Azimuthally sensitive correlations in nucleus-nucleus collisions

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We define a set of model-independent observables which generally characterize the azimuthal dependence of two-particle correlations in nucleus-nucleus collisions. We explain how they can be analyzed, and show to what extent such analyses are model dependent. We discuss specific applications to the anisotropic flow of decaying particles, azimuthally sensitive Hanbury-Brown Twiss interferometry, and correlations between particles at large transverse momentum. A quantitative prediction is made for jet quenching with respect to the reaction plane.

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I. INTRODUCTION

In noncentral nucleus-nucleus collisions, azimuthal angles of outgoing particles are generally correlated with the direction of the impact parameter. This phenomenon, called "anisotropic flow," has been known for 20 years [1], and has raised particular interest at the Brookhaven Relativistic Heavy Ion Collider (RHIC) where it is thought to provide unique evidence for quark-gluon plasma (QGP) formation [2].

Most often, one studies the azimuthal dependence of *single-particle* production [3]. Here, we would like to discuss the azimuthal dependence of *two-particle* correlations. This is of interest in various situations.

(1) *Anisotropic flow of short-lived particles*. The flow of unstable particles (for instance Λ baryons) is studied through their decay products. One must first identify a correlation between daughter particles, typically through an invariant mass plot; then study how this correlation depends on the azimuthal angle of the decaying particle [4–9].

(2) *Azimuthally sensitive two-particle interferometry*. Bose-Einstein correlations between identical particles are commonly used to measure the size and shape of the emitting source [10]. In noncentral collisions, the source projection on the transverse plane is no longer circular [11], and this can directly be seen in Hanbury-Brown Twiss (HBT) studies of two-particle correlations, as already observed at the Brookhaven Alternating Gradient Synchrotron [12] and at the RHIC [13].

(3) *Jet quenching with respect to the reaction plane*. The energy loss of hard partons traversing a deconfined medium [14,15] is a crucial signature of QGP formation at the RHIC [16]. In particular, it results in a modification of the pattern of azimuthal correlations between high-p_T hadrons, compared to *pp* collisions: the back-to-back correlation is suppressed [17,18]. In a noncentral collision, the average length of matter traversed by a parton depends on its azimuth [19,20], which results in azimuthally dependent two-particle correlations [21,22].

In this paper, we give for the first time a unified presentation of these phenomena, which have so far been discussed separately. In all cases, analyzing azimuthally dependent correlations involves two distinct operations: (1) measuring the distribution of a pair of particles with respect to the reaction plane; (2) isolating the "true" correlation from the uncorrelated part. Both issues can be discussed independently, on a fairly general footing.

The first operation is discussed in Sec. II, where the observables associated with two-particle anisotropic flow are defined. These observables are model independent and can in principle be measured accurately. In particular, it will clearly appear that any method used to measure the single-particle anisotropic flow can also be used to analyze azimuthally sensitive correlations, modulo minor modifications. In that view, we recall in Appendix A the main features of existing methods for analyzing one-particle flow, and we introduce the changes necessary to measure pair flow. While existing methods all require one to estimate the reaction plane on an event-by-event basis [23,24] (see, for instance, Refs. [25] for Λ flow, [26] for azimuthally sensitive HBT interferometry, and [27] for correlations between high-momentum particles), this step is by no means necessary with the procedure we suggest. This opens the possibility to apply the improved methods of flow analysis recently devised in Refs. [28–32] and to resolve an inconsistency of present analyses: on the one hand, one studies a correlation (between decay products, due to quantum statistics, from jet fragmentation) which is essentially a "nonflow" correlation; on the other hand, one uses the event-plane method which relies on the assumption that all correlations between particles are due to flow [23].

The second operation is discussed in Sec. III. Unlike the first one, it will be shown to be always model dependent. Several specific applications are discussed in Sec. IV, together with predictions regarding the pair-flow coefficients. Our results are summarized in Sec. V.

II. OBSERVABLES FOR TWO-PARTICLE ANISOTROPIC FLOW

We first recall definitions for single-particle distributions. For particles of a given type in a given rapidity (y) and transverse momentum (p_T) window, the probability distribu-

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tion of the azimuthal angle ϕ (measured with respect to a fixed direction in the laboratory) reads

$$p(\phi - \Phi_R) = \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} v_n e^{in(\phi-\Phi_R)}, \tag{1}$$

where Φ_R is the (unknown) azimuth of the reaction plane (impact parameter) in the laboratory frame.

The Fourier coefficients [33] in this expansion are given by $v_n = \langle e^{-in(\phi-\Phi_R)}\rangle$, where angular brackets denote an average over particles and events. Given the normalization choice in Eq. (1), $v_0=1$. Since $p(\phi-\Phi_R)$ is real, $v_{-n}=(v_n)^*$, where the asterisk denotes the complex conjugate. If, in addition, the system is symmetric with respect to the reaction plane $[-(\phi-\Phi_R)$ is equivalent to $\phi-\Phi_R]$, as in a collision between spherical (although not necessarily identical) nuclei when parity is conserved, Eq. (1) reads

$$p(\phi - \Phi_R) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{+\infty} v_n \cos n(\phi - \Phi_R)\right] \tag{2}$$

with $v_n = \langle \cos n(\phi-\Phi_R)\rangle$, i.e., v_n is real.

The Fourier coefficients v_n have by now become familiar in the study of anisotropic flow in (ultra)relativistic heavy-ion collisions. Nevertheless, it is instructive to recall why they are the proper tools to parametrize azimuthal anisotropies. The key feature is that even though the reaction plane Φ_R is unknown on an event-by-event basis, the first Fourier coefficients v_n can be accurately reconstructed from a statistical analysis of azimuthal correlations between outgoing particles (see Appendix A 1 for a review of the methods for analyzing single-particle flow). However, the higher the value of n, the larger the uncertainty on v_n [34]. Therefore the probability $p(\phi-\Phi_R)$ at a specific azimuth cannot be measured in practice. Furthermore, since v_n is defined as an average, it is also easier to compute in theoretical studies—in particular, in Monte Carlo models—than the probability distribution itself.

The above definitions can readily be generalized to the distribution of particle *pairs* with respect to the reaction plane. A pair of particles of given species is characterized by six kinematic variables p_{T_1}, y_1, ϕ_1, p_{T_2}, y_2, ϕ_2. It is convenient to combine ϕ_1 and ϕ_2 into the relative angle $\Delta\phi \equiv \phi_2 - \phi_1$ (or any similar observable that does not depend on the overall orientation of the pair in the transverse plane, as, e.g., the invariant mass) and a "pair angle"

$$\phi_{\rm pair} \equiv x\phi_1 + (1-x)\phi_2, \tag{3}$$

where $0 \leqslant x \leqslant 1$. One can restrict $\phi_{\rm pair}$ and $\Delta\phi$ to the ranges $-\pi \leqslant \phi_{\rm pair} < \pi$ and $-\pi \leqslant \Delta\phi < \pi$. If $x=\frac{1}{2}$, $\phi_{\rm pair}$ is the mean angle. The choice of x depends on the problem under study: most often, one chooses for $\phi_{\rm pair}$ the azimuthal angle of the total transverse momentum $\mathbf{p}_{T_1}+\mathbf{p}_{T_2}$ (see Secs. IV A and IV B); in studies of azimuthal correlations between high-momentum particles, $x=1$ is a more common choice (see Sec. IV C).

Consider now a sample of pairs of particles in some range of p_{T1}, p_{T2}, y_1, y_2, and $\Delta\phi$. To study the probability distribution of the pair angle $\phi_{\rm pair}$ within this sample, we write its

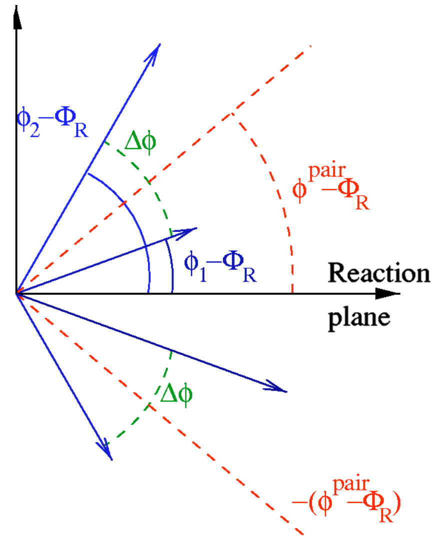

FIG. 1. Illustration of the various azimuthal angles ϕ_1, ϕ_2, $\phi_{\rm pair}$, $\Delta\phi$, with $x=1/2$.

probability distribution in a way analogous to Eq. (1):

$$p(\phi_{\rm pair} - \Phi_R) = \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} v_n^{\rm pair} e^{in(\phi_{\rm pair}-\Phi_R)}. \tag{4}$$

Like the usual v_n's, the "pair-flow" coefficients $v_n^{\rm pair}$ are defined by $v_n^{\rm pair} = \langle e^{-in(\phi_{\rm pair}-\Phi_R)}\rangle$, with the normalization $v_0^{\rm pair}=1$. Since the probability distribution is real valued, the coefficients also satisfy the property

$$v_{-n}^{\rm pair} = (v_n^{\rm pair})^* . \tag{5}$$

But unlike the single-particle flow v_n, the pair-flow coefficient $v_n^{\rm pair}$ is in general not a real number. The underlying reason, exemplified in Fig. 1, is that the transformation $\phi_{\rm pair}-\Phi_R \to -(\phi_{\rm pair}-\Phi_R)$ for a constant $\Delta\phi$ is *not* a symmetry of the system.[1] As a consequence, sine terms are also present in the real form of the Fourier expansion, and Eq. (2) is replaced by

$$p(\phi_{\rm pair} - \Phi_R) \equiv \frac{1}{2\pi}\Bigg(1 + 2\sum_{n=1}^{+\infty}[v_{c,n}^{\rm pair}\cos n(\phi_{\rm pair}-\Phi_R) + v_{s,n}^{\rm pair}\sin n(\phi_{\rm pair}-\Phi_R)]\Bigg), \tag{6}$$

where the *real* coefficients $v_{c,n}^{\rm pair} = \langle \cos n(\phi_{\rm pair}-\Phi_R)\rangle$ and $v_{s,n}^{\rm pair} = \langle \sin n(\phi_{\rm pair}-\Phi_R)\rangle$ are related to the complex $v_n^{\rm pair}$ by the relation $v_n^{\rm pair} = v_{c,n}^{\rm pair} - i v_{s,n}^{\rm pair}$.

The existence of such sine terms was already noted in Ref. [26] in the context of azimuthally sensitive HBT studies. Note that it does not imply parity violation, as it would in the case of single-particle flow [35]. The physical meaning of these additional terms will be illustrated in Sec. IV in

[1] The actual symmetry is under the simultaneous transformation $\phi_{\rm pair}-\Phi_R \to -(\phi_{\rm pair}-\Phi_R)$, $\Delta\phi \to -\Delta\phi$. Its consequences for the coefficients $v_n^{\rm pair}$ are discussed in Appendix B.

various physical situations. In particular, we shall show that they may yield insight on the mechanism responsible for the deficit in high-p_T particles.

It may be interesting to note that the pair-flow coefficients $v_n^{\rm pair}(\Delta\phi)$, when viewed as functions of the relative angle $\Delta\phi$, have peculiar properties which are derived in Appendix B. Checking that measured values of the coefficients possess these properties then provides a way to evaluate the errors affecting the measurement.

In experimental analyses, any method that can be used to measure the single-particle flow v_n can be applied to extract the cosine terms $v_{c,n}^{\rm pair}$, without any modification: one simply considers the pair as a single particle with azimuthal angle $\phi_{\rm pair}$. The generalizations required in order to extract the sine terms $v_{s,n}^{\rm pair}$ are quite straightforward. They are summarized in Appendix B 2 for various methods of flow analysis. In particular, some of these methods can safely correct for nonflow effects, others for acceptance anisotropies. This extends the possibility of studying azimuthally dependent correlations to detectors with partial azimuthal coverage.

To conclude this section, let us emphasize that the new characterization of azimuthally sensitive two-particle correlations which we propose, with pair-flow Fourier coefficients, represents in our view an improvement over previous parametrizations, in the same way as v_n is an improvement over older observables for anisotropic flow. The reason is simply that $v_{c,n}^{\rm pair}$ and $v_{s,n}^{\rm pair}$ are model-independent and detector-independent observables.

III. ISOLATING THE CORRELATED PART

Subtracting the "trivial" uncorrelated part in order to isolate the "true" correlation is far from trivial. In this section, we discuss this issue in as simple and general a way as possible. For the sake of simplicity, we start with the case when there is no anisotropic flow. In Sec. III A, we explain why the subtraction always involves some degree of arbitrariness, most often in the form of an arbitrary constant. Although this is to some degree well known, at least to those who actually perform correlation analyses, we think it is worth recalling, since the literature on the subject is rather confusing. In Sec. III B, we recall the various ways of normalizing the correlation, depending on the observable under study.

In practice, however, anisotropic flow is most often present, and makes the background subtraction more difficult in heavy-ion collisions than in elementary collisions. Note that this applies to all correlation analyses, not only to the azimuthally dependent ones: the correlation of single particles with the reaction plane induces a correlation between them, which must be always subtracted, at least in principle, in order to isolate other effects (see, for instance, [36]). In Sec. III C, we explain how this can be done, and show that this subtraction implies further approximations.

A. A model-dependent issue

In a given event, let N_1 and N_2 denote the numbers of particles in two phase-space bins, (p_{T_1}, y_1, ϕ_1) and (p_{T_2}, y_2, ϕ_2). To simplify the discussion, we assume that the two bins are separated. If they overlap, one need only replace N_1N_2 by the number of pairs in what follows. The simplest definition of the correlation between the two bins is

$$\mathcal{C} = \langle N_1N_2\rangle - \langle N_1\rangle\langle N_2\rangle, \tag{7}$$

where angular brackets mean an average over many events.[2]

Such a definition is not satisfactory in practice because the sample of events used in the analysis always contains events with different centralities: in particular, the total multiplicity may have sizeable fluctuations within the sample of events considered, and these fluctuations alone induce a correlation between any two phase-space bins. This correlation is of a rather trivial nature, but it may well overwhelm the interesting ones [37]. A simple way out of this problem would be to normalize the two terms in Eq. (7) by the total number of pairs of correlated and uncorrelated particles, respectively, that is, to define instead the correlation as

$$\mathcal{C} = \langle N_1N_2\rangle - \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}\langle N_1\rangle\langle N_2\rangle, \tag{8}$$

where N is the number of particles in a large enough phase-space bin (typically, the total number of charged pions seen in the detector for interferometry analyses). This definition is also unsatisfactory for two reasons: first, it obviously introduces some degree of arbitrariness in the definition of the correlation, depending on the choice of the phase space for N; second, part of the fluctuations in N may be meaningful for the correlation analysis, as in the case of Bose-Einstein correlations [38], so that there is no point in subtracting them.

In actual analyses, the correlation is rather defined as

$$\mathcal{C} = \langle N_1N_2\rangle - c\langle N_1\rangle\langle N_2\rangle, \tag{9}$$

where c is some free coefficient. This coefficient is kept constant throughout the correlation analysis (which typically involves varying the invariant mass, the relative momentum, or the relative azimuth between the two particles). It is then fitted in such a way that the correlation $\mathcal{C}$ vanishes when it is expected to: at large relative momentum in HBT analyses [38], and in some range of $\Delta\phi$ in correlations between high-p_T particles [17,18]. In addition, as mentioned above, the background subtraction is often complicated by the existence of anisotropic flow, as we shall discuss in Sec. III C.

B. Normalizations

There are essentially three ways of normalizing yields of particle pairs in azimuthally independent analyses, depending on the observable under study.

(1) One simply computes the average number of pairs per event, $\langle N_1N_2\rangle$. For instance, in order to measure Λ production, one plots the number of (p,π^-) pairs per event as a function of the invariant mass M of the pair. The number of pairs in the peak around the Λ mass gives the yield of Λ baryons, modulo acceptance corrections.

[2] In other terms, $\langle N_1\rangle$, $\langle N_2\rangle$, and $\langle N_1N_2\rangle$ are the one- and two-particle inclusive cross sections, divided by the total inelastic nucleus-nucleus cross section.

(2) One divides the number of pairs, $\langle N_1 N_2\rangle$, by the number of uncorrelated pairs, $c\langle N_1\rangle\langle N_2\rangle$. This is the standard observable for Bose-Einstein correlations, where the ratio varies ideally between 2 and 1 as the relative momentum of the pair increases.

(3) The third, intermediate choice is to divide the number of pairs, $\langle N_1 N_2\rangle$, by the number of "trigger particles," $\langle N_1\rangle$ [17]. After subtraction of the uncorrelated part, $c\langle N_2\rangle$, one thus obtains the mean number of particles, N_2, correlated with a trigger particle, which is independent of the system size (i.e., the same for a nucleus-nucleus and for a proton-proton collision) if there is no final-state interaction.

C. Subtracting the correlation due to flow

When the particles in the pair are individually correlated with the reaction plane Φ_R, this induces a trivial correlation between them, which must also be subtracted.

This subtraction is easy in principle: one simply repeats the operations of Secs. III A and III B for a fixed orientation of the reaction plane Φ_R. Then, the following substitutions hold:

$$\langle N_1 N_2\rangle \to \langle N_1 N_2\rangle(2\pi)p(\phi_{\rm pair}-\Phi_R),$$

$$\langle N_1\rangle \to \langle N_1\rangle(2\pi)p_1(\phi_1-\Phi_R),$$

$$\langle N_2\rangle \to \langle N_2\rangle(2\pi)p_2(\phi_2-\Phi_R). \quad (10)$$

In these equations, $\langle N_1 N_2\rangle$, $\langle N_1\rangle$, and $\langle N_2\rangle$ denote quantities averaged over Φ_R; $p(\phi_{\rm pair}-\Phi_R)$ is the distribution of the pair angle, defined in Eq. (6), and $p_1(\phi_1-\Phi_R)$, $p_2(\phi_2-\Phi_R)$ denote the single-particle azimuthal distributions of each particle, defined as in Eq. (2).

Once the azimuthal distributions of pairs and single particles with respect to the *true* reaction plane have been properly reconstructed, extracting the correlation, and its azimuthal dependence, is straightforward.

Strictly speaking, however, the one- and two-particle probabilities $p(\phi-\Phi_R)$ and $p(\phi_{\rm pair}-\Phi_R)$ at a specific azimuth relative to the reaction plane cannot be reconstructed. As already mentioned in Sec. II, only the first few Fourier coefficients v_n or $v_n^{\rm pair}$ can be reconstructed, due to larger absolute uncertainties on higher-order coefficients. On the other hand, the Fourier coefficients of a smooth function of $\phi-\Phi_R$ are expected to decrease quickly as the order increases (this expectation is supported by recent experimental single-particle flow data [39]), so that one can reasonably truncate the series, keeping only the measured coefficients. This truncation is always required in order to estimate the correlation from anisotropic flow [17,27]. At the RHIC, for instance, the error on the azimuthal distribution at midrapidity is likely to be dominated by the error on the fourth harmonic v_4, and one can take $(1/\pi)\delta v_4$ as the error on $p(\phi-\Phi_R)$.

IV. APPLICATIONS

We shall now discuss specific applications, with emphasis on the details of the experimental procedure.

A. Anisotropic flow of short-lived particles

Let us begin with the measurement of the anisotropic flow of particles that are seen through their decay products, such as $\Lambda\to p\pi^-$ [4,7–9], $\pi^0\to\gamma\gamma$, $\eta\to\gamma\gamma$ [5], $K_S^0\to\pi^+\pi^-$ [6,9]. We shall illustrate the recipe by discussing the flow of Λ baryons.

For each event, one sorts (π^-,p) pairs into bins of invariant masses M. The first step is then to analyze the total Λ yield. Following the standard procedure, one counts the number of pairs in each invariant-mass bin, irrespective of the pair azimuth; let $N_{\rm pairs}(M)$ denote this number. One then separates this distribution into an uncorrelated part [the background $N_b(M)$] and a correlated part [the peak $N_\Lambda(M)$, centered around the expected Λ mass]:

$$N_{\rm pairs}(M) = N_b(M) + N_\Lambda(M). \quad (11)$$

The integral of the correlated part $N_\Lambda(M)$ over M is the Λ yield.

In most cases, the peak is well above the background: to perform the above decomposition, one need not go through the whole procedure of the previous section: instead, one simply assumes that the background $N_b(M)$ is a smooth function of M [25]. Please note that the anisotropic flow of π^- and p correlates their azimuthal angles, and therefore distorts the background. However, the distorted background remains smooth, so that this effect need not be taken into account.

Next, one defines the azimuthal angle of the pair, $\phi_{\rm pair}$, as the azimuthal angle of the total transverse momentum $\mathbf{p}_{T_1}+\mathbf{p}_{T_2}$, and one analyzes the pair flow coefficients $v_{c,n}^{\rm pair}(M)$ and $v_{s,n}^{\rm pair}(M)$ in each bin. One then performs a decomposition similar to Eq. (11) for the azimuthally dependent part of the pair yield:

$$N_{\rm pairs}(M)v_{c,n}(M) = N_b(M)v_{c,n}^{(b)}(M) + N_\Lambda(M)v_{c,n}^{\Lambda},$$

$$N_{\rm pairs}(M)v_{s,n}(M) = N_b(M)v_{s,n}^{(b)}(M) + N_\Lambda(M)v_{s,n}^{\Lambda}. \quad (12)$$

This decomposition is performed assuming that the background components $N_b(M)v_{c,n}^{(b)}(M)$ and $N_b(M)v_{s,n}^{(b)}(M)$ are smooth functions of M. In this particular case, symmetry with respect to the reaction plane for Λ particles implies $v_{s,n}^{\Lambda}=0$, except for experimental biases and fluctuations. This identity can be used in order to check the accuracy of the experimental procedure, as in the case of single-particle flow [24]. If the background consists of uncorrelated particles, one also has $v_{s,n}^{(b)}(M)=0$.

In most analyses so far, the decomposition between the background and the peak is performed independently for several bins (typically, 20) in $\phi_{\rm pair}-\Psi_R$ [8,9,25], where Ψ_R is an estimate of the reaction plane. With the above procedure, the decomposition is only performed twice, in Eqs. (11) and (12).

When the peak-to-background ratio is low, finally, mixed events can be used to define the background [5]. However, the above-mentioned distortion of the background due to anisotropic flow must then be taken into account, as we shall see in more detail in Sec. IV B.

B. Quantum correlations

Azimuthally dependent Bose-Einstein (or, more generally, short-range) correlations are analyzed in two steps. The first step is to perform a Fourier expansion of the pair yield with respect to the reaction plane, for each relative momentum **q** [26]. As explained in Sec. II, any method of flow analysis can be used to extract the Fourier coefficients $v_{c,n}(\mathbf{q})$ and $v_{s,n}(\mathbf{q})$. Even with the event-plane method, no binning in $\phi_{\rm pair}-\Psi_R$ is required, in contrast to present analyses [12,13]. Once the coefficients are known, one easily builds the distribution of pairs relative to the reaction plane (up to the truncation issue mentioned in Sec. III C).

Next comes the difficult part of the analysis: one must divide the number of pairs per event by the number of uncorrelated pairs, as explained in Sec. III B. For a fixed orientation of the reaction plane Φ_R, this number depends on Φ_R (see Sec. III C):

$$N_{\rm uncor}(\Phi_R) = 2\pi\langle N_1\rangle p_1(\phi_1-\Phi_R)2\pi\langle N_2\rangle p_2(\phi_2-\Phi_R). \tag{13}$$

The Φ_R-independent part, $\langle N_1\rangle\langle N_2\rangle$, can be obtained using a standard mixed-event analysis. The Φ_R-dependent part, however, involves the (first) flow coefficients v_n of both particles in the pair.

To avoid this complication, the procedure suggested in Ref. [26] is to used mixed events with *aligned event planes*. This procedure, however, is only approximate, because one mixes events with different *reaction* planes, although the estimated planes are the same. To be specific, let us compare in a simple case the distribution of uncorrelated pairs following the exact procedure, Eq. (13), and using mixed events with aligned event planes. To simplify the calculation, we assume that only elliptic flow v_2 is present, and that it has the same value for both particles in the pair; we further assume that the pair angle is the mean angle, $x=\frac{1}{2}$ in Eq. (3). Then the exact result is

$$\frac{N_{\rm uncor}(\Phi_R)}{\langle N_1\rangle\langle N_2\rangle} = 1 + 2v_2^2\cos 2\Delta\phi + 4v_2\cos 2\Delta\phi\cos 2(\phi_{\rm pair} - \Phi_R) + 2v_2^2\cos 4(\phi_{\rm pair}-\Phi_R). \tag{14}$$

This is to be compared with the result obtained following the method of Ref. [26]:

$$\frac{N_{\rm mixed}(\Phi_R)}{\langle N_1\rangle\langle N_2\rangle} = 1 + 2v_2^2\langle\cos 2\Delta\Psi_R\rangle^2\cos 2\Delta\phi + 4v_2\cos 2\Delta\phi\cos 2(\phi_{\rm pair}-\Phi_R) + 2v_2^2\frac{\langle\cos 2\Delta\Psi_R\rangle^2}{\langle\cos 4\Delta\Psi_R\rangle}\cos 4(\phi_{\rm pair}-\Phi_R), \tag{15}$$

where $\Delta\Psi_R\equiv\Psi_R-\Phi_R$ is the difference between the estimated event plane and the true reaction plane. As expected, both results coincide when $\Delta\Psi_R=0$. Quite remarkably, the mixed-event method is correct to leading order in v_2 even when $\Delta\Psi_R\neq 0$. However, it misses the coefficients of order v_2^2. Typical values of the correction factors for the STAR experiment at the RHIC are $\langle\cos 2\Delta\Psi_R\rangle^2\simeq 0.6$ and $\langle\cos 2\Delta\Psi_R\rangle^2/\langle\cos 4\Delta\Psi_R\rangle\simeq 1.3$.

In addition to the systematic uncertainty we just discussed, the price to pay for aligned mixed events is that one must essentially perform the whole correlation analysis for fixed values of both the pair angle $\phi_{\rm pair}$ *and* the estimated reaction plane Ψ_R. We suggest instead the following method.

(1) Place particle pairs in bins according to their rapidities y_1, y_2, total transverse momentum $\mathbf{K}\equiv\mathbf{p}_{T_1}+\mathbf{p}_{T_2}$, and relative momentum **q**.

(2) In each such bin, build the correlation function $C(\mathbf{q})$ as in the standard, azimuthally insensitive HBT analysis.

(3) Reconstruct the azimuthal distributions of pairs, $p(\phi_{\rm pair}-\Phi_R)$, and of single particles, $p_1(\phi_1-\Phi_R)$ and $p_2(\phi_2-\Phi_R)$, with respect to the *actual* reaction plane; that is, measure the first Fourier coefficients $v_n^{\rm pair}$ and v_n for each particle in the pair (at RHIC energies, measuring the second and fourth harmonics v_2 and v_4 should be enough to guarantee that the distributions are reasonably well reconstructed).

(4) With the help of the substitution Eq. (10), build the azimuthal dependence of the correlation function.

One then eventually extracts azimuthally dependent HBT radii using standard techniques (in particular, including correction for Coulomb effects) which are beyond the scope of this paper.

C. Two-particle azimuthal correlations

Two-particle azimuthal correlations at large transverse momentum are under intense investigation in ultrarelativistic nucleus-nucleus collisions, since it has been realized that they yield direct evidence for hard scattering [36]. In that case, one correlates a high-p_T particle, the "trigger" particle, hereafter labeled 1, with a lower-p_T particle, hereafter labeled 2. We assume for simplicity that particles 1 and 2 belong to separate p_T intervals. This is not the case in the STAR analysis [17], where particle 2 can be any particle with momentum lower than p_{T_1} above some cut. This difference, however, is not crucial for the following discussion.

The following quantities must be measured: the average number of pairs per event, as a function of the relative angle $\Delta\phi$, $\langle N_{\rm pairs}(\Delta\phi)\rangle$ (in practice, pairs are naturally sorted into equal-size bins of $\Delta\phi$), and the average numbers of particles per event, $\langle N_1\rangle$ and $\langle N_2\rangle$.

In studying the azimuthal dependence of the correlation, a natural choice is to take the azimuthal angle of the trigger particle ϕ_1 as the pair angle $\phi_{\rm pair}$, i.e., one chooses $x=1$ in Eq. (3). One needs to reconstruct the azimuthal distribution of pairs (for a given $\Delta\phi$ bin), $p^{\Delta\phi}(\phi_1-\Phi_R)$, and the azimuthal distributions of both trigger and associated particles, $p_1(\phi_1-\Phi_R)$ and $p_2(\phi_2-\Phi_R)$. One may then reconstruct the whole correlation function for a fixed value of $\phi_1-\Phi_R$. In particular, the correlation functions for the specific values $\phi_1=\Phi_R$ (in plane) and $\phi_1=\Phi_R+\pi/2$ (out of plane) are given by

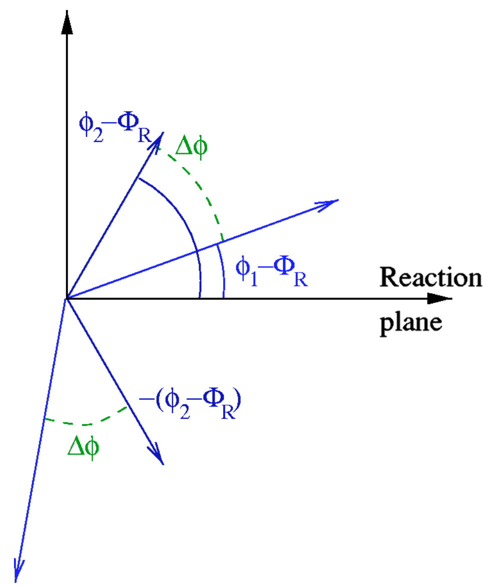

FIG. 2. Illustration of the prediction made in Eq. (17). The long arrows represent the momenta of trigger particles, while the shorter arrows represent the momenta of associated particles. If the modification of the correlation is due to the quenching of the associated particle, it must be unchanged under the transformation $\phi_2-\Phi_R \to -(\phi_2-\Phi_R)$.

$$C^{\rm out}(\Delta\phi) = \frac{\langle N_{\rm pairs}(\Delta\phi)\rangle}{\langle N_1\rangle}\frac{p^{\Delta\phi}(\pi/2)}{p_1(\pi/2)} - 2\pi c\langle N_2\rangle p_2\left(\frac{\pi}{2}+\Delta\phi\right),$$

$$C^{\rm in}(\Delta\phi) = \frac{\langle N_{\rm pairs}(\Delta\phi)\rangle}{\langle N_1\rangle}\frac{p^{\Delta\phi}(0)}{p_1(0)} - 2\pi c\langle N_2\rangle p_2(\Delta\phi), \quad (16)$$

where c is a constant close to unity, as explained in Sec. III A. It is independent of $\Delta\phi$ and $\phi_1-\Phi_R$.

Let us briefly compare the above procedure with the one suggested by Bielcikova *et al.* [27]: these authors show how to analyze correlations in and out of an event plane, which is not exactly the reaction plane. Since the event-plane resolution is a detector-dependent quantity, this prevents quantitative comparisons between different experiments. In addition, the algebra to subtract the uncorrelated part is much simpler with our method.

The standard interpretation of the modification of the correlation function in nucleus-nucleus collisions, compared to proton-proton collisions [17], is that the associated parton loses energy on its way through nuclear matter. If this interpretation is correct, then for a given $\Delta\phi$ the number of pairs per trigger particle depends only on the path followed by the associated particle. Symmetry with respect to the reaction plane implies that it is unchanged if $\phi_2-\Phi_R$ is changed into its opposite. As illustrated in Fig. 2, this symmetry is by no means trivial since the path followed by the trigger particle is now different. This gives us, for arbitrary $\phi_2-\Phi_R$ and $\Delta\phi$, the prediction

$$\frac{p^{\Delta\phi}(\phi_2-\Phi_R-\Delta\phi)}{p_1(\phi_2-\Phi_R-\Delta\phi)} = \frac{p^{\Delta\phi}(-\phi_2+\Phi_R-\Delta\phi)}{p_1(-\phi_2+\Phi_R-\Delta\phi)}. \quad (17)$$

If the only nonvanishing Fourier harmonic in the single-particle and pair azimuthal distributions is v_2, a simple calculation shows that the previous identity is equivalent to

$$v_{s,2}^{\rm pair}(\Delta\phi) = (v_{c,2}^{\rm pair}(\Delta\phi) - v_2^{(1)})\tan 2\Delta\phi, \quad (18)$$

where $v_2^{(1)}$ is the elliptic flow for the trigger particle, and the pair-flow coefficients $v_{c,2}^{\rm pair}$ and $v_{s,2}^{\rm pair}$ have been defined in Sec. II. This prediction is consistent with the general symmetry property (B3).

V. SUMMARY AND PERSPECTIVES

We have introduced novel, model-independent observables that describe the dependence in azimuth of two-particle correlations in heavy-ion collisions. These observables, namely, the coefficients $v_{c,n}^{\rm pair}$ and $v_{s,n}^{\rm pair}$ in the Fourier expansion of the azimuthal distribution (6) of the pair-angle $\phi_{\rm pair}$ that characterizes (together with the relative azimuth) particle pairs, generalize in a natural way the Fourier coefficients v_n for single-particle anisotropic flow. As the latter, the pair-flow coefficients can easily be measured in experiments, using any "usual" method of flow analysis (modulo minor modifications for the measurement of the sine terms, $v_{s,n}^{\rm pair}$): event-plane, two-particle correlations, cumulants, and Lee-Yang zeros can equally be applied. We recommend the last two, however, when possible, in order to disentangle flow from nonflow effects.

A main point of this paper is that these observables should replace, in future analyses, quantities that are defined for a given azimuth relative to the event plane. Much in the same way, the Fourier coefficients v_n have now replaced earlier observables such as the "flow angle," the "squeeze-out ratio," in most if not all analyses of single-particle flow. The reason is that the event plane is not exactly the reaction plane, and the dispersion varies from one experiment to the other. Therefore, observables defined with respect to an event plane yield only qualitative information. $v_n^{\rm pair}$, on the other hand, allows studies of azimuthally sensitive correlations to enter the quantitative era.

In a second part, we have briefly shown how to relate our observables to physical quantities of interest in three different cases: anisotropic flow of decaying particles, interferometry, and azimuthal correlations of high-momentum particles. It is important to stress that, unlike the measurement of the pair-flow coefficients $v_{c,n}^{\rm pair}$ and $v_{s,n}^{\rm pair}$, this second step does depend on the underlying physical picture. This model dependence leads to some arbitrariness, which in practice takes the form of the introduction of a normalization constant and (for HBT and high-p_T-particle studies) a necessary truncation of the Fourier expansion of the single-particle distribution.

A striking difference with the usual studies of anisotropic flow is the general occurrence of sine terms in the Fourier series expansion. The relevance of such sine terms was already discussed in the context of single-particle flow [24,35] and azimuthally sensitive HBT interferometry [26]. Here, we have shown for the first time that in the case of correlations between high-p_T particles, jet quenching would result in a specific value for the sine term, given by Eq. (18).

Azimuthally sensitive correlations are among the subtlest analyses in our field, and have already given valuable insight on the physics of high-energy nuclear collisions. We hope that the observables and methods introduced in this paper will help to improve future analyses. Thanks to the high statistics now available at the RHIC, or that one can anticipate at the CERN Large Hadron Collider, new measurements will become possible, for instance, the azimuthal dependence of nonidentical-particle interferometry or the anisotropic flow of various "new" particle types, while probing new regions of phase space.

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APPENDIX A: METHODS FOR ANALYZING SINGLE- AND TWO-PARTICLE FLOW

As stated in Sec. II, the measurement of the pair-flow coefficients $v_{c,n}^{\text{pair}}$ and $v_{s,n}^{\text{pair}}$ involves the same methods of analysis as for single-particle-flow coefficients v_n (modulo a small modification when measuring $v_{s,n}^{\text{pair}}$). This prompts us to recall briefly the various methods that have been proposed in the literature, indicating the modification necessary to measure the sine coefficient.

1. Analysis of one-particle flow

The most employed method of flow analysis at (ultra)relativistic energies is the event-plane method [23,24,34], which relies on the event-by-event determination of an estimate of the reaction plane, the so-called "event plane." Once the latter has been estimated (and various procedures to correct for acceptance issues have been performed), one correlates its azimuth with that of each outgoing particle, assuming that *all* correlations between the event plane and a given particle (that is, actually, all two-particle correlations) are due to flow. Eventually one must correct for the event-plane dispersion (computed with the help of "subevents") on a statistical basis:

$$v_n(p_T,y) = \frac{\langle \cos n(\psi - \Psi_n)\rangle}{\langle \cos n(\Psi_R - \Phi_R)\rangle}, \tag{A1}$$

where ψ, Ψ_R, and Φ_R, respectively, denote the azimuths of the particle under study, the event plane, and the real reaction plane, while the denominator measures the event-plane dispersion.

In addition to this first method, it has long been known that anisotropic flow can be analyzed with two-particle azimuthal correlations [40], without having to estimate the reaction plane in each event. The procedure consists in building a two-particle correlator, similar to that employed in interferometry studies [10], by forming the ratio of the number of "real" pairs (of particles in a same event) with relative angle $\Delta\phi$ over the number of "background" pairs (mixing particles from different events) with $\Delta\phi$:

$$C_2(\Delta\phi) \equiv \frac{N_{\text{pairs}}(\Delta\phi)}{N_{\text{mixed}}(\Delta\phi)}. \tag{A2}$$

As usual, dividing by "mixed events" automatically corrects for acceptance anisotropies, so that one can even work with a detector having only limited azimuthal coverage [41], while the event-plane method requires an almost perfect azimuthal symmetry. The key point in constructing $C_2(\Delta\phi)$ is that its Fourier coefficients, which can be deduced by fitting the function, are precisely $\langle\cos(n\Delta\phi)\rangle = (v_n)^2$ and $\langle\sin(n\Delta\phi)\rangle = 0$. Letting first both particles in the pair run over the whole phase space covered by the detector, one obtains an estimate of "integrated flow," v_n, corresponding to some (detector-dependent) average of the coefficient. Restricting then one, and only one, of the particles in the pair (whose azimuth will be denoted by ψ) to some definite particle type, transverse momentum p_T, and rapidity y, while letting the other (azimuth ϕ) be any particle in the event, one constructs a correlator whose Fourier coefficients are

$$\langle \cos n(\psi - \phi)\rangle = v_n v_n(p_T,y), \tag{A3a}$$

$$\langle \sin n(\psi - \phi)\rangle = 0. \tag{A3b}$$

The second identity reflects the evenness of $C_2(\psi-\phi)$ (when parity is conserved), while the first yields the "differential flow" $v_n(p_T,y)$. Please note that since only one particle per pair belongs to a small phase-space bin while it is correlated to all other particles in the event, it follows that statistical errors are the same as with the event-plane method. Finally, the bias from "nonflow" effects [42] is of the same order of magnitude within both event-plane and two-particle correlation methods, but it is easier to subtract, when it is possible, in the latter, as exemplified in the case of unwanted correlations due to global momentum conservation in Ref. [43].

As already stated, the main limitation of both the event-plane and the two-particle methods is their relying on the assumption that all azimuthal correlations between particles result from their correlation with the reaction plane [23]. In other words, they neglect nonflow correlations, whose magnitude is known to be large at ultrarelativistic energies [44]. One may try to subtract part of the nonflow effects by performing cuts in phase space, correlating together only particles that are widely separated (which certainly accounts for short-range correlations), but this results in larger statistical errors, while not removing entirely all unwanted effects. The only systematic way to remedy the problem of nonflow correlations in the flow analysis is to apply improved methods of analysis, based on multiparticle correlations [28–32], which have been implemented at the CERN Super Proton Synchrotron [45] and at the RHIC [46]. The essence of these methods is that the relative magnitude of nonflow effects decreases, while that of collective anisotropic flow grows, when one considers the cumulants of correlations between an increasing number of particles. Measuring cumulants of four- and six-particle correlations, one thus minimizes the systematic error due to nonflow effects, the ultimate case being the use of Lee-Yang zeros [31,32], equivalent to "infinite-order cumulants," which isolate *collective* behaviors in the system, i.e., flow effects. The price to pay is an increase in statistical uncertainties, but the latter is moderate at RHIC and LHC energies, especially if one uses all detected particles in the analysis.[3]

In practical analyses, these improved methods necessitate the computation of a generating function of multiparticle correlations, $G(z)$, where z is a complex variable [see Eq. (5) in Ref. [28] or Eqs. (3), (5) in Ref. [32]]. One then derives estimates of integrated flow: in the cumulant approach, by extracting the successive derivatives of $\ln G(z)$ at $z=0$ and identifying them with the corresponding derivatives of $\ln I_0(2v_n|z|)$ [28]; when using Lee-Yang zeros, simply by looking for the location of the first zero of $G(z)$ in the complex plane [see Ref. [31], Eq. (9)]. Once estimates of integrated flow have been obtained, they are used to compute values of differential flow $v_n(p_T,y)$ for particles in a small (p_T,y) bin, whose azimuthal angle we shall denote by ψ. This is done by correlating ψ to the generating function [see Eq. (26) in Ref. [28] or Eq. (9) in Ref. [32]].

2. Analysis of two-particle flow

Any of the methods recalled in the previous section can also be employed to measure the anisotropic flow of pairs as well, modulo small modifications. Whatever the method, the first step is strictly the same, namely, the construction of the event plane (and the computation of its statistical dispersion) in the event-plane method, or the measurement of estimates of integrated flow in the two-particle and multiparticle methods.

We shall now describe the changes that must be made to the measurement of differential single-particle flow in order to analyze the coefficients $v_{c,n}^{\rm pair}$, $v_{s,n}^{\rm pair}$. In short, the first necessary modification is the obvious replacement of ψ (the azimuth of "differential" particles) by the pair angle $\phi_{\rm pair}$; then no further change is needed to obtain the cosine coefficient $v_{c,n}^{\rm pair}$, whose measurement strictly parallels that of $v_n(p_T,y)$, while for $v_{s,n}^{\rm pair}$ one should replace the "$\cos n\psi$" term that is correlated to either the event plane or the other particles or a generating function with a "$\sin n\phi^{\rm pair}$" term. Let us be more explicit.

(1) In the event-plane method, the pair-flow Fourier coefficients are given by the averages

$$v_{c,n}^{\rm pair} = \frac{\langle \cos n(\phi^{\rm pair} - \Psi_R)\rangle}{\langle \cos n(\Psi_R - \Phi_R)\rangle}, \qquad \text{(A4a)}$$

$$v_{s,n}^{\rm pair} = \frac{\langle \sin n(\phi^{\rm pair} - \Psi_n)\rangle}{\langle \cos n(\Psi_R - \Phi_R)\rangle}, \qquad \text{(A4b)}$$

where Ψ_R is the event-plane azimuthal angle and the averages run over pairs and events. Note the analogy between Eqs. (A1) and (A4a).

(2) When using two-particle correlations, one builds a two-point correlator $C_2(\phi^{\rm pair}-\phi)$, where ϕ is any particle in the same event as those involved in the pair, with the trivial exception of the pair particles to avoid autocorrelations. In opposition to the correlator used in single-particle flow studies, $C_2(\phi^{\rm pair}-\phi)$ is no longer an even function, so that its Fourier expansion has nonvanishing both cosine and sine coefficients which are related to the pair-flow coefficients, namely,

$$\langle \cos n(\phi^{\rm pair} - \phi)\rangle = v_n v_{c,n}^{\rm pair}, \qquad \text{(A5a)}$$

$$\langle \sin n(\phi^{\rm pair} - \phi)\rangle = v_n v_{s,n}^{\rm pair}, \qquad \text{(A5b)}$$

where v_n is the integrated flow while the averages run over all $(\phi^{\rm pair},\phi)$ in each event, then over events. Once again, Eq. (A5a) is reminiscent of Eq. (A3a), while the difference between Eqs. (A3b) and (A5b) is due to the fact that, whereas single-particle emission is symmetric with respect to the reaction plane, pair emission, on the other hand, is not symmetric (see Sec. II and Fig. 1).

(3) To measure the Fourier coefficient $v_{s,n}^{\rm pair}$ in the Lee-Yang zeros method, one should replace $\cos n(\psi-\theta)$ by $\sin n(\phi_{\rm pair}-\theta)$ in the numerator of Eq. (12) [Eq. (9)] in Ref. [31] [Ref. [32]].[4]

(4) Finally, in the cumulant method, the relevant cumulants when measuring $v_{s,n}^{\rm pair}$ are the *imaginary* parts in the power-series expansion of Eqs. (29) and (27) in Ref. [28], while the real parts are needed for the analysis of single-particle differential flow $v_n(p_T,y)$ or of the cosine coefficients $v_{c,n}^{\rm pair}$. As a result, the interpolation formula that allows one to extract the cumulants is similar to Eq. (B7) of Ref. [28] [Eq. (11) of Ref. [29]], modulo the replacement $(X_{p,q},Y_{p,q})\to(Y_{p,q},-X_{p,q})$, where $X_{p,q}$ and $Y_{p,q}$ are still given by Eq. (B6) in Ref. [28].

Even if every method of single-particle flow analysis can in principle also be used to measure pair flow, modulo the modifications we described above, there exists a clear difference between the two-particle methods (both event-plane and two-particle correlation methods) on the one hand, and the multiparticle approaches on the other hand. As a matter of fact, we already mentioned that both two-particle methods rely on the assumption that all correlations between two arbitrary particles are due to anisotropic flow; in other words, that other sources of two-particle correlations are absent, or at most weak [23]. Now, if the purpose of using one of these methods is precisely to measure some azimuthally dependent two-particle effect, the procedure is somehow self-contradictory. Such an inconsistency does not affect the measurement of pair flow through multiparticle methods, since the latter do not assume that two-particle correlations are nonexistent, they merely minimize their effect.

APPENDIX B: SYMMETRY PROPERTIES OF PAIR-FLOW COEFFICIENTS

In this appendix, we list a few mathematical properties of the pair-flow coefficients $v_n^{\rm pair}$ for the sake of completeness.

[3]One should not worry about possible double particle countings, which amount to (unphysical) nonflow effects, and are thus automatically taken care of in the methods.

[4]We assume for simplicity that m takes the value 1 in the cited equations. Values $m>1$ correspond to higher harmonics, for which the same modifications apply.

The invariance of the two-particle distribution under the transformation $(\phi_1,\phi_2)\to(\phi_1+2\pi,\phi_2)$ translates into the (pseudo)periodicity property

$$v_n^{\text{pair}}(\Delta\phi+2\pi)=v_n^{\text{pair}}(\Delta\phi)e^{-2i\pi nx}, \tag{B1}$$

where x has been defined in Eq. (3). If x is changed to x' in Eq. (3), $v_n^{\text{pair}}(\Delta\phi)$ is changed to $v_n'^{\text{pair}}(\Delta\phi)=v_n^{\text{pair}}(\Delta\phi)e^{in(x-x')\Delta\phi}$.

If the system has symmetry with respect to the reaction plane (no parity violation), the two-particle distribution is unchanged under the joint transformation $(\phi^{\text{pair}},\Delta\phi)\to(-\phi^{\text{pair}},-\Delta\phi)$. At the level of the Fourier coefficients, this symmetry gives $v_n^{\text{pair}}(-\Delta\phi)=v_{-n}^{\text{pair}}(\Delta\phi)$. Together with property (5), this yields

$$v_n^{\text{pair}}(-\Delta\phi)=v_{-n}^{\text{pair}}(\Delta\phi)=[v_n^{\text{pair}}(\Delta\phi)]^*. \tag{B2}$$

The corresponding properties for the real Fourier coefficients $v_{c,n}^{\text{pair}}$ and $v_{s,n}^{\text{pair}}$ are

$$v_{c,n}^{\text{pair}}(-\Delta\phi)=v_{c,-n}^{\text{pair}}(\Delta\phi)=v_{c,n}^{\text{pair}}(\Delta\phi),$$

$$v_{s,n}^{\text{pair}}(-\Delta\phi)=v_{s,-n}^{\text{pair}}(\Delta\phi)=-v_{s,n}^{\text{pair}}(\Delta\phi). \tag{B3}$$

These various properties may prove useful to check that measured estimates of the Fourier coefficients behave "properly."

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