Superconvergence relations and parity violating analog of Gerasimov-Drell-Hearn sum rule

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Sum rule of superconvergence type for parity violating amplitudes (p.v. analogue of Gerasimov-Drell-Hearn sum rule) is discussed. Elementary processes initiated by polarized photons in the lowest order of electroweak theory are calculated as examples illustrating the validity of the p.v. sum rules. The parity violating polarized photon-induced processes for the proton target are considered in the frame of effective low energy theories and phenomenological models based on p.v. nucleon-meson effective interactions. Assuming the saturation of p.v. sum rule, bounds on the range of parameters, poorly known from existing experimental data and used in these models, are found. The asymmetries for p.v. π^0 and π^+ production are discussed. It is argued that verification of the sum rule in future high intensity polarized photon beam experiments is feasible.

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I. INTRODUCTION

The GDH sum rule [1] is recently intensively measured [2,3] and considered as a clean and important test of the knowledge of the nucleon spin structure, especially in the resonance region [4]. The rising interest in GDH (and its Q^2 dependence generalization) and more generally in the spin structure of nucleon has started with the new generation of precise spin experiments [5,6]. First direct data for real photons taken at MAMI [3] are especially important in understanding the subject, new data at higher energies are now available and expected in the future from ELSA, GRAAL, JLab and Spring-8 [7–9].

The experiments based on intense polarized beams of photons [3,10] give also the opportunity to test the weak (parity violating) part of photon-hadron interactions. The knowledge of p.v. couplings in nucleon-meson and nucleon-nucleon forces is a very important point for understanding the physics of nonleptonic weak p.v. hadronic interactions. In addition the $\gamma\rho\pi$ and $\gamma\Delta N$ p.v. couplings, very poorly known, can also play a role in photon-induced reactions.

It was shown in paper [11] that the polarized photon asymmetry in π^+ photoproduction near the threshold could be a good candidate to measure p.v. pion-nucleon coupling h_π^1. Similar expectations are connected with the low energy Compton scattering [12,13]. Let us mention here that the h_π^1 coupling has been measured in nuclear [14] and atomic [15] systems. However, the extraction of h_π^1 from such experiments is difficult due to poor understanding of many-body systems. In fact, the disagreement between ^{18}F and ^{133}Cs experiments is seen [14,15].

The experimental observation of p.v. effects in photonic reactions is generally difficult because the expected asymmetries are very small. However, it seems sound to expect that the new high luminosity machines, generating intense polarized photon beams can change the situation in the nearest future [11,16,17]. Having this in mind a set of sum rules for parity violating part of Compton amplitudes has been recently derived by one of us (L.L.) [18]. In particular, p.v. analogue of GDH sum rule for the asymmetric amplitude, based on low energy theorems [19] and under assumption of superconvergence of type, $f(z)/z \to 0$ at infinity, has been formulated there. In the past a number of superconvergence type sum rules for parity conserving Compton scattering (one example of which reduces to GDH sum rule) has been obtained and discussed [20–22]. The superconvergence relations have been also studied in detail in the very general context of axiomatic local field theory and its analyticity properties [23].

In this paper we shall discuss p.v. analogue of GDH sum rule having in mind possible phenomenological implications.

The general formulas exploited in the paper and the saturation hypothesis for p.v. sum rule are discussed in Sec. II. In Sec. III the models of p.v. low energy photon-proton interactions [i.e., heavy baryon chiral perturbation theory ($HB\chi PT$) [13,24] and low energy phenomenological models [25–28]] are briefly described. Assuming the saturation for p.v. sum rule (similar to observed quick saturation in GDH integral) we are able to narrow down the allowed values of the p.v. photon-meson and photon-Δ-nucleon couplings (poorly known) and select the models with small high energy contributions (Sec. IV). For these selected models, the energy dependence of the asymmetries for pion photoproduction is calculated according to approach proposed in [25]. In the same section it is shown that measurement of the photon energy dependence of the asymmetries from threshold up to energy large enough to saturate p.v. sum rule (saturation energy) allows to distinguish between the different models which obey quick saturation feature. We conclude the paper with Sec. V.

II. GENERAL FORMULAS AND THE SATURATION HYPOTHESIS

Let us consider p.v. Compton amplitudes. For polarized photons scattered off unpolarized targets the following (crossing-antisymmetric) dispersion relation holds [compare Eq. (3.18) in [18]]:

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$$\mathrm{Re}\, f_h^{(-)\gamma} = \frac{\omega}{\pi} P \int_{\omega_{\mathrm{th}}}^{\infty} \frac{\omega'}{\omega'^2 - \omega^2} (\sigma_h^T - \sigma_{-h}^T) d\omega' + (\mathrm{subtr.}), \quad (1)$$

where ω is the photon energy in laboratory system. $f_h^{(-)\gamma}$ and σ_h^T are amplitude and relevant total cross section averaged over target particle spin, respectively; h indicates photon helicity eigenvalue.

It was pointed out in [18] that the assumption of superconvergence for amplitude $f_h^{(-)\gamma}$ [i.e., no subtractions in Eq. (1)]:

$$\left. \frac{f_h^{(-)\gamma}(\omega)}{\omega} \right|_{\omega \to \infty} \to 0, \quad (2)$$

together with the low energy theorem (LET) [19] leads to the p.v. analogue of GDH sum rule [once the limit $\omega \to 0$ is taken in unsubtracted form of Eq. (1)]:

$$\int_{\omega_{\mathrm{th}}}^{\infty} \frac{\sigma_h^T - \sigma_{-h}^T}{\omega'} d\omega' = 0. \quad (3)$$

In the paper we consider 1/2 spin targets and in this case Eq. (3) is equivalent to

$$\int_{\omega_{\mathrm{th}}}^{\infty} \frac{(\sigma_{1/2,+}^T - \sigma_{1/2,-}^T) - (\sigma_{-1/2,-}^T - \sigma_{-1/2,+}^T)}{\omega'} d\omega' = 0, \quad (4)$$

where $\pm 1/2$ denotes target spin projection on photon's momentum and $\pm$ denotes photon's helicity.

The parity conserving (p.c.) GDH sum rule has been obtained by Almond [29] and, neglecting T-violation, it reads

$$\int_{\omega_{\mathrm{th}}}^{\infty} \frac{(\sigma_{1/2,+}^T - \sigma_{1/2,-}^T) + (\sigma_{-1/2,-}^T - \sigma_{-1/2,+}^T)}{\omega'} d\omega' = \frac{4\pi^2\alpha}{m^2}\kappa^2, \quad (5)$$

where κ is the anomalous magnetic moment.

Equations (4) and (5) are equivalent to the following pair of GDH sum rules:

$$\int_{\omega_{\mathrm{th}}}^{\infty} \frac{\sigma_{\pm 1/2,\pm}^T - \sigma_{\pm 1/2,\mp}^T}{\omega'} d\omega' = \frac{2\pi^2\alpha}{m^2}\kappa^2. \quad (6)$$

Let us emphasize that only if the p.v. sum rule (3) is true, the sum rules (6) become equivalent and identical with p.c. sum rule (5). In such a case the photon momentum direction can be ignored and sum rules (6) reduce to standard form of GDH sum rule used in literature.

In contrast to the nonzero contribution to standard GDH integral in higher order of perturbation theory, the integrals in p.v. sum rules (3) should be zero at any order of perturbation theory. The photon-charged lepton reactions have been studied in the past in the frame of electroweak theory. The contribution to GDH sum rule from the processes mediated by weak bosons in the lowest order of perturbation theory has been calculated in [30] and found to be zero. Quite recently it was shown in [31] that GDH sum rule for the electron evaluated at order of α^3 agreed with the Schwinger contribution to the anomalous magnetic moment.

As illustrative examples of the elementary parity violating processes we have calculated cross sections in Born approximation for three different inelastic polarized photon scatterings off lepton targets: the photon-neutrino reaction into W boson and electron, the photon-electron reactions into neutrino and W boson and into electron and Z^0 boson production. The calculations of cross sections have been done using *FeynArts 3* and *HighEnergyPhysics* (HEP) [32] packages.

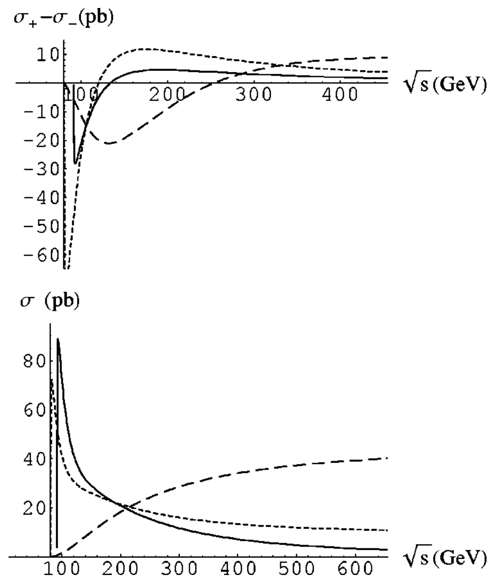

FIG. 1. The difference of the polarized photon cross sections $\sigma_+ - \sigma_-$ and unpolarized cross section σ as a function of CMS energy for the reactions: $\gamma e \to Z^0 e$ (solid line), $\gamma\nu \to We$ (dotted line, multiplied by factor 0.1), and $\gamma e \to \nu W$ (dashed line, in $\sigma_+ - \sigma_-$ multiplied by factor 5).

We have verified that p.v. sum rule (3) is satisfied both for $\sigma_{\nu\gamma}^T = \sigma_{\nu\gamma \to eW}$ and $\sigma_{e\gamma}^T = \sigma_{e\gamma \to \nu W} + \sigma_{e\gamma \to eZ}$. In considered Born approximation stronger results hold, namely the integrals $\int_{\omega_{\mathrm{th}}}^{\infty} [(\sigma_{1/2,+} - \sigma_{1/2,-})/\omega'] d\omega'$ and $\int_{\omega_{\mathrm{th}}}^{\infty} [(\sigma_{-1/2,+} - \sigma_{-1/2,-})/\omega'] d\omega'$ are zero separately for all the considered processes in accordance with the results of Ref. [30].

The behavior of the differences of cross sections as a function of CMS energy is presented in Fig. 1. It is seen in Fig. 1 that the saturation of the p.v. sum rule requires the high energy contribution because the convergence of the differences of the cross sections to zero is slow.

On the other hand, one of the most interesting features of GDH sum rule for nucleon targets is a quick saturation of the GDH integral. The dominant contribution (about 90%) to the GDH sum rule comes from the photon's energy range from the threshold up to 0.55 GeV [2,3,7,8]. The saturation hypothesis in analogy with the feature observed in the GDH sum rule, is an important point in the analysis presented in Sec. III. Therefore we are now going to formulate the criterion of the saturation of integral in p.v. sum rule (3). It is relatively easy to define the saturation when the integral in the sum rule has non zero value, as it is in the case of GDH sum rule where the value of integral is determined by the anomalous magnetic moment of target particle. However, the problem appears when the integral in sum rule should be zero. Below we shall formulate the saturation criterion valid for both situations. Given any superconvergence sum rule of the form

$$a = \int_{\omega_{\rm th}}^{\infty} \frac{\Delta\sigma(\omega')}{\omega'} d\omega', \qquad (7)$$

we define the quantity F:

$$F(\omega) = \frac{I_0}{I_1}, \qquad (8)$$

where

$$I_0 = \left| a - \int_{\omega_{\rm th}}^{\omega} \frac{\Delta\sigma(\omega')}{\omega'} d\omega' \right| \qquad (9)$$

and

$$I_1 = \int_{\omega_{\rm th}}^{\omega} \frac{|\Delta\sigma(\omega')|}{\omega'} d\omega'. \qquad (10)$$

Requirement that $F(\omega)$ does not exceed the prescribed small value at $\omega = \omega_{\rm sat}$ determines the saturation energy. The usefulness of such definition of saturation is based on the assumption that there is no large contribution to the integral from photons with energies higher than $\omega_{\rm sat}$.

For the GDH sum rule on proton, where the experimental data [3,7] exist we can estimate $\omega_{\rm sat}$ to be 0.55 (i.e., $E_{\rm sat}/E_{\rm th} = 1.5$ in CMS) for $F(\omega_{\rm sat}) = 0.1$.

As there are no experimental data for p.v. sum rules on the proton we shall use the values $\omega_{\rm sat} = 0.55$ and $F(\omega_{\rm sat}) = 0.1$ in the discussion of phenomenological consequences (Sec. III).

III. PROTON TARGET: THE MODELS OF P.V. LOW ENERGY PHOTON-PROTON INTERACTIONS

In this section we shall discuss two different approaches to p.v. low energy photon-nucleon interactions. We begin with p.v. Compton amplitude on proton calculated in the frame of *HBχPT* [12,13,24]. According to [13] the p.v. Compton amplitude can be written in CMS as follows:

$$\begin{aligned} M^{(-)s_f,s_i}_{h_f,h_i}(\vec{k},\vec{k}') = \overline{N_{s_f}}[&F_1 \vec{\sigma} \cdot (\hat{\vec{k}} + \hat{\vec{k}}')\vec{\epsilon}_i \cdot \overline{\vec{\epsilon}'_f} \\ &- F_2(\vec{\sigma} \cdot \overline{\vec{\epsilon}'_f}\hat{\vec{k}}' \cdot \vec{\epsilon}_i + \vec{\sigma} \cdot \vec{\epsilon}_i \hat{\vec{k}}' \cdot \overline{\vec{\epsilon}'_f}) \\ &- F_3 \hat{\vec{k}} \cdot \overline{\vec{\epsilon}'_f}\hat{\vec{k}}' \cdot \vec{\epsilon}_i \vec{\sigma} \cdot (\hat{\vec{k}} + \hat{\vec{k}}') \\ &- iF_4 \vec{\epsilon}_i \times \overline{\vec{\epsilon}'_f} \cdot (\hat{\vec{k}} + \hat{\vec{k}}')]N_{s_i}, \end{aligned} \qquad (11)$$

so that

$$f^{(-)p}_{1/2} = 2F_1, \qquad (12)$$

$$f^{(-)\gamma}_{h=+1} = -2F_4. \qquad (13)$$

To discuss p.v. sum rule and the superconvergence relations the interesting quantity is F_4, according to Eq. (13). The calculations based on *HBχPT* analysis in NLO [13] provide the following value of coefficient F_4:

$$F_4^{\rm NLO} = -\frac{e^2 g_A h_\pi^1 \mu_n}{8\sqrt{2}\pi^2 M F_\pi}\left[\omega - \frac{m_\pi^2}{\omega}\arcsin^2\left(\frac{\omega}{m_\pi}\right)\right]. \qquad (14)$$

It is easy to check that at high energies the real part of $F_4^{\rm NLO}/\omega$ converges to a constant so the superconvergence condition (3) is violated. Therefore the p.v. sum rule (4) does not hold.

A similar formula with 6 independent structure functions A_i can be written for the p.c. Compton amplitude [13,33]. In this case the *HBχPT* gives a similar result also for A_3 forward scattering amplitude. According to [33] A_3 is equal to

$$A_3^{\rm NLO}|_{\Theta=0} = -\frac{e^2\omega\kappa_p^2}{2M^2} - \frac{e^e g_A^2}{8\pi^2 F_\pi^2}\left[\omega - \frac{m_\pi^2}{\omega}\arcsin^2\left(\frac{\omega}{m_\pi}\right)\right] \qquad (15)$$

which again violates superconvergence relation of the type (3) as in p.v. case.

Discussing *HBχPT* it is important to note the fact that the spin-dependent p.c. polarizability $\gamma_{p,n}$ (expressed by the integral similar to GDH integral but with higher power of energy in the denominator of integrand) essentially depends on loop corrections and that the contribution from the Δ and the lowest order result differs not only in the value but also in the sign from result [34,35]. Therefore *a priori* it is not excluded that p.v. sum rule (3) might be satisfied if more corrections were taken into account in the frame of *HBχPT*. To our knowledge there is no *χPT* based model for p.v. Compton amplitude, which obeys the superconvergence relation (3).

Having this fact in mind we will consider an existing in the literature low energy phenomenological model of pion photoproduction based on so-called pole approximation and effective Lagrangians [25] (compare also [36]) and [26–28]. The model discussed in [25] is relevant in the low energy regime so we will limit ourselves to energies below 0.55 GeV. The upper bound on energy is high enough to discuss and apply the saturation hypothesis as it was said in the previous section. The contribution from the high energy region will be ignored for a moment, assuming that it is unimportant.

The detailed description of the approach can be found in [25]. The asymmetries of the polarized photon cross sections for π^+ and π^0 production are expressed by the sum of the p.v. coupling constants multiplied by the relevant form factors (see Figs. 11–15 in [25]). In our calculations the ρNN couplings $(h^0_\rho, h^1_\rho, h^2_\rho)$, ωNN couplings (h^0_ω, h^1_ω) and $\pi N\Delta$ coupling (f_Δ) have to be taken into account. For π^+ production the important contribution follows from p.v. πNN coupling (h^1_π). In addition, there are two extra contributions from Δ directly coupled to photon and nucleon ($\gamma\Delta N$ coupling -μ^*) and from the interaction between photon, pion and ρ meson ($\gamma\rho\pi$ coupling -h_E). The last two parameters are directly related to the p.v. photon-mesons and photon-Δ-nucleon interactions while the previous ones are related to the strong sector (p.v. meson-nucleon couplings).

The knowledge of the values of p.v. couplings is rather limited; practically only ranges of values are known from experimental data (for review of the situation see [27] and references therein). On the other hand, the strong sector meson-nucleon couplings can be calculated in different approaches and models as reviewed in [27]; we shall exploit

TABLE I. Predictions for the p.v. meson-nucleon coupling constants after [27]. All couplings are in units 10^{-7}.

Model	h_π^1	h_ρ^0	h_ρ^1	h_ρ^2	h_ω^0	h_ω^1
1. DDH, range (K=6); Ref. [26]	0.0	11.4	0.0	−7.6	5.7	−1.9
2. DDH, range (K=6); Ref. [26]	11.4	−30.8	0.4	−11.0	−10.3	−0.8
3. DDH, (“best”); Ref. [26]	4.6	−11.4	−0.2	−9.5	−1.9	−1.1
4. D, range (K=3); Ref. [28]	1.3	8.3	0.0	−8.2	−0.5	−1.8
5. D, range (K=3); Ref. [28]	2.0	−23.1	−0.3	−8.2	−10.6	−2.2
6. D, range (K=1); Ref. [28]	0.5	7.0	−0.2	−10.3	2.5	−2.0
7. D, range (K=1); Ref. [28]	0.4	−29.5	0.0	−10.3	−10.2	−1.1
8. KM; Ref. [24]	0.2	−3.7	−0.1	−3.3	−6.2	−1.0

these in the next section. Especially difficult situation exists for p.v. $\gamma\Delta N$ coupling μ^* and $\gamma\rho\pi(h_E)$ which are not given in the models from [27]. Only quite large limits $\mu^* \in (-15,15)$ and $h_E \in (-17,17)$ in units 10^{-7} can be given for these, based on data analysis couplings.

The μ^* and h_E couplings can be calculated if some extra assumptions are added. The vector-meson dominance model have been used in [25] to estimate the p.v. $\gamma\Delta N$ coupling. Neglecting ω and ϕ meson contributions and assuming that p.v. $\rho\Delta N$ coupling is of magnitude similar to p.v. ρNN coupling the following relation has been formulated [25]:

$$\frac{\mu^*}{M} = \frac{h_\rho^0}{g_\rho m_\rho}. \tag{16}$$

Taking g_ρ equal to 0.434π (after [25]), the $\mu^* \simeq 0.55 h_\rho^0$ is uniquely defined by the h_ρ^0 coupling.

The h_E coupling can be calculated according to the analysis described in [37]. Assuming so-called factorization [37] and taking into account the present data [38] on the widths of a_1 and b resonances, their masses and couplings, the two possible solutions have been found for h_E:

$$h_E \sim 10 \times 10^{-7}, \quad h_E \sim 1 \times 10^{-7}. \tag{17}$$

The results are close to the ones obtained in $SU(6)_W$ based calculations in [37].

In the next section the p.v. sum rule (4) together with the saturation criterion will be used to select the models which posses the quick saturation feature.

IV. PHENOMENOLOGICAL CONSEQUENCES OF SATURATION

The p.v. meson-nucleon coupling constants calculated from the flavor-conserving part of the quark weak interactions are reviewed in [27]. The eight sets of numerical predictions for the p.v. meson-nucleon coupling constants, calculated with different assumptions and models have been summarized in Table I, taken from [27] (Table 1).

The predictions of the p.v. couplings depend on the method of calculations (as in [27]). The strong effects are partially incorporated in the meson's and nucleon's wave functions and partially in bare quark interactions. This part of strong interaction corrections manifests themselves via dependence on K parameter (K=1 in the absence of corrections) in effective quark interactions. The first three models (DDH), are based on the calculations from [26]. Models 1 and 2 contain strong corrections characterized by K=6 range parameter. The models 4–7 (D) have been calculated in [28]. The models 4 and 5 are corrected for strong interactions (K=3) while models 6 and 7 (K=1) have no strong corrections taken into account. The predictions for model 4 and 6 correspond to the factorization approximation used in the calculations, while 5 and 7 are the results obtained assuming the validity of the $SU(6)_W$ symmetry. The last set of couplings (model 8, KM) is based on $HB\chi PT$ calculations, [24]. $h_\rho^{1'}$ coupling, (originally listed in Table 1 in [27]) has been omitted; in fact it is zero for all models except KM. $h_\rho^{1'}$ is not used in our calculations as it does not enter in approach of Henley *et al.* [25].

The values of p.v. couplings listed in the Table I will be used to verify the quick saturation feature in the approach discussed in the previous section and in [25]. The μ^* and h_E couplings will be treated as free parameters in the range limited by the experimental knowledge: $\mu^* \in (-15,15)$ and $h_E \in (-17,17)$ in units 10^{-7}. The contribution related to f_Δ parameter is very small in the considered approach; we have fixed this coupling to be 10^{-7} after [25]. Taking the values of p.v. coupling constants from Table I the differences of polarized photon cross sections have been calculated and used for an estimation of F defined in Eq. (8). The saturation expressed by the condition $F < 0.1$ limits the allowed values of μ^* and h_E couplings.

Apart from the model 2 and the “best fit” model 3 from DDH group, a quick saturation can be achieved for all other models from Table I by limiting range of free parameters μ^* and h_E. The values of μ^* and h_E from Eqs. (16) and (17) allow us to have the saturation property for models 4–7; the model 8 (KM) with these values of μ^* and h_E has no saturation.

It should be emphasized that the nonsaturating models (2 and 3) are characterized by large values of h_π^1. In consequence, it is impossible to find any pair of μ^* and h_E couplings in the allowed range to satisfy the saturation condition. This observation leads to the conclusion that for these models some additional structure should be observed for

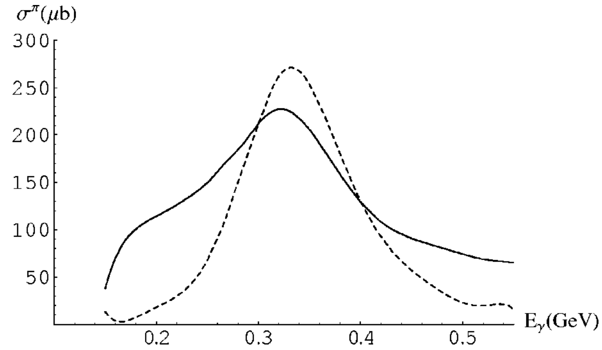

FIG. 2. The unpolarized cross section for pion production: π^+ (solid line) and π^0 (dotted line) according to model from [25].

higher than 0.55 GeV photon energy [compare discussion and Eq. (4.14) in Ref. [18][1]].

Considering quick saturation as an universal feature related to the complexity of hadronic targets, the absence of saturation can be treated as an argument against large value of h_π^1.

We are now going to discuss the experimental consequences of the models satisfying quick saturation, i.e., the π^+ and π^0 p.v. asymmetries defined as

$$A^\pi(\omega) = \frac{\sigma_+^\pi(\omega) - \sigma_-^\pi(\omega)}{2\sigma^\pi(\omega)}, \tag{18}$$

where $\sigma^\pi(\omega)$ denotes unpolarized pion production cross section and ω is the photon energy in laboratory frame.

The energy dependence of $\sigma^\pi(\omega)$ is the same for all the considered models and agrees well with the experimental data. It is shown in Fig. 2.

In Fig. 3 the pion photoproduction asymmetries as a function of energy are shown for the strong interaction corrected model 5 based on $SU(6)_W$ symmetry. The π^+ asymmetry is positive at threshold, large and negative at photon energies close to 0.5 GeV. The π^0 asymmetry at threshold are negative and relatively large. The saturation limits the parameter space (h_E and μ^*) for model 5 allowing to predict the asymmetries in a rather narrow corridor of uncertainties. For the rest of the saturated models the asymmetries are similar in shape, however smaller and less constrained. The π^0 asymmetries at threshold are sensitive to the assumptions under which the predictions for couplings have been calculated; the factorization (e.g., model 4) prefers zero or very small and rather positive asymmetry while $SU(6)_W$ symmetry based models (e.g., model 5) lead to larger and negative asymmetries. It allows to split all models into two categories. The first category includes models with large, negative asymmetry for π^0 production at threshold (models 5 and 7); the second models with π^0 asymmetries at threshold close to zero or positive (models 1, 4, 6, and 8). The π^+ asymmetries at threshold are positive as is the case of all models reaching 2×10^{-7}. The measurement of the π^+ asymmetry close to saturation energy can give also the opportunity to distinguish

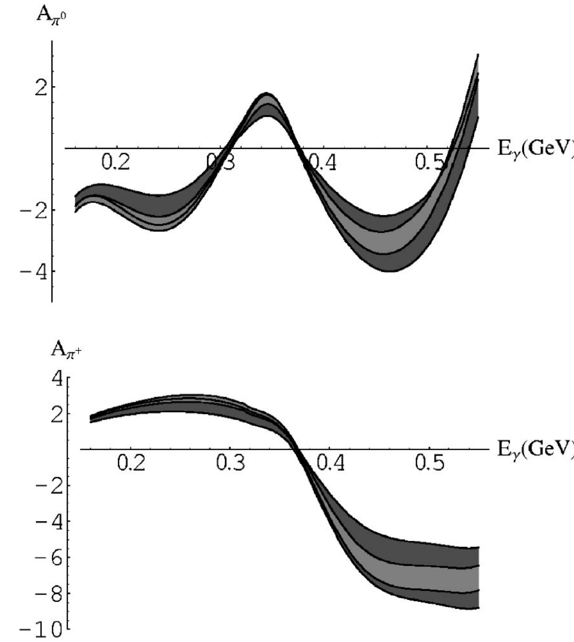

FIG. 3. The asymmetries for π^+ and π^0 photoproduction (in units 10^{-7}) as a function of the photon energy. The shadowed bands reflect freedom of the values of p.v. photon-meson couplings μ^* and h_E allowed by saturation condition for model 5. The darker band is for positive, lighter for negative values of h_E. μ^* is always negative in the shadowed bands.

between models; large, negative value -8×10^{-7} indicate models 5 or 4, smaller value is a feature of models 1, 6, 7, 8 and also model 4 is not excluded. Combining the two measurements (π^0 asymmetry at threshold and π^+ asymmetry at energy close to saturation 0.55 GeV) together would allow to select particular model or group of models (model 5, models 4 and 6, or models 1 and 8). The most interesting is model 5; the predicted values of pion photoproduction asymmetries are relatively large.

We shall discuss now the experimental feasibility for checking p.v. sum rule (3). The intensity and polarization of the electron beam at JLab allow to produce an intense, circularly polarized beam of photons from the bremsstrahlung process [16,17]. Taking 60 μA current of 12 GeV electron beam [17] and 1 mm Au plate target [16] we calculate the photon bremsstrahlung spectrum:

$$\frac{dN_\gamma(\omega)}{d\omega} = \frac{1.5\times10^{14}}{\text{sec}}\left(\frac{1}{\omega} - \frac{0.08}{\text{GeV}}\right) \simeq \frac{1.5\times10^{14}}{\text{sec}}\frac{1}{\omega}. \tag{19}$$

Hence for 1 cm long liquid hydrogen target the number of events/sec is $(6/\text{pb/sec})\int([\sigma^{\pi^+}(\omega)+\sigma^{\pi^0}(\omega)]/\omega)d\omega$, i.e., for energy range from threshold to 0.55 GeV it reads 1.7×10^9 events/sec; the region 0.137 to 0.3 GeV contributes 7×10^8 events/sec, while the region close to saturation point (0.4–0.55 GeV) contributes 2.7×10^8 events/sec. The rates 10^8-10^9 events/sec seem to be large, but the rate of 10^9 events/sec is expected in LHC and the relevant detection techniques are feasible (see Table 1 in [39]). To overcome

[1]On the left-hand side of Eq. (4.14) in Ref. [18] the factor 1/2 is missing.

statistics a large number of events is needed (compare discussion in Ref. [11]):

$$N > 2\left(\frac{k}{\eta}\right)^2\left(\int \frac{\sigma^{\pi}(\omega)}{\omega}d\omega \Bigg/ \int \frac{\Delta\sigma^{\pi}(\omega)}{\omega}d\omega\right)^2. \quad (20)$$

The measured result of the integral: $(\int[\Delta\sigma^{\pi}(\omega)/\omega]d\omega)_{\text{meas.}}$ will lie in the range of $(1\pm\eta)\int[\Delta\sigma^{\pi}(\omega)/\omega]d\omega$ at k standard deviations' confidence level.

To verify quick saturation hypothesis the sum rule (3) should be measured up to the photon energy of 0.55 GeV. If the result comes out 40–110 pb, the hypothesis is not satisfied; in this case one needs (taking $k=1$ and $\eta=1$) $10^{13}-10^{14}$ events which corresponds to $6\times10^3-6\times10^4$ sec of the beam time.

A much smaller result would indicate the possibility of quick saturation. Models with saturation, see Fig. 3, exhibit different signs of contributions from different energy regions; in model 5 low energy contribution (up to 0.3 GeV) is positive (22–28 pb) while the region close to saturation point (0.4–0.55 GeV) yields (−10)–(−14) pb. It demands $4\times10^{13}-6\times10^3$ and $1.5\times10^{12}-4.5\times10^{12}$ events for the threshold and saturation point regions, respectively. The corresponding beam times are $6\times10^4-8.5\times10^4$ sec and $6\times10^3-1.7\times10^4$ sec.

V. CONCLUDING REMARKS

We have discussed p.v. superconvergent sum rule formulated in [18] and we have examined its phenomenological consequences. The sum rule has been checked within the lowest perturbative order of electroweak theory for the photon induced processes with elementary lepton targets. It would be interesting to check this sum rule in higher perturbative orders as it was recently done for GDH in QED in [31].

In analogy with the GDH sum rule for nucleon the saturation hypothesis has been formulated and the eight models with different sets of p.v. couplings [27] have been analyzed using p.v. photoproduction approach proposed in [25]. Models with the largest leading p.v. pion-nucleon coupling h_{π}^1 do not saturate p.v. sum rule integral below energies of photon less than 0.55 GeV and the contributions from higher energies cross sections are needed. It suggests some structure in the difference of the cross sections to be observed for higher photon's energy for these models.

The other models considered in this paper have enough freedom in parameter space, defined by data and calculations, to saturate the p.v. sum rule integral below photon energy of 0.55 GeV.

The verification of our essential predictions is experimentally feasible with the beam time of the order of 10^5 sec in the near future experimental facilities.

It is an open question whether the sum rule (4) could be satisfied separately for different izospin components of weak interactions in the case of the nucleon target. It would resemble an observation made in Sec. II that in Born approximation the p.v. sum rule (4) holds separately for different reactions with elementary targets. If it were the case, the more stringent limits for p.v. couplings might be obtained.

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