Peculiar properties of the cluster-cluster interaction induced by the Pauli exclusion principle

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The role of the Pauli principle in the formation of both the discrete spectrum and multichannel states of the two-cluster nuclear systems is studied in the algebraic version of the resonating-group method. Solutions of the Hill-Wheeler equations in the discrete representation of a complete basis of the Pauli-allowed states are discussed for the ^{4}He+n, ^{3}H+^{3}H, and ^{4}He+^{4}He binary systems. An exact treatment of the antisymmetrization effects related to the kinetic energy exclusively is shown to result in either an effective repulsion or attraction of the clusters. It also yields a change in the intensity of the centrifugal potential. Both factors significantly affect the scattering phase behavior. Special attention is paid to the ^{6}He+^{6}He multichannel two-cluster system as well as to the coupled-channel calculation of the ^{12}Be nucleus (provided that ^{6}He+^{6}He and ^{4}He+^{8}He clusterings are taken into account). In the latter case, the cluster-cluster interaction derived from the kinetic-energy operator modified by the Pauli principle leads to inelastic processes and ensures the existence of both the bound state and a resonance in the ^{12}Be compound nucleus.

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I. INTRODUCTION

The Pauli exclusion principle is known to significantly influence the interaction of composite particles. The concept of Pauli forbidden states has important consequences for the structure of wave functions of relative motion of two nuclei leading to damping the function in the interior region of internuclear distances. Furthermore, the so-called "partly Pauli-forbidden states" (the Pauli-allowed states whose eigenvalues are not equal to unity) play an important role in two-cluster scattering affecting both positions and widths of resonances [1–3].

The effect of the Pauli principle on the nucleus-nucleus interaction can be consistently described by microscopic methods such as the resonating-group method (RGM) [4] and the generator-coordinate method (GCM) [5]. However, the evaluation of exchange integrals is known to be a very tedious part of two-cluster RGM calculations. That is why in the commonly employed nuclear models the Pauli exclusion principle is to some extent neglected.

There are several different approaches which approximately account for the influence of the Pauli principle in collisions between composite nuclei (clusters). One of them (see, for instance, Refs. [6–9]) is based on the fact that the Pauli principle does not allow identical nucleons to be at the same point in space, and hence the action of the antisymmetrizer is simulated by introducing an additional repulsive potential between clusters. Such a potential is not uniquely determined. In Refs. [6–8], where the problems of α-α scattering and the scattering of the α particle by the ^{12}C, ^{16}O, ^{20}Ne nuclei were considered, the Pauli principle was simulated with a phenomenological repulsive potential of infinite strength. The parameters of this potential have been chosen so as to reproduce the energies and widths of the resonance states in the corresponding compound system. The authors of paper [9] simulated the Pauli principle for a few-cluster nuclear system within the method of hyperspherical functions. As in Refs. [6–8], they used a phenomenological repulsive potential. But within such an approximation a complete and accurate exclusion of the forbidden states is not ensured.

To eliminate the forbidden states, Saito suggested the orthogonality condition model [10]. In this model the allowed states are found from the requirement of their orthogonality to the forbidden states. In Refs. [11–13] phenomenological potentials of a special kind were used. Such potentials contain the operators of projection onto the forbidden states, and in the limit of great potential depth these states are suppressed. To construct these potentials, an explicit form of the wave functions forbidden by the Pauli principle should be used. The latter functions can be easily found for systems of two closed shell or $(0s)$-shell nuclei. However, the influence of the partly Pauli-forbidden states on nucleus-nucleus scattering is disregarded. Nevertheless, the latter states may have tangible effect on physically measurable quantities [2].

These approaches turned to be fruitful and provided important information about discrete and continuum states of a number of few-cluster systems. However, the fundamental problem of constructing a complete basis of the Pauli-allowed states in the generator-parameter space has not been resolved yet. The understanding of the influence of these states on the dynamics of cluster systems (especially, multichannel ones) is still a matter of dispute.

The effects of antisymmetrization on the effective potential between two light nuclei have been studied in some detail by other authors [1–3,14,15]. The importance of Pauli effects apart from the elimination of the Pauli forbidden states was pointed out in the fish-bone model by Schmid [1,2]. In Ref. [14] the structure of the contributions of kinetic, Coulomb, and nuclear potential energy to the nucleus-nucleus interaction is discussed for the example of the α-α system in an orthogonalized representation of the RGM

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equation. The "renormalized RGM potentials" are calculated for this system and compared with simple local nucleus-nucleus potentials appearing in phenomenological models. Lemere *et al.* [15] and Baldock and Robson [3] studied the corrections brought in the nucleus-nucleus scattering by successively increasing the number of nucleons exchanged between the clusters. In Ref. [15] resonating-group calculations of $^3\mathrm{He}+\alpha$ and $^{16}\mathrm{O}+\alpha$ systems have been performed. In Ref. [3] an application of the natural boundary condition approach to $^{16}\mathrm{O}+^{16}\mathrm{O}$ elastic scattering was reported.

The authors of all quoted papers concluded that the influence of the Pauli principle on the structure of nucleus-nucleus potential is very sensitive to the choice of the nucleon-nucleon interaction. Hence, it is difficult to make general conclusions about the main features of the interaction between clusters arising from the exchange potentials. The study of the influence of the exchange effects on the kinetic energy can, however, provide such information. But careful analysis of the effect of the antisymmetrization on the physical observables in the interaction between clusters arising solely from the kinetic energy has not been performed yet. As the kinetic and potential energy enter the Hamiltonian additively, the influence of the Pauli principle on the former can be treated separately. Naturally, it is of importance to know the magnitude and energy dependence of elastic and inelastic scattering cross sections, phase shifts, wave functions, etc., formed by the modified kinetic energy operator exclusively. Such estimations allow one to judge when the contribution from the kinetic energy is essential.

As mentioned above, the requirements of the Pauli principle can be accurately met within the resonating-group method. But the commonly used form of the norm kernel and the Hamiltonian kernel entering the RGM dynamical equations complicates an analysis of the effects induced by the influence of the Pauli principle, and thus many peculiarities of the cluster behavior in the collision may be overlooked. A detailed analysis of the exchange effects can be performed with the use of the algorithm outlined in Refs. [16,17].

We assert that the contribution from the kinetic-energy operator to exchange effects is significant, and that analyzing the antisymmetrization effects related to the kinetic energy one can obtain an important information about collisions of composite nuclear systems. Of course, within the range of nuclear forces the potential energy (with its exchange part) plays an important role, and in this region it can change (weaken or strengthen) the cluster-cluster interaction derived from the kinetic-energy operator modified by the Pauli principle. However, it is reasonable to expect that the basic features of the antisymmetrization effects on the nucleus-nucleus interaction may be learned by studying the exchange effects on the kinetic energy operator exclusively. The analysis of the antisymmetrization effects by Lemere *et al.* [15] and Schmid [1] supports this point of view. Furthemore, the range of the influence of the Pauli principle on the kinetic energy appears to be significantly larger than that of the cluster-cluster interaction generated by the nucleon-nucleon potential, especially for heavier clusters or clusters with an open shell. We thus conclude that there is a region where the cluster-cluster interaction is dominated by the kinetic energy operator modified by the Pauli principle.

In this paper, an algorithm of taking into account the Pauli principle in the calculation of the matrix elements of the Hamiltonian is proposed within the formalism of the generator-coordinate method and the Hill-Wheeler equation. For a given cluster structure of the system studied, or for several coupled cluster configurations, a complete basis of the Pauli-allowed states [classified with the use of the SU(3) symmetry indices] is considered along with a complete set of their eigenvalues. The eigenvalues are shown to indicate the existence of the leading SU(3) irreducible representations (irreps) that dominate in the wave function of the binary cluster system.

The effective nucleus-nucleus potential induced by the kinetic energy operator modified by the Pauli principle is derived here for a binary cluster system. We also discuss results obtained for a zero-range nuclear force, when the potential is switched on in the most compact configuration for each of the systems studied, as a first step for the inclusion of the potential energy in the calculations.

In a consistent microscopic approach, the forbidden states do not enter an expression for the norm and the Hamiltonian kernel. Therefore, an effective potential related to the antisymmetrization affects only the allowed states; and, as will be shown later, it may be not a repulsive potential. A repulsion arises in the states whose eigenvalues are less than unity, whereas an attraction appears in the states with the eigenvalues exceeding unity. It has been noted before [3] that the exchange terms behave repulsively or attractively depending on the choice of the nucleon-nucleon interaction. However the possibility for the kinetic exchange terms to contribute attractively is found for the first time.

The paper is organized as follows. In Sec. II, we define a complete discrete basis of the harmonic-oscillator states allowed by the Pauli principle, following the procedure described in Refs. [16,17]. Then we derive the Hill-Wheeler equations in the representation of the discrete basis (Sec. III). The general properties of the solutions of the latter equations are also discussed in Sec. III. In Sec. IV, the range of the influence of the Pauli principle on the kinetic energy is compared with that of the cluster-cluster interaction generated by the nucleon-nucleon potential. Some features of the effective internuclear potential are analyzed. By considering some examples of the binary cluster systems with one open channel, a physical interpretation of the phenomena directly related to the antisymmetrization of the wave function is suggested (Sec. V). Antisymmetrization effects in a multi-channel binary cluster system are studied at an example of continuum states of $^{12}\mathrm{Be}$ that are able to decay through $^6\mathrm{He}+^6\mathrm{He}$ and $^8\mathrm{He}+^4\mathrm{He}$ channels (Sec. VI). The resonance structure of the $^{12}\mathrm{Be}$ nucleus is still unclear and the problem is only likely to be resolved by a microscopic coupled-channel calculation [18]. That is why this system is chosen as the main example of a real and nontrivial application of our approach. Concluding remarks are made in Sec. VII.

II. COMPLETE BASIS OF THE ALLOWED STATES

To derive the Hill-Wheeler integral equations, the transformation from the coordinate (or momentum) representation

to the representation in terms of generator parameters should be done. Hence we begin with determining a generating function of the system under consideration.

Let $\mathbf{\Phi}(\mathbf{R},\mathbf{r})$ be the generator function of the Hill-Wheeler method, antisymmetric with respect to a permutation of the nucleonic coordinates. Here $\mathbf{r}$ and $\mathbf{R}$ are the sets of the single-particle position vectors and generator parameters, respectively. These parameters describe the dynamics of the degrees of freedom which are of interest for us. We construct this function as the Slater determinant composed of the single-particle orbitals to ensure its proper permutational symmetry. Instead of constructing the basis of orthonormal states $\{\phi_n(\mathbf{r})\}$ defined in the coordinate space, it is expedient to introduce its map $\{\psi_n(\mathbf{R})\}$ in the generator parameter representation. Then an explicit form of the Pauli-allowed basis functions can easily be found along with a set of quantum numbers n. The simplification is attained due to the fact that the number of generator parameters $\mathbf{R}$ is significantly smaller than the number of single-particle variables $\mathbf{r}$ of the functions $\{\phi_n(\mathbf{r})\}$.[1]

In order to construct functions $\psi_n(\mathbf{R})$, let us introduce an expression

$$I(\mathbf{S},\mathbf{R}) = \int \Phi(\mathbf{S},\mathbf{r})\Phi(\mathbf{R},\mathbf{r})d\tau, \tag{1}$$

which is usually called the norm kernel (the overlap integral of the antisymmetric generating functions of the Hill-Wheeler method). Integration in Eq. (1) is over all single-particle vectors. The norm kernel is symmetric with respect to permutations of the generator parameters $\mathbf{R}$ and $\mathbf{S}$. Hence it can be treated as a kernel of the integral equation

$$\Lambda\psi(\mathbf{R}) = \int I(\mathbf{S},\mathbf{R})\psi(\mathbf{S}^*)d\mu_{\mathbf{S}}. \tag{2}$$

The symmetry of the kernel ensures the existence of its nontrivial eigenfunctions $\psi_n(\mathbf{R})$ and eigenvalues Λ_n. Here n is a set of quantum numbers of the basis functions. All that remains is to define the integration domain of the generator parameters $\mathbf{R}$ and $\mathbf{S}$ as well as the measure $d\mu_{\mathbf{R}}$.

Both problems can be solved, provided the Slater determinant $\Phi(\mathbf{R},\mathbf{r})$ is composed of the Bloch-Brink orbitals which are known to be the generating functions for the single-particle harmonic-oscillator basis. For the first time the determinant $\Phi(\mathbf{R},\mathbf{r})$ was used as a trial function (but in a slightly different form) in the Brink's α-cluster model, that is, in the variational calculation of the spectrum of excited states in ^{8}Be. There are two notable differences between the Brink's determinant and the one used here [16]. Firstly, the latter is the generating function for the many-particle harmonic-oscillator basis of the Pauli allowed states. Secondly, its vector generator parameters take complex values.

If we restrict ourselves to one complex vector

$$\mathbf{R} = \frac{\boldsymbol{\xi} + i\boldsymbol{\eta}}{\sqrt{2}},$$

then the expression for the Bargmann measure takes the form

$$d\mu_{\mathbf{R}} = \exp\{-(\mathbf{R}\cdot\mathbf{R}^*)\}\frac{d\boldsymbol{\xi}d\boldsymbol{\eta}}{(2\pi)^3}.$$

Here $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are real vectors which are treated as independent variables of the allowed states defined in the Fock-Bargmann representation [19]. The eigenvalues and eigenfunctions of the kernel (1) are uniquely defined. Moreover, kernel (1) is a sum of orthogonal degenerate kernels. Each of them corresponds to definite values of the number of oscillator quanta ν and SU(3) symmetry indices (λ,μ), and therefore, these kernels are orthogonal. Therefore, solving Eq. (2) is reduced to a standard algebraic procedure for an integral equation with a degenerate kernel. In the most general case, and only because the generator functions are constructed as Slater determinants composed of the Bloch-Brink orbitals, the eigenfunctions of the norm kernel are labeled by the total number of the oscillator quanta ν, SU(3) symmetry indices (λ,μ), a multiplicity index $\alpha_{(\lambda,\mu)}$ when several different (λ,μ) multiplets exist, the angular momentum L, its projection M, and, if necessary, one more additional quantum number α_L. The latter is needed to label the states with the same L in a given (λ,μ) multiplet. Then the Hilbert-Schmidt expansion of the kernel of the integral equation (2) is

$$I(\mathbf{S},\mathbf{R}) = \sum_n \Lambda_n\psi_n(\mathbf{S})\psi_n(\mathbf{R}), \tag{3}$$

where each of the eigenvalues Λ_n of the norm kernel corresponds to the eigenfunction $\psi_n(\mathbf{R})$. Naturally, the eigenfunctions of kernel (1) are orthogonal with respect to the Bargmann measure and normalized to the dimensionality of the irrep (λ,μ) [20].

The second-order Casimir operator of the SU(3) group commutes with the operator of permutation of the nucleon position vectors. Hence, the SU(3) symmetry indices naturally appear as the quantum numbers of the eigenfunctions $\psi_n(\mathbf{R})$. Any other classification of the basis states spoils the diagonal form of Hilbert-Schmidt expansion (3). For instance, keeping ν as a quantum number, quantum numbers of the angular-momentum-coupled ("physical") basis can be introduced instead of the SU(3) symmetry indices (λ,μ). The states of this new basis (referred to as the "l basis" in what follows) are labeled by the number of quanta ν, the angular momenta of each of the clusters l_1 and l_2, and the angular momentum of their relative motion l (see, for example, Refs. [16,17]). Then a unitary transformation should be applied to the functions $\psi_n(\mathbf{R})$. But it results in an off-diagonal form of the expansion (3), because not all the eigenvalues Λ_n are equal to unity. It is only in the asymptotic limit of the large number of oscillator quanta ν that the eigenvalues are close to unity. The unitary transformation from the SU(3) basis to the l-basis leaves the expansion (3) intact, thus making the calculations in the continuum possible in either basis [16].

In the absence of SU(3) degeneracy the eigenfunctions of the kernel (1) can be constructed straightforwardly with the

[1]Indeed, the problem of the reduction of the number of independent variables of the wave function without violation of the Pauli principle was solved on the basis of the generator-coordinate method suggested by Griffin and Wheeler [32].

use of algebraic methods. In the case of SU(3) degeneracy, the eigenfunctions are found by solving an integral equation with the degenerate kernel (see, for example, Ref. [17]), which is reduced to a set of homogeneous algebraic equations with the rank equal to the degree of the SU(3) degeneracy.

The eigenvalues Λ_n appear to be nonzero starting with some minimal number of quanta $\nu_{\min}$, take only positive values and limit to unity when $\nu \to \infty$. They are equal for all the states belonging to the same SU(3) irreducible representation (λ, μ), except for the case of SU(3) degeneracy. In the latter case, the eigenvalues belonging to multiplets of the same irrep (λ, μ) do differ, and an additional index is required to distinguish between them.

We need to bring some order into this variety of SU(3) multiplets and find the leading irreps that dominate in the wave function of the binary cluster system. Here we shall deal with irreps with even symmetry indices λ and μ. The lowest allowed SU(3) multiplet (λ_0, μ_0) appears at $\nu = \nu_{\min}$. If $\nu = \nu_{\min} + 2$, then the allowed states belong to several (in the simplest case, two) irreps: $(\lambda_0 + 2, \mu_0)$ and $(\lambda_0, \mu_0 + 4)$. With ν increasing, the number of the Pauli-allowed SU(3) irreps grows to the greatest allowed value, which is the same for all ν starting with ν_1. In a three-cluster (or multicluster) system the number of the Pauli-allowed SU(3) irreps infinitely increases with ν.

Irreps with different numbers of quanta can be arranged into several branches, with all the states of the same branch having the same symmetry index μ and differing only in value of the first index λ. That is, the irreps $(\lambda_0 + \nu - \nu_{\min}, \mu_0)$ are assumed to belong to the first branch, $(\lambda_0 + \nu - \nu_{\min} - 2, \mu_0 + 4)$ to the second, etc. A hierarchy among these irreps is established by the magnitude of the eigenvalues $\Lambda_{\nu,(\lambda,\mu)}$. The irreps with the maximal values of $\Lambda_{\nu,(\lambda,\mu)}$ are the leading ones. In particular, the irreps $(\lambda_0 + \nu - \nu_{\min}, \mu_0)$ belong to the leading irreps.

For a number of binary cluster systems (for instance, ^{8}He + ^{4}He and ^{6}He + ^{6}He), although not for all, the least symmetric SU(3) irreps correspond to the branch $(\lambda_0 + \nu - \nu_0, \mu_0)$. As for the most symmetric ones, they appear at $\nu = \nu_1$.

III. SOLUTION OF THE HILL-WHEELER EQUATION

With the generating functions $\Phi(\mathbf{S}, \mathbf{r})$ and $\Phi(\mathbf{R}, \mathbf{r})$ having been constructed, we can express the Hamiltonian kernel of the cluster system under consideration in terms of the basis functions as follows:

$$H(\mathbf{S},\mathbf{R}) = \int \Phi(\mathbf{S},\mathbf{r})\hat{H}\Phi(\mathbf{R},\mathbf{r})d\tau = \sum_n \sum_{\tilde{n}} \psi_n(\mathbf{S})\langle n|\hat{H}|\tilde{n}\rangle\psi_{\tilde{n}}(\mathbf{R}). \tag{4}$$

Let the wave function $\Phi(\mathbf{r})$ of the generator-coordinate method be defined as the Hill-Wheeler integral (see, for example, Ref. [21])

$$\Phi(\mathbf{r}) = \int C(\mathbf{R}^*)\Phi(\mathbf{R},\mathbf{r})d\mu_{\mathbf{R}},$$

containing a new unknown function $C(\mathbf{R}^*)$. The equation for the latter follows from the variational principle for the functional

$$\int\int C(\mathbf{S}^*)[H(\mathbf{S},\mathbf{R}) - EI(\mathbf{S},\mathbf{R})]C(\mathbf{R}^*)d\mu_{\mathbf{S}}d\mu_{\mathbf{R}} = 0, \tag{5}$$

where the Lagrange multiplier E has the meaning of the energy.

In order to reduce the functional to an algebraic expression, let us expand the unknown functions $C(\mathbf{R}^*)$ $[C(\mathbf{S}^*)]$ in the basis of the Pauli-allowed states

$$C(\mathbf{R}^*) = \sum_n C_n^* \psi_n(\mathbf{R}^*), \quad C(\mathbf{S}^*) = \sum_{\tilde{n}} C_{\tilde{n}} \psi_{\tilde{n}}(\mathbf{S}^*).$$

Then making use of the expansions (3) and (4), we arrive at the algebraic expression

$$\sum_n \sum_{\tilde{n}} C_n^*(\langle n|\hat{H}|\tilde{n}\rangle - E\Lambda_n\delta_{n,\tilde{n}})C_{\tilde{n}} = 0. \tag{6}$$

Variation of the functional (6) brings us to a set of the algebraic equations for the coefficients C_n,

$$\sum_{\tilde{n}} \langle n|\hat{H}|\tilde{n}\rangle C_{\tilde{n}} - E\Lambda_n C_n = 0. \tag{7}$$

Certainly, n takes all the values allowed for the Pauli-allowed basis functions. Alternatively, the two quadratic forms on the left-hand side of Eq. (6) can be diagonalized. Redefining the coefficients C_n^* $(C_{\tilde{n}})$ as

$$\overline{C_n^*} = \sqrt{\Lambda_n} C_n^*, \quad \overline{C_{\tilde{n}}} = \sqrt{\Lambda_{\tilde{n}}} C_{\tilde{n}},$$

the following equation is obtained for the coefficients:

$$\sum_n \sum_{\tilde{n}} \left\{ \overline{C_n^*} \frac{\langle n|\hat{H}|\tilde{n}\rangle}{\sqrt{\Lambda_n\Lambda_{\tilde{n}}}} \overline{C_{\tilde{n}}} - E\delta_{n,\tilde{n}} \overline{C_n^*}\,\overline{C_{\tilde{n}}} \right\} = 0 \tag{8}$$

instead of Eq. (6). Then it remains to reduce the renormalized matrix of the Hamiltonian to a diagonal form by means of the unitary transformation.

Now let us discuss the general properties of the solutions of the set of equations (7). For a binary cluster system the components of discrete eigenstates with energy $E_\kappa = -\kappa^2/2 < 0$ decrease exponentially with the number of radial quanta $\nu = 2k$ following the law

$$C_n^\kappa = A_n^\kappa \frac{\sqrt{2}\exp(-\sqrt{2|E_\kappa|}\sqrt{4k+2l+3})}{\sqrt{r_0}\sqrt[4]{4k+2l+3}}.$$

Here n denotes the set of the quantum numbers of the l basis, l is the angular momentum of the cluster relative motion, r_0 is the oscillator length, and A_n^κ is the asymptotic normalization coefficient [22]. The internal cluster functions are assumed to be fixed and described by the translation-invariant shell model wave functions with the same oscillator length for both clusters.

The asymptotic behavior (at large values of the number of quanta ν) of the continuum eigenstates $\{C_n(E)\}$ with energy $E>0$ is expressed in terms of Hankel functions of the first and second kind, and the scattering S-matrix elements or Bessel and Neumann functions, and the K-matrix elements [16,17]. Although the eigenvectors are orthonormalized, the orthonormality conditions contain the eigenvalues Λ_n as the weight coefficients.

With the eigenvectors determined, it is easy to proceed either to the eigenfunctions $\Phi_\kappa(\mathbf{r})$, $\Phi_E(\mathbf{r})$ in the coordinate representation or to the functions $\Psi_\kappa(\mathbf{R})$, $\Psi_E(\mathbf{R})$ in the Fock-Bargmann space:

$$\Psi_{\kappa(E)}(\mathbf{R}) = \sum_n \sqrt{\Lambda_n} C_n^{\kappa(E)} \psi_n(\mathbf{R}). \tag{9}$$

An expression for the density matrix in the Fock-Bargmann representation can be written as

$$\rho(\mathbf{S},\mathbf{R}) = \sum_\kappa \left\{ \sum_n \sqrt{\Lambda_n} C_n^{\kappa^*} \psi_n(\mathbf{S}) \sum_{\tilde{n}} \sqrt{\Lambda_{\tilde{n}}} C_{\tilde{n}}^{\kappa} \psi_{\tilde{n}}(\mathbf{R}) \right\} + \int dE \left\{ \sum_n \sqrt{\Lambda_n} C_n^*(E) \psi_n(\mathbf{S}) \sum_{\tilde{n}} \sqrt{\Lambda_{\tilde{n}}} C_{\tilde{n}}(E) \psi_{\tilde{n}}(\mathbf{R}) \right\}. \tag{10}$$

The summation in Eq. (10) is over the discrete states, while the integration is over the continuum states. The density matrix provides us with the information about the behavior of the probability distribution function in the phase space. Note that the number of independent variables in Eq. (10) is significantly smaller than in the distribution function $\rho(\{\mathbf{r}\})$ defined in the coordinate space. This is the main advantage of the generator-coordinate method and, in particular, of the Fock-Bargmann representation.[2]

Integrating $\rho(\mathbf{R},\mathbf{R}^*)$ over the phase space, we can reduce it to the following sum

$$\int \rho(\mathbf{R},\mathbf{R}^*) d\mu_{\mathbf{R}} = \sum_\kappa \sum_n |C_n^\kappa|^2 \Lambda_n,$$

provided that all the states of the system belong to the discrete part of the spectrum. Here

$$(C_n^\kappa)^2 \Lambda_n = \left| \int \psi_n(\mathbf{R}^*) \Psi_\kappa(\mathbf{R}) d\mu_{\mathbf{R}} \right|^2$$

is the realization probability of the state $\psi_n(\mathbf{R})$ in the wave function of the cluster system $\Psi_\kappa(\mathbf{R})$.

Thus, employing the basis of the Pauli-allowed states, we deal with a discrete representation of any state of the system, whether belonging to the discrete or to the continuum spectrum. The expansion coefficients of the wave function in the basis of the Pauli-allowed states are $\{\sqrt{\Lambda_n} C_n\}$. The absolute value of a coefficient squared gives us the probability for the corresponding Pauli-allowed basis state; and a convergence holds both for the bound states and the continuum.

[2]Thus, the Fock-Bargmann representation provides us with the simplest means of realization of the generator-coordinate method and opens new prospects of analysis of cluster systems.

IV. HOW DOES THE ANTISYMMETRIZATION OPERATOR ACT

The RGM wave function belonging to the continuous spectrum of a few-cluster system has a remarkably simple form at large intercluster distances. In this region, the potential energy of the cluster-cluster interaction can be neglected; and there is also no need for the antisymmetrizer. Then the wave function is given by the product of the intrinsic cluster wave functions and the wave function of the free motion of the centers of the clusters.

With the intercluster distance decreasing, the wave function of the system is modified and no longer reducible to such a simple form. Nucleons belonging to different clusters are not isolated from each other any more, and hence, the cluster radii can, in effect, change. The change of the radii of the nuclear clusters results in changing both their internal energies and the energy of their relative motion, even if the potential energy of the cluster-cluster interaction is disregarded.

Among the factors coming into play at this stage, the most important are those which are directly related to the influence of the Pauli exclusion principle. As a result, a general picture of the phenomenon becomes intricate, and to interpret it completely, each factor determining the final result should be carefully analyzed. Further we shall concentrate our attention on the phenomena that reveal the characteristic features of the fully antisymmetric RGM wave function at relatively small intercluster distances. With that end in view, some considerable simplifications will be introduced for the description of the potential energy of the cluster-cluster interaction.

All in all, there are, at least, two questions to be answered. (1) What is the range of the nucleus-nucleus interaction generated by the Pauli exclusion principle? (2) What are the main features of such an interaction?

As discussed in the Introduction, the kinetic energy modified by the Pauli principle is an important ingredient in the formation of the observables. Consider the set of the algebraic equations (7) in which only the operator of the kinetic energy of the relative motion (in the c.m. frame) is retained:

$$\sum_{\tilde{n}} \langle n|\hat{T}|\tilde{n}\rangle C_{\tilde{n}} - E\Lambda_n C_n = 0. \tag{11}$$

Due to the particular simplicity of the kinetic energy operator, its generating matrix element can be written in the form

$$T(\mathbf{R},\mathbf{S}) = \hat{T}_{\mathbf{R}} I(\mathbf{R},\mathbf{S}) = \hat{T}_{\mathbf{R}} \sum_n \Lambda_n \psi_n(\mathbf{R}) \psi_n(\mathbf{S}),$$

where

$$\hat{T}_{\mathbf{R}} = -\frac{\hbar^2}{4mr_0^2}[\mathbf{R}^2 - 2(\mathbf{R}\cdot\nabla_{\mathbf{R}}) - 3 + \nabla_{\mathbf{R}}^2] \tag{12}$$

is the Fock-Bargmann map of the kinetic energy operator (see, for example, Ref. [16]). m stands for the nucleon mass from now on. From Eq. (12) it immediately follows that the kinetic energy matrix in the harmonic-oscillator representation is tridiagonal, whence

$$T(\mathbf{R},\mathbf{S}) = \sum_{\nu} \{\Lambda_{\nu-2} T_{\nu-2,\nu} \psi_{\nu-2}(\mathbf{R}) + \Lambda_{\nu} T_{\nu,\nu} \psi_{\nu}(\mathbf{R}) + \Lambda_{\nu} T_{\nu,\nu+2} \psi_{\nu+2}(\mathbf{R})\} \psi_{\nu}(\mathbf{S}).$$

Here $T_{\nu,\tilde{\nu}}$ are the matrix elements of the operator $\hat{T}_{\mathbf{R}}$ between the functions $\psi_{\nu}(\mathbf{R})$ (the latter functions are normalized to unity with the Bargmann measure),

$$T_{\nu,\tilde{\nu}} = \int d\mu_{\mathbf{R}} \psi_{\tilde{\nu}}(\mathbf{R}^{*}) \hat{T}_{\mathbf{R}} \psi_{\nu}(\mathbf{R}).$$

A nonlocal kinetic energy operator is obtained. For simplicity, the single-channel case is considered here, when the basis functions differ only in the number of oscillator quanta ν.

A typical set of the equations (11) can be written as

$$\Lambda_{\nu-2} T_{\nu,\nu-2} C_{\nu-2} + \Lambda_{\nu}(T_{\nu,\nu} - E) C_{\nu} + \Lambda_{\nu} T_{\nu,\nu+2} C_{\nu+2} = 0. \tag{13}$$

A standard set of the discrete representation which disregards the Pauli principle

$$T_{\nu,\nu-2} C_{\nu-2} + (T_{\nu,\nu} - E) C_{\nu} + T_{\nu,\nu+2} C_{\nu+2} = 0 \tag{14}$$

differs from the set of equation (13) in two respects. First, only the matrix elements between the allowed states enter the latter. Therefore, there is no need to introduce any additional potential in the initial Hamiltonian in order to remove the Pauli-forbidden states. Such a term would give no contribution to Eq. (11).

Secondly, the elimination of the forbidden states still does not resolve the problem in total. Indeed, the matrix of the set (13) contains the eigenvalues Λ_{ν} belonging to the Pauli-allowed states, and, for this reason, it is not identical to the matrix (14) of the kinetic-energy operator of the free motion of clusters.

Equation (14) take asymptotic form at large values of ν and are reduced to the Schrödinger differential equation of free motion with definite angular momentum l (see Ref. [23] for details). At large ν, Eq. (13) take the same form as Eq. (14), because there all eigenvalues Λ_{ν} equal unity. But at small values of ν the eigenvalues differ from unity, and the asymptotic form of the equations becomes complicated. As a result, Eq. (13) tend to the equations of motion in the field of the "kinetic energy exchange potential." Our purpose is to find these equations in order to reveal the main features of the cluster-cluster interaction in their collision.

An equation of the set (13) for a collision of clusters with angular momentum l can be written in the form of the finite-difference equation

$$-\frac{1}{2}\left\{\left(1+\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right)\left(\nu+\frac{3}{2}-\frac{(2l+1)^2}{8\nu}\right)+1-\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right\} \times \frac{1}{4}(C_{\nu+2}-2C_{\nu}+C_{\nu-2}) - \frac{1}{2}\left\{1+\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}+\left(1-\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right) \times\left(\nu+\frac{3}{2}-\frac{(2l+1)^2}{8\nu}\right)\right\}\frac{1}{4}(C_{\nu+2}-C_{\nu-2}) + \left\{\left(1+\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right)\frac{(2l+1)^2}{32\nu}+\frac{1}{4}\left(1-\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right)\left(\nu+\frac{1}{2}\right)\right\}C_{\nu} = \frac{mr_0^2}{\hbar^2}EC_{\nu}.$$

The latter takes the form of the Bessel differential equation in the limit $\nu \gg 1$, where the eigenvalues Λ_{ν} can be set to unity:

$$\left(\frac{d^2}{dy^2}+\frac{1}{y}\frac{d}{dy}-\frac{(2l+1)^2}{4}\frac{1}{y^2}+\frac{mr_0^2}{\hbar^2}2E\right)C(y)=0,$$

$$y=\sqrt{2\nu+2l+3}. \tag{15}$$

The diagonal matrix $\|U^{\mathrm{Pauli}}_{\nu,\tilde{\nu}}\delta_{\nu,\tilde{\nu}}\|$

$$U^{\mathrm{Pauli}}_{\nu,\nu} = \frac{1}{2}\left(1+\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right)\frac{(2l+1)^2}{16\nu}+\frac{1}{4}\left(1-\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right)\left(\nu+\frac{1}{2}\right) \tag{16}$$

can be considered the matrix of the operator of the effective cluster-cluster interaction derived from the kinetic-energy operator modified by the Pauli principle. The physical meaning of the first term in Eq. (16) is quite simple. It is the centrifugal potential that is renormalized with the factor

$$\frac{1}{2}\left(1+\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right).$$

The latter, along with the eigenvalues, limits to unity as $\nu \to \infty$. If the eigenvalues approach unity from below as ν increases, the renormalization factor does not exceed unity. Therefore, partial suppression of the centrifugal potential is observed in this case. If the eigenvalues approach unity from above, the centrifugal potential gets stronger.

The second term,

$$\frac{1}{4}\left(1-\frac{\Lambda_{\nu-2}}{\Lambda_{\nu}}\right)\left(\nu+\frac{1}{2}\right),$$

represents a finite-range potential generated by the exchange effects on the kinetic energy (referred to as "effective potential" in what follows). Its intensity decreases in magnitude as the difference $\Lambda_{\nu}-\Lambda_{\nu-2}$ vanishes. If the latter remains negative as ν grows (the eigenvalues monotonically approach unity from above), then the effective potential turns to be attractive. If the difference of the eigenvalues remains positive as ν increases (the eigenvalues monotonically approach unity from below), then the effective potential is repulsive. Obviously, the range of such an interaction depends on the width of the interval where the eigenvalues deviate from unity.

Indeed, as was shown in Sec. III, the wave function of a binary cluster system can be presented in the form of the expansion (9). If $\Lambda_{\nu}<1$, the eigenvalues suppress the terms with small values of ν in the expansion (9); that can be naturally interpreted as the action of effective repulsive forces at small intercluster distances. This leads to a decrease of the ground state energy in absolute value and to a corresponding change in the scattering phase shifts due to the

appearance of the additional effective repulsion. On the contrary, if $\Lambda_\nu > 1$, then the terms with small ν are more preferred; that can be considered an effective attraction.

Consequently, the antisymmetrization effects result not only in the elimination of the forbidden states, but also in changing the relative kinetic energy as clusters approach each other. In the case of the free motion of two composite particles in a state with definite angular momentum l, a centrifugal potential is the sole term that changes the relative velocity of the clusters in collision. It decreases the kinetic energy of the relative motion of the clusters until they stop at the turning point $r_\nu = r_0\sqrt{2\nu+2l+3}$ and then begin to fly apart. The antisymmetrization renormalizes the strength of the centrifugal potential, which is the first of the factors affecting the relative velocity of the clusters. With the intercluster distance decreasing, the effective potential also comes into play.

If the eigenvalue Λ_ν is a monotonically increasing function of ν, then the relative velocity of the clusters decreases to a lesser degree than in the field of the centrifugal potential of the free motion. But then, the clusters come within the range of the repulsive effective potential which decreases the kinetic energy of the relative motion. If the eigenvalue Λ_ν is a monotonically decreasing function of ν, then a strengthened centrifugal potential slows down the cluster motion to a greater degree than in the case of the free motion. But at a smaller distance the relative velocity of the clusters increases due to the action of the effective attractive potential.

It should be noted also that in the low-energy region the elastic scattering phase-shift is affected by the effective potential related to antisymmetrization. The change in the centrifugal barrier caused by the Pauli principle affects, mainly, the asymptotic behavior of the phase shift.

To estimate the range of influence of the antisymmetrizer on the structure of the wave functions, it is appropriate to consider two extreme values of the number of quanta $\nu_{\min}$ and $\nu_{\max}$. As long as $\nu < \nu_{\min}$, all the eigenvalues equal zero. If the number of quanta satisfy the inequality $\nu_{\min} \leqslant \nu \leqslant \nu_{\max}$, then the eigenvalues become positive, but deviate from unity. Finally, provided that the inequality $\nu > \nu_{\max}$ holds, the eigenvalues can be considered to be approximately equal to unity. The value of $\nu_{\max}$ is defined in a rather approximate way, which demonstrates the diffuseness of the range of the antisymmetrization operator. The quantity r_{eff},

$$r_{\text{eff}} = \sqrt{\frac{A_1+A_2}{A_1A_2}}\, r_0\sqrt{(2\nu_{\max}+2l+3)},$$

defines the distance between the centers of the two nuclear clusters (composed of A_1 and A_2 nucleons, respectively) where the antisymmetrization effects come into play. It is appropriate to compare the range of the kinetic energy exchange potential with that of the cluster-cluster interaction generated by the nucleon-nucleon potential.

The matrix elements of the potential energy operator, which enter the equations of the algebraic version of the RGM, can be extracted from the potential energy kernel $U(\mathbf{R},\mathbf{S})$ defined in the Fock-Bargmann space. This kernel is an entire function of $\mathbf{R}$ and $\mathbf{S}$. Similar to the kinetic energy kernel, it can be considered to be the result of the action of some operator representing the nucleon-nucleon potential on the norm kernel. To obtain the potential energy matrix elements $\langle \nu, l|U|\tilde{\nu}, l\rangle$, the interaction kernel $U(\mathbf{R},\mathbf{S})$ needs to be projected to the states with definite values of the oscillator quanta, the SU(3) labels, and the angular momentum. This can be done analytically. The antisymmetrizer eliminates the matrix elements of the potential energy operator between the forbidden states as well as those coupling the allowed and the forbidden states.

At large values of the number of quanta ν the matrix of the potential energy operator of the cluster-cluster interaction is known to be equivalent to the diagonal matrix $\|\delta_{\nu,\tilde{\nu}}U(r_\nu)\|$ (see Ref. [24]), the elements of which rapidly decrease with ν similar to the cluster-cluster potential $U(r)$ itself in the coordinate space:

$$\sum_{\nu_{\min}}^{\infty} \langle \nu, l|U|\tilde{\nu}, l\rangle C_{\tilde{\nu},l} = U(r_\nu)C_{\nu,l}. \tag{17}$$

Hence, for a short-range cluster-cluster potential we conclude that matrix elements of the equivalent diagonal matrix rapidly decrease. For a central nucleon-nucleon potential having a Gaussian form we will have

$$U(r_\nu) \sim U_0 \exp\left\{-\frac{r_0^2(2\nu+2l+3)}{b_0^2}\frac{A_1+A_2}{A_1A_2}\right\}, \tag{18}$$

where b_0 is the range of the Gaussian potential and U_0 is its strength. $U(r_\nu)$ contains both the direct and exchange interaction term. They are not given separately, because this would destroy the symmetry of the potential energy kernel.

Strictly speaking, expression (18) also depends implicitly on the eigenvalues Λ_ν of the norm kernel. However, this dependence cannot increase the range of the potential energy operator of the cluster-cluster interaction estimated by extrapolating the asymptotic relation (18) to the region of smaller values of ν. With the number of nucleons in each cluster increasing, the number of forbidden states grows. Then the asymptotic estimate (18) appears to be valid even for the minimal number of quanta allowed by the Pauli principle.[3] The number of partly forbidden states also increases, which leads to the suppression of the potential energy matrix elements.

At large values of the number of oscillator quanta ν the effective cluster-cluster interaction derived from the kinetic energy operator modified by the Pauli principle vanishes as the eigenvalues Λ_ν tend to unity. Thus, the range of such an interaction can be estimated with the help of the relation

$$\Lambda_\nu - 1 \sim \beta(\nu)\alpha^{-\nu} = \beta(\nu)\exp(-\nu \ln \alpha). \tag{19}$$

Parameter $\alpha > 1$ is completely determined by the type of clustering of the nuclear system considered; and that is why it is the same for all the branches of the SU(3) irreps. As for the parameter $\beta(\nu)$, its dependence on the number of quanta

[3]If the value of oscillator lenght r_0 is chosen in accordance with the experimental values of the cluster radii.

follows a power law and differs for different SU(3) irreps even within one cluster configuration.

It follows from Eq. (19) that the kinetic energy exchange potential decreases exponentially with the number of quanta, and so is the cluster-cluster interaction generated by the nucleon-nucleon forces. Comparing the decrements of the expressions (18) and (19), it is possible to establish which of the potentials has larger range. It appears to be that

$$\frac{A_1+A_2}{A_1A_2}\frac{2r_0^2}{b_0^2} \gg \ln\alpha$$

even for the simplest binary cluster systems composed of s clusters, such as ^{4}He+n, ^{4}He+^{4}He, etc. This means that as the clusters approach each other, they first experience an influence of the effective interaction caused by the exchange effects on the kinetic energy. And the potential of the cluster-cluster interaction generated by the nucleon-nucleon forces comes into play only at the smallest intercluster distances.

Further we shall explain how the eigenvalues (and, therefore, the parameters of the effective cluster-cluster interaction) depend on the number of nucleons of a compound system and on the type of clustering by considering some examples of binary nuclear systems composed of s and p clusters.

V. EXAMPLES OF THE EFFECTIVE POTENTIALS: SINGLE-CHANNEL CLUSTER SYSTEMS

In this section, we will consider the simplest examples of two-cluster systems with one open channel in order to demonstrate the influence of the kinetic-energy exchange effects on the scattering phase shifts and on the wave functions in the discrete representation. To reveal the main features of the nucleus-nucleus interaction affected by the Pauli principle, we shall concentrate our attention on studying the exchange effects on the kinetic energy operator exclusively, although we shall also discuss the results obtained in the approximation of zero-range nuclear force.

In addition, we shall compare our results with those which can be obtained by simulating the Pauli principle with some phenomenological potentials of the optical model. Not aiming at a complete discussion of all the factors influencing the choice of a nucleus-nucleus potential, we shall focus on those features of optical model potentials which follow the requirements of the Pauli principle.

Conventional optical model descriptions of nucleus-nucleus scattering usually employ two kinds of effective local nucleus-nucleus potentials which are phase-equivalent with those derived on microscopic grounds: deep or shallow potentials. Deep potentials are energy independent and vary very gradually with angular momentum of the nuclei involved. These potentials produce a number of unphysical bound states which simulate the forbidden states of the microscopic approach. Indeed, it is exactly the number of the forbidden states that determines the phase-shift variation between zero and infinite energy, and in optical model potentials this must be accounted for. In contrast, shallow optical potentials are often found to be strongly angular-momentum dependent and do not have nonphysical states in their discrete spectrum. However, they should be singular to reproduce the high-energy behavior of the microscopic phase shifts [25]. In our approach, neither a deep potential with unphysical states, nor a singular shallow potential is employed, in order to ensure the correct phase-shift variation.

In our equations, the forbidden states are excluded from the very beginning, because their eigenvalues are equal to zero, i.e., $\Lambda_\nu=0$ for all $\nu<\nu_{\min}$. The known asymptotic estimate establishes a relation between the wave function $\Phi_l(\mathbf{q})$ in the coordinate representation and coefficients C_ν^l,

$$\Phi_l(|\mathbf{q}|=r_0\sqrt{2\nu+3})=\frac{\sqrt{\Lambda_\nu}C_\nu^l}{\sqrt{2r_0}(2\nu+3)^{1/4}}, \quad \nu\gg 1.$$

From this it follows that the wave function Φ_l is rather close to zero within the interval

$$0<|\mathbf{q}|<r_0\sqrt{2\nu_{\min}+3}.$$

Such a wave function can be obtained in the calculations with a model repulsive-core potential (see, for example, Refs. [6,8]). The number of the forbidden states with angular momentum $l<\nu_{\min}$ equals $[(\nu_{\min}-l)/2]$. Hence, the elastic scattering phase shift $\delta_l(E)$ should be counted from $[(\nu_{\min}-l)/2]\pi$, in order to make it vanish at high energy. Such energy dependence of the scattering phase shift can also be reproduced by means of a soft core, with its height being determined by the phase-shift variation. The radius of the core depends not only on $\nu_{\min}$, but also on the angular momentum l; and can be estimated with the help of the relation $R_{\text{core}}=r_0\sqrt{2\nu_{\min}+3}$.

Note that for the states with angular momentum $l\geqslant\nu_{\min}$ the forbidden states are absent, and hence no core need to be introduced. Certainly, the larger is $\nu_{\min}$, the more accurate are these considerations.

All this, however, does not mean that a core entirely reproduces the action of the Pauli principle. As was explained in the previous section, the eigenvalues of the norm kernel deviate from unity at $\nu>\nu_{\min}$ and affect the RGM Hamiltonian in the discrete basis representation.

The two-cluster systems considered in this section have a remarkable property: all states with a given number of quanta ν belong to the same SU(3) multiplet $(\nu,0)$. That simplifies noticeably the explicit form of the Pauli-allowed orthonormal basis functions in the Fock-Bargmann space (they are constructed from even powers of one complex vector), and provides only two sets of the eigenvalues: one for the even states, and the other for the odd ones. In addition, the matrix elements of the kinetic energy operator take a particularly simple form in this case.

In the absence of the operator of the nucleon-nucleon interaction two different modes of behavior of the scattering phases are observed in the energy region up to 50 MeV (in the c.m. frame). Later we shall show that at low energies the eigenvalues approaching unity from above produce a positive scattering phase, while those approaching unity from below, a negative one.

A. The $n+{}^4$He system

The scattering of a neutron by an α-particle is one of the simplest examples considered in the framework of the Hill-Wheeler method. We wish to show that even for this system the scattering phase cannot be reproduced in full by any optical model potential which simulates the action of the Pauli principle.

The norm kernel for the $\alpha+n$ system generating a complete basis of the Pauli-allowed states or, in other words, the kernel of an integral equation reads

$$I_{n+\alpha}(\mathbf{R},\mathbf{S}) = \exp(\mathbf{R}\cdot\mathbf{S}) - \exp\left[-\frac{1}{4}(\mathbf{R}\cdot\mathbf{S})\right]. \qquad (20)$$

The norm kernel does not contain the Pauli-forbidden states. They are already excluded. Therefore, it can be compared with the result of action of the model repulsive potential which eliminates the forbidden states. But we should also take account of the eigenvalues belonging to the allowed states. Hence, provided that the Pauli principle is simulated by some phenomenological potential, at this stage an additional interaction should be introduced to reproduce the dynamics of the system after the elimination of the forbidden states. Later the main features of this additional phenomenological potential will be discussed.

The eigenvalues of the norm kernel for $\alpha+n$ system are equal to

$$\Lambda_\nu = 1 - \left(-\frac{1}{4}\right)^\nu.$$

Obviously, in the limit $\nu\to\infty$ the eigenvalues approach unity from below, if the number of quanta is even ($\nu=2k$); and from above, if the number of quanta is odd ($\nu=2k+1$). The minimal number of quanta $\nu_{\min}$ which corresponds to the lowest Pauli-allowed basis state is equal to 1. This means that the branch which belongs to the SU(3) irrep $(2k+1,0)$ appears first and its eigenvalues take the largest values.

To illustrate our conclusions about the characteristic features of the effective interaction induced by the Pauli principle, it is appropriate to consider the phase-shifts of the elastic scattering of a neutron by the α particle[4] in the states with $L^\pi=0^+$ [Fig. 1(a)] and $L^\pi=1^-$ [Fig. 1(b)]. Zero angular momentum corresponds to the eigenvalues $\Lambda_{2k}<1$, while the eigenvalues Λ_{2k+1} which exceed unity correspond to the momentum $L=1$. The behavior of the scattering phases $\delta_{L=0}$ and $\delta_{L=1}$ for the case when only antisymmetrization effects on the kinetic energy are taken into account and in the approximation of zero radius for the nuclear force is presented in Fig. 1. In this approximation the potential energy of the interaction of the neutron and α particle is simulated with the single parameter, the diagonal matrix element of the potential energy operator $\hat{U}$ in the state with the minimal number of even (for $L=0$) or odd (for $L=1$) quanta, i.e.,

[4]These phase shifts have already been calculated earlier [33], but our purpose is to show to what extent they are influenced by the antisymmetrization operator.

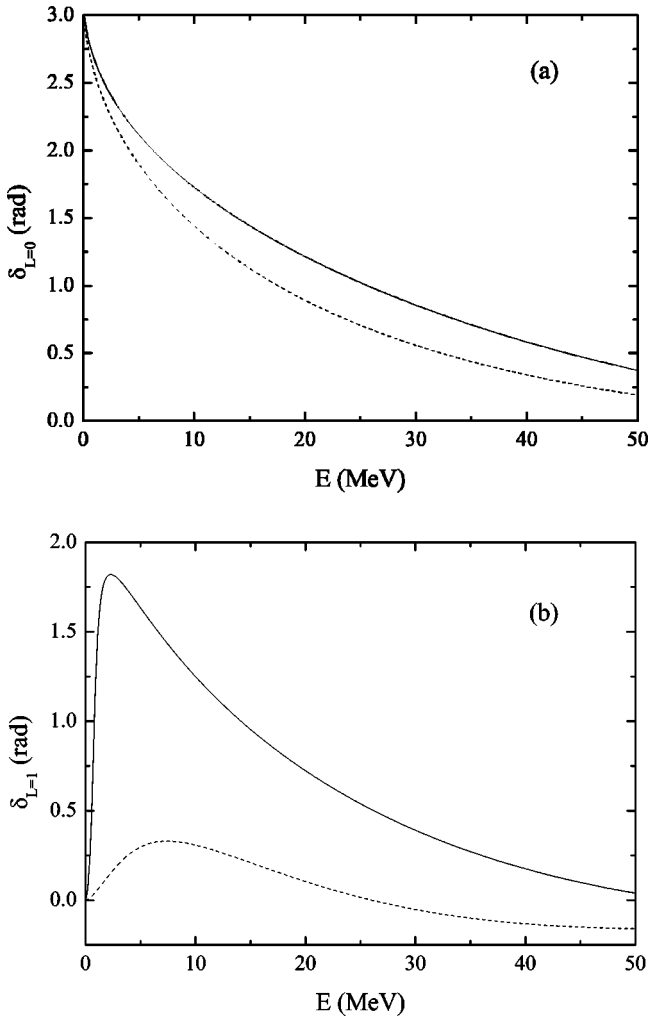

FIG. 1. Phase shifts of the $\alpha+n$ scattering for the states with (a) $L=0$ and (b) $L=1$. Solid curve: phase shifts obtained in the zero-range approximation for the nuclear force. Dashed curve: phase shifts obtained by keeping the kinetic energy exchange potential only (see text for details).

$$\langle(\nu,0)|U|(\nu',0)\rangle = \begin{cases} U_0 = -5.52 \text{ MeV} & \text{if } \nu=\nu'=\nu_{\min}=2,\ L=0, \\ \overline{U_0} = -11.05 \text{ MeV} & \text{if } \nu=\nu'=\nu_{\min}=1,\ L=1, \\ 0 & \text{otherwise.} \end{cases}$$

The parameter $\overline{U_0}$ was fitted to reproduce position of the maximum of the total elastic scattering cross section $E_r = 0.92\pm0.04$ MeV [26]. The half width $\Gamma=1.3$ MeV also agrees well with the experimental value of $\Gamma=1.2$ MeV, although the maximum value of the total cross section is twice as large as its experimental value of 7.6 b [26]. As regards the parameter U_0, it was chosen to provide a reasonable description of the experimentally observed phase of the $\alpha+n$ elastic scattering with $L=0$ at low energies [34].

As long as the energy E of the relative motion of the clusters is small, the phase shift of the elastic scattering with angular momentum L obeys the law

$$\delta_L \sim \pi n - a_L (2E)^{L+1/2},$$

i.e., as for a standard short-range potential. Here n is the number of bound states. If $L=0$, a_0 is the scattering length. The system $\alpha+n$ has no bound states, but there is one Pauli-forbidden state at $k=0$ with angular momentum $L=0$, which is known to have the same influence on the behavior of the scattering phase δ_0 as a bound state [27]. Therefore, at zero energy the scattering phase δ_0 is naturally counted from π. There are no forbidden states having odd number of quanta, and hence the scattering phase δ_1 should be set to zero at zero energy.

The positive sign of the scattering length a_0 conforms to the known general consideration that Pauli principle can be simulated by a repulsive potential. Of course, the attraction $\overline{U_0}$ appears not to be strong; otherwise it would change the signs of the scattering phase and the scattering length.

As it can be seen from Fig. 1(b), $a_{L=1}<0$. Therefore, the effective interaction caused by the kinetic energy exchange potential is attractive for the states with angular momentum $L=1$. The intensity of the attraction induced by the antisymmetrization effects arising solely from the kinetic energy is not high enough to assure the existence of the experimentally observed $L^\pi=1^-$ resonance in the continuum of the ^{5}He nucleus.[5] But a contribution from the kinetic energy exchange potential is not neglible, because the range of the latter potential exceeds that of the nuclear forces.

It should be stressed that this energy dependence of the scattering phase δ_1 cannot be reproduced by a simulation of the action of the Pauli principle with a soft or hard core. Such an approximation would be inappropriate, because it will not be able to explain the positive sign of the phase shift at low energies.

B. The ^{3}H+^{3}H system

Now let us consider a collision of the two nuclei ^{3}H with opposite spins. As the total isospin of the system is equal to unity, the even and odd number of quanta, obviously, corresponds to the singlet and triplet states, respectively. The eigenvalues of the norm kernel are given by

$$\Lambda^{S=0}_{\nu=2k} = 1 - \left(\frac{1}{3}\right)^{\nu}, \quad \Lambda^{S=1}_{\nu=2k+1} = 1 - \left(\frac{1}{3}\right)^{(\nu-1)}.$$

In contrast to the system $n+\alpha$, the eigenvalues with the even and odd number of quanta are identical and tend to unity from below. The minimal number of quanta allowed by the Pauli principle increased by one compared with the previous case.

Again, as for the system $n+\alpha$ in the states with $\Lambda_{2k}<1$, the scattering phase shift determined only by the antisymmetrized kinetic energy operator corresponds to the repulsion of the clusters at small distances between them. Because of the existence of the sole forbidden state in both channels, the scattering phases at zero energy are counted from π (see Fig. 2). The singlet scattering phase falls faster than the triplet one, because the former has lower value of the angular momentum. For the singlet state and $L=0$ a version with zero-range attractive potential has also been considered, assuming that the interaction can be reproduced by just one diagonal matrix element

$$\langle (2,0)|\hat{U}|(2,0)\rangle = -34.76 \text{ MeV}.$$

This model potential provides the experimental value of the $^6\text{He}\rightarrow{}^3\text{H}+{}^3\text{H}$ decay threshold (12.3 MeV [26]) and a reasonable value of the r.m.s. radius of the ^{6}He nucleus, equal to 2.24 fm. The singlet phase of the elastic scattering for the potential which assures the existence of a bound state is counted from 2π (Fig. 2). In the energy range being considered, this scattering phase is larger than those obtained in the potential-free version.

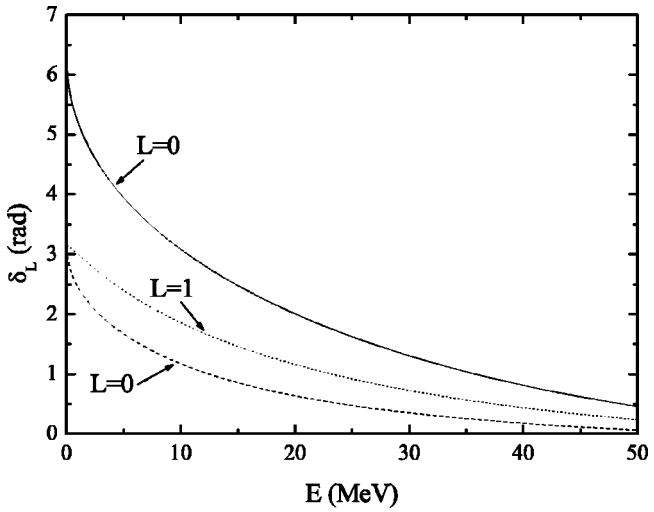

FIG. 2. Phases of the ^{3}H+^{3}H scattering. Solid curve: the phase obtained in the zero-range approximation for nuclear force. Dotted and dashed curves: the phases obtained by granting the kinetic energy exchange potential exclusively.

C. The $\alpha+\alpha$ system

Finally, let us address a well-known example of the scattering of two α particles; in other words, let us consider the ^{8}Be nuclear system in the α-cluster model. The norm kernel for the α-α system only contains basis functions with even number of quanta, because the wave function of two identical bosons must be symmetric with respect to the interchange of the clusters

$$\Lambda_{2k} = 1 - 4\left(\frac{1}{4}\right)^{k}, \quad \Lambda_{2k+1} = 0.$$

$\Lambda_{2k}<1$ for any given number of quanta, and the minimal allowed number of quanta takes the maximum value $\nu_{\min}=4$ among all the cases considered in this section. As a result, the Pauli principle leads to the repulsion of clusters; and the behavior of the scattering phase is similar to the one discussed above for the states with $L=0$.

[5] As is well known, taking into account a spin-orbit interaction leads to the splitting of the state $L^\pi=1^-$ and, as a result, two scattering phases $\delta_{3/2}$ and $\delta_{1/2}$ appear, each showing a resonance behavior. In our case we have only one resonance.

TABLE I. Parameters of the effective interaction related to the antisymmetrization for s-cluster systems. $\Lambda_k - 1 = \beta \exp\{-k \ln \alpha\}$.

			$\nu = 2k$		$\nu = 2k+1$	
	$\nu_{\min}$	$\Lambda_{\nu_{\min}} - 1$	α	β	α	β
$n+\alpha$	1	1/4	16	−1	16	1/4
$^3\mathrm{H}+^3\mathrm{H}$	2	−1/9	9	−1	9	−1
$\alpha+\alpha$	4	−1/4	4	−4	0	0

We would like to note that the phase shifts of elastic α-α scattering have been calculated [28] using the algebraic version of the RGM. In that work, the exchange effects both on the kinetic and potential energy have been taken into account, but the authors focused their attention on a possibility to use the oscillator basis for the solution of the problems in the continuum. A contribution to the phase shifts of the kinetic energy matrix modified by the Pauli principle was not studied there. Now we can state that the high-energy behavior of the phase shifts formed by the kinetic energy operator (with its exchange part) is in perfect agreement with Levinson's theorem. Moreover, the latter phase shifts are of the same order of magnitude as those obtained with the inclusion of the potential energy operator.

D. Comparison of the eigenvalues for different nuclear systems

As has already been discussed in the previous section, for a large number of quanta, the behavior of an effective potential related to the antisymmetrization is determined by the expression (19). In a general case, at small intercluster distances an effective potential has rather a cumbersome form and depends on several exponentially decreasing terms

$$\Lambda_\nu^{(\lambda,\mu)} - 1 = \sum_j \beta_j^{(\lambda,\mu)}(\nu) \exp\{-\nu \ln \alpha_j^{(\lambda,\mu)}\},$$

$$\alpha_j^{(\lambda,\mu)} > 1. \tag{21}$$

However, in the problem of scattering of two s clusters at a given ν the SU(3) irrep $(\nu,0)$ appears to be the sole possible representation; and in the sum (21) only one term survives.

In Table I the parameters of an effective interaction induced by the kinetic energy operator modified by the Pauli principle are presented for a number of binary nuclear systems composed of s clusters. By analyzing the data listed in Table I, some general conclusions about the dependence of such an effective interaction on the number of nucleons in the interacting clusters can be drawn.

The range of the antisymmetrizer can be determined by the minimal number of quanta $\nu_{\min}$ corresponding to the first nonvanishing eigenvalue. It is easy to understand that $\nu_{\min}$ increases with the number of nucleons in the system under consideration, because the number of the occupied states grows. Indeed, in the $n+\alpha$ system the first allowed state appears already at $\nu_{\min}=1$, while in the case of the two α particles it appears only at $\nu_{\min}=4$. On the contrary, the parameter α defining the decrement of the effective potential essentially decreases with the number of nucleons. At the same time, the parameter β increases, which is an evidence of the increasing range of exchange effects. A negative sign of β indicates that an effective interaction is repulsive. It is attractive only for the states with odd number of quanta in the system $^4\mathrm{He}+n$. Note that in the latter case it is exactly the basis states corresponding to the eigenvalues $\Lambda_\nu > 1$ that dominate in the wave function of $^4\mathrm{He}+n$.

Finally, one more important parameter determining the strength of an effective potential related to antisymmetrization is the value of this potential at the minimal number of quanta. As seen from Table I, the strength of this effective interaction is maximum in the states with $\nu=\nu_{\min}$ and increases with the number of nucleons in the compound nucleus.

VI. MULTICHANNEL BINARY CLUSTER SYSTEMS

A. The ^{6}He+^{6}He system

The cluster configuration $^6\mathrm{He}+^6\mathrm{He}$ is a relatively simple multichannel system, which well illustrates the role of the Pauli principle in the coupling between different channels. The ^{6}He clusters have open p shell, and, therefore, a possibility of excitation (to the 2^+ resonance state) of these clusters should be properly taken into account. We did not have this feature in the previous examples. As a consequence, now at a given even value of the total number of quanta $\nu=2k>8$ a basis of the Pauli-allowed states with the total orbital angular momentum $L=0$ belongs to five SU(3) irreps with even symmetry indices (λ,μ): $(2k-2,0)$, $(2k,2)_1$, $(2k-4,4)$, $(2k,2)_2$, $(2k+4,0)$. Notice that the multiplets $(2k,2)_1$ and $(2k,2)_2$ have the same SU(3) symmetry indices, but different eigenvalues $\Lambda_{(2k,2)_1}$ and $\Lambda_{(2k,2)_2}$.

As a result, the norm kernel $I_{^6\mathrm{He}+^6\mathrm{He}}$ for the states with $L=0$ takes the form

$$\begin{aligned} I_{^6\mathrm{He}+^6\mathrm{He}} = & \sum_{k=2}^{\infty} \Lambda_{(2k-2,0)} \psi_{(2k-2,0)} \tilde{\psi}_{(2k-2,0)} \\ & + \sum_{k=3}^{\infty} \Lambda_{(2k,2)_1} \psi_{(2k,2)_1} \tilde{\psi}_{(2k,2)_1} \\ & + \sum_{k=3}^{\infty} \Lambda_{(2k-4,4)} \psi_{(2k-4,4)} \tilde{\psi}_{(2k-4,4)} \\ & + \sum_{k=4}^{\infty} \Lambda_{(2k,2)_2} \psi_{(2k,2)_2} \tilde{\psi}_{(2k,2)_2} \\ & + \sum_{k=5}^{\infty} \Lambda_{(2k+4,0)} \psi_{(2k+4,0)} \tilde{\psi}_{(2k+4,0)}. \end{aligned}$$

These eigenfunctions, along with their eigenvalues, were given in Ref. [17]. Hence, here we will restrict ourselves to only those results from Ref. [17] which are relevant to the problem discussed.

First, we present in Fig. 3 the dependence of the eigenvalues belonging to the five different branches (according to their definition introduced in Sec. II) on k. As seen from this

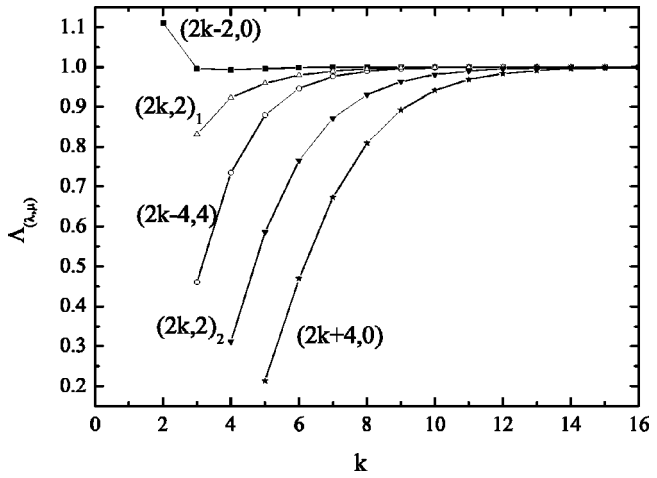

FIG. 3. Eigenvalues $\Lambda_{(\lambda,\mu)}$ of the norm kernel for the system ^{6}He+^{6}He versus the number of quanta k. Curves are labeled by the SU(3) symmetry indices (λ,μ).

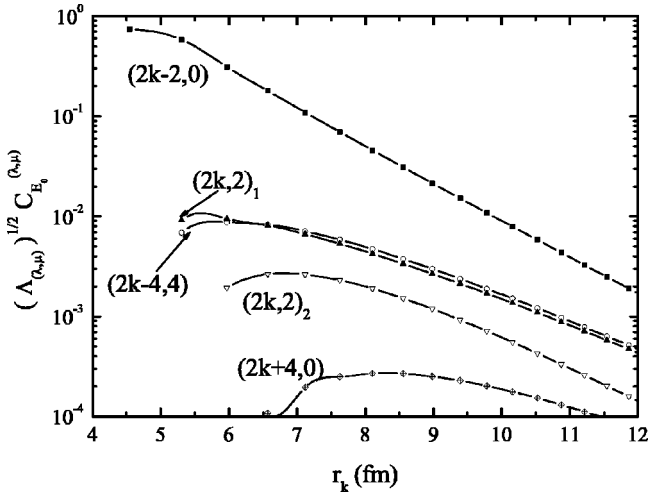

FIG. 4. Ground state of ^{12}Be=^{6}He+^{6}He: coefficients $\sqrt{\Lambda_{(\lambda,\mu)}}C^{(\lambda,\mu)}_{E_0}(r_k)$ of the WF expansion in the SU(3) basis, half-logarithmic scale. Curves are labeled by the SU(3) symmetry indices (λ,μ).

figure, all the eigenvalues except $\Lambda_{(2,0)}$ are less than unity and, therefore, generate a repulsive effective potential. The largest eigenvalues belong to the branch $(2k-2,0)$; while the smallest ones, to the branch $(2k+4,0)$. In the states of the latter branch a cluster repulsion caused by the action of the Pauli principle is maximal, as well as the range of the antisymmetrization effects. The eigenvalues are close to unity only if $k \geqslant 14$. In addition, this branch starts with $k=5$, i.e., later than the others. The repulsion in the states of the branch $(2k,2)_2$, for which the minimal number of k equals 4, is somewhat less intensive; and its eigenvalues can be set to unity, if $k \geqslant 13$. The repulsion for the branches $(2k-4,4)$ and $(2k,2)_1$, which appear at $k=3$, is even less pronounced. The eigenvalues of these branches are rather close to unity, if $k \geqslant 10$. Of interest is the fact that in the absence of degeneracy the higher U(3) symmetry [the larger the eigenvalues of the second-order Casimir operator of U(3) group], the smaller the eigenvalues.

The basis of the Pauli-allowed states for the ^{6}He+^{6}He system corresponds to five different channels. Above some threshold energy ($E=3.6$ MeV) all these channels are open. But there is an energy range in the continuous part of the spectrum where some channels are closed. The Pauli principle manifests itself in making all the five channels coupled at small intercluster distances. The range of this domain is determined by the requirement that at its border all five different eigenvalues of the allowed states are almost equal to unity. As soon as all eigenvalues approach unity, a unitary transformation from the SU(3) basis to the l basis, which allows us to decouple the channels of the latter basis, becomes possible [17]. Below we shall specify at what values of k (and, therefore, r_k) it occurs.

Coupling of the channels via the kinetic energy operator directly results in the appearance of off-diagonal elements of the S matrix and, hence, in inelastic processes in the collision of two ^{6}He nuclei. Certainly, the potential energy of the cluster-cluster interaction can also influence the inelastic scattering cross sections. However, as before, we shall restrict our analysis to the contribution of the kinetic energy exchange potential. A simple potential energy operator used here does not couple different channels.

Information about the magnitude of the repulsion in the states belonging to different branches is provided by the expansion coefficients (Fig. 4) of the ground state wave function of ^{12}Be=^{6}He+^{6}He calculated in the zero-range approximation [17]. We assume that the interaction can be reproduced by just two diagonal matrix elements in the SU(3) representation $(2k-2,0)$, i.e.,

$$\langle (2k-2,0)|U|(2k'-2,0)\rangle = \begin{cases} U_0 = -44.2 \text{ MeV} & \text{if } k=k'=2, \\ U_1 = -28.7 \text{ MeV} & \text{if } k=k'=3, \\ 0 & \text{otherwise.} \end{cases}$$

These values were fitted to the experimental values of the r.m.s. radius of ^{12}Be (2.59 ± 0.06 fm [29]) in its ground state, and the ^{6}He+^{6}He decay threshold energy (10.11 MeV [30]). The oscillator length was fixed to 1.37 fm.

Comparing the coefficients belonging to different branches, we come to the conclusion that the following inequality holds for them as well as for their eigenvalues

$$\Lambda_{(2k-2,0)} \geqslant \Lambda_{(2k,2,)_1} \geqslant \Lambda_{(2k-4,4)} \geqslant \Lambda_{(2k,2,)_2} \geqslant \Lambda_{(2k+4,0)},$$

with a single exception: the coefficients of the branch $(2k-4,4)$ are somewhat larger than the coefficients of the branch $(2k,2)_1$, if $r_k > 6.5$ fm. Let us consider now the wave function in the continuum [Fig. 5(a)]. The energy $E=0.885$ MeV of this state is above the threshold of the ^{12}Be decay into two ^{6}He nuclei in their ground state ($E=0$), but less than the threshold energy ($E=1.8$ MeV) of the decay of ^{12}Be into the channel where one of the clusters is in its ex-

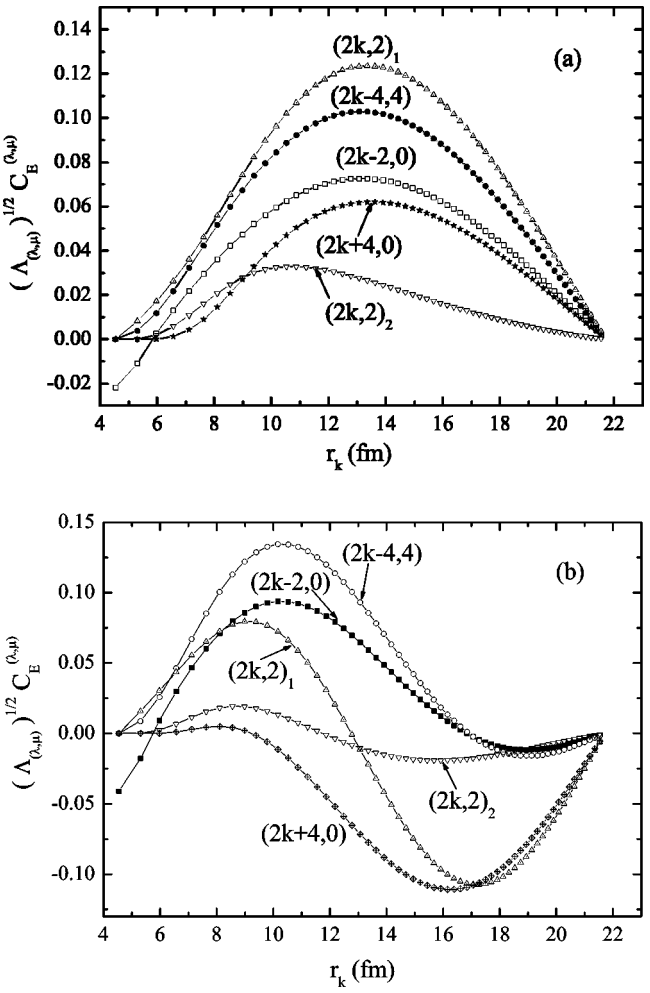

FIG. 5. Coefficients $\sqrt{\Lambda_{(\lambda,\mu)}}C_E^{(\lambda,\mu)}(r_k)$ of the expansion of the continuum states of ^{12}Be=^{6}He+^{6}He in the SU(3) basis at (a) E =0.885 MeV and (b) E=3.3 MeV. Curves are labeled by the SU(3) symmetry indices (λ,μ).

cited 2^+ state.[6] As long as $r_k<6$ fm, the behavior of the coefficients is determined by the magnitude of repulsion in the corresponding SU(3) branches, as in the g.s. wave function. However, if $r_k>6$ fm, then a rearrangement of their values occurs. Namely, the coefficients of the irreps $(2k,2)_1$ take the lead, followed by the irreps $(2k-4,4)$ and $(2k-2,0)$. An hierarchy among the coefficients is established by the weights of corresponding SU(3) irreps in the wave function of the l basis belonging to an open channel. Thus, at large r_k the basis functions of the irreps $(2k,2)_2$ contain only those functions of the l basis which correspond to the closed channels. Therefore, the expansion coefficients belonging to the branch $(2k,2)_2$ fall exponentially with r_k increasing. Finally, consider the state with the energy E=3.3 MeV [Fig. 5(b)]. This state is above the threshold for the decay of ^{12}Be into the two ^{6}He nuclei, with one of them excited. For the expansion coefficients the same behavior is observed as in the previous case, but the coefficients are nonzero only for

[6]Its experimental width is only about 113 keV, so we treat it as a bound state here.

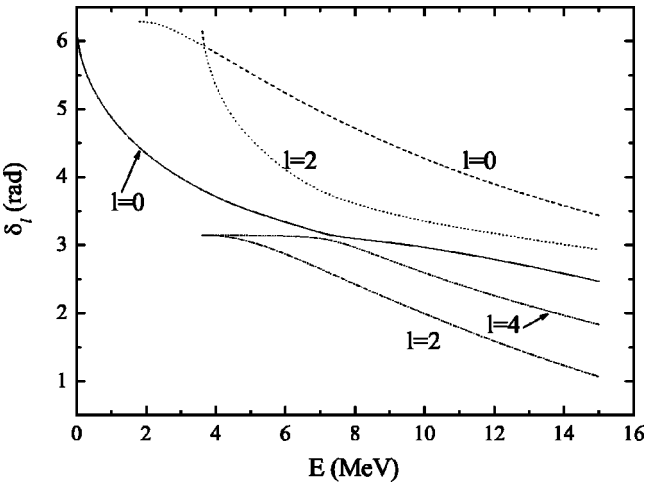

FIG. 6. Eigenphases $\delta_l(E)$ of the ^{6}He+^{6}He system formed by the kinetic energy operator modified by the Pauli principle. The values of the angular momentum l of the cluster relative motion are shown near the curves.

those SU(3) irreps which contain the basis functions of the two open channels.

Important information about the multichannel continuous spectrum of the ^{6}He+^{6}He system is provided by its five (according to the maximal number of the open channels) eigenphases, presented in Fig. 6. Just above the corresponding threshold $E_{\rm thr}$ an eigenphase δ_l obeys the law

$$\delta_l(E) = n\pi + \text{const}\, E^{l+1/2}, \quad n = 0,1, \ldots .$$

Here n is the number of forbidden states in the corresponding channel. In Fig. 6 two quasicrossings of the phase curves are seen. Eigenphases cannot cross, because a crossing would contradict the unicity theorem for a solution of the wave equation. Beyond the crossing point an eigenphase follows the path that its counterpart had before the point. The fall of the eigenphases with energy indicates a repulsion due to the antisymmetrization effects, and is not compatible with the assumption that a resonance exists in the system.

B. ^{4}He+^{8}He and ^{6}He+^{6}He clustering: coupled-channel approach

Along with the ^{6}He+^{6}He clustering of ^{12}Be, let us consider also the ^{4}He+^{8}He cluster structure. The latter allows an excitation of the ^{8}He nucleus to its 2^+ state. Taking into account the ^{4}He+^{8}He clustering makes essential corrections with respect to the results with the ^{6}He+^{6}He clustering.

First of all, the number of the allowed states increases and an additional SU(3) degeneration appears. As a result, the number of the channels with the total angular momentum L=0, total spin S=0, and isospin T=2 increases to 7. The ^{4}He+^{8}He clustering provides for the two additional branches of the basis states having SU(3) symmetry $(2k-2,0)$ and $(2k,2)$. Basis states of different cluster configurations with the same SU(3) symmetry indices are not orthogonal, while the two states (2,0) are identical. The eigenfunctions of the norm kernel for two coupled cluster configurations are the

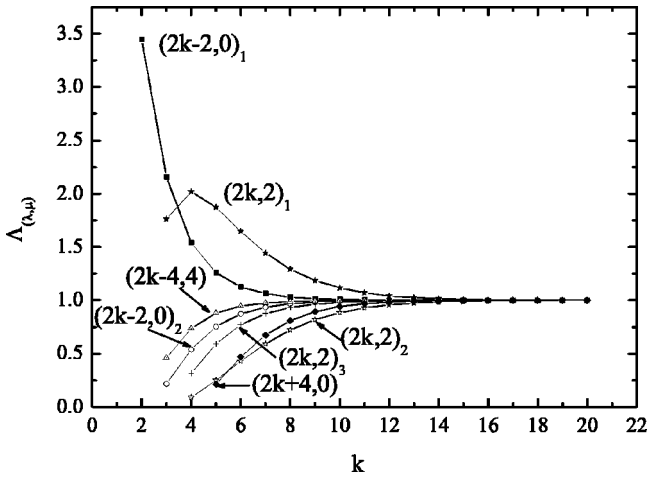

FIG. 7. Eigenvalues $\Lambda_{(\lambda,\mu)}$ of the norm kernel for the ^{12}Be system in the coupled-channel approach (cf. Fig. 3; see text for details).

superpositions of the basis functions of the ^{4}He+^{8}He and ^{6}He+^{6}He channels.

Let us start with the Hilbert-Schmidt expansion of the new norm kernel $I_{(8,4)+(6,6)}$ for the states with $L=0$. Seven different sums are written down in descending order of the eigenvalues $\Lambda_{(\lambda,\mu)}$,

$$I_{(8,4)+(6,6)} = \sum_{k=2}^{\infty} \Lambda_{(2k-2,0)_1} \psi_{(2k-2,0)_1} \tilde{\psi}_{(2k-2,0)_1} + \sum_{k=3}^{\infty} \Lambda_{(2k,2)_1} \times \psi_{(2k,2)_1} \tilde{\psi}_{(2k,2)_1} + \sum_{k=3}^{\infty} \Lambda_{(2k-4,4)} \psi_{(2k-4,4)} \tilde{\psi}_{(2k-4,4)} + \sum_{k=3}^{\infty} \Lambda_{(2k-2,0)_2} \psi_{(2k-2,0)_2} \tilde{\psi}_{(2k-2,0)_2} + \sum_{k=4}^{\infty} \Lambda_{(2k,2)_2} \times \psi_{(2k,2)_2} \tilde{\psi}_{(2k,2)_2} + \sum_{k=4}^{\infty} \Lambda_{(2k,2)_3} \psi_{(2k,2)_3} \tilde{\psi}_{(2k,2)_3} + \sum_{k=5}^{\infty} \Lambda_{(2k+4,0)} \psi_{(2k+4,0)} \tilde{\psi}_{(2k+4,0)}.$$

Now the states $(2k-2,0)$ become twofold degenerate, while the degree of degeneracy for the states $(2k,2)$ is increased to 3. The dependence on the number of quanta of the eigenvalues belonging to the different branches is shown in Fig. 7. Its remarkable feature is that now the eigenvalues of the two branches, $\Lambda_{(2k-2,0)_1}$ and $\Lambda_{(2k,2)_1}$, exceed unity. In the states of these branches an effective potential related to antisymmetrization corresponds to an attraction. This directly affects the structure of the g.s. wave function of the ^{12}Be nucleus causing the expansion coefficients belonging to the SU(3) irreps $(2k-2,0)_1$ to dominate in the latter function. The basis functions of these irreps contain the ^{6}He+^{6}He and the ^{4}He +^{8}He clustering on equal footing.

Now in order to reproduce the threshold energy (8.95 MeV [31]) of the ^{12}Be break up into ^{4}He and ^{8}He in their ground states and the r.m.s. radius of the ^{12}Be nucleus, $r_{\text{r.m.s.}}=2.66$ fm, the magnitude of the parameter U_0 of the model zero-range potential should be reduced to 42.2 MeV, while the parameter U_1 should be set to zero. Such a change of the potential parameters seems to be natural, because the basis of the ^{6}He+^{6}He configuration is supplemented with the basis states of the ^{4}He+^{8}He configuration. Contribution of the latter to the energy of the ground state appears to be rather considerable.

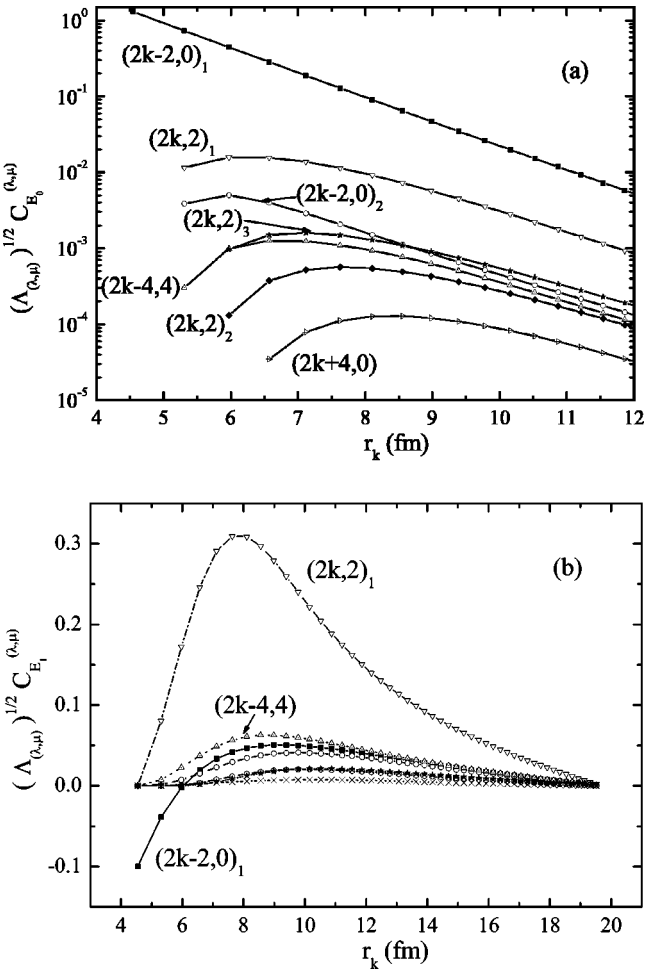

FIG. 8. Coefficients of the expansion of the discrete states of the ^{12}Be nucleus in the SU(3) basis, when the ^{6}He+^{6}He and the ^{8}He +^{4}He coupled cluster configurations are included and a zero-range nuclear force is used. (a) g.s. wave function, $E_0=-8.95$ MeV; (b) excited state, $E_1=-0.386$ MeV. The SU(3) symmetry indices (λ,μ) are shown near the curves.

The coefficients $\sqrt{\Lambda_{(\lambda,\mu)}} C^{(\lambda,\mu)}_{E_0}(r_k)$ are presented in Fig. 8(a). Comparing Fig. 8(a) with Fig. 7, we can conclude that the smaller are the eigenvalues, the smaller are the corresponding coefficients.

The version of the potential chosen provides for an existence of the second bound state at the energy $E_1=-0.386$ MeV. The coefficients $\sqrt{\Lambda_{(\lambda,\mu)}} C^{(\lambda,\mu)}_{E_1}(r_k)$ are presented in Fig. 8(b). The coefficients of the irreps $(2k,2)_1$ appear to be dominating for this state, while the coefficients of the $(2k-2,0)_1$ irreps take the second position, where they compete with the coefficients of the irreps $(2k-4,4)$. The

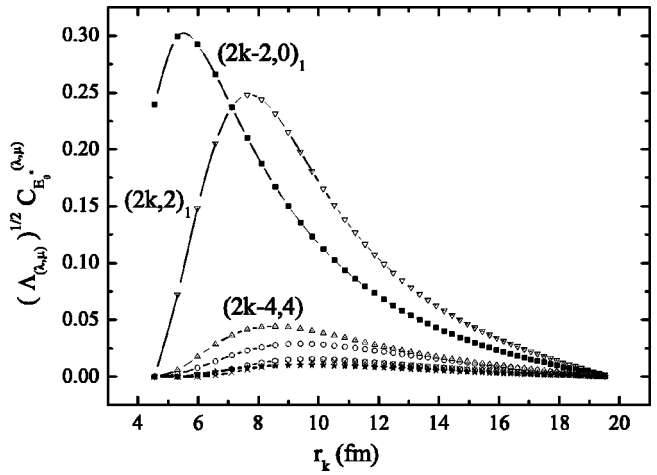

FIG. 9. Coefficients $\sqrt{\Lambda_{(\lambda,\mu)}}C_{E_0^*}^{(\lambda,\mu)}(r_k)$ of the expansion of the g.s. wave function in the SU(3) basis, calculated in the coupled-channel approach and with due regard to the exchange effects in the kinetic energy exclusively. The SU(3) symmetry indices (λ,μ) are shown near the curves.

contribution of the other coefficients again correlates with the magnitude of their eigenvalues. Moreover, now even without any cluster-cluster potential generated by the nucleon-nucleon forces, a bound state of the ^{12}Be nuclear system appears, with an energy $E_0^*=-0.75$ MeV and r.m.s. radius $r_{\rm rms}=4$ fm. In Fig. 9 the expansion coefficients of the wave function for this state are shown. At small r_k, the coefficients that correspond to the irreps $(2k-2,0)_1$ and $(2k,2)_1$ possess the largest values among all the coefficients. The bound state appears due to the attraction, for which the latter two SU(3) branches are responsible. For simplicity, the five threshold energies are assumed to be equal. Now the question of the correct assignment of the seven eigenphases at zero energy remains to be clarified. To answer this question we calculate the total number of the forbidden states of the l basis. The ^{8}He+^{4}He configuration supplies three forbidden states. One of them corresponds to the channel with ^{8}He and ^{4}He clusters being in their ground states, zero angular momentum of their relative motion and zero number of quanta, i.e., $k=0$. The other forbidden state corresponds to the same channel, but with $k=1$. The last forbidden state belongs to the channel with $k=1$ and the ^{8}He nucleus excited. We therefore conclude that there are eleven forbidden states. At zero energy, the eigenphases of the three channels with $l=0$ as well as the eigenphase for one of the three channels with $l=2$ should be set to 3π. The eigenphases for the other channels with $l=2$ and the eigenphase of the channel with $l=4$ begin at 2π. Note that the eigenphases discussed above are calculated with due regard to the antisymmetrization effects in the kinetic energy solely. The energy dependence of the eigenphases is presented in Fig. 10.

Let us recall that, for the ^{6}He+^{6}He clustering, all eigenphases decrease monotonically with energy. For the case of the coupled ^{6}He+^{6}He and ^{8}He+^{4}He configurations, but without a nucleon-nucleon potential [Fig. 10(a)], two of the seven eigenphases with $l=2$ first ascend, reach their maxima within

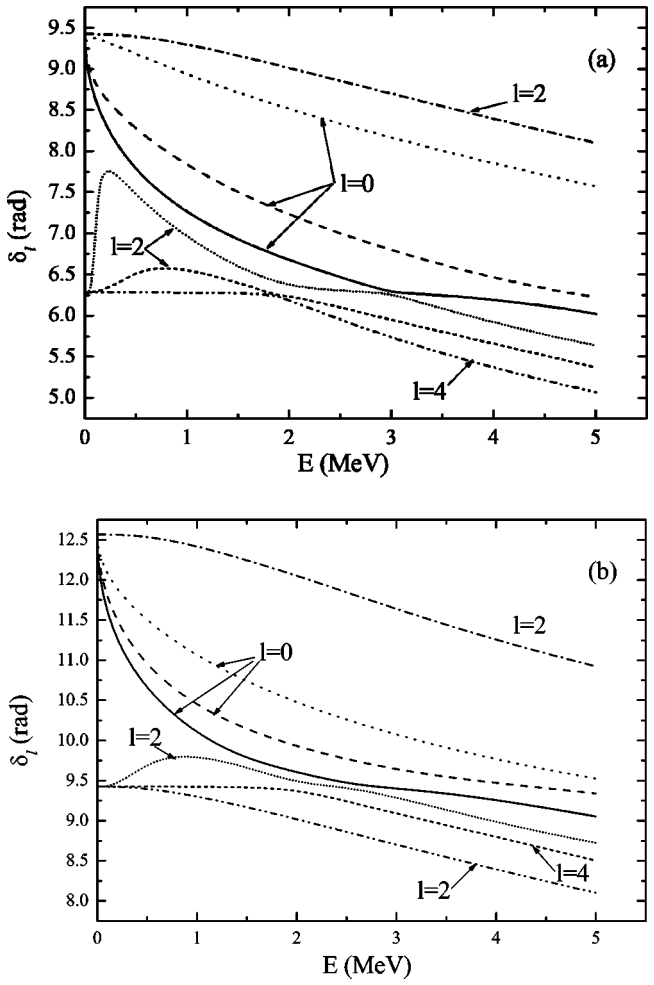

FIG. 10. Eigenphases $\delta_l(E)$ of the ^{12}Be system. (a) The eigenphases obtained by allowing only for the antisymmetrization effects on the kinetic energy. (b) The eigenphases obtained in the zero-range approximation for the nuclear force. The values of the angular momentum l of the cluster relative motion are shown near the curves.

the energy range up to 1 MeV, and only beyond they begin to decrease monotonically. We have already observed such a behavior for the $\alpha+n$ scattering phase with the angular momentum $L=1$. In order to try to link these maxima to possible resonances, let us consider the wave function in the continuum for the energy of 0.22 MeV (see Fig. 11). The chosen energy is close to that at which one of the eigenphases has a pronounced peak. In this wave function the states corresponding to the channels with $l=2$ prevail, and that is compatible with the assumption on the existence of the resonance.

The inclusion of the zero-range potential pulls down the resonance under the break-up threshold of ^{12}Be into ^{8}He +^{4}He [Fig. 10(b)]. As a result, along with the ground state of the ^{12}Be nucleus ($E_0=-8.95$ MeV, an excited state appears at the energy $E_1=-0.386$ MeV. The energy behavior of the eigenphases is otherwise similar to that which has already been discussed for the ^{6}He+^{6}He clustering.

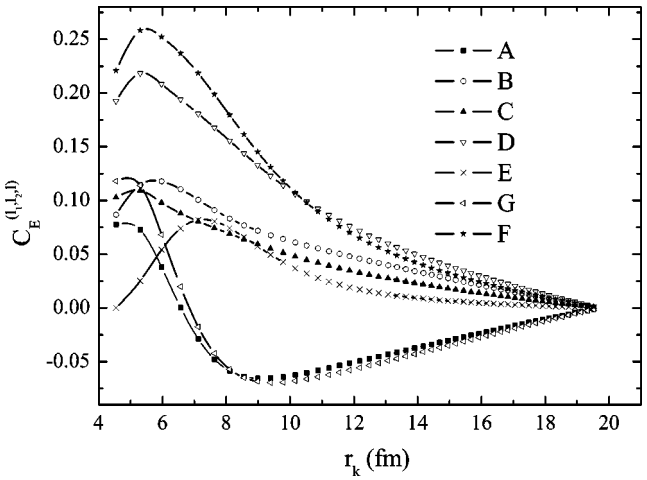

FIG. 11. Coefficients $C_E^{(l_1,l_2,l)}(r_k)$ of the expansion of the continuum states of the ^{12}Be system in the l basis at $E=0.22$ MeV. ^{6}He+^{6}He clustering: line A—$l_1=l_2=l=0$; line B—$l_1=l_2=2$, $l=0$; line C—$l_1=2(0)$, $l_2=0(2)$, $l=2$; line D—$l_1=l_2=l=2$; line E—$l_1=l_2=2$, $l=4$; ^{8}He+^{4}He clustering: line G—$l_1=l_2=l=0$; line F—$l_1=l=2$, $l_2=0$.

VII. CONCLUSIONS

The influence of the Pauli principle on the structure of the continuum states of the compound systems populated at the intermediate stage of collisions between light nuclei was studied within the algebraic version of the resonating group method. The exchange effects on the kinetic energy operator were analyzed by the use of the discrete representation of the complete basis of the Pauli-allowed many-particle harmonic-oscillator states classified with the use of the SU(3) symmetry indices. The eigenvalue problem for the norm kernel was reduced to degenerate integral equations in the Fock-Bargmann space.

The influence of the Pauli exclusion principle on the collision of clusters through the kinetic energy was shown to be reducible to three effects which affect the dynamics of the cluster-cluster interaction. In the innermost region of intercluster distances, the nucleus-nucleus interaction is dominated by the elimination of the Pauli forbidden states that drastically increases the scattering phase-shift variation and may be simulated by a repulsive potential at small intercluster distances. The larger is the number of the forbidden states, the larger should be the strength and the range of such a model potential. Outside of the latter region, the cluster-cluster interaction derived from the kinetic energy (with its exchange part) is shown to be repulsive or attractive, depending on whether the eigenvalues of the antisymmetrization operator approach unity from below or from above. Such an effective interaction can significantly affect the scattering phase behavior. Finally, a decrease or an increase of the centrifugal potential occurs in the same region. It also influences the phase shift, especially at high energies. The range of the influence of the Pauli principle on the kinetic energy appears to be significantly larger than that of the cluster-cluster interaction generated by the nucleon-nucleon potential, especially for heavier clusters or clusters with an open shell.

If there are several open channels, the exchange effects arising from the kinetic energy influence the cross sections of inelastic scattering channels belonging to the excitation of the clusters or to their rearrangement. The eigenphases of the multichannel systems that define the nature of inelastic collisions have been calculated and quasiintersection of the eigenphases was established.

A considerable intensification of the Pauli effects is observed for coupled cluster configurations, such as ^{6}He+^{6}He and ^{4}He+^{8}He which are relevant to the ^{12}Be compound nucleus. This phenomenon relates to the appearance of new branches of excitation with the especially large eigenvalues, greater than unity, belonging to the allowed states. As a result, an effective attraction arising from the kinetic energy operator modified by the Pauli principle appears to be strong enough to ensure the existence of both a bound state and a resonance even without an interaction between nucleons of different clusters.

ACKNOWLEDGMENTS

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