QCD sum rule description of nucleons in asymmetric nuclear matter

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We calculate the nucleon parameters in isospin asymmetric nuclear matter using the QCD sum rules. The nucleon self-energies Σ_v and Σ_s^* are expressed in terms of the in-medium values of QCD condensates. Simple approximate expressions for the self-energies are obtained in terms of these condensates. The relation between successive inclusion of the condensates and meson-exchange picture of the nucleon interaction with medium is analyzed. The values of the self-energies and symmetry energy agree with those obtained by the methods of nuclear physics.

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I. INTRODUCTION

In this paper we investigate the vector and scalar self-energies of nucleons in nuclear matter composed of neutrons and protons, distributed with densities ρ_n and ρ_p. We calculate the dependence on the total density

$$\rho = \rho_p + \rho_n \tag{1}$$

and on the asymmetry parameter

$$\beta = \frac{\rho_n - \rho_p}{\rho_p + \rho_n} \tag{2}$$

[another conventional presentation of the asymmetry parameter is $\beta=(N-Z)/(N+Z)=1-2Z/A$ with N and Z standing for the total number of neutrons and protons, while $A=N+Z$]. We present also the equations for the single-particle potential energies. The results are obtained by the QCD sum rule approach.

The QCD sum rules were introduced to express the hadron parameters in terms of the vacuum expectation values of QCD operators [1–3]. The approach succeeded in describing the static characteristics as well as some of the dynamical characteristics of the hadrons in vacuum—see, e.g., the reviews in Refs. [4] and [5]. Later the QCD sum rules were applied for an investigation of modified nucleon parameters in symmetric nuclear matter [6–8]. They were based on the Borel-transformed dispersion relation for the function $\Pi_m(q)$ describing the propagation of the system with the quantum numbers of the nucleon (the proton) in nuclear matter. Considering nuclear matter as a system of A nucleons with momenta p_i, one introduces the vector

$$p = \frac{\Sigma p_i}{A}, \tag{3}$$

which is thus $p \approx (m,0)$ in the rest frame of the matter.

The spectrum of the function $\Pi_m(q)$ is much more complicated than that of the vacuum function $\Pi_0(q)$. The separation of the singularities connected with the nucleon in matter from those connected with the excitations of matter itself can be achieved by keeping the variable

$$s = (p+q)^2 \tag{4}$$

as a constant and by fixing [7–9]

$$s = 4E_{0F}^2, \tag{5}$$

with E_{0F} being the total nucleon energy at the Fermi surface.

This condition ensures that the nucleon pole in the dispersion relations

$$\Pi_m^j(q^2,s) = \frac{1}{\pi}\int \frac{\mathrm{Im}\,\Pi_m^j(k^2,s)}{k^2-q^2}dk^2 \tag{6}$$

for the components $\Pi_m^j(q^2,s)$ $(j=q,p,I)$ of the function

$$\Pi_m(q) = q_\mu\gamma^\mu\Pi_m^q(q^2,s) + p_\mu\gamma^\mu\Pi_m^p(q^2,s) + I\Pi_m^I(q^2,s) \tag{7}$$

corresponds to the nucleon with the three-dimensional momentum, $|\mathbf{q}|$ being equal to the Fermi momentum p_F. This approach was employed in Refs. [7–10] and in Ref. [11]. Another version of the finite-density QCD sum rules based on dispersion relations in the time component q_0 at $|\mathbf{q}|$ fixed have been reviewed in Ref. [12]. In vacuum $\Pi_0^p=0$, while the functions $\Pi_0^{q,I}$ depend on q^2 only.

By using Eq. (6) the characteristics of the nucleon in nuclear matter can be expressed through the in-medium values of QCD condensates. The possibility of an extension of the "pole+continuum" model [1,2] to the case of finite densities was shown in Refs. [7–10].

The lowest order of the operator product expansion (OPE) of the left-hand side (LHS) of Eq. (6) can be presented in terms of the vector and scalar condensates [7–10]. Vector condensates $v_\mu^i=\langle M|\bar{q}^i\gamma_\mu q^i|M\rangle$ of the quarks with flavor i ($|M\rangle$ denotes the ground state of the matter) are the linear functions of the nucleon densities ρ_n and ρ_p. In the asymmetric matter both SU(2) symmetric and asymmetric condensates

$$v_\mu = \langle M|\bar{u}(0)\gamma_\mu u(0) + \bar{d}(0)\gamma_\mu d(0)|M\rangle = v_\mu^u + v_\mu^d,$$

$$v_\mu^{(-)} = \langle M|\bar{u}(0)\gamma_\mu u(0) - \bar{d}(0)\gamma_\mu d(0)|M\rangle = v_\mu^u - v_\mu^d \tag{8}$$

obtain nonzero values. In the rest frame of matter, $v_\mu^i=v^i\delta_{\mu 0}$, $v_\mu=v\,\delta_{\mu 0}$, $v_\mu^{(-)}=v^{(-)}\delta_{\mu 0}$. We can present $v^i=\langle n|\bar{q}^i\gamma_0 q^i|n\rangle\rho_n+\langle p|\bar{q}^i\gamma_0 q^i|p\rangle\rho_p$. The values $\langle N|\bar{q}^i\gamma_0 q^i|N\rangle$ are just the numbers of valence quarks in the nucleons $\langle n|\bar{u}\gamma_0 u|n\rangle=\langle p|\bar{d}\gamma_0 d|p\rangle=1$, $\langle p|\bar{u}\gamma_0 u|p\rangle=\langle n|\bar{d}\gamma_0 d|n\rangle=2$, and thus $v^u=\rho_n+2\rho_p=\rho(\frac{3}{2}-\frac{\beta}{2})$, $v^d=2\rho_n+\rho_p=\rho(\frac{3}{2}+\frac{\beta}{2})$. Hence, we obtain

$$v(\rho)=v_N\rho, \quad v^{(-)}(\rho,\beta)=\beta v_N^{(-)}\rho, \tag{9}$$

with

$$v_N=3, \quad v_N^{(-)}=-1. \tag{10}$$

The LHS of Eq. (6) also contains the SU(2) symmetric and asymmetric scalar condensates

$$\kappa_m(\rho)=\langle M|\bar{u}(0)u(0)+\bar{d}(0)d(0)|M\rangle,$$

$$\zeta_m(\rho,\beta)=\langle M|\bar{u}(0)u(0)-\bar{d}(0)d(0)|M\rangle. \tag{11}$$

These condensates can be presented as

$$\kappa_m(\rho)=\kappa_0+\kappa(\rho),$$

where $\kappa_0=\kappa_m(0)$ is the vacuum value,

$$\kappa(\rho)=\kappa_N\rho+\ \ldots, \quad \kappa_N=\langle N|\bar{u}u+\bar{d}d|N\rangle, \tag{12}$$

and

$$\zeta_m(\rho,\beta)=-\beta(\zeta_N\rho+\ \ldots), \quad \zeta_N=\langle p|\bar{u}u-\bar{d}d|p\rangle. \tag{13}$$

The dots on the RHS of Eqs. (12) and (13) denote the terms which are nonlinear in ρ. In the gas approximation such terms should be omitted. The SU(2) invariance of vacuum was assumed in Eq. (13). The expectation value κ_N is related to the πN sigma term $\sigma_{\pi N}$—i.e., [13]

$$\kappa_N=\frac{2\sigma_{\pi N}}{m_u+m_d}, \tag{14}$$

with $m_{u,d}$ standing for the current masses of the light quarks, while the value of $\sigma_{\pi N}$ can be extracted from experimental data on low-energy πN scattering [14,15]. However, the value ζ_N should be calculated under certain model assumptions on the quark structure of the nucleon. These were the condensates of dimension $d=3$.

Turning to the condensates of dimension $d=4$, we find, for the gluon condensate,

$$g_m(\rho)=\left\langle M\left|\frac{\alpha_s}{\pi}G^2(0)\right|M\right\rangle=g_0+g(\rho), \quad g_0=g_m(0),$$

$$g(\rho)=g_N\rho+\ \ldots, \tag{15}$$

with the nucleon expectation value $g_N=\langle N|(\alpha_s/\pi)G^2|N\rangle\approx-\frac{8}{9}m$ obtained in Ref. [16] in a model-independent way. Another contribution of the dimension $d=4$ comes from the nonlocal vector condensate $\langle N|\bar{q}(0)\gamma_\mu q(x)|N\rangle$. It can be expressed in terms of the higher moments of the nucleon structure function [9]. Next come the expectation values of the four-quark operators with dimension $d=6$. The importance of these contributions was analyzed in Ref. [17]. In the gas approximation they can be presented in terms of nucleon expectation values. The latter can be obtained in the framework of certain models.

We shall analyze the sum rules in the gas approximation. It is a reasonable starting point, since the nonlinear contributions to the most important scalar condensate $\kappa(\rho)$ are relatively small at densities of the order of the phenomenological saturation density $\rho_0=0.17\ \text{fm}^{-3}$ of symmetric matter [10]. The QCD sum rule approach, applied to a description of the nucleon self-energies in symmetric matter [18], provided results which are consistent with those obtained by the methods of nuclear physics.

In the case of symmetric matter ($\beta=0$), the leading OPE terms ($d=3,4$) can be either calculated or expressed in terms of the observables. In the asymmetric case we need a model for the calculation of the expectation value ζ_N, defined by Eq. (13). In most quark models the nucleon is treated as a system of valence quarks and an isospin symmetric sea of quark-antiquark pairs. Under this assumption the condensate ζ_N is determined by the valence quarks. In models with nonrelativistic valence quarks, $\zeta_N=1$. In more realistic relativistic models $\zeta_N<1$ due to the relativistic reduction.

The calculations of the four-quark condensates require model assumptions on the structure of the nucleon. The complete set of the four-quark condensates was obtained in Ref. [19] by using the perturbative chiral quark model (PCQM). The chiral quark model, originally suggested in Ref. [20], was developed further in Refs. [21–23].

Thus, in the calculations, which include terms of the order of $1/q^2$ of the OPE obtained in the framework of PCQM we must use the PCQM value $\zeta_N=0.54$ [21].

On the RHS of the sum rules we describe the nucleon by the relativistic in-medium propagator [24]

$$G_N^{-1}=q_\mu\gamma^\mu-m-\Sigma, \tag{16}$$

with the total self-energy

$$\Sigma=q_\mu\gamma^\mu\Sigma_q+p_\mu\gamma^\mu\frac{\Sigma_p}{m}+\Sigma_s. \tag{17}$$

We shall use the QCD sum rules for the calculation of the nucleon characteristics:

$$\Sigma_v=\frac{\Sigma_p}{1-\Sigma_q}, \quad m*=\frac{m+\Sigma_s}{1-\Sigma_q}, \quad \Sigma_s^*=m*-m, \tag{18}$$

identified with the vector self-energy, Dirac effective mass, and the effective scalar self-energy—see, e.g., Ref. [24]. Two other parameters to be determined in the approach are the in-medium shifts of the effective values of the nucleon residue $\delta\lambda_m^2$ and of the continuum threshold δW_m^2. We present also the result for the single-particle potential energies:

$$U=\Sigma_s^*+\Sigma_v. \tag{19}$$

We trace the dependence of these characteristics on the total density ρ and on the asymmetry parameter β—Eqs. (1) and (2).

Our approach includes only the strong interactions between the nucleons. The electromagnetic and weak interactions are neglected.

In asymmetric matter the characteristics obtain different values for the proton and neutron. Considering the asymmetry parameter in the interval $-1\leqslant\beta\leqslant1$, one can see that the

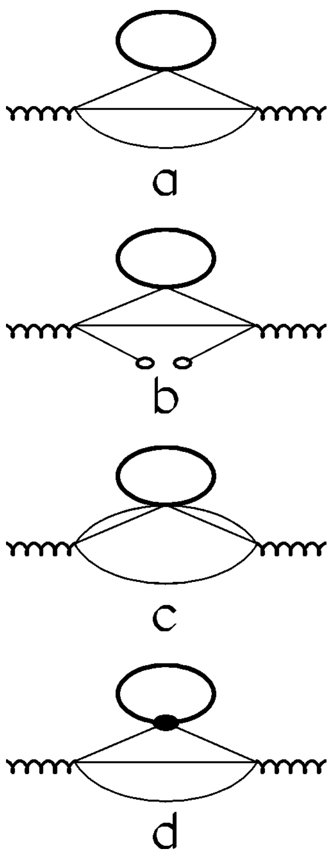

FIG. 1. Successive inclusion of the OPE terms. The helix lines denote the current (23). Solid lines denote the quarks. Thick lines show the matter. (a) The local vector and scalar condensates are included (contribution of the gluon condensate is not shown). (b) The factorized part of the four-quark condensates is included. One of the $\bar{q}q$ operators acts on vacuum, while another $\bar{q}q$ operator acts inside the nucleon. Small circles stand for averaging over vacuum. (c) The internal parts of the four-quark operators are included. All the quark operators act inside the nucleon. (d) The nonlocality of the vector condensate is included. The dark blobs denote the nonlocal condensates.

values of any parameter P_N for the proton (p) and neutron (n) are connected as

$$P_n(\beta) = P_p(-\beta). \tag{20}$$

We shall present all values for the proton using Eq. (20) to obtain those for the neutron.

We carry out the calculations in the gas approximation, including only terms linear in ρ on the LHS of the sum rules. Thus we can neglect the Fermi motion of the nucleons of the matter, which manifest themselves in higher order of Fermi momentum, and put

$$s = 4m^2 \tag{21}$$

in Eq. (4). Having in mind future extensions of the approach, we shall keep the dependence on s, using Eq. (21) for the specific computations.

In the simplest Hartree approximation (without multiparticle forces) Σ_p is linear in density. If one neglects the Fermi motion of the nucleon, as we did, the same is true for the

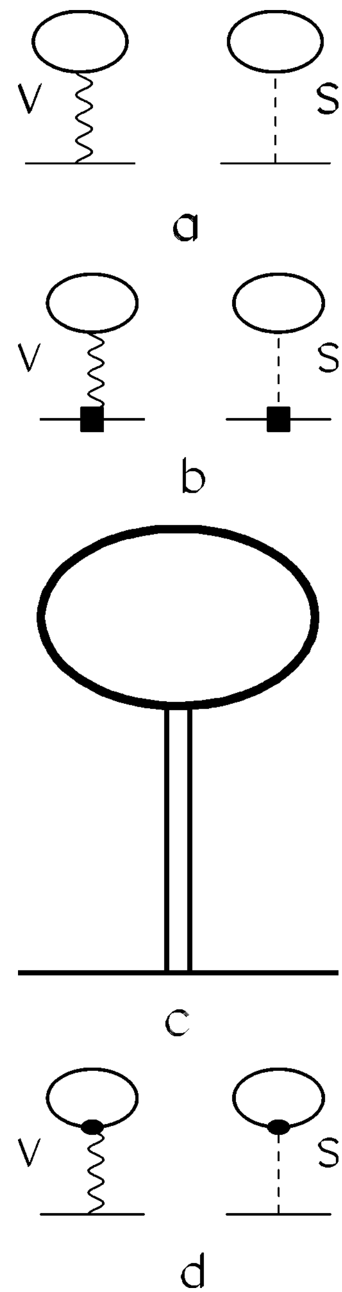

FIG. 2. The analogs of the OPE terms in the meson-exchange interaction of the nucleon with matter. (a)–(d) correspond to Figs. 1(a)–1(d). The solid line denotes the nucleon. Thick lines show the matter. Wavy and dashed lines stand for vector and scalar mesons. (a) Exchanges by vector and scalar ("effective") mesons with the pion vertices. (b) Exchanges by these mesons with the anomalous Lorentz structures of the vertices (the dark squares). (c) Exchanges by the $\bar{q}q$ pairs with strong correlations between them, presumably local interactions with two mesons. The double line denotes the two-meson systems. (d) Inclusion of the nonlocal structure of the vertices of interaction between the vector mesons and the nucleons of the matter. The dark blobs denote the form factors of the vertices.

scalar self-energy Σ_s. Thus, the parameters Σ_v and m^*, Eq. (18), exhibit nonlinear behavior due to the nonzero value of Σ_q ($\Sigma_q = 0$ in the mean-field approximation). In a similar way, the characteristics Σ_v and m^* are not linear in ρ and β in our approach due to the relativity large LHS of the sum rules for the structure Π_m^q. This reflects the distribution of the baryon charge between the pole and continuum. Another reason for the nonlinear effects (although numerically less important) is the nonlinear structure of the sum rule equations.

The in-medium nucleon QCD sum rules present nucleon interactions with matter in terms of an exchange by uncorrelated quark-antiquark pairs. The latter exchanges correspond to meson exchanges with the same quantum numbers. Thus the contributions of the lowest-order OPE terms can be viewed to the exchange by the isoscalar and isovector vector and scalar mesons, with the pointlike vertices of the interactions, having the standard Lorentz structure. Inclusion of nonlocal effects in the quark condensates corresponds to inclusion of nonlocal effects in the vertices of the interactions between the nucleons of matter and the mesons. The four-quark condensates, in which two of the quark operators are averaged over the vacuum state, correspond to the anomalous Lorentz structures of the vertices of the nucleon-meson interactions. In terms of the hadronic degrees of freedom four-quark condensates, in which all the quark operators act inside the nucleons, correspond to the exchanges by strongly correlated four-quark systems. Such exchanges can be viewed, e.g., as those by two mesons, interacting with the nucleon at the same point. This is illustrated by Figs. 1 and 2.

We compare our results for the nucleon self-energies and potential energies with those obtained by the traditional methods of nuclear physics. We find a promising consistency of the results.

We present the general equations in Sec. II, calculate the LHS of the sum rules in Sec. III, and provide the solutions in Sec. IV. We compare the results to those obtained by the methods of nuclear physics in Sec. V. We summarize in Sec. VI.

II. GENERAL EQUATIONS

The equations presented in this section are similar to those for symmetric matter obtained in Ref. [18]. The function $\Pi_m(q)$ defined by Eq. (7) (often referred to as the "polarization operator") is presented as

$$\Pi_m(q) = i\int d^4x e^{i(qx)}\langle M|Tj(x)\overline{j}(0)|M\rangle, \qquad (22)$$

with j being the three-quark local operator (often referred to as the "current") with proton quantum numbers. The usual choice is the current [2]

$$j(x) = \varepsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma_\mu d^c(x), \qquad (23)$$

with definite isospin $I=1/2$—see, e.g., Ref. [25]. Here T denotes a transpose and C is the charge conjugation matrix. The upper indices denote the colors.

The dispersion relations for the functions $\Pi_m^j(q^2,s)$ $(j=q,p,I)$ can be presented as

$$\Pi_m^{j,OPE}(q^2,s) = \frac{\lambda_m^{*2}b_j}{m_m^2-q^2} + \frac{1}{2\pi i}\int_{W_m^2}^{\infty}\frac{\Delta_{k^2}\Pi^{j,OPE}(k^2,s)}{k^2-q^2}, \qquad (24)$$

with $b_q=1$, $b_p=-\Sigma_v$, and $b_I=m^*$. The new position of the nucleon pole is

$$m_m^2 = \frac{(s-m^2)\Sigma_v/m - \Sigma_v^2 + m^{*2}}{1+\Sigma_v/m}.$$

The Borel-transformed sum rules (see Appendix A) take the form

$$L_m^q(M^2,W_m^2) = \Lambda_m(M^2), \qquad (25)$$

$$L_m^p(M^2,W_m^2) = -\Sigma_v\Lambda_m(M^2), \qquad (26)$$

$$L_m^I(M^2,W_m^2) = m^*\Lambda_m(M^2), \qquad (27)$$

with

$$\Lambda_m(M^2) = \lambda_m^{*2}e^{-m_m^2/M^2}, \qquad (28)$$

while λ_m^{*2} is the effective value of the nucleon residue in nuclear matter.

Varying in Eq. (24) the value of s defined by Eq. (4) we can find the position of the nucleon poles with different values of $|\mathbf{q}|$. The choice (5) leads to $|\mathbf{q}|=p_F$. In the simplified case expressed by Eq. (21), $|\mathbf{q}|=\mathbf{0}$. We shall investigate this very case. Inclusion of finite values of the Fermi momentum should be done together with inclusion of nonlinear terms in the scalar condensates. It was noted earlier [9] that in this approach the contributions of the nucleon and antinucleon poles are separated. The antinucleon pole corresponding to $q_0\approx -m$ generates the pole $q^2=5m^2$ shifted far to the right from the nucleon one. Thus it is among the contributions included in the second term on the RHS of Eq. (24).

Actually, we shall solve the sum rule equations, subtracting the vacuum effects:

$$L^q(M^2,W_m^2,W_0^2) = \Lambda_m(M^2) - \Lambda_0(M^2), \qquad (29)$$

$$L^p(M^2,W_m^2) = -\Sigma_v\Lambda_m(M^2), \qquad (30)$$

$$L^s(M^2,W_m^2,W_0^2) = m^*\Lambda_m(M^2) - m\Lambda_0(M^2), \qquad (31)$$

with $L^j=L_m^j-L_0^j$, while $L_0^{q,I}$ stand for the LHS of the QCD sum rules in vacuum $(L_0^p=0)$ and $\Lambda_0(M^2)$ is the vacuum value of the function $\Lambda_m(M^2)$. The vacuum values of the parameters are $m=0.94$ GeV, $\lambda_0^2=1.9$ GeV^6, and $W_0^2=2.2$ GeV^2. Recall that the matching of the LHS and RHS of the vacuum QCD sum rule has been achieved in the domain

$$0.8\ \text{GeV}^2 < M^2 < 1.4\ \text{GeV}^2. \qquad (32)$$

Thus Eqs. (29)–(31) should be solved in the same interval of the values of M^2.

III. CONTRIBUTIONS TO THE LEFT-HAND SIDE OF THE SUM RULES

We shall include terms of the order of $q^2 \ln q^2$, $\ln q^2$, and $1/q^2$ on the LHS of the sum rules (24). This corresponds to the terms of the order of M^4, M^2, and 1 on the LHS of the Borel-transformed equations (29)–(31). We shall include subsequently contributions of three types. The terms $\ell_m^j(M^2)$ stand for the lowest-order local condensates. They contribute to $q^2 \ln q^2$ and $\ln q^2$ terms on the LHS of Eq. (24). These contributions correspond to simple exchanges by isovector vector and scalar mesons between the nucleon on the RHS of Eq. (24) and the nucleons of matter, Figs. 1(a) and 2(a). The terms $u_m^j(M^2)$ are caused by the nonlocalities of the vector condensate, Fig. 1(d), contributing to the terms of the order of $\ln q^2$ and $1/q^2$. They correspond to the account of the form factors in the vertices between the isovector mesons coupled to the nucleons, Fig. 2(d). Finally, $\omega^j(M^2)$ describes contributions of the four-quark condensates, Figs. 1(b) and 1(c), being of the order of $1/q^2$. They correspond to two-meson exchanges (or to exchanges by four-quark mesons, if there are any) and to a somewhat more complicated structure of the meson-nucleon vertices, Fig. 2(c). Thus we present the LHS of Eqs. (25)–(27) as

$$L_m^j = \ell_m^j + u_m^j + \omega_m^j \tag{33}$$

and the LHS of Eqs. (29)–(31) as

$$L^j = \ell^j + u^j + \omega^j, \tag{34}$$

with $\ell^j = \ell_m^j - \ell_0^j$, $u^j = u_m^j$, and $\omega^j = \omega_m^j - \omega_0^j$, while ℓ_0^j and ω_0^j are the corresponding contributions in the vacuum case.

A. Local condensates of the lowest dimensions

The contributions of the local condensates of the lowest dimensions expressed by Eqs. (10)–(13) and (15) can be presented as

$$\ell^q = f_v^q(M^2, W_m^2)v^q(\rho) + f_g^q(M^2, W_m^2)g(\rho), \tag{35}$$

$$\ell^p = f_v^p(M^2, W_m^2)v^p(\rho,\beta), \tag{36}$$

$$\ell^I = f_\kappa^I(M^2, W_m^2)t^I(\rho,\beta), \tag{37}$$

with the dependence on ρ and β being contained in the factors

$$v^q(\rho) = 3\rho, \quad v^p(\rho,\beta) = 3\rho\left(1 - \frac{\beta}{4}\right), \quad t^I(\rho,\beta) = \rho(\kappa_N + \zeta_N\beta), \tag{38}$$

while the condensate $g(\rho)$ is given by Eq. (15). The functions f_a^j are [18]

$$f_v^q(M^2, W_m^2) = -\frac{8\pi^2}{3}\frac{(s-m^2)M^2E_{0m} - M^4E_{1m}}{mL^{4/9}},$$

$$f_g^q(M^2, W_m^2) = \frac{\pi^2 M^2 E_{0m}}{L^{4/9}},$$

$$f_v^p(M^2, W_m^2) = -\frac{8\pi^2}{3}\frac{4M^4E_{1m}}{L^{4/9}},$$

$$f_\kappa^I(M^2, W_m^2) = -4\pi^2 M^4 E_{1m}. \tag{39}$$

The notation E_{km} $(k=0,1)$ in Eq. (39) means that the functions $E_0(x) = 1 - e^{-x}$, $E_1(x) = 1 - (1+x)e^{-x}$ depend on the ratio $x = W_m^2/M^2$ and the factor $L(M^2) = (\ln M^2/\Lambda_{QCD}^2)/(\ln \nu^2/\Lambda_{QCD}^2)$ accounts for the anomalous dimension—i.e., the most important corrections of the order α_s enhanced by the "large logarithms." We put $\Lambda_{QCD} = 0.15$ GeV, while $\nu = 0.5$ GeV is the normalization point of the characteristic involved.

Now we must find the β dependence of the nucleon self-energies and also of the parameters W_m^2 and λ_m^{*2}. Note that there is a simple solution of Eqs. (25)–(27):

$$\sum\nolimits_v (\rho,\beta) = \sum\nolimits_v (\rho,0)\left(1 - \frac{\beta}{4}\right), \tag{40}$$

$$m^*(\rho,\beta;\kappa_N,\zeta_N) = m^*(\rho,0;\kappa_N + \beta\zeta_N,0), \tag{41}$$

$$W_m^2(\rho,\beta) = W_m^2(\rho,0), \tag{42}$$

which is true with the same accuracy as solutions for symmetric matter $(\beta=0)$ [18]. Indeed, assuming that W_m^2 does not change with β we find that the function $\Lambda_m(M^2)$ on the LHS of Eq. (25) also should not depend on β.[1] This leads to Eqs. (40)–(42).

B. Inclusion of the nonlocal condensates

It was shown in Ref. [9] that only vector nonlocal condensate contributes to the OPE of the LHS of the sum rules. Thus, for a flavor i,

$$\theta_\mu^i(x) = \langle M|\bar{q}^i(0)\gamma_\mu q^i(x)|M\rangle = \frac{p_\mu}{m}\Phi_a^i((px),x^2) + ix_\mu m\Phi_b^i((px),x^2). \tag{43}$$

Each of the functions on the RHS of Eq. (43) can be presented as the sum of the proton and neutron contributions:

$$\Phi_{a(b)}^i = \rho_p\phi_{n,a(b)}^i + \rho_n\phi_{p,a(b)}^i. \tag{44}$$

As a result of the SU(2) invariance, they can be presented in terms of the proton functions $\phi_{a(b)}^i = \phi_{p,a(b)}^i$:

$$\Phi_{a(b)}^i = \rho_p\phi_{a(b)}^i + \rho_n\phi_{a(b)}^{i'}, \tag{45}$$

with $i = u,d$, $i' \neq i$.

Expansion in powers of x^2 corresponds to expansion of the function $\Pi_m(q)$ in powers of q^2. To obtain terms of the order of q^{-2} it is sufficient to include the two lowest terms of the expansions in powers of x^2. One can present [9,26]

[1]The numerical solution, which will be considered in Sec. IV, provides indeed $W_m^2(\rho,\beta) \approx W_m^2(\rho,0)$ with an accuracy of 10%.

$$\phi^i_{a(b)}((px),x^2) = \int_0^1 d\alpha e^{-i\alpha(px)} f^i_{a(b)}(\alpha,x^2), \tag{46}$$

with

$$f^i_{a(b)}(\alpha,x^2) = \eta^i_{a(b)}(\alpha) + \frac{1}{8}x^2 m^2 \xi^i_{a(b)}(\alpha). \tag{47}$$

Here $\eta^i_a(\alpha)=f^i_a(\alpha,0)$ is the contribution of the quarks with the flavor i to the asymptotic of the proton structure function $\eta(\alpha)=\eta^u_a(\alpha)+\eta^d_a(\alpha)$. Their moments are well known—at least those which are numerically important. The lowest moments of the functions η^i_b can be expressed in terms of the moments of the functions η^i_a and ξ^i_a [9].

In asymmetric matter the two combinations $\Phi^u_{a(b)}+\Phi^d_{a(b)}$ and $\Phi^u_{a(b)}-\Phi^d_{a(b)}$ contribute, while at $\beta=0$ only the former one survives. Thus we can present

$$u^j(M^2) = [u^j_{N,1}(M^2) + \beta u^j_{N,2}(M^2)]\rho \tag{48}$$

($j=q,p$) for the functions $u^j(M^2)$ to the RHS of Eq. (34). In symmetric matter $u^j(M^2)=u^j_{N,1}(M^2)\rho$. These functions were calculated in Ref. [18]. In asymmetric matter we must include

$$u^q_{N,2}(M^2) = \frac{8\pi^2}{3L^{4/9}m}\left[\frac{3}{2}m^2M^2E_{0m}\langle(\eta^u - \eta^d)\alpha\rangle\right] \tag{49}$$

and

$$u^p_{N,2}(M^2) = \frac{8\pi^2}{3L^{4/9}}\Bigg[3[M^4E_{1m} - (s-m^2)M^2E_{0m}]\langle(\eta^u - \eta^d)\alpha\rangle + \frac{9}{5}m^2M^2E_{0m}\langle(\eta^u - \eta^d)\alpha^2\rangle - \frac{27}{10}m^2M^2E_{0m}(\langle\xi^u\rangle - \langle\xi^d\rangle)\Bigg], \tag{50}$$

while $u^I(M^2)=0$, as well as in symmetric matter. Here we denote $L=L(M^2)$, $E_{1m}=E_1(W^2_m/M^2)$, and $E_{0m}=E_0(W^2_m/M^2)$—see Sec. III A. The values of $\langle\xi^u\rangle=-0.24$ and $\langle\xi^d\rangle=0.09$ were calculated in Ref. [27]. We defined $\langle F\rangle=\int_0^1 d\alpha F(\alpha)$ for any function $F(\alpha)$.

C. Inclusion of the four-quark condensates

The exchange by two quark-antiquark pairs between the current (23) and matter is described in terms of the four-quark expectation values

$$H^{XY}_m(\rho) = \langle M|\bar{u}\Gamma^X u\bar{u}\Gamma^Y u|M\rangle, \quad R^{XY}_m(\rho) = \langle M|\bar{d}\Gamma^X d\bar{u}\Gamma^Y u|M\rangle, \tag{51}$$

with $\Gamma^{X,Y}$ being the basic 4×4 matrices $\Gamma^I=I$, $\Gamma^{Ps}=\gamma_5$, $\Gamma^V=\gamma_\mu$, $\Gamma^A=\gamma_\mu\gamma_5$, and $\Gamma^T=(i/2)(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$, acting on the Lorentz indices of the quark operators. We did not display the color indices in Eq. (51), keeping in mind that the quark operators are color antisymmetric.

In the gas approximation,

$$H^{XY}_m(\rho,\beta) = H^{XY}_m(0) + \rho_n h^{XY}_n + \rho_p h^{XY}_p,$$

$$R^{XY}_m(\rho,\beta) = R^{XY}_m(0) + \rho_n r^{XY}_n + \rho_p r^{XY}_p. \tag{52}$$

The characteristics h^{XY}_N and r^{XY}_N can be presented as

$$h^{XY}_N = \frac{5}{6}(\langle 0|\bar{u}\Gamma^X u|0\rangle\langle N|\bar{u}\Gamma^Y u|N\rangle + \langle 0|\bar{u}\Gamma^Y u|0\rangle\langle N|\bar{u}\Gamma^X u|N\rangle) + \langle N|(\bar{u}\Gamma^X u\cdot\bar{u}\Gamma^Y u)_{int}|N\rangle, \tag{53}$$

$$r^{XY}_N = \frac{2}{3}(\langle 0|\bar{d}\Gamma^X d|0\rangle\langle N|\bar{u}\Gamma^Y u|N\rangle + \langle 0|\bar{u}\Gamma^Y u|0\rangle\langle N|\bar{d}\Gamma^X d|N\rangle) + \langle N|(\bar{d}\Gamma^X d\cdot\bar{u}\Gamma^Y u)_{int}|N\rangle. \tag{54}$$

The first and second "factorized" terms on the RHS of Eqs. (53) and (54) describe two quark operators, acting on the vacuum state, while the other two operators act inside the nucleon. Of course, these terms obtain nonvanishing values only in the scalar case $\Gamma^X=I$ (or $\Gamma^Y=I$). The last terms describe the "internal" action of all four operators inside the nucleon. This is shown by the subscript *int*. The coefficients $5/6$ and $2/3$ on the RHS of Eqs. (53) and (54) present the weights of the color-antisymmetric states [18].

Note that there are the terms which depend on the quark masses explicitly. In a somewhat straightforward approach one substitutes the current quark masses. Following more sophisticated models of pions [28] one should substitute the constituent quark masses, thus obtaining the values, which are negligibly small in our scale.

Thus we obtain

$$(\Pi)_{4q} = \left(A^q_{4q}(\beta)\frac{q_\mu\gamma^\mu}{q^2} + A^p_{4q}(\beta)\frac{(pq)}{m^2}\frac{p_\mu\gamma^\mu}{q^2} + A^I_{4q}(\beta)m\frac{I}{q^2}\right)\frac{a}{(2\pi)^2}\rho, \tag{55}$$

with the coefficients $A^j_{4q}(\beta)$ being determined by the nucleon four-quark expectation values [19] while

$$a = -(2\pi)^2\langle 0|\bar{u}u|0\rangle. \tag{56}$$

We use the value $\langle 0|\bar{u}u|0\rangle=(-241\text{ MeV})^3$, corresponding to $a=0.55\text{ GeV}^3$, employed in Ref. [3]. Note that a is just a convenient scale for presentation of the results. It does not reflect the chiral properties of $(\Pi)_{4q}$.

The coefficients A^q_{4q} results mainly as the sum of the expectation value of the product of the four u-quark operators, described by the first ("factorized") term on the RHS of Eq. (53) and that of the product of two u- and two d-quark operators in the vector channel—Eq. (54). The former contributions depend on β, while the latter do not. Thus, the coefficient A^q_{4q} depends on β strongly. The factor A^p_{4q} is determined mostly by the expectation value (54) in the vector channel, with the protons and neutrons providing the equal contributions. This explains the weak dependence of the parameter A^p_{4q} on β. The coefficient A^I_{4q} is dominated by the first term on the RHS of Eq. (54), providing stronger dependence on β. The calculations give

$$A_{4q}^{q} = -0.11 - 0.21\beta, \quad A_{4q}^{p} = -0.57 + 0.09\beta,$$
$$A_{4q}^{I} = 1.90 - 0.92\beta. \tag{57}$$

In the simplified model of the pion, which does not include renormalization of the quark masses by the interactions, the value of the coefficient A_{4q}^{q} is somewhat different:

$$A_{4q}^{q} = 0.25 - 0.22\beta, \tag{58}$$

while the values of A_{4q}^{p} and A_{4q}^{I} remain unchanged.

The contributions of the four-quark condensates to the LHS of the Borel-transformed sum rules (29)–(31) can be presented in the same way as for the symmetric case:

$$\omega^{j} = \omega_{N}^{j}\rho, \quad \omega_{N}^{j} = A_{4q}^{j}(\beta) f_{4q}^{j},$$

$$f_{4q}^{q} = -8\pi^{2}a, \quad f_{4q}^{p} = -8\pi^{2}\frac{s-m^{2}}{2m}a, \quad f_{4q}^{I} = -8\pi^{2}ma. \tag{59}$$

IV. SOLUTIONS OF THE SUM RULE EQUATIONS

Now we present the solutions of the sum rule equations, focusing on the functions $\Sigma_{v}(\rho,\beta)$ and $m^{*}(\rho,\beta)$. We shall include the terms ℓ^{j}, u^{j}, and ω^{j} [Eq. (34)] in the succession on the LHS of Eqs. (29)–(31). Two lowest-order OPE contributions to the vector structures Π_{m}^{q} and Π_{m}^{p} are presented in terms of the vector and gluon condensates and of the nucleon structure functions. These characteristics are either calculated in a model-independent way or determined in the experiments. The lowest-order OPE terms in the scalar channel are expressed in terms of isotope-symmetric and isotope-asymmetric scalar condensates $\kappa_{N} = \langle p|\bar{u}u + \bar{d}d|p\rangle$ and $\zeta_{N} = \langle p|\bar{u}u - \bar{d}d|p\rangle$—Eqs. (12) and (13). Here the situation becomes somewhat more complicated.

The expectation value κ_{N} is related to the πN sigma term $\sigma_{\pi N}$ by Eq. (14). The value of $\sigma_{\pi N}$ can be extracted from the data on low-energy πN scattering. The procedure consists in subtracting the high-order chirality-violating terms σ' from the experimental value $\Sigma_{\pi N}$—i.e., $\sigma_{\pi N} = \Sigma_{\pi N} - \sigma'$. The value $\sigma' \approx 15$ MeV was obtained in Ref. [29] by the dispersion relation technique. However, there are some uncertainties in deducing the value of $\Sigma_{\pi N}$ from the experimental data. The canonical value $\Sigma_{\pi N} = (60 \pm 8)$ MeV [14] is now challenged by the higher values 77 ± 6 MeV [15]. Assuming $m_{u} + m_{d} = 11$ MeV [13], we find that $\Sigma_{\pi N} = 64$ MeV corresponds to $\kappa_{N} = 8$. Additional uncertainties emerge, since the true value of the sum $m_{u} + m_{d}$ may be somewhat larger.

There are no experimental data on the expectation value ζ_{N}. If the nucleon is treated as a system of valence quarks and an isospin-symmetric sea of the quark-antiquark pairs, the expectation value ζ_{N} is determined by the contribution of the valence quarks. Thus, reasonable values are $\zeta_{N} = 1$ for nonrelativistic models and $\zeta_{N} < 1$ in the relativistic case. Until we include only the leading OPE terms ℓ^{j}, we can solve the sum rule equations for any values of κ_{N} and ζ_{N}. However, the four-quark condensates are obtained in the framework of a specific perturbative chiral quark model. Within this model, $\sigma_{\pi N} = 45$ MeV [22], leading to $\kappa_{N} = 8$. Thus, to be self-consistent, we must use this value as the basic one in the general equations, which include the contributions ω^{j}. Note also that the values of $\Sigma_{\pi N}$ extracted from the experimental data are correlated with the assumption on the strange quark content $y_{N} = 2\langle p|\bar{s}s|p\rangle / \langle p|\bar{u}u + \bar{d}d|p\rangle$ [15]. The values $\Sigma_{\pi N} \approx 77$ MeV correspond to $y_{N} \approx 0.35$, with a large part of the nucleon mass being due to the strange quarks. The smaller values of $\Sigma_{\pi N}$ require much smaller values of y_{N}. In the PCQM one finds $y_{N} = 0.08$ [22], in agreement with the smaller values of $\Sigma_{\pi N}$. The PCQM value $\zeta_{N} = 0.54$ can be obtained by using the results of Ref. [21].

We shall present most of the numerical results for the values

$$\kappa_{N} = 8, \quad \zeta_{N} = 0.54. \tag{60}$$

Anyway, in Sec. IV B the nucleon characteristics will be presented as an explicit function of the condensates—e.g., of the parameters κ_{N} and ζ_{N}.

We shall find the values of the parameters which minimize the relative difference between the RHS and LHS of Eqs. (29)–(31) at values of M^{2} in the interval (32).

A. Solution of the general equations

Here we present solutions of the general equations (29)–(31), which are identical to Eqs. (25)–(27). Recall that we approximate the in-medium condensates by the functions which are linear both in ρ and β. However, the solutions $\Sigma_{v}(\rho,\beta)$ and $m^{*}(\rho,\beta)$ are not linear. One can demonstrate this by presenting Eqs. (26) and (27) as

$$\Sigma_{v} = -\frac{L_{m}^{p}}{L_{m}^{q}}, \quad m^{*} = \frac{L_{m}^{I}}{L_{m}^{q}}, \tag{61}$$

with the density and β dependence of L_{m}^{q} leading to nonlinear behavior of Σ_{v} and m^{*} (even if we assume $W_{m}^{2} = W_{0}^{2}$). The nonlinear dependence of the RHS of Eq. (61) on W_{m}^{2} also cause the nonlinear contributions to Σ_{v} and m^{*}. However, they are numerically less important.

Now we include the terms ℓ^{j}, u^{j}, and ω^{j} in succession on the LHS of Eqs. (29)–(31).

1. Role of the lowest-order local condensates

As we have seen, these are the contributions which contain the vector condensate $v(\rho)$, gluon condensate $g(\rho)$, and scalar condensates κ_{m} and ζ_{m}—Eqs. (9)–(13) and (15). An account of these terms corresponds to one-meson exchanges between the nucleon under consideration and the nucleon of matter, with pointlike structures of the meson-nucleon vertices, Fig. 2(a). The solution can be obtained by using Eqs. (35)–(37) for the functions ℓ^{j}. As we have seen, there is a simple solution expressed by Eqs. (40)–(42). The procedure of minimization of the difference between the LHS and RHS of Eqs. (29)–(31) indeed prefers the values $W_{m}(\rho,\beta) \approx W_{m}(\rho,0)$. Thus Eqs. (40) and (41) appear to be true with good accuracy. Hence,

$$\Sigma_v^{(p)}(\rho,\beta) = \Sigma_v(\rho,0)\left(1 - \frac{\beta}{4}\right),$$

$$\Sigma_v^{(n)}(\rho,\beta) = \Sigma_v(\rho,0)\left(1 + \frac{\beta}{4}\right),$$

$$m^{*(p)}(\rho,\beta,\kappa_N,\zeta_N) = m*(\rho,0,\kappa_N+\beta\zeta_N,0),$$

$$m^{*(n)}(\rho,\beta,\kappa_N,\zeta_N) = m*(\rho,0,\kappa_N-\beta\zeta_N,0). \qquad (62)$$

Thus, in matter with an excess of neutrons ($\beta>0$), we obtain $\Sigma_v^{(n)}>\Sigma_v^{(p)}$ and $m^{*(n)}>m^{*(p)}$. For example, using the value $\Sigma_v(\rho,0)$ obtained by the sum rule approach in Ref. [18] [$\Sigma_v(\rho,0)=335$ MeV], we find $\Sigma_v^{(n)}-\Sigma_v^{(p)}=170$ MeV, $m^{*(n)}-m^{*(p)}=50$ MeV for neutron matter ($\beta=1$) at $\rho=\rho_0$.

The minimization procedure chooses $W_m^2(\beta=-1)=2.50\ \mathrm{GeV}^2$ and $W_m^2(\beta=0)=2.30\ \mathrm{GeV}^2$, $W_m^2(\beta=1)=2.05\ \mathrm{GeV}^2$, $\Sigma_v^{(n)}-\Sigma_v^{(p)}=170$ MeV, and $m^{*(n)}-m^{*(p)}=50$ MeV. Thus, Eqs. (62) work well indeed. However, this approximation is not sufficient for a description of the potential energy $U(\rho,\beta)$, Eq. (19), providing $U>0$ for both symmetric and asymmetric cases.

2. Role of the four-quark condensates

Now we include the four-quark condensate; i.e., we use Eq. (33) for $L^j=\ell^j+\omega^j$, with ω^j described by Eqs. (59). Inclusion of these terms mimics several contributions on the RHS of the dispersion relations (24). In the condensates $\bar{u}\Gamma^X u\bar{u}\Gamma^X u$, presented by Eq. (53), the first term, which has a nonvanishing value only if $\Gamma^X=I$, generates a contribution to the Π_m^q structure due to the anomalous Lorentz structure of the interaction between the scalar field and the nucleon, caused by the chiral-odd vacuum condensate $\langle 0|\bar{q}q|0\rangle$. In a similar way the first term on the RHS of Eq. (54) describes the contribution of the vector meson exchange to the scalar structure of the nucleon propagator. The anomalous Lorentz structures emerge if the nucleon-meson vertices are treated beyond the lowest order. These contributions are illustrated by Fig. 2(b). The last terms on the RHS of Eqs. (53) and (54) describe exchanges by the four-quark strongly correlated system [see Fig. 2(c)]. The condensates presented by Eq. (54) have the same values for the proton and neutron. Thus their contributions do not depend on β.

The differences between the neutron and proton characteristics in neutron matter at $\rho=\rho_0$ become $\Sigma_v^{(n)}-\Sigma_v^{(p)}=140$ MeV and $m^{*(n)}-m^{*(p)}=-110$ MeV. In the simplified model for the pion with the current masses of the constituent quarks, where $A_{4q}^q(\beta)$ is given by Eq. (58), we obtain $\Sigma_v^{(n)}-\Sigma_v^{(p)}=145$ MeV, while $m^{*(n)}-m^{(p)}=-110$ MeV at these values of ρ and β.

3. Final results

The nonlocal contributions come from the account of the x dependence of the expectation values of the vector operators $\langle M|\overline{q^i}(0)\gamma_\mu q^i(x)|M\rangle$ with $q^i(x)=(1+x_\mu D^\mu+\cdots)q^i(0)$. As we have seen, the nonlocality of the scalar condensates is not important in our case. The nonlocality is included by putting $L^j=\ell^j+\omega^j+u^j$ with u^j defined by Eqs. (49) and (50). We use the structure functions obtained in Ref. [30] for the calculation of the terms u^q and u^p.

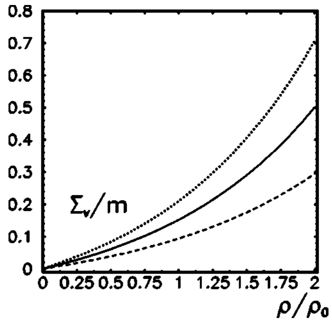

FIG. 3. The density dependence of the vector self-energy Σ_v [solution of Eqs. (29)–(31)]. The solid line shows the results for symmetric matter ($\beta=0$). The dashed and dotted lines show the proton and neutron self-energies $\Sigma_v^{(p)}$ and $\Sigma_v^{(n)}$ in neutron matter ($\beta=1$).

An account of the nonlocality of the vector condensate corresponds to inclusion of the form factor of the vertex of the interaction between the vector meson and the nucleon of the matter—Fig. 2(d). Recall that a similar contribution for the effective scalar meson exchanges vanishes in our approximation—Sec. III B.

We find the dependence of the nucleon vector self-energy Σ_v and of the effective mass $m*$ on the density of matter and on the asymmetry parameter β and show the results in Figs. 3 and 4. For example, the differences between the neutron and proton characteristics in the neutron matter at $\rho=\rho_0$ are $\Sigma_v^{(n)}-\Sigma_v^{(p)}=110$ MeV, and $m^{*(n)}-m^{*(p)}=-70$ MeV. In the simplified model for the pion with current quark masses, where $A_{4q}^q(\beta)$ is given by Eq. (58), we obtain $\Sigma_v^{(n)}-\Sigma_v^{(p)}=115$ MeV and $m^{*(n)}-m^{*(p)}=-65$ MeV at these values of ρ and β. The minor change is due to the small change in the β dependence of the condensate A_{4q}^q. The nucleon residue λ_m^2 and the spectrum threshold W_m^2 exhibit very weak dependence on β. Thus we can assume

$$\lambda_m^2(\rho,\beta)=\lambda_m^2(\rho,0), \quad W_m^2(\rho,\beta)=W_m^2(\rho,0). \qquad (63)$$

The consistency between the RHS and LHS of Eqs. (29)–(31) is illustrated by Fig. 5.

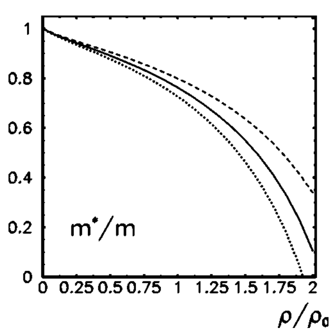

FIG. 4. The density dependence of the effective mass $m*$ [solution of Eqs. (29)–(31)]. The solid line shows the results for symmetric matter ($\beta=0$). The dashed and dotted lines show the proton and neutron effective masses $m^{*(p)}$ and $m^{*(n)}$ in neutron matter ($\beta=1$).

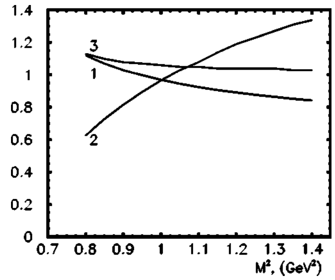

FIG. 5. The consistency of the LHS and RHS of Eqs. (29)–(31) for neutrons in neutron matter at $\rho=\rho_0$. The lines 1, 2, and 3 show the LHS to RHS ratio of Eqs. (29)–(31) correspondingly in the interval (32) of the values of M^2.

As a result of the nonlinear character of Eqs. (29)–(31), the self-energies $\Sigma_v(\rho,\beta)$ and $\Sigma_s^*(\rho,\beta)=m^*(\rho,\beta)-m$ could have been nonlinear in both ρ and β. The nonlinear behavior of these characteristics with ρ manifests itself explicitly. However, the dependence on β appears to be linear in the framework of the accuracy of our computations (see the next subsection). Thus both Σ_v and Σ_s^* can be approximated by linear functions of β:

$$\Sigma_v(\rho,\beta) = \frac{\rho}{\rho_0}[V_1(\rho) + \beta\tau_z V_2(\rho)],$$

$$\Sigma_s^*(\rho,\beta) = \frac{\rho}{\rho_0}[S_1(\rho) + \beta\tau_z S_2(\rho)], \qquad (64)$$

with $\tau_z=1$ for the proton and $\tau_z=-1$ for the neutron. The functions $V_{1,2}(\rho)$ and $S_{1,2}(\rho)$ are shown in Fig. 6. They can be approximated by polynomials of the second order—see Appendix B.

The single-particle potential energy is expressed by Eq. (19). At $\rho=\rho_0$ the neutron-proton difference of the potential energy caused by the isovector interaction is $\Delta U_{np} \approx 38\beta$ MeV at small β. In Fig. 7 we show the dependence $U(\rho)$ for several values of β for both neutrons and protons. Recall that the potential energy is determined with an accuracy lower than the self-energies.

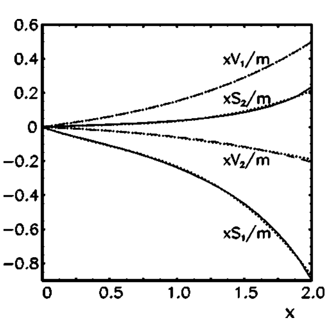

FIG. 6. The density dependence of the functions xV_1, xV_2, xS_1, and xS_2 introduced by Eqs. (64), $x=\rho/\rho_0$. The dotted lines demonstrate the quality of fitting with the simple functions on ρ, as described in Appendix B.

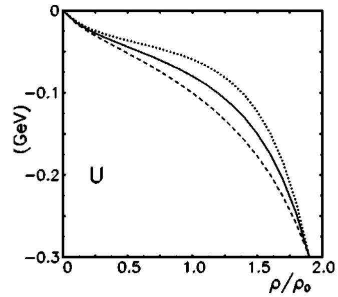

FIG. 7. The density dependence of the single-particle potential energy U, Eq. (19). The solid line shows the results for symmetric matter ($\beta=0$). The dashed and dotted lines show the proton and neutron self-energies $U^{(p)}$ and $U^{(n)}$ in neutron matter ($\beta=1$).

B. Explicit expression for the nucleon parameters in terms of QCD condensates

We can present an approximate solution of Eqs. (29)–(31), in which the nucleon self-energies are expressed in terms of the QCD condensates explicitly. We see from Eq. (63) that $W_m^2(\rho,\beta)\approx W_m^2(\rho,0)$, while $W_m^2(\rho,0)$ is close to its vacuum value W_0^2 [18]. Thus, we can put $W_m^2=W_0^2$ on the RHS of Eqs. (25)–(27) and (61). This enables us to present the proton characteristics Σ_v and m^* as explicit functions of the quark condensates and of the Borel mass M^2:

$$\Sigma_v = -\frac{T_v^p\big(v_N + 3/4\beta v_N^{(-)}\big) + T_{u1}^p + \beta T_{u2}^p + T_\omega^p A_{4q}^p(\beta)}{1 + F^q(\rho,\beta)}\frac{\rho}{\rho_0}, \qquad (65)$$

$$m^* = \left[m + [T_\kappa^I(\kappa_N + \beta\zeta_N) + T_\omega^I A_{4q}^I(\beta)]\frac{\rho}{\rho_0}\right]\frac{1}{1 + F^q(\rho,\beta)}, \qquad (66)$$

with

$$F^q(\rho,\beta) = [T_v^q v_N + mT_g^q g_N + T_{u1}^q + \beta T_{u2}^q + T_\omega^q A_{4q}^q(\beta)]\frac{\rho}{\rho_0}. \qquad (67)$$

We denote $T_\omega^j = T_\omega^j(M^2)$ and $T_k^j = T_k^j(M^2,W_0^2)$ for other k and introduce, for $j=q,p,I$,

$$T_k^j(M^2,W_0^2) = \rho_0 f_k^j(M^2,W_0^2)\frac{e^{m^2/M^2}}{\lambda_0^2} \quad (k=v,g,\kappa),$$

$$T_{ur}^j(M^2,W_0^2) = \rho_0 u_{N,r}^j(M^2,W_0^2)\frac{e^{m^2/M^2}}{\lambda_0^2} \quad (r=1,2),$$

$$T_\omega^j(M^2) = \rho_0 f_{4q}^j\frac{e^{m^2/M^2}}{\lambda_0^2}, \qquad (68)$$

with the functions f_k^q and f_{4q}^q defined by Eqs. (39) and (59). It is instructive to present the density ρ in units of the observable saturation density of symmetric matter, $\rho_0=0.17$ fm^{-3}.

Note that in the interval determined by Eq. (32) the functions $T_k^j(M^2)$ defined by Eq. (68) ($k=v,g,\kappa,u1,u2$; $j=q,p,I$) depend on M^2 rather weakly. Thus, approximating

$$T_k^j(M^2) = C_k^j, \tag{69}$$

we can replace the functions $T_k^j(M^2)$ on the LHS of Eqs. (68) by the constant coefficients C_k^j. Numerically the most important functions $T_v^p(M^2)$ and $T_\kappa^I(M^2)$ can be approximated by the constant values with the errors of about 4% and 7%. The largest errors of about 25% emerge in the averaging of the functions T_ω^j. This solves the problem of expressing the in-medium change of nucleon parameters through values of the condensates. For the proton,

$$\Sigma_v = -[C_v^p v_N + \beta C_{v^{(-)}}^p v_N^{(-)} + mC_{u1}^p + \beta m C_{u2}^p + mC_\omega^p A_{4q}^p(\beta)]\frac{\rho}{\rho_0}\frac{1}{(1-\mathcal{F}_q)}, \tag{70}$$

$$m^* = \left[m + [C_\kappa^I \kappa_N + \beta C_\zeta^I \zeta_N + C_\omega^I A_{4q}^I(\beta)]\frac{\rho}{\rho_0}\right]\frac{1}{(1-\mathcal{F}_q)}, \tag{71}$$

with

$$\mathcal{F}_q = -[C_v^q v_N + mC_g^q g_N + mC_{u1}^q + \beta m C_{u2}^q + mC_\omega^q A_{4q}^q(\beta)]\frac{\rho}{\rho_0}. \tag{72}$$

The coefficients on the RHS of Eqs. (70)–(72) are

$$C_v^q = -0.062, \quad C_g^q = 0.011\ \text{GeV}^{-1}, \quad C_\omega^q = -0.070,$$

$$C_{u1}^q = -0.074, \quad C_{u2}^q = 0.008,$$

$$C_\kappa^I = -0.042\ \text{GeV}, \quad C_\zeta^I = -0.042\ \text{GeV},$$

$$C_\omega^I = -0.063\ \text{GeV},$$

$$C_v^p = -0.090\ \text{GeV}, \quad C_{v^{(-)}}^p = -0.068\ \text{GeV}, \quad C_\omega^p = -0.095,$$

$$C_{u1}^p = 0.094, \quad C_{u2}^p = -0.020. \tag{73}$$

Note that the dependence of $\mathcal{F}_q$ on β is very weak. (Recall that the lowest-order OPE terms in the Π_m^q structure of the polarization operator do not depend on β.) Thus, on the LHS of Eqs. (70) and (71) only the dependence of the numerators on β is important. This explains the linear dependence of the self-energies Σ_v and Σ_s^* on β.

The values of Σ_v, m^*, and $\mathcal{F}_q$ for the neutron are described by Eqs. (70)–(72) with β changed to $-\beta$. Equations (70) and (71) enable us to obtain Σ_v and m^* during successive inclusion of condensates of higher dimensions. If only the leading OPE terms are included the values provided by Eqs. (70) and (71) actually coincide with the solutions of Eqs. (29)–(31). If all the contributions are included, Eqs. (70) and (71) reproduce the values of Σ_v and m^* with an accuracy of 15% and 10% correspondingly for symmetric matter. The precision of Eqs. (70) and (71) changes with β. For neutron matter these equations provide the values $\Sigma_v^{(n)} - \Sigma_v^{(p)} = 120$ MeV and $m^{*(n)} - m^{*(p)} = -60$ MeV, comparing to the values of 110 MeV and −70 MeV, obtained in the previous subsection from the general solution of Eqs. (29)–(31).

V. DISCUSSION

Now we compare our results to those obtained by nuclear physics methods. The lowest-order OPE terms on the LHS of the sum rules describe mainly the exchanges by localized $\bar{q}q$ pairs. This corresponds to the vector and (effective) scalar meson exchanges between the nucleon and nucleons of matter. These exchanges have pointlike vertices and standard Lorentz structures.

Inclusion of the higher-order OPE terms corresponds to a more complicated picture of the meson exchanges between the nucleons. Turning to the four-quark condensates, we can separate the two types of terms. In the "factorized" contributions one of the $\bar{q}q$ operators is averaged over vacuum. In the "internal" terms both $\bar{q}q$ pairs act inside the nucleons, Eqs. (53) and (54). The first (factorized) term on the RHS of Eq. (53) describes the contribution to the structure Π_m^q of the polarization operator, which contains the scalar expectation value $\langle N|\bar{u}u|N\rangle$. In a similar way the first (factorized) term on the RHS of Eq. (54) contributes to the scalar structure of the polarization operator, being proportional to the vector expectation value $\langle N|\bar{u}\gamma_\mu u|N\rangle$. These terms correspond to the anomalous Lorentz structures of the nucleon-meson vertices. As to the "internal" terms—i.e., the last terms on the RHS of Eqs. (53) and (54)—they can be interpreted as exchanges by two-meson systems with their local interactions with the nucleon or as the exchanges by four-quark mesons (if there are any).

Inclusion of the nonlocal vector condensates $\bar{q}(0)\gamma_\mu q(x)$ means that the vertices of the interactions between the nucleons of the matter and the vector mesons do not have a pointlike structure, requiring rather a description by form factors. The nonlocality of the scalar condensate does not influence the results in our approach.

A usual subject of calculation is the difference between the characteristics of the neutron and proton. If only the lowest OPE terms are included, the vector self-energies are determined by the vector condensates. The neutron-proton difference $\Sigma_v^{(n)} - \Sigma_v^{(p)}$, usually attributed to ρ meson exchange, is 170 MeV at $\rho = \rho_0$ and $\beta = 1$. Inclusion of the four-quark condensates and of the nonlocalities subtract 30 MeV and 28 MeV from this value. The lowest-order OPE terms provide the difference of the effective masses $m^{*(n)} - m^{*(p)} = 50$ MeV at the same values of ρ and β. Inclusion of the four-quark condensates and of the nonlocalities adds (−160 MeV) and 40 MeV. This leads to $\Sigma_v^{(n)} - \Sigma_v^{(p)} = 110$ MeV and $m^{*(n)} - m^{*(p)} = -70$ MeV in neutron matter at $\rho = \rho_0$.

The structure of the equations for Σ_v and m^*, Eqs. (70) and (71), is similar to that, Eq. (18), employed in nuclear physics. Recall that in the Hartree approximation dependence of Σ_v on the density deviates from the linear law due to the term Σ_q in denominator of the first ratio on the RHS of Eq. (18). (The same refers to Σ_s^* if the nucleon Fermi motion is neglected.) In our approach the nonlinear behavior of the self-energies is due to nonzero values of $\mathcal{F}_q$.

Now we compare the numerical results. Considering the papers containing relativistic calculations, we can compare the vector and scalar self-energies Σ_v and $\Sigma_s^* = m^* - m$. In the case of works carried out in the nonrelativistic approximation, we can compare the nucleon potential energies $U^{n,p}$. We analyze also the contribution to the parameter, conventionally denoted as a_4 [31], which is defined as

$$\varepsilon(\rho_0,\beta) = \varepsilon(\rho_0,0) + \beta^2 a_4 + O(\beta^4), \tag{74}$$

being thus the lowest-order term of the β^2 expansion of the averaged binding energy ε per nucleon at the saturation value of density.

Of course, we cannot expect very good agreement, since our calculations are carried out in the gas approximation. Future and more sophisticated calculations should include scalar and four-quark condensates beyond the gas approximation. This would correspond to an account of the renormalization of the nucleon interactions with matter by particle-hole excitations on the RHS of the sum rules. Another reason is that the results should be corrected for the effects of antisymmetrization of the total final-state wave function ("exclusion effect") [32].

The general feature of the relativistic calculations is that they provide the positive value of the difference $\Sigma_v^{(n)} - \Sigma_v^{(p)} > 0$ in the matter with the neutron excess. Also the proton effective masses are above the neutron ones in this case—i.e., $m^{*(n)} - m^{*(p)} < 0$. Our calculations show the same tendency. As to the quantitative results, our values of $\Sigma_v^{(n)} - \Sigma_v^{(p)}$ and $m^{*(n)} - m^{*(p)}$ appear to be about twice smaller than those obtained in Ref. [33]. Our value of the effective mass splitting is also about 2 times smaller than the result of Ref. [34] but is only 30% smaller than that of Ref. [35]. However, we find a somewhat smaller discrepancy with the relativistic Brueckner-Hartree-Fock (RBHF) calculations presented in Ref. [36]. They found $\Sigma_v^{(n)} - \Sigma_v^{(p)} \approx 30$ MeV and $m^{*(n)} - m^{*(p)} \approx -25$ MeV at $\beta = 0.2$ (Fig. 3 of Ref. [36]), while our results are $\Sigma_v^{(n)} - \Sigma_v^{(p)} \approx 20$ MeV and $m^{*(n)} - m^{*(p)} \approx -15$ MeV. Another RBHF analysis [37] provided results which are very close to ours. One can extract the values $\Sigma_v^{(n)} - \Sigma_v^{(p)} \approx 80$ MeV and $m^{*(n)} - m^{*(p)} \approx -50$ MeV at $\beta = 0.75$ from Figs. 10 and 11 of Ref. [37]. Our values are 80 MeV and -55 MeV correspondingly. Note, however, that the split of the effective masses, obtained in Ref. [37], is due to exchange effects only.

The nonrelativistic calculations, carried out in various approaches [38,39], provide $U^{(n)} - U^{(p)} \approx 60$ MeV at $\rho = \rho_0$ and $\beta = 1$. This is consistent with the earlier calculations [40]. Our value is 40 MeV at $\rho = \rho_0$ and $\beta = 1$.

Another important parameter is the symmetry energy—Eq. (74). Note that we calculate the quantity

$$\Delta\varepsilon = \frac{1}{2\rho}(U^{(n)}\rho_n + U^{(p)}\rho_p), \tag{75}$$

which is the true contribution to the energy per nucleon. The value

$$\Delta\varepsilon(\rho_0,\beta) - \Delta\varepsilon(\rho_0,0) = \beta^2 \Delta a_4 + O(\beta^4), \tag{76}$$

thus being the true contribution of the isovector forces. Our value is $\Delta a_4 \approx 10$ MeV. This is close to the one obtained in Ref. [32]. The β^2 law is true up to $\beta^2 = 1$ with 10% accuracy in agreement with Refs. [36], [38], and [39]. Using Eq. (64) we can express $\Delta\varepsilon = \frac{1}{2}[V_1 + S_1 - \beta^2(V_2 + S_2)]$. To find the total contribution of the potential energy one must include the term caused by the exclusion effect, mentioned above. This adds 9 MeV to a_4 [32]. Including also the contribution of the kinetic energy, we obtain $a_4 = 29$ MeV. The various calculations of this parameter provide values around 30 MeV [33–41]. Thus, our result agrees with those obtained by nuclear physics methods.

VI. SUMMARY

We expressed the vector and scalar self-energies of a nucleon in asymmetric nuclear matter as a function of density ρ and of the asymmetry parameter β. We presented the nucleon characteristics in terms of the in-medium expectation values of QCD operators. The main ingredients are the nonlocal vector condensates $\langle M|\bar{u}(0)\gamma_0 u(x) \pm \bar{d}(0)\gamma_0 d(x)|M\rangle$, the scalar condensates $\langle M|\bar{u}(0)u(0) \pm \bar{d}(0)d(0)|M\rangle$, and the four-quark condensates. The local vector condensates are calculated easily. The nonlocality of the vector condensates is expressed in terms of the nucleon structure functions. The scalar condensate $\kappa(\rho) = \langle M|\bar{u}u + \bar{d}d|M\rangle$ is presented in terms of the observable sigma term. The scalar condensate $\zeta(\rho,\beta) = \langle M|\bar{u}u - \bar{d}d|M\rangle$ and the four-quark condensates are calculated in the framework of the perturbative chiral quark model (PCQM).

Although we treat the condensates in the gas approximation, the nucleon characteristics are not linear in density. The corresponding equations (70) and (71) are analogous to the equations of nuclear physics beyond the mean-field approximation. Also Eqs. (70) and (71) provide an explicit expression of the nucleon characteristics in terms of the QCD condensates.

The successive inclusion of OPE terms on the LHS of the sum rules finds direct analogs in the meson-exchange description of the interactions of the nucleon in nuclear matter. The lowest-order OPE terms correspond to the exchanges by the vector and (effective) scalar mesons with pointlike vertices of the interactions. The higher-order terms correspond to the nonlocal structure of nucleon-meson vertices, including anomalous Lorentz structures, and to the exchanges by strongly correlated four-quark systems. A possible interpretation of the latter contributions is a local two-meson exchange (or the exchanges by the four-quark mesons, if there are any [42]), Figs. 1 and 2.

We obtained the functions $\Sigma_v(\rho,\beta)$ and $m^*(\rho,\beta)$ for all values of β. We calculated also the single-particle potential energy $U(\rho,\beta)$. The results are presented in Figs. 3–7.

Note that we did not need phenomenological parameters of the nucleon-meson interactions. We used the condensates which have been either calculated or expressed in terms of the observables. For the four-quark condensates we used, however, the already known input parameters of the PCQM. While including condensates of higher dimension in succession, we found direct analogs of the meson-nucleon exchange mechanisms of nuclear physics.

Our results for the nucleon self-energies are in reasonable agreement with the results of nuclear physics. The value of the symmetry energy is close to the one obtained by nuclear physics methods.

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APPENDIX A

To improve the overlap of OPE and phenomenological descriptions one usually applies the Borel transform defined as

$$Bf(q^2) = \lim_{Q^2,n\to\infty} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n f(q^2) \equiv \tilde{f}(M^2),$$

$$Q^2 = -q^2, \quad M^2 = Q^2/n \tag{A1}$$

with M called the Borel mass. It is important in applications to the sum rules that the Borel transform eliminates the polynomials and emphasizes the contribution of the lowest state on the RHS of Eq. (24) due to the relation

$$B\frac{1}{m^2 - q^2} = e^{-m^2/M^2}. \tag{A2}$$

Thus, the terms on the RHS of Eqs. (25)–(27) are just Borel transforms of the first term on the RHS of Eq. (24).

TABLE I. The values of the coefficients b_i (GeV) of the polynomials P_2 defined by Eq. (B1), which approximate the nucleon self-energies.

	b_0	b_1	b_2
V_1	0.102	0.015	0.026
V_2	−0.036	−0.008	−0.011
S_1	−0.254	0.150	−0.114
S_2	0.034	−0.035	0.034

APPENDIX B

The functions $V_{1,2}(\rho)$ and $S_{1,2}(\rho)$ introduced by Eq. (64) can be approximated by polynomials of the second order:

$$P_2(x) = b_0 + b_1 x + b_2 x^2, \tag{B1}$$

with $x=\rho/\rho_0$. The values of the coefficients b_i are presented in Table I. This leads to parametrization of the proton potential energy:

$$U(x) = [x(-0.15 + 0.17x - 0.09x^2) + \beta x(-0.04x + 0.02x^2)]\text{GeV}. \tag{B2}$$

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