studying (μ^-, e^+) conversion in ^{27}Al within shell-model approach where it was found that the imaginary part for the light neutrino exchange dominates over the real one by the factor of about 20. However, it is notable that the relative contribution of the imaginary part is model dependent and can vary from one nucleus to another. In this situation, the role of the imaginary part in (μ^-, e^+) conversion requires further study for other nuclear systems.

From the view point of nuclear structure theory it is instructive to compare the values of (μ^-, e^+) conversion nuclear matrix elements with the corresponding values of $0\nu\beta\beta$-decay matrix elements of $A=48$ nuclear system. For $0\nu\beta\beta$ decay, this system is represented by ^{48}Ca with the matrix elements

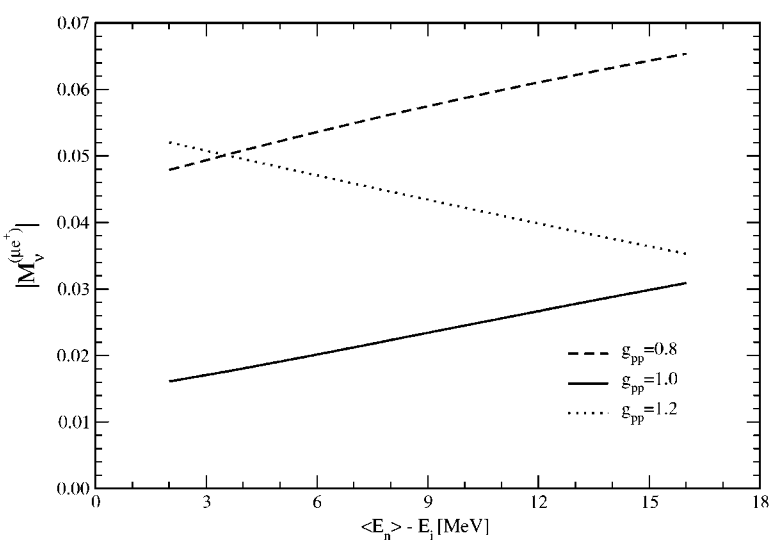

FIG. 4. The nuclear matrix elements of the light Majorana neutrino exchange mechanisms of the (μ^-, e^+) conversion in ^{48}Ti as a function of the average value of energy difference $\langle E_n\rangle - E_i$.

$$|\mathcal{M}_{\nu}^{(ee)}| = 0.82, \quad |\mathcal{M}_{N}^{(ee)}| = 24.2 \tag{53}$$

derived within the *pn*-RQRPA approach in Ref. [4]. As seen, the matrix elements of the (μ^-,e^+) conversion (55) are strongly suppressed in comparison with those of $0\nu\beta\beta$ decay (53) by the factors of about 17 and 5 for the light and heavy Majorana neutrino exchange mechanisms, respectively. As we commented above the explanation of this difference between the two processes mostly resides in the large momentum of outgoing positron produced in (μ^-,e^+) conversion.

VI. (μ^-,e^+) CONVERSION AND EFFECTIVE NEUTRINO MASSES

Now, let us discuss the possible issues of (μ^-,e^+) conversion experiments for neutrino physics. From Eq. (50) we obtain the (μ^-,e^+) conversion branching ratios in ^{48}Ti for the light and heavy Majorana neutrino exchange mechanisms

$$R_i^{(\mu e^+)} \equiv \frac{\Gamma_i^{(\mu e)}}{\Gamma^{(\mu\nu)}} = 2.6 \times 10^{-22} |\mathcal{M}_i^{(\mu e)}|^2 |\eta_i^{(\mu e)}|^2 \ (i = \nu, N). \tag{54}$$

Here we use the known experimental value $\Gamma^{(\mu\nu)} = 2.60 \times 10^6$ s^{-1} [55] of ordinary muon capture rate in ^{48}Ti. For the further discussion we choose the following sample values of nuclear matrix elements of ^{48}Ti from Table I:

$$|\mathcal{M}_{\nu}^{(\mu e^+)}| = 0.025, \quad |\mathcal{M}_{N}^{(\mu e^+)}| = 5.2 \tag{55}$$

corresponding to $g_{pp} = 1.0$ with the presence of the two-nucleon short-range correlations.

Substituting these numerical values of nuclear matrix elements to Eq. (54), we obtain

$$R_{\nu}^{(\mu e^+)} = 1.6 \times 10^{-25} \times \frac{|\langle m\rangle_{\mu e}|^2}{m_e^2}, \tag{56}$$

$$R_{N}^{(\mu e^+)} = 7.0 \times 10^{-21} \times |\langle M_N^{-1}\rangle_{\mu e}|^2 m_p^2. \tag{57}$$

From the existing experimental upper bound in Eq. (2) one obtains the following limits for the effective masses of light and heavy Majorana neutrinos

$$|\langle m\rangle_{\mu e}| \leqslant 1.3 \times 10^6 \text{ MeV},$$

$$|\langle M_N^{-1}\rangle_{\mu e}|^{-1} \geqslant 3.3 \times 10^{-2} \text{ MeV}. \tag{58}$$

Obviously, these limits have no physical sense since they do not satisfy the consistency condition in Eq. (11) with the characteristic energy scale $q_0 \sim m_\mu = 105$ MeV of (μ^-,e^+) conversion. Meaningful limits on the parameters $\langle m\rangle_{\mu e}$, $\langle M_N^{-1}\rangle_{\mu e}$, which may have some impact on neutrino physics, could be reached if the (μ^-,e^+) conversion experiments would improve their sensitivities by at least 10 orders of magnitude. Clearly, such a tremendous improvement is unrealistic for the near-future experiments.

On the other hand, we can estimate the expected branching ratios of (μ^-,e^+) conversion induced by the light and heavy Majorana neutrino exchange using the estimates of $\langle m\rangle_{\mu e}$, $\langle M_N^{-1}\rangle_{\mu e}$ made in Sec. III from the present neutrino data. Substituting the values of these parameters in Eqs. (56) and (57) we obtain the following results. For the *light Majorana neutrino exchange contribution* with different neutrino mass hierarchies we have the following:

(i) Normal neutrino mass hierarchy, $|\langle m\rangle_{\mu e}| \simeq (0.35-5.3) \times 10^{-3}$ eV

$$R_{\nu}^{(\mu e^+)} \simeq (0.008 - 1.7) \times 10^{-41}. \tag{59}$$

(ii) Inverted neutrino mass hierarchy, $|\langle m\rangle_{\mu e}| \simeq (0.3-3.3) \times 10^{-2}$ eV

$$R_{\nu}^{(\mu e^+)} \simeq (0.05 - 6.7) \times 10^{-40}. \tag{60}$$

(iii) Quasidegenerate mass hierarchy

$$R_{\nu}^{(\mu e^+)} \lesssim 1.3 \times 10^{-36}, \quad \langle m_\nu\rangle < 1.46 \text{ eV} \tag{61}$$

for the Troitsk ^{3}H experiment [44],

$$R_{\nu}^{(\mu e^+)} \lesssim 1.5 \times 10^{-36}, \quad \langle m_\nu\rangle < 1.56 \text{ eV} \tag{62}$$

for the Mainz ^{3}H experiment [45],

$$R_{\nu}^{(\mu e^+)} \lesssim 1.6 \times 10^{-38}, \quad \langle m_\nu\rangle < 0.16 \text{ eV} \tag{63}$$

for the cosmological data [46], and

$$R_{\nu}^{(\mu e^+)} \sim 1.3 \times 10^{-38}, \quad \langle m_\nu\rangle \sim 0.14 \text{ eV} \tag{64}$$

for the cosmological data [47].

Let us note that the cosmological data based limits (63) and (64), albeit more stringent, are more model dependent than the laboratory ones (61) and (62).

For the *heavy Majorana neutrino contribution* we obtain the following upper limit:

$$R_{N}^{(\mu e^+)} \leqslant 3.8 \times 10^{-24}. \tag{65}$$

All the values of (μ^-,e^+) conversion branching ratio in Eqs. (59)–(65) are hopelessly low for being detected even in a distant future. Thus, searching for (μ^-,e^+) conversion cannot have any direct impact on neutrino physics. On the other hand, any observation of (μ^-,e^+) conversion at branching ratios above the limits in Eqs. (59)–(65) would be an unambiguous signal of new physics beyond the simplest extension of the SM with massive Majorana neutrinos and would imply the presence of new interactions.

This conclusion is in sharp contrast with $0\nu\beta\beta$-decay experiments, which already provide important information on neutrino properties and are expected to detect neutrino contributions in the near future. This is due to their unique sensitivities to the $0\nu\beta\beta$-decay signal. In order to give an impression as to what extent $0\nu\beta\beta$-decay experiments overcome insensitivities the experiments searching for (μ^-,e^+) conversion let us compare, as an example, the rates of (μ^-,e^+) conversion in ^{48}Ti and the $0\nu\beta\beta$ decay of ^{48}Ca. To this end it is sufficient to consider only light Majorana neutrino exchange contributions in both cases. For the rate of $0\nu\beta\beta$ decay we have the well-known formula

$$\Gamma_\nu^{(ee)} = \ln 2 G_{01}\left|\frac{\langle m_\nu\rangle_{ee}}{m_e}\right|^2 |\mathcal{M}_\nu^{(ee)}|^2, \tag{66}$$

where $G_{01}=8.031\times10^{-14}$ year^{-1} [56] and

$$\langle m_\nu\rangle_{ee} = \sum_{k=\text{light}} (U_{ek})^2 m_k. \tag{67}$$

Using the value of the $0\nu\beta\beta$-decay nuclear matrix element $\mathcal{M}_\nu^{(ee)}$ from Eq. (53) we estimate the ratio of the (μ^-,e^+) conversion to $0\nu\beta\beta$-decay rates

$$\frac{\Gamma_\nu^{(\mu e^+)}}{\Gamma_\nu^{(ee)}} = 9.7\times10^4 \times \frac{|\mathcal{M}_\nu^{(\mu e)}|^2}{|\mathcal{M}_\nu^{(ee)}|^2}\left|\frac{\langle m_\nu\rangle_{\mu e}}{\langle m_\nu\rangle_{ee}}\right|^2 = 351\left|\frac{\langle m_\nu\rangle_{\mu e}}{\langle m_\nu\rangle_{ee}}\right|^2. \tag{68}$$

The the (μ^-,e^+) conversion receives a significant enhancement mostly due to the larger available energy of this process. Thus, for $\langle m_\nu\rangle_{\mu e}\sim\langle m_\nu\rangle_{ee}$ the (μ^-,e^+) conversion rate $\Gamma_\nu^{(\mu e^+)}$ is by more than 2 orders-of-magnitude larger than the rate $\Gamma_\nu^{(ee)}$ of $0\nu\beta\beta$ decay. Nevertheless the experimental prospects for searching for $0\nu\beta\beta$ decay are incomparably better than those for (μ^-,e^+) conversion. This is mainly because the number of potentially $0\nu\beta\beta$-decaying nuclei monitored in $0\nu\beta\beta$ experiments is by many orders of magnitude larger than the number of mesoatoms created by muon beams in the muon-conversion experiments.

VII. SUMMARY AND OUTLOOK

In summary, the light and heavy Majorana neutrino exchange mechanisms of (μ^-,e^+) conversion have been studied. Special emphasis was made on the nuclear structure aspects of this process. We have performed the realistic calculations of the corresponding nuclear matrix elements for ^{48}Ti nucleus used as a stopping target in the current [31] and the forthcoming [33] (μ^-,e^+) conversion experiments. Our analysis is based on the *pn*-RQRPA approach and limited to the case of $0^+_{\text{g.s.}}\rightarrow0^+_{\text{g.s.}}$ transition channel, which is most relevant for experimental searches for (μ^-,e^+) conversion. The effects of the ground state and two-nucleon short-range correlations have been properly taken into account. We pointed out that their inclusion results in the significant reduction of (μ^-,e^+) conversion matrix elements.

Our detailed analysis confirmed the conjecture of Refs. [22,23] on the importance of the imaginary part of the nuclear matrix elements for the case of the light Majorana neutrino exchange mechanism of (μ^-,e^+) conversion. A similar result was recently obtained in Ref. [24] for (μ^-,e^+) conversion in ^{27}Al.

We also derived the limits on the effective masses of light $\langle m\rangle_{\mu e}$ and heavy $\langle M_N^{-1}\rangle_{\mu e}$ Majorana neutrinos from the neutrino oscillations, tritium beta decay, accelerator, and cosmological data. Using these limits we estimated the expected rates of (μ^-,e^+) conversion induced by Majorana neutrino exchange. Their values were found to be so small that even within a quite distant future the (μ^-,e^+) conversion experiments will hardly be able to detect the neutrino contribution and, thus, to have a direct impact on neutrino physics. On the other hand, the eventual observation of (μ^-,e^+) conversion at larger rates would be an unambiguous signal of physics beyond the standard model implying nonstandard interactions. Moreover, this observation, independently of the (μ^-,e^+) conversion rate, would definitely prove that neutrinos are Majorana particles as follows from the "black box"-type theorem [1], establishing the fundamental relation between LNV processes and the Majorana nature of neutrinos. In view of this, it is important to study possible scenarios of physics beyond the SM consistent with the values of (μ^-,e^+) conversion rates within the reach of the present and near-future experiments.

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APPENDIX A: BOUND MUON WAVE FUNCTION

The bound muon wave function ($1S$ state) is given by the expression

$$\Psi(x) = \Phi(r)e^{-iE_{\mu^-}x_0}\frac{u_\mu^s}{\sqrt{2E_{\mu^-}}}, \tag{A1}$$

where the radial $\Phi(r)$ and the spinorial u_μ^s parts have the forms

$$\Phi(r) = \frac{Z^{3/2}}{(\pi a_\mu^3)^{1/2}}e^{-Zr/a_\mu} \tag{A2}$$

and

$$u_\mu^s = \sqrt{2E_{\mu^-}}\begin{pmatrix}\chi^s\\0\end{pmatrix}, \tag{A3}$$

with $a_\mu=4\pi/(m_\mu e^2)$ $(a_\mu/a_e\approx m_e/m_\mu\approx5\times10^{-3})$, m_μ is the reduced mass of muon atom, and Z is nuclear charge.

APPENDIX B: MUON AVERAGE PROBABILITY DENSITY OVER NUCLEUS

Muon average probability density over nucleus is defined as

$$\langle\Phi\rangle^2 \equiv \frac{\int|\Phi(\vec{x})|^2\rho(\vec{x})d^3x}{\int\rho(\vec{x})d^3x}, \tag{B1}$$

where $\rho(\vec{x})$ is the nuclear charge density. To a good approximation it can be written in the following compact form [19]:

$$\langle\Phi\rangle^2 = \frac{\alpha^3 m_\mu^3}{\pi}\frac{Z_{\text{eff}}^4}{Z}. \tag{B2}$$

Here the effective charge for $Z=22$ nuclear system is Z_{eff} is $Z_{\text{eff}}=17.5$ [19].

APPENDIX C: NUCLEAR MODEL

Here we shortly outline our approach to the nuclear structure calculations.

We introduce particle (quasiparticle) creation operators as $c^{\dagger}_{\tau m_\tau}$ ($a^{\dagger}_{\tau m_\tau}$) for $\tau = p, n$. The indices $p \equiv (n_p, l_p, j_p)$ and $n \equiv (n_n, l_n, j_n)$ denote proton and neutron quantum numbers in a particular shell. Transformation from the particle to quasiparticle basis is realized by the Bogolyubov transformation

$$\begin{pmatrix} c^{\dagger}_{\tau m_\tau} \\ \tilde{c}_{\tau m_\tau} \end{pmatrix} = \begin{pmatrix} u_\tau & -v_\tau \\ v_\tau & u_\tau \end{pmatrix} \begin{pmatrix} a^{\dagger}_{\tau m_\tau} \\ \tilde{a}_{\tau m_\tau} \end{pmatrix}, \tag{C1}$$

where the tilde denotes time reversal, $\tilde{a}_{\tau m_\tau} = (-1)^{j_\tau - m_\tau} a_{\tau - m_\tau}$.

Occupation amplitudes u_τ, v_τ and quasiparticle energies E_τ are obtained by solving the BCS equation [57]

$$\begin{pmatrix} \varepsilon_\tau - \lambda_\tau & \Delta_\tau \\ \Delta_\tau & -\varepsilon_\tau + \lambda_\tau \end{pmatrix} \begin{pmatrix} u_\tau \\ v_\tau \end{pmatrix} = E_\tau \begin{pmatrix} u_\tau \\ v_\tau \end{pmatrix}, \tag{C2}$$

where ε_τ is the energy of single particle state derived from the Wood-Saxon potential. The pairing potential takes the form

$$\Delta_\tau = (2j_\tau + 1)^{-1/2} \sum_a (2j_a + 1)^{1/2} G(aa, \tau\tau; J=0) u_a v_a. \tag{C3}$$

Here $G(aa, \tau\tau; J)$ is the particle-particle matrix element defined, e.g., in Ref. [58]. The value of the Lagrange multiplier λ is fixed by the particle number N in noncorrelated BCS vacuum

$$\langle N_\tau \rangle = \sum_\tau (2j_\tau + 1) v_\tau^2. \tag{C4}$$

After the diagonalization, the BCS equation (C2) reads

$$E_\tau = \sqrt{(\varepsilon_\tau - \lambda_\tau)^2 + \Delta_\tau^2}, \quad v_\tau^2 = \tfrac{1}{2}\left(1 - \frac{\varepsilon_\tau - \lambda_\tau}{E_\tau}\right), \quad u_\tau^2 = 1 - v_\tau^2. \tag{C5}$$

This system of equations can be solved by the iteration of the parameter λ_τ with the condition $N = \langle N_\tau \rangle$.

The nuclear Hamiltonian in quasiparticle representation takes after the BCS transformation in the form

$$H = \sum_{\tau m_\tau} E_\tau a^{+}_{\tau m_\tau} a_{\tau m_\tau} + H_{22} + H_{40} + H_{04} + H_{31} + H_{13}, \tag{C6}$$

where H_{ij} is the normally ordered part of residual interaction with i creation and j annihilation operators.

Within the pn-RQRPA, the mth nuclear excited state $|m, JM\rangle$ with the angular momentum J and its projection M is obtained from the RPA vacuum $|0^+_{\mathrm{RPA}}\rangle$

$$|m, JM\rangle = Q^{m\dagger}_{JM^\pi} |0^+_{\mathrm{RPA}}\rangle, \tag{C7}$$

where the RPA vacuum is defined by the condition

$$Q^{m}_{JM^\pi} |0^+_{\mathrm{RPA}}\rangle = 0 \tag{C8}$$

and the phonon operator $Q^{m}_{JM^\pi}$ is defined as

$$Q^{m\dagger}_{JM^\pi} = \sum_{pn} X^{m}_{(pn,J^\pi)} A^{\dagger}_{(pn,JM)} - Y^{m}_{(pn,J^\pi)} \tilde{A}_{(pn,JM)}, \tag{C9}$$

where $A^{\dagger}_{(pn,J^\pi)} [\tilde{A}_{(pn,J^\pi)}]$ is a two-particle creation (annihilation) operator that couples quasiparticles to the angular momentum J with the projection M

$$A^{\dagger}(pn, JM) = \sum_{m_p, m_n} C^{JM}_{j_p m_p j_n m_n} a^{\dagger}_{p m_p} a^{\dagger}_{n m_n}, \tag{C10}$$

$$\begin{aligned}\tilde{A}(pn, JM) &= (-1)^{J-M} A(pn, J\,M) \\ &= (-1)^{J-M} \sum_{m_p, m_n} C^{J-M}_{j_p m_p j_n m_n} a_{p m_p} a_{n m_n}.\end{aligned} \tag{C11}$$

Here $C^{JM}_{j_p m_p j_n m_n}$ are Clebsh-Gordan coefficients.

The commutator $[A, A^{\dagger}]$ is replaced within the pn-RQRPA by its mean value in the QRPA vacuum

$$\begin{aligned}[A, A^{\dagger}] &\to \langle 0^+_{\mathrm{RPA}} | [A(pn, JM), A(p'n', JM)] | 0^+_{\mathrm{RPA}} \rangle \\ &= \delta_{pp'} \delta_{nn'} \left\{ 1 - \frac{1}{\hat{j}_p} \langle 0^+_{\mathrm{RPA}} | [a^{\dagger}_p \tilde{a}_p]_{00} | 0^+_{\mathrm{RPA}} \rangle - \frac{1}{\hat{j}_n} \langle 0^+_{\mathrm{RPA}} | [a^{\dagger}_n \tilde{a}_n]_{00} | 0^+_{\mathrm{RPA}} \rangle \right\} \\ &\equiv \delta_{pp'} \delta_{nn'} D_{pn, J^\pi},\end{aligned} \tag{C12}$$

where $\hat{j}_p \equiv \sqrt{2j_p + 1}$ and

$$[a^{\dagger}_p \tilde{a}_p]_{00} \equiv \sum_{m_p} C^{00}_{j_p m_p j_p - m_p} a^{\dagger}_{p m_p} a_{p - m_p}. \tag{C13}$$

Within the quasiboson approximation, the RPA vacuum $|0^+_{\mathrm{RPA}}\rangle$ in Eq. (C12) is replaced by the noncorrelated BCS vacuum $|0^+_{\mathrm{BCS}}\rangle$ (i.e., $D_{pn,J^\pi} = 1$). The quasiboson approximation violates the Pauli exclusion principle.

From the Schrödinger equation,

$$[H, Q^{m\dagger}_{JM^\pi}] |0^+_{\mathrm{RPA}}\rangle = \Omega^{m}_{J^\pi} Q^{m\dagger}_{JM^\pi} |0^+_{\mathrm{RPA}}\rangle, \tag{C14}$$

with the excitation energy $\Omega^{m}_{J^\pi}$, we obtain the RQRPA equation,

$$\begin{pmatrix} \overline{\mathcal{A}} & \overline{\mathcal{B}} \\ \overline{\mathcal{B}} & \overline{\mathcal{A}} \end{pmatrix} \begin{pmatrix} \overline{X^m} \\ \overline{Y^m} \end{pmatrix} = \Omega^{m}_{J^\pi} \begin{pmatrix} \overline{X^m} \\ -\overline{Y^m} \end{pmatrix}. \tag{C15}$$

Here matrices $\overline{\mathcal{A}}$, $\overline{\mathcal{B}}$ have the form

$$\begin{aligned}\overline{\mathcal{A}}^{J^\pi}_{pn,p'n'} &= (E_p + E_n) \delta_{pp'} \delta_{nn'} - 2[G(pn, p'n'; J)(u_p u_n u_{p'} u_{n'} \\ &\quad + v_{p'} v_n v_{p'} v_{n'}) + F(pn, p'n'; J)(u_p v_n u_{p'} v_{n'} \\ &\quad + v_p u_n v_{p'} u_{n'})] D^{1/2}_{pn,J^\pi} D^{1/2}_{p'n',J^\pi},\end{aligned} \tag{C16}$$

$$\overline{\mathcal{B}}^{J^\pi}_{pn,p'n'} = (E_p+E_n)2[G(pn,p'n';J)(u_pu_nv_{p'}v_{n'} + v_{p'}v_nu_{p'}u_{n'}) - F(pn,p'n';J)(u_pv_nv_{p'}u_{n'} + v_pu_nu_{p'}v_{n'})]D^{1/2}_{pn,J^\pi}D^{1/2}_{p'n',J^\pi}, \quad (C17)$$

and amplitudes $\overline{X}^m_{(pn,J^\pi)}$, $\overline{X}^m_{(pn,J^\pi)}$ are

$$\overline{X}^m_{(pn,J^\pi)} = D^{1/2}_{pn,J^\pi}X^m_{(pn,J^\pi)}, \quad \overline{Y}^m_{(pn,J^\pi)} = D^{1/2}_{pn,J^\pi}Y^m_{(pn,J^\pi)}, \quad (C18)$$

where $F(pn,p'n';J)$ is the particle-hole interaction matrix element. From the mapping procedure (C12) we obtain for the coefficients $D_{pn,J}$ the system of nonlinear equations [35]

$$D_{pn,J} = 1 - \frac{1}{\hat{j}_p^2}\sum_{n'J'm} D_{pn',J'^\pi}|\overline{Y}^m_{(pn',J^\pi)}|^2 - \frac{1}{\hat{j}_n^2}\sum_{p'J'm} D_{p'n,J'^\pi}|\overline{Y}^m_{(p'n,J^\pi)}|^2. \quad (C19)$$

The amplitudes $\overline{X}^m_{(pn,J^\pi)}$, $\overline{Y}^m_{(pn,J^\pi)}$ and the excitation energies $\Omega^m_{J^\pi}$ are obtained by iterating the coupled equations (C19) and (C15).

The (μ^-,e^+) conversion nuclear matrix elements within the pn-RQRPA are transformed to the sum of the two-particle matrix elements

$$M^{\rm type} = \sum_{\substack{pnp'n'\\ J^\pi m_i m_f \mathcal{J}}} (-1)^{j_n+j'_p+J+\mathcal{J}}(2\mathcal{J}+1)\begin{Bmatrix} j_p & j_n & J\\ j'_n & j'_p & \mathcal{J}\end{Bmatrix}\langle p(1),p'(2);\mathcal{J}|f(r_{12})\tau_1^+\tau_2^+\mathcal{O}^{\rm type}_{12}f(r_{12})|n(1),n'(2);\mathcal{J}\rangle\langle 0_f^+||[c^\dagger_{p'},\tilde{c}_{n'}]_J||J^\pi m_f\rangle \times\langle J^\pi m_f|J^\pi m_i\rangle\langle J^\pi m_i||[c^\dagger_p,\tilde{c}_n]_J||0_i^+\rangle. \quad (C20)$$

Here $\{\cdots\}$ is the Wigner $6j$ symbol and $\mathcal{O}^{\rm type}_{12}$ is the space- and spin-dependent part of the matrix element. The single particle densities are defined as

$$\frac{\langle 0_f^+||[c^\dagger_p,\tilde{c}_n]_J||0_i^+\rangle}{\sqrt{2J+1}} = (u_p^{(i)}v_n^{(i)}\overline{X}^{m_i}_{(pn,J^\pi)} + v_p^{(i)}u_n^{(i)}\overline{Y}^{m_i}_{(pn,J^\pi)})\sqrt{D^{(i)}_{pn,J^\pi}}, \quad (C21)$$

$$\frac{\langle 0_f^+||[\widetilde{c^\dagger_p,\tilde{c}_n}]_J||0_i^+\rangle}{\sqrt{2J+1}} = (v_p^{(f)}u_n^{(f)}\overline{X}^{m_f}_{(pn,J^\pi)} + u_p^{(f)}v_n^{(f)}\overline{Y}^{m_f}_{(pn,J^\pi)})\sqrt{D^{(f)}_{pn,J^\pi}}, \quad (C22)$$

where the indices (i) and (f) indicate that the excitations are defined with respect to the ground state of the initial and final nucleus, respectively. When these states are not the same, the overlap factor

$$\langle J^\pi m_f|J^\pi m_i\rangle \approx \sum_{pn}(\overline{X}^{m_i}_{(pn,J^\pi)}\overline{X}^{m_f}_{(pn,J^\pi)} - \overline{Y}^{m_i}_{(pn,J^\pi)}\overline{Y}^{m_f}_{(pn,J^\pi)}) \times(u_n^{(i)}u_n^{(f)} + v_n^{(i)}v_n^{(f)}). \quad (C23)$$

must be introduced [59]. Repulsion between the nucleons at short distances is described by the short-range correlation factor $f(r_{12})$ of the form

$$f(r_{12}) = 1 - e^{-\alpha r_{12}^2}(1-br_{12}^2), \quad (C24)$$

where α=1.1 fm^2 a b=0.68 fm^2 [54]. Particle-particle and particle-hole channels of the nuclear Hamiltonian are renormalized by the parameters g_{pp} and g_{ph}

$$F(pn,p'n';J) \rightarrow g_{ph}F(pn,p'n';J), \quad (C25)$$

$$G(pn,p'n';J) \rightarrow g_{pp}G(pn,p'n';J). \quad (C26)$$

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