Dipole and quadrupole excitations in ^{88}Sr up to 6.8 MeV

L. Käubler,[1] H. Schnare,[1] R. Schwengner,[1] H. Prade,[1] F. Dönau,[1] P. von Brentano,[2] J. Eberth,[2] J. Enders,[3] A. Fitzler,[2] C. Fransen,[2] M. Grinberg,[4] R.-D. Herzberg,[2,*] H. Kaiser,[3,†] P. von Neumann-Cosel,[3] N. Pietralla,[2,5] A. Richter,[3] G. Rusev,[1,4] Ch. Stoyanov,[4] and I. Wiedenhöver[2,‡]

[1]*Institut für Kern- und Hadronenphysik, FZ Rossendorf, Postfach 510119, 01314 Dresden, Germany*

[2]*Institut für Kernphysik, Universität zu Köln, Zülpicher Straße 77, 50937 Köln, Germany*

[3]*Institut für Kernphysik, Technische Universität Darmstadt, Schlossgartenstraße 9, 64289 Darmstadt, Germany*

[4]*Institute for Nuclear Research and Nuclear Energy, BAS, 72 Tzarigradsko Chaussee, 1784 Sofia, Bulgaria*

[5]*Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, New York 11794, USA*

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Dipole and quadrupole excitations in the semimagic $N=50$ nucleus ^{88}Sr were investigated at the superconducting Darmstadt electron linear accelerator S-DALINAC with bremsstrahlung of an end-point energy of 6.8 MeV. Many new dipole excitations could be identified, and their reduced excitation probabilities were determined. The experimental findings are discussed in the context of quasiparticle-phonon-model and shell-model calculations. A breaking of the $N=50$ core is essential to describe the structure of the observed excitations. The two-phonon quadrupole-octupole $J^{\pi}=1^{-}$ state exhibits unusual features which are presently not understood.

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I. INTRODUCTION

The structure of the stable $N=50$ isotones is governed by the filling of the the $p_{1/2}$, $p_{3/2}$, and $g_{9/2}$ proton shells. Neither ^{88}Sr nor ^{90}Zr represents hereby a good doubly closed core, and several orbitals contribute to the ground-state (g.s.) wave functions; see, e.g., [1–3]. Both the breaking of the $N=50$ core and its possible influences on the proton distributions will affect the electric $(E1)$ and magnetic $(M1)$ dipole strength functions. The gross and the fine structure of $E1$ and $M1$ distributions below the particle threshold have been subject of recent investigations [4–8] in various nuclei. In many of the experimental studies, the main focus was on the low-energy $E1$ "pygmy" dipole resonance [9], and the main fraction of the detected dipole strength has been found to have electric character [10]. From studying the details of the low-lying $E1$ strength distributions one is able to learn—through comparison with nuclear models—whether or not these excitation modes arise from the collective motion of many valence nucleons, e.g., from an out-of-phase oscillation of the excess neutrons with respect to a proton-neutron core. For a detailed discussion, we refer to the experimental analyses of [6–8] as well as the theoretical work cited in these papers. Another characteristic feature in many semimagic or vibrational nuclei is a low-lying $E1$ excitation due to the coupling of the quadrupole and octupole vibrations; see, e.g., Ref. [4] and references therein. On the other hand, low-lying $M1$ spin-flip excitations have been identified in light and medium-mass nuclei. As examples for experimental and theoretical studies we refer to Refs. [11–13] and the literature cited therein.

In the present study the fragmentation of dipole strength in the $N=50$ nucleus ^{88}Sr is investigated by means of a nuclear-resonance-fluorescence experiment. On the basis of a comparison of the experimental data with the results of shell-model and quasiparticle-phonon-coupling calculations conclusions on the structures of some of the observed excitations are given. A quadrupole-octupole-coupled $E1$ excitation, the distribution of electric dipole excitations above 5 MeV, the problem of magnetic dipole strength at energies below the particle threshold, as well as the structure of quadrupole excitations, are discussed.

II. EXPERIMENTAL METHODS

Nuclear resonance fluorescence (NRF) represents a very sensitive technique to study low-lying electric and magnetic dipole and to a certain extent also electric quadrupole excitations in heavy nuclei (see Ref. [4] and references therein). Excitation energies E_x, integrated photon-scattering cross sections I_s, ground-state transition widths Γ_0, and branching ratios Γ_0/Γ can be extracted from the spectra of the scattered photons. These quantities are transformed into reduced transition probabilities $B(E1)$, $B(M1)$, $B(E2)$, or half-lives $T_{1/2}$ as is shown below.

A (γ,γ') experiment on ^{88}Sr has been performed at the superconducting electron accelerator S-DALINAC [14] of the Technische Universität Darmstadt. Bremsstrahlung with an end-point energy of 6.8 MeV has been collimated onto a $^{88}SrCO_3$ target with an enrichment of 99.9% in ^{88}Sr, a diameter of 18 mm, and a mass of 2.732 g. For photon-flux-calibration purposes the target has been covered at the front and back with disks of natural boron of masses of 0.271 and 0.313 g, respectively. Two EUROBALL CLUSTER detec-

*Present address: Oliver Lodge Laboratory, University of Liverpool, Liverpool L69 7ZE, U.K.

†Present address: Ingenieurbüro Fritz, 64683 Einhausen, Germany.

‡Present address: Physics Department, Florida State University, Tallahassee, FL 32306.

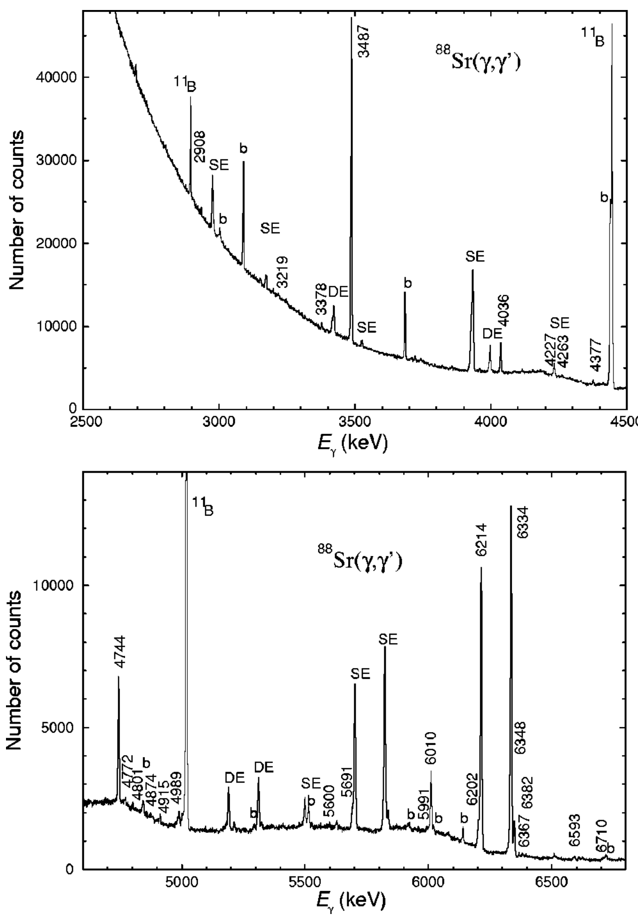

FIG. 1. Total γ-ray spectra observed in the $^{88}Sr(\gamma,\gamma')$ reaction with two CLUSTER detectors. Peaks marked by their γ-ray energy belong to ^{88}Sr. The symbols SE, DE, and b stand for single-escape, double-escape, and background peaks, respectively.

tors [15] placed at angles of $\Theta=94°$ and $\Theta=132°$ with respect to the incident photon beam have been used to detect the scattered γ rays. Data have been taken for 170 h with an average electron-beam current of about 30 μA. Details of the experimental arrangement are given in Refs. [8,13,16].

The total γ-ray spectrum taken with both EUROBALL CLUSTER detectors is given in Fig. 1. The total spectrum of one CLUSTER detector contains single events detected in each of the seven crystals individually as well as so-called add-back events reconstructing full-energy signals from the signals of neighboring crystals where part of the energy was deposited in the case of Compton scattering or pair production and subsequent annihilation of the positron. For the energy and efficiency calibration up to 3.5 MeV a ^{56}Co source has been used. A Monte Carlo simulation provided the shape of the bremsstrahlung photon flux, and the product of photon flux and efficiency above $E_\gamma > 3.5$ MeV could be determined from the known integrated scattering cross sections of the ^{11}B lines. The knowledge of the relative-efficiency curve is needed when decay branches to states other than the ground state are observed. As we have no direct information on the detection efficiency for $E_\gamma > 3.5$ MeV, the experimental branching ratios Γ_0/Γ have comparatively large uncertainties due to the necessary extrapolation of the efficiency. However, this applies to a few cases only.

The total scattering cross section I_s for a decay of the photoexcited state to the g.s., integrated over the resonance and the full solid angle, is given by

$$I_s = g\left(\pi\frac{\hbar c}{E_\gamma}\right)^2\frac{\Gamma_0^2}{\Gamma}, \qquad (1)$$

where Γ_0 is the partial decay width of the photoexcited state with spin J to the g.s. with spin J_0, and Γ is the total width. The factor $g=(2J+1)/(2J_0+1)$ represents the statistical weight. The integrated scattering cross section I_s is related to the reduced excitation probabilities $B(E1)\uparrow$, $B(M1)\uparrow$, or $B(E2)\uparrow$ through the width Γ_0 for decay into the g.s.:

$$B(E1)\uparrow = gB(E1)\downarrow = 2.866\times10^{-3}\frac{\Gamma_0}{E_\gamma^3}\ e^2\ \mathrm{fm}^2, \qquad (2)$$

$$B(M1)\uparrow = gB(M1)\downarrow = 0.2598\frac{\Gamma_0}{E_\gamma^3}\ \mu_N^2, \qquad (3)$$

$$B(E2)\uparrow = gB(E2)\downarrow = 6201\frac{\Gamma_0}{E_\gamma^5}\ e^2\ \mathrm{fm}^4. \qquad (4)$$

The analysis of photon scattering experiments is outlined in detail in Refs. [4,17–19]. For the determination of I_s and Γ_0, the weighted average of the experimental results obtained from the total γ-ray spectra of both CLUSTER detectors has been calculated. To get information about the angular momenta J of the populated levels, the efficiency-corrected intensity ratios $I_\gamma(94°)/I_\gamma(132°)$ have been calculated using normalization factors obtained from the measured intensities of known γ transitions. For the used detector arrangement angular distribution ratios of 0.72 for the case $0\to1\to0$ and 1.74 for the case $0\to2\to0$ are expected.

III. RESULTS

The experimental results are summarized in Tables I and II. Typical uncertainties of the given excitation energies are about 1 keV. The excitation of 23 states in ^{88}Sr has been observed in the present NRF experiment, where 15 levels are clearly identified as dipole excitations (Table I). Also the states at 4263 and 6367 keV are very likely populated by dipole excitations, but the measured angular distribution ratios deviate from the expected values. A firm assignment of the multipole order of the transition was made when one theoretical angular distribution ratio was within two standard deviations of the measured value and the other theoretical ratio was excluded by at least three standard deviations. If one of these two criteria was violated, we have given the spin assignment in parentheses. All the hitherto known [20] dipole excitations up to $E_x \leqslant 7.0$ MeV have been observed. In contrast to a NRF experiment on the $N=50$ nucleus ^{87}Rb [3], we observe candidates for quadrupole excitations in ^{88}Sr. The 2^+ states at 1836 and 3219 keV are mainly populated indirectly from higher-lying states. We did not observe any known 2^+ level between $3.5 \leqslant E_x \leqslant 6.6$ MeV [20].

In the following a detailed comparison of the data for some states observed in our experiment with the previously

TABLE I. Experimental results of the present (γ,γ') experiment on ^{88}Sr. The excitation energy E_x, the integrated cross section I_s, the intensity ratio $I_\gamma(94°)/I_\gamma(132°)$, the angular momentum and parity J^π, the value $g\Gamma_0^2/\Gamma$, the total width Γ, and the reduced excitation probabilities $B(M1)\uparrow$, $B(E1)\uparrow$, and $B(E2)\uparrow$ are given.

E_x (keV)	I_s (eV b)	$I_\gamma(94°)/I_\gamma(132°)$	J^{π} [a]	$g\Gamma_0^2/\Gamma$ (meV)	Γ [b] (meV)	$B(M1)\uparrow$ (μ_N^2)	$B(E1)\uparrow$ ($10^{-3}\ e^2\ \mathrm{fm}^2$)	$B(E2)\uparrow$ ($e^2\ \mathrm{fm}^4$)
3219.2 [c]		1.01(29)	2^{+} [d]					14(7) [d]
3378.1	2.1(3)	0.70(18)	1	6.2(9)	2.1(3)	0.014(2)	0.15(2)	
3487.1	162(18)	0.68(3)	1^+	514(57)	171(19)	1.05(12)		
4036.2	26(5)	1.86(16)	2^+	112(20)	34(6)			160(29)
4226.6	1.9(5)	0.66(28)	1	8.9(24)	3.0(8)	0.010(3)	0.11(3)	
4262.8	1.6(5)	1.05(41)	$(1,2^+)$	7.8(25)		0.0086(28)	0.096(31)	6.8(22)
4743.8	55(7)	0.67(4)	1^{-} [e]	322(42)	173(24)		3.7(5)	
4771.6	2.2(6)	1.27(42)	$(1,2^+)$	13.1(33)		0.010(3)	0.12(3)	6.6(17)
4801.3	1.8(3)	0.54(25)	1	10.6(21)	3.5(7)	0.0082(16)	0.091(18)	
4914.5	3.9(6)	0.65(22)	1	24.4(38)	8.1(13)	0.018(3)	0.20(3)	
4989.4	3.5(21)	2.95(167) [f]	$(1,2^+)$	23(14)		0.016(10)	0.17(10)	9.1(55)
5600.4	1.5(7)		$(1,2^+)$	12(6)		0.0060(30)	0.066(33)	2.7(14)
5691.1	4.2(10)	0.41(29) [g]	1	36(8)	12(3)	0.017(4)	0.19(4)	
5991.2	7.5(19)	1.57(44)	$(1,2^+)$	69(18)	14(4)	0.017(5)	0.19(5)	11(3)
6009.8	94(22)	0.71(5)	1	880(200)	294(68)	0.35(8)	3.9(9)	
6201.7	37(9)	0.48(11) [g]	(1)	374(95)	125(32)	0.136(34)	1.5(4)	
6213.6	547(136)	0.77(3)	1^{-} [d]	5490(1360)	1920(480)		22.4(56)	
6334.4	730(187)	0.73(3)	1^{-} [d]	7620(1960)	2540(650)		28.7(74)	
6347.5	77(20)	0.71(3)	1	807(207)	269(69)	0.273(70)	3.0(8)	
6366.8	4.2(13)	1.32(16)	$(1,2^+)$	44(14)		0.015(5)	0.16(5)	5.3(16)
6381.8	7.3(21)	0.74(17)	1	77(22)	26(7)	0.026(7)	0.28(8)	
6593.1	8.6(25)	0.63(16)	1	97(28)	32(9)	0.029(9)	0.32(9)	
6710.0	15(7)	0.83(29)	1	176(81)	175(85)	0.087(43)	0.96(48)	

[a] Angular momentum as determined from this work. For $J=2$ positive parity is assumed.
[b] Assuming $\Gamma_0/\Gamma=1$ except for measured branching ratios from Table II.
[c] This state is populated mainly from higher-lying states.
[d] Taken from [20].
[e] Parity from [27].
[f] Estimated intensity from double-escape peak subtracted.
[g] Estimated intensity from single-escape peak subtracted.

available experimental information is presented.

The 3487 keV level is reported to have spin $J=1$ as a result of a NRF experiment [21], which has not been adopted in the compilation [20], where $J^\pi=(2)^+$ is given. The spin $J=1$ and positive parity [22] have been confirmed in agreement with the results of a form-factor measurement in electron scattering [23,24]. The width of $\Gamma=0.171(19)$ eV deduced from our experiment agrees within the experimental error with $\Gamma=0.150(24)$ eV given in Ref. [21].

The 4036 keV level is a 2^+ state [20] with a half-life of $T_{1/2}=20(10)$ fs or $T_{1/2}=14(3)$ fs, deduced from the $B(E2)$ values obtained in inelastic-electron-scattering experiments by van der Bijl *et al.* [24] and Peterson and Alster [25], respectively. This state has not been observed in a NRF experiment up to now. The assignment $J^\pi=2^+$ is confirmed by our experiment, but we observed additionally for the first time a γ-decay branch from this state to the 2_1^+ state (Table II). We obtain $T_{1/2}=13(3)$ fs for the 4036 keV level in agreement with the results given above.

The 4744 keV level with angular momentum $J=1$ has been observed in a previous NRF experiment [26], where any γ branches other than to the ground state were not found. In that work the parity of this level was not determined, but it has been suggested to be a 1^- state arising from the coupling of the vibrational 2^+ and 3^- states. While clear evidence exists for such 1^- two-phonon states in the semimagic $Z=28$, $Z=50$, and $N=82$ nuclei, the situation in $N=50$ isotones was much less clear at the beginning of our investigation. Therefore, it was one of our goals to determine the parity of the 4744 keV level using the EUROBALL CLUSTER detector placed at $\Theta=94°$ as a Compton polarimeter for the measurement of the γ-ray linear polarization. Surprisingly, positive parity was found [22] with a significance of about three standard deviations. A later experiment with a

TABLE II. Branching ratios Γ_1/Γ and Γ_2/Γ of decay transitions to the 2_1^+ state at 1836 keV and the 2_2^+ state at 3219 keV, respectively, obtained in the present ^{88}Sr(γ,γ') experiment at $E_0=6.8$ MeV. The symbols E_i, E_f, J_i^π, and J_f^π denote the energies, angular momenta, and parities of the initial and final states, respectively.

E_i (keV)	J_i^π	E_f (keV)	J_f^π	$\Gamma_{1,2}/\Gamma$	$B(M1)\downarrow$ (μ_N^2)	$B(E1)\downarrow$ ($10^{-4}\ e^2\ \mathrm{fm}^2$)
4036.2	2^+	1835.8	2^+	0.190(39)	$0.054^{+0.012}_{-0.008}$	
4743.8	1^-	1835.8	2^+	0.035(14)		$2.5^{+0.4}_{-0.3}$
4743.8	1^-	3219.2	2^+	0.176(54)		87^{+14}_{-10}
6213.6	1^-	1835.8	2^+	0.024(4)		$5.5^{+1.8}_{-1.1}$
6710.0	(1)	1835.8	2^+	0.42(11)	$0.057^{+0.054}_{-0.019}$	$6.4^{+6.0}_{-2.1}$

completely polarized incident γ-ray beam generated by Laser-Compton backscattering of relativistic electrons [10], however, clearly identified negative parity with high accuracy [27] in conflict with our earlier finding. Possible reasons for the wrong assignment of positive parity given in [22] have already been discussed in Ref. [27]: the polarization sensitivity of the Compton polarimeter at 4.7 MeV amounts to a few percent only and the experimental error of the Compton scattering asymmetry for the statistically weak 4744 keV γ ray may be underestimated. In addition to Metzger [26] two γ branches deexciting the 4744 keV level were found (Table II). The value $g\Gamma_0^2/\Gamma=322(42)$ meV obtained in the present investigation agrees with the value of $g\Gamma_0^2/\Gamma=285(60)$ meV of Ref. [26].

The 6010 keV level has not been identified in a NRF experiment so far. The nuclear data sheets [20] list a level at an excitation energy of 6011.14(8) keV and tentatively assign $J^\pi=(2^+)$ on the basis of the results of a (t,p) reaction. This is in conflict with results from a (p,p') experiment that suggest $J=(1)$. The latter finding is supported by the angular distribution ratio in the present experiment.

The 6214 keV level has been detected in a photon-scattering experiment by Isoyama *et al.* [28]. A width of $\Gamma=1.81(22)$ eV and an angular momentum $J=1$ have been obtained. Negative parity was concluded from an investigation of ^{88}Sr with linearly polarized photons [29]. The present results confirm $J=1$, but also a decay transition from this state to the 2_1^+ level (Table II) has been found. Our result of $\Gamma=1.92(48)$ eV agrees with that of the earlier investigation.

The 6334 keV level with negative parity has been observed together with the 6214 keV level in the same experiments [28,29]. Also for this level $J=1$ is confirmed, and our result of $\Gamma=2.54(65)$ eV agrees with $\Gamma=2.83(29)$ eV obtained in [28].

The 6710 keV level is tentatively assigned to the 6708.6(3) keV level observed in the (d,p) and (p,p') reactions where no spin has been deduced [20]. We obtain $J=1$ for this state.

We found evidence for a dipole transition at 3378 keV which despite its weakness clearly stands out from the background. We did not find any possible γ ray which could be produced by target contaminations, surrounding materials, etc. Such a transition was also not observed during the irradiation of other nuclides in the same course of experiments [8,13,16,30,31]. As the energy of this transition does not fit the level spacing between any higher-lying state and a low-lying state, as e.g., the first 2^+ state, we consider it as a ground-state transition. The new observation of a state at 3378 keV is surprising as this is the fifth excited state in a nucleus as thoroughly studied as ^{88}Sr. If this state had negative parity, one should be able to observe it, e.g., in the β decay of ^{88}Rb, but there is no evidence for such a transition [32]. However, if positive parity is assumed, the state could easily escape detection in a β-decay experiment. Such an assignment could also explain the absence of a signal in proton scattering [33] and is not in conflict with the data where unnatural-parity states have been searched for [34]. Its low strength also precludes observation in (e,e') experiments [23,24]. The models discussed below do not predict a $J=1$ state around this energy, but several 2^+ states between 3.0 and 3.7 MeV.

IV. SHELL-MODEL CALCULATIONS

The parameters of the shell-model calculation for ^{88}Sr are adopted from Ref. [35]. The shell-model space includes the active proton orbitals $\pi(0f_{5/2},1p_{3/2},1p_{1/2},0g_{9/2})$ and neutron orbitals $\nu(1p_{1/2},0g_{9/2},1d_{5/2})$ relative to a hypothetic ^{66}Ni core. Since an empirical set of effective interaction matrix elements for this model space is not available up to now, various empirical sets have been combined with the matrix elements of a modified surface-delta interaction. Details of this procedure are described in Refs. [36,37]. The effective interaction in the proton shells was taken from Ref. [1]. In that work, the residual interaction and the single-particle energies of the proton orbitals were deduced from a least-squares fit to 170 experimental level energies in $N=50$ nuclei with mass numbers between 82 and 96. The data given in Ref. [38] have been used for the proton-neutron interaction between the $\pi(1p_{1/2},0g_{9/2})$ and the $\nu(1p_{1/2},0g_{9/2})$ orbitals. These data were derived from an iterative fit to 95 experimental level energies of $N=48$, 49, and 50 nuclei. The matrix elements of the neutron-neutron interaction of the $\nu(1p_{1/2},0g_{9/2})$ orbitals have been assumed to be equal to the isospin $T=1$ component of the proton-neutron interaction given in Ref. [38]. For the $(\pi 0f_{5/2},\nu 0g_{9/2})$ residual interaction, the matrix elements proposed in Ref. [39] have been used.

TABLE III. Single-particle energy levels used in the shell-model calculations.

Orbital	$\epsilon_{\text{s.p.}}$ (MeV)
$\pi 0f_{5/2}$	−9.106
$\pi 1p_{3/2}$	−9.033
$\pi 1p_{1/2}$	−4.715
$\pi 0g_{9/2}$	−0.346
$\nu 1p_{1/2}$	−7.834
$\nu 0g_{9/2}$	−6.749
$\nu 1d_{5/2}$	−4.144

The single-particle energies relative to the ^{66}Ni core have been derived from the single-particle energies of the proton orbitals given in Ref. [1] with respect to the ^{78}Ni core and from the neutron single-hole energies of the $1p_{1/2}$, $0g_{9/2}$ orbitals [38]. The transformation of these single-particle energies to those relative to the ^{66}Ni core has been performed [40] on the basis of the effective residual interactions described above. The obtained values of the single-particle energies $\epsilon_{\text{s.p.}}$ are listed in Table III and have been used together with the strengths of the residual interactions to calculate level energies as well as $M1$ and $E2$ transition strengths. For the latter, effective g factors of $g_s^{\text{eff}}=0.7g_s^{\text{free}}$ and effective charges of $e_\pi=1.72e$ and $e_\nu=1.44e$ [41] have been applied.

The nucleus ^{88}Sr has 10 protons and 12 neutrons in the considered configuration space. To make the calculations feasible, a truncation of the occupation numbers was necessary. At most three protons are allowed to occupy the $(1p_{1/2},0g_{9/2})$ subshell and at most one $0g_{9/2}$ neutron can be lifted to the $1d_{5/2}$ orbital. With these restrictions, configuration spaces with dimensions up to 7000 have been obtained. The calculations were carried out with the code RITSSCHIL [42]. The results are discussed in relation to the experimental findings in Sec. VI. The predicted structure of the lowest-lying positive parity states is shown in Table IV.

TABLE IV. Main components of shell-model wave functions of the lowest-lying 0^+, 1^+, and 2^+ states in ^{88}Sr.

J^π	E_x (keV)	Configuration
0_1^+	0	94% $\pi(fp)_0$
		6% $\pi(fp)_2\ \nu(0g_{9/2}^{-1}1d_{5/2})_2$
1_1^+	3697	79% $\pi(1p_{3/2}^{-1}1p_{1/2})$
2_1^+	1657	73% $\pi(1p_{3/2}^{-1}1p_{1/2})$
		21% $\pi(fp)_2$
		6% $\pi(fp)_0\ \nu(0g_{9/2}^{-1}1d_{5/2})_2$
2_2^+	3210	59% $\pi(0f_{5/2}^{-1}1p_{1/2})$
		5% $\nu(0g_{9/2}^{-1}1d_{5/2}^{1})$
2_3^+	4036	56% $\pi(1p_{3/2}^{-2})$
		42% $\pi(fpg)$
		2% $\pi(fpg)\ \nu(0g_{9/2}^{-1}1d_{5/2})_2$

V. QUASIPARTICLE-PHONON-MODEL CALCULATIONS

The nuclear quasiparticle-phonon model (QPM) [43] allows one to study the gross and fine structure of elementary excitations through coupling of quasiparticle states with low-lying phonon modes. This has been used in the past to compare the model predictions on $E1$-strength distributions with the results of photon-scattering experiments. The model is explained in detail in Ref. [44]. Here, only a brief presentation of the model will be given. The model Hamiltonian can be written in the form

$$H = H_{\text{av}} + H_{\text{pair}} + H_M^{\text{ph}} + H_{\text{SM}}^{\text{ph}} + H_M^{\text{pp}}, \qquad (5)$$

where H_{av} is the Woods–Saxon potential and H_{pair} represents the monopole pairing interaction. H_M^{ph} stands for the separable multipole-multipole interaction in the particle-hole channel. The separable spin-multipole interaction in the particle-hole channel is denoted by $H_{\text{SM}}^{\text{ph}}$, and H_M^{pp} stands for the residual interaction in the particle-particle channel.

The model basis wave functions are constructed out of quasiparticle random-phase approximation (RPA) phonons [43]. We will refer to the RPA phonon states using the notation $[\lambda_i^\pi]$, where λ denotes the multipolarity and i stands for the root number. The phonons are of different degree of collectivity, from collective ones (e.g., $[2_1^+]$) to pure two-quasiparticle configurations. The wave function of an excited state of the considered nucleus is taken as a superposition of one-, two-, and three-phonon components [44]. The multiphonon components in the wave function lead to a violation of the Pauli principle. This has been accounted for by using exact commutation relations for the phonons (considering them as superpositions of two-quasiparticle creation and annihilation operators). The result for the norm of the excited-state wave function can be found in Ref. [45].

The model Hamiltonian in terms of RPA phonons has two parts, a harmonic one, which is in fact the RPA Hamiltonian, and an anharmonic one, which accounts for the interaction of phonons and quasiparticles. The latter couples multiphonon components differing by one phonon, i.e., one- and two-phonon components, two- and three-phonon components, and so on. Detailed expressions for the Hamiltonian can be found in Refs. [44,46].

The monopole-pairing interaction H_{pair} is treated in the BCS approximation. It is parametrized for the given basis of single-particle states by the constant matrix elements $G_n=0.193$ MeV and $G_p=0.210$ MeV. In the present work, H_M^{ph} includes multipole terms of isoscalar and isovector type, with multipolarity $\lambda=1-6$. The parameters for the multipole-multipole terms H_M^{ph}, with angular momenta and parities J^π other than 2^+ or 3^-, are chosen in such a way that in the RPA the first state for a given J^π is a pure two-quasiparticle state. The parameters associated with $J^\pi=2^+$ and $J^\pi=3^-$ are fixed to fit the experimentally measured energies of the 2_1^+ and 3_1^- states, respectively, together with the corresponding $B(E\lambda)$ values to the ground state. For 2^+ excitations the particle-particle channel is included. The strength of the interaction in this channel between protons and between neutrons was chosen to be $G_p^{(2)}=G_n^{(2)}=0.70\ \kappa_0^{(2)}$, where $\kappa_0^{(2)}$ is the isoscalar quadrupole constant for the particle-hole separable residual

TABLE V. Structure of the lowest RPA phonons predicted by QPM calculations for ^{88}Sr in terms of two-quasiparticle components. Only the main contributions are shown.

λ_i^π	E (keV)	Structure	Contribution (%)
1_1^+	3246	$\pi(1p_{3/2}^{-1}1p_{1/2})$	100
2_1^+	2100	$\nu(0g_{9/2}^{-1}1d_{5/2})$	11
		$\pi(1p_{3/2}^{-1}1p_{1/2})$	29
		$\pi(0f_{5/2}^{-1}1p_{1/2})$	28
		$\pi(0g_{9/2}^{-1}0g_{9/2})$	9
		$\pi(0f_{5/2}^{-1}0f_{5/2})$	8
		$\pi(1p_{3/2}^{-1}1p_{3/2})$	7
3_1^-	2850	$\nu(0g_{9/2}^{-1}0h_{11/2})$	8
		$\pi(1p_{3/2}^{-1}0g_{9/2})$	66
		$\pi(0f_{5/2}^{-1}0g_{9/2})$	8

interaction. The spin-multipole terms of the Hamiltonian H_{SM}^{ph} have been used to generate phonons with $\lambda^\pi = 1^+, 2^-, 3^+$, and 4^-. RPA phonons with energies up to 7.5 MeV have been taken into account.

The transition operator of the QPM has one- and two-phonon parts [43,44]. The one-phonon part gives the contribution from components in the wave function with a difference of one phonon. The two-phonon part couples components with the same number of phonons and for the case of a transition to the ground state couples the two-phonon components with the ground state. The latter makes it possible to describe transitions to the ground state due to two-phonon components as in the case of the $E1$ transition from the two-phonon 1^- state.

The effective charges for the $E1$ transitions have to be reduced according to Ref. [47]. In the calculations the influence of the giant dipole resonance is taken into account explicitly. For the two-phonon part of the interaction the polarization of the core is taken into account by applying an additional factor $(1+\chi)$, where $\chi=-0.7$ [47].

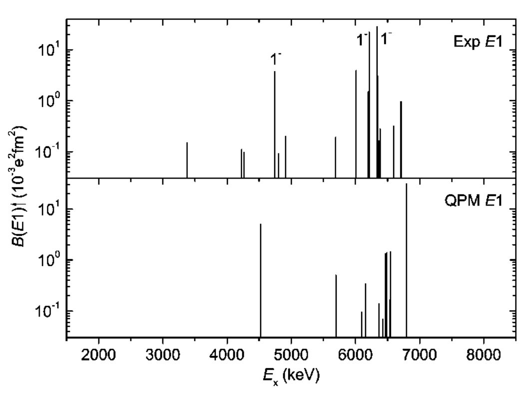

FIG. 2. Electric dipole strength distribution in ^{88}Sr detected in this experiment, assuming all excitations with unknown multipole character to be of electric nature, compared with the results of QPM calculations. Excitations with experimentally known negative parity are indicated. Note that further data exist [20,27–29] for 1^- states above 6.8 MeV that are not shown here.

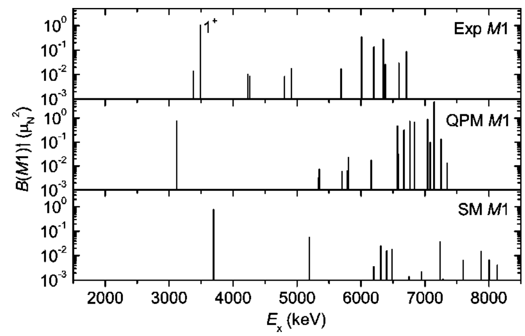

FIG. 3. Magnetic dipole strength distribution in ^{88}Sr, assuming all excitations with unknown multipole character to be of magnetic nature, compared with the results of QPM and shell-model calculations. Excitations with experimentally known positive parity are indicated.

The structure of the lowest RPA phonons in the QPM calculations for ^{88}Sr in terms of dominant particle-hole configurations is presented in Table V.

VI. DISCUSSION

In Figs. 2–4 the experimentally obtained results are compared with the predictions of shell-model and QPM calculations. Based on the angular correlation ratios (Table I), the levels at 4263 and 6367 keV are tentatively considered to be $J=1$ states, and the levels at 4772, 4989, and 5991 keV to be quadrupole excitations. All calculated states with $B(E1)\uparrow \geq 0.03\times10^{-3}\ e^2\ \mathrm{fm}^2$, $B(M1)\uparrow \geq 0.001\mu_N^2$, and $B(E2)\uparrow \geq 1.0\ e^2\ \mathrm{fm}^4$ are plotted. The configuration space of our shell-model calculation does not allow $E1$ transitions to be computed. Therefore, only the results of the QPM calculations are compared to tentative excitations of 1^- states.

A. Electric dipole excitations in ^{88}Sr

1. The two-phonon 1^- *state*

Two-phonon 1^- states are well established in many vibrational nuclei near closed shells [4,5] and are formed by the coupling of the first quadrupole and octupole phonons (2_1^+

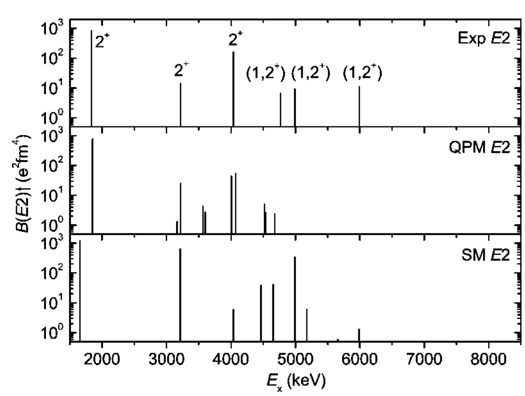

FIG. 4. Electric quadrupole strength distribution in ^{88}Sr obtained in the present experiment up to 6.8 MeV, compared with the results of QPM and shell-model calculations.

$\otimes 3_1^-$). As already discussed in Sec. III, the state at 4744 keV shows characteristics of a two-phonon 1^- state in ^{88}Sr [27]. The value of $B(E1)\uparrow = 3.7(5)\times 10^{-3}\ e^2\,\mathrm{fm}^2$ for the state in ^{88}Sr is close to $B(E1)\uparrow = 3.2(2)\times 10^{-3}\ e^2\,\mathrm{fm}^2$ obtained for the 4634 keV state in the $N=50$ nucleus ^{92}Mo [48], which is the best candidate for a two-phonon 1^- state in that nuclide. Regarding also the $B(E1)\uparrow$ values in $N=82$ nuclei [6], an approximate constancy of the $B(E1)$ values seems to be a feature of the transitions to the 1^- two-phonon states at a given shell closure. The corresponding values for the $Z=50$ nuclei amount to $B(E1)\uparrow \approx 7\times 10^{-3}\ e^2\,\mathrm{fm}^2$ [5] and for the $N=82$ nuclei to $B(E1)\uparrow \approx 17\times 10^{-3}\ e^2\,\mathrm{fm}^2$ [6]. Another criterion for the quadrupole-octupole coupled two-phonon 1^- state is the correlation [49] of its $E1$ decay strength to the g.s. with the $E1$ strength between the constituent one-phonon modes. Using the quadrupole-octupole-coupled $E1$ operator an $E1$-strength ratio of $B(E1;1^-\rightarrow 0_1^+)/B(E1;3_1^-\rightarrow 2_1^+)=7/3$ is expected for an ideal two-phonon coupling [49]. Our new data lead to a slightly smaller experimental value of $B(E1;1^-\rightarrow 0_1^+)/B(E1;3_1^-\rightarrow 2_1^+)=1.6(2)$ as expected due to the influence of the collectivity of the single phonons and g.s. correlations [50].

As shown in Fig. 2, the QPM calculations reproduce the observed 1^- state at 4744 keV relatively well. The model calculations predict that the first 1^- state consists to 89% of the two-phonon structure $[2_1^+\otimes 3_1^-]$. Considering the configuration space of the QPM calculations, the only allowed particle-hole $E1$ transition in the model is $\nu 0g_{9/2}\rightarrow \nu 0h_{11/2}$, i.e., a breakup of the $N=50$ core is required. The shell-model wave function of the 0^+ ground state (Table IV) contains a 6% admixture of the type $\pi(fp)_2\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_2$, in principle facilitating such an $E1$ transition. However, the restrictions of the model space did not allow us to include the $0h_{11/2}$ orbit.

In the $N=50$ nucleus ^{92}Mo, very similar neutron components of the g.s. wave function are obtained, providing a qualitative understanding of the approximate constancy of the $B(E1)$ values. Thus, the nearly equal $B(E1;0^+\rightarrow 1^-)$ values for the two-phonon 1^- states in the $N=50$ nuclei ^{88}Sr and ^{92}Mo can be understood. In the same manner the strengths observed in the $Z=50$ and $N=82$ semimagic nuclei suggest a similar role of the $\pi 0g_{9/2}\rightarrow \pi 0h_{11/2}$ and $\nu 0h_{11/2}\rightarrow \nu 0i_{13/2}$ transitions, respectively. A simple model to describe the two-phonon excitation strength from the g.s. along these lines is discussed in Refs. [51,52]. Within the QPM, the mixing of the dominant $1p1h$ configuration of the giant dipole resonance GDR into the $[2_1^+\otimes 3_1^-]1^-$ wave function reproduces the observed $B(E1)$ strengths quantitatively [53].

Considering the systematics of $[2_1^+\otimes 3_1^-]1^-$ states in medium-mass and heavy nuclei, the 1^- state at 4744 keV in ^{88}Sr exhibits unusual features. (i) As already discussed in Ref. [27], the value $E_x(1^-)/[E_x(2_1^+)+E_x(3_1^-)]$, which is an indication for the anharmonicity of a state, is slightly larger than 1 (1.038). This is in contrast to most of the observed two-phonon 1^- states [5] where values smaller than 1 have been found. Also the QPM calculations predict a value of about 0.85. Recent investigations [54] of the low-spin level schemes of the nuclei ^{92}Zr and ^{96}Mo indicate positive values of the anharmonicities for the quadrupole-octupole-coupled two-phonon 1^- states in these nuclei, as well, which may be a more typical phenomenon in the $A\approx 90$ mass region. (ii) A significant discrepancy is observed for the γ decay to the quadrupole vibrational 2^+ state between the experimental result $[B(E1)\downarrow = 2.5(4)\times 10^{-4}\ e^2\,\mathrm{fm}^2]$ and the model expectation $[B(E1)\downarrow = 7.7\times 10^{-7}\ e^2\,\mathrm{fm}^2]$ (iii) A candidate for a γ branch from the 1^- state to the second 2^+ state at 3219 keV is observed (Table II). Such a decay has not been reported for any other quadrupole-octupole two-phonon state and is also not predicted by the QPM.

2. The distribution of electric dipole excitations above 5 MeV

The distribution of electric dipole excitations between about 5 MeV and the particle threshold, which have been observed in magic or semimagic nuclei with $N=Z=20$, $N=28$ [55], $Z=50$ [56], $N=82$ [6], and $Z=82$ nuclei [7,8] are a subject of current interest. An overview about the discussed structures is given in Refs. [6–8]. The distributions are interpreted as a pygmy dipole resonance originating from surface density oscillations of the neutron skin relative to an approximately isospin-saturated core, or as a toroidal $E1$ mode which is an example for a vortex collective motion in nuclei predicted in different models (see Refs. [6,7,57,58] and references therein).

As shown in Fig. 2, also in the $N=50$ nucleus ^{88}Sr considerable $E1$ strength is found. In this graph all experimentally known 1^- states and other dipole excitations with unknown parity with energies below 6.8 MeV are included. Additional information on $E1$ strength above the experimentally accessed region is provided by [20,27–29].

The total $E1$ strength up to $E_x=8.1$ MeV has an energy centroid of $E_{\mathrm{centr}}=7.0(9)$ MeV and amounts to $\Sigma B(E1)\uparrow = 141(15)\times 10^{-3}\ e^2\,\mathrm{fm}^2$ exhausting 0.38(3)% of the isovector 1^- energy-weighted sum rule (EWSR) [47]. These values can be compared with $E_{\mathrm{centr}}=7.7(2)$ MeV, $\Sigma B(E1)\uparrow = 238(4)\times 10^{-3}\ e^2\,\mathrm{fm}^2$, and EWSR=0.54(1)% for the $N=50$ nucleus ^{92}Mo [48]. The QPM calculations for ^{88}Sr predict a cumulative strength between 6 and 6.5 MeV which is in accordance with the experiment. However, the total strength is significantly underestimated.

B. Magnetic dipole excitations in ^{88}Sr

The only unambiguously identified magnetic dipole excitation in ^{88}Sr up to now is the transition to the 1^+ state at 3487 keV, which has been observed in photon [21,22] and electron [23,24] scattering. Both the QPM and the shell-model calculations reproduce the energy of the level and the $B(M1)\uparrow$ strength fairly well; see Fig. 3. The structure of the shell-model wave function given in Table IV supports a dominant proton $1p_{3/2}\rightarrow 1p_{1/2}$ spin-flip character of this rather strong $M1$ transition as does the structure of the 1_1^+ phonon (see Table V) which contributes 95% to the wave function of this state within the QPM. In passing we note that the form factor of the transition [23,24] could provide a unique testing ground for the role of weak admixtures to the dominant configuration which gain importance at larger momentum transfers.

In the upper part of Fig. 3 all experimental dipole states with unknown parity are included. This is probably strongly overestimating the possible $B(M1)$ strength because a survey of the experimental data near closed shells suggests a dominance of $E1$ strength in the region below the spin-flip $M1$ resonance setting in at $E_x \approx 7$ MeV. Comparing the model results, the shell model predicts less $M1$ strength than the QPM. However, this is due to the model-space limitations which do not include most of the configurations relevant for the spin-flip $M1$ resonance. Above 6.5 MeV the onset of the spin-flip resonance is visible in the QPM results, but the full model space extends to 7.5 MeV only. The QPM results suggest that some of the weaker transitions between 5 and 6.5 MeV may be of $M1$ nature, but their fraction of the total reduced dipole strength is probably small. In fact, the existence of strong $M1$ excitations below 8 MeV with $M1$ excitation strengths larger than about $0.2\mu_N^2$ [28] has been excluded from negative-parity assignments using polarized photon beams [27,29].

C. Electric quadrupole excitations in ^{88}Sr

As demonstrated in Fig. 4, additionally to the well known quadrupole vibrational state at 1836 keV further $E2$ transitions were found in ^{88}Sr. The experimental findings are similar to those observed in NRF experiments on the $N=50$ isotone ^{92}Mo, where above 3.5 MeV also some 2^+ states have been observed [48] with excitation strengths $B(E2)\uparrow$ in the order of some 10 e^2 fm^4.

Both the QPM and the shell model describe the lowest experimentally observed transitions reasonably well. In detail, both models reproduce the $B(E2)$ strength of the transition to the first excited 2^+ state, while the shell model somewhat underestimates the excitation energy. In the case of the QPM, the model parameters are fitted to the properties of the experimental 2_1^+ level. The structure obtained in both models (Tables IV and V) show a predominance of proton configuration with a small admixture of the $\nu(0g_{9/2}^{-1}1d_{5/2}^{1})$ configuration of the order of 6–7 %, similar to the shell-model ground state.

The energies and strengths of the 2^+ states at 3219 keV and 4036 keV are reasonably well reproduced by the QPM, while the shell model predicts both transitions but overestimates the lower one and underestimates the upper one. The QPM gives a 2^+ state at 3220 keV with dominant one-phonon character. In the shell model a 2^+ state at 3210 keV is predicted, which shows the character of a $\pi(1f_{5/2}^{-1}2p_{1/2}^{1})$ proton excitation with a small $\nu(0g_{9/2}^{-1}1d_{5/2}^{1})$ neutron contribution of 5% (see Table IV) in the wave function.

Above 4 MeV, neither of the calculations resembles the data. However, also the 2^+ character of some of the observed states is questionable because of very large error bars in the angular distributions.

VII. SUMMARY

As a result of a nuclear-resonance-fluorescence experiment on the $N=50$ nucleus ^{88}Sr with bremsstrahlung of an end-point energy of 6.8 MeV, the knowledge on the structure of dipole and quadrupole excitations in this nucleus could be considerably extended. The experimentally obtained results are compared with the predictions of shell-model and QPM calculations. The $E1$ transition from the 1^- two-phonon state to the ground state can be interpreted to result from an admixture of the dominant $\nu 0h_{11/2} \rightarrow \nu 0g_{9/2}$ configuration of the GDR into the two-phonon wave function similar to the $N=50$ nucleus ^{92}Mo, explaining the nearly equal values of $B(E1)\uparrow \approx 3.5 \times 10^{-3}$ e^2 fm^2 in both nuclides. However, the two-phonon quadrupole-octupole 1^- state in ^{88}Sr exhibits unusual features compared to most states with similar character, which at present cannot be interpreted within the model calculations.

As in many spherical nuclei, also in ^{88}Sr a concentration of (presumably) electric dipole strength above 5 MeV is observed. However, an extension of the present data toward higher excitation energies is needed to draw conclusions on their nature and the possible observation of a pygmy resonance. Also, knowledge of the parity of the observed dipole states would be important for an in-depth understanding. Such experiments will become feasible at the NRF facility of the ELBE accelerator at Rossendorf which has recently started operation [59].

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