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Proton emission from triaxial nuclei

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Proton decay from triaxially deformed nuclei is investigated. The deformation parameters corresponding to the mother nucleus are determined microscopically and the calculated decay widths are used to probe the mean-field wave function. The proton wave function in the mother nucleus is described as a resonant state in a coupled-channel formalism. The decay width, as well as the angular distribution of the decaying particle, are evaluated and their dependence upon the triaxial deformation parameters is studied in the decay of ^{161}Re and ^{185}Bi. It is found that the decay width is very sensitive to the parameters defining the triaxial deformation while the angular distribution is a universal function which does not depend upon details of the nuclear structure.

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The investigation of nuclei at the proton drip line is a very active field of present nuclear physics [1–3]. One important facet of these investigations is the possibility it offers to probe nuclear mean fields in proton rich nuclei. Many spherical as well as deformed nuclei in the region $50<Z<82$ are proton emitters. Most of the measured transitions connect ground states [4–9], but in the last years also decays to excited states have been detected [10,11].

The study of proton emission has so far been performed mainly for decays from odd-even nuclei. The emitted proton has been described by using two very different approaches which, nevertheless, provide similar results if the decay width is very small. In one of these approaches one describes the wave function corresponding to the decaying proton as an outgoing solution of the Schrödinger equation. The proton energy thus obtained is complex, corresponding to the resonant pole of the S matrix in the complex energy plane [12–14]. In the other approach one uses a coupled-channel formalism on the real-energy axis, i.e., by means of real scattering states [15–19]. Since decay widths which can be measured correspond to narrow resonances (living a relatively long time) stationarity is a very good approximation and the decay width can be determined in a standard way by evaluating the outgoing current at large distances. A critical comparison between the two approaches was performed in Ref. [20]. Recently, also odd-odd nuclei have been measured [21,22] and theoretically analyzed [23,24]. However, in all these cases it has been assumed that the mother and the daughter nuclei are spherical or, if deformed, have cylindrical symmetry.

During our work on proton decay from triaxially deformed nuclei, a study with the same topic appeared by Davids and Esbensen [25]. Their work presents a formalism for proton decay from triaxial nuclei, with particular emphasis in the intrinsic (K) and laboratory (R) representations. However, the case of ^{141}Ho studied in their work did not allow for sizable effects induced by triaxiality.

In a very recent paper [26] the influence of the γ vibrations upon the proton decay rate was also analyzed.

The aim of this paper is to investigate the influence of triaxial deformation on the decay width as well as on the angular distribution of the emitted protons. We will show that triaxial deformation is crucial to understand certain cases of proton decay. Since the existence of triaxial deformation is one of the long-standing issues in nuclear structure, the study of proton decay may evolve as an important tool to analyze and determine those deformations. The formalism to be used, based on the expansion of the wave function corresponding to the triaxial field in spherical waves, will be presented very briefly. Details are well known and given even in text books, e.g., in Ref. [27]; see also the recent work in Ref. [25].

Special emphasis will be given to the features which are specific to the description of the resonance in the mother nucleus. An important ingredient in this type of calculations is, besides the triaxiality of the mean field, the microscopic description of the decay process, including the shape of the mean field itself. We will achieve this by finding the minima of the potential-energy surface (minimal energy), including both deformations and pairing interactions [28].

Let us then consider the proton emission from a mother nucleus which is odd in protons. The Schrödinger equation describing the motion of the emitted proton in the deformed field of a rotating nucleus is given by

$$\mathbf{H}\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) \equiv \left[-\frac{\hbar^2}{2\mu}\Delta_r + \mathbf{T}(\omega) + V(\mathbf{r},\mathbf{s})\right]\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) = E_p\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}), \quad (1)$$

where μ denotes the reduced mass, E_p is the energy of the emitted proton, $\mathbf{T}$ describes the core rotation, and V is the nuclear plus Coulomb potential. Here ω are the Euler angles. Thus, we suppose that the rotational and single-particle motions are decoupled.

In the intrinsic representation (with coordinate $\mathbf{r}'$) the wave function is given by [29]

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$$\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r}',\mathbf{s}) = \frac{1}{\sqrt{2}}\sum_{K_i} A(\alpha,J_i,K_i)[\mathcal{D}^{J_i*}_{M_i,K_i}(\omega)\Phi^{(\alpha\rho=1)}(\mathbf{r}',\mathbf{s}) - (-)^{J_i+K_i}\mathcal{D}^{J_i*}_{M_i,-K_i}(\omega)\Phi^{(\alpha\rho=-1)}(\mathbf{r}',\mathbf{s})]. \quad (2)$$

Here $\mathcal{D}^{J_i*}_{M_i,K_i}$ denotes the normalized Wigner function and $A(\alpha,J_i,K_i)$ is the mixing coefficient, labeled by the state number α of the triaxial core Hamiltonian, i.e.,

$$\mathbf{T}(\omega)\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) \equiv \sum_{k=1}^{3}\frac{\hat{J}_k^2}{2\mathcal{I}_k}\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) = \epsilon_\alpha\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}), \quad (3)$$

where $\mathcal{I}_k$ denote the moments of inertia in the intrinsic nuclear system. The corresponding eigenvalue linear system of equations can be easily derived by using standard matrix elements of the angular momentum operators. Using the notation $\rho=\pm 1$ for the eigenvalues $\pm i$ of the symmetry operator $R_3(\pi)$ and expanding the intrinsic wave function in spherical waves one obtains

$$\Phi^{(\alpha\rho)}(\mathbf{r}',\mathbf{s}) = \sum_{ljK}\frac{g^{(\alpha\rho)}_{ljK}(r)}{r}\mathcal{Y}^{[l(1/2)]}_{jK}(\hat{r}',\mathbf{s}),$$
$$\mathcal{Y}^{(l1/2)}_{jK}(\hat{r}',\mathbf{s}) \equiv [i^l Y_l(\hat{r}') \otimes \chi_{1/2}(\mathbf{s})]_{jK}. \quad (4)$$

In order to determine the decay width we have to evaluate the asymptotic behavior of the outgoing proton wave function in the laboratory system of coordinates, where the proton is assumed to move with spin j which couples to the spin J_f of some rotational band of the even-even core, i.e., the daughter nucleus. The total spin of the system has to be conserved, i.e., it is the spin J_i of the mother nucleus. By changing from the intrinsic to laboratory system of coordinates one obtains the total wave function from the outgoing channel viewpoint, i.e.,

$$\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) = \frac{1}{\sqrt{2}}\sum_{J_f}\sum_{K_iKK_f}\sum_{lj}\overline{A}(\alpha,;J_i,j,J_f;K_i,K,K_f) \times\Bigg\{[\mathcal{D}^{J_f*}_{K_f}(\omega)\otimes\mathcal{Y}^{[l(1/2)]}_{j}(\hat{r},\mathbf{s})]_{J_iM_i}\frac{g^{(\alpha\rho=1)}_{ljK}(r)}{r} + (-)^{J_f+K-K_i}[\mathcal{D}^{J_f*}_{-K_f}(\omega)\otimes\mathcal{Y}^{[l(1/2)]}_{j} \times(\hat{r},\mathbf{s})]_{J_iM_i}\frac{g^{(\alpha\rho=-1)}_{ljK}(r)}{r}\Bigg\}, \quad (5)$$

where we introduced the following shorthand notation,

$$\overline{A}(\alpha;J_i,j,J_f;K_i,K,K_f) \equiv (-)^{J_i-J_f-K}A(\alpha,J_i,K_i) \times\langle J_i,K_i;j,-K|J_f,K_f\rangle. \quad (6)$$

Here we used a formal summation over three projections K_i, K, K_f, with the condition $K_f=K_i-K$. In order to compute the decay width we rewrite the above wave function (5) by using the following ansatz:

$$\Psi^{(\alpha)}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) = \sum_{J_flj}[\Psi^{(\alpha')}_{J_f}(\omega)\otimes\mathcal{Y}^{[(l1/2)]}_{j}(\hat{r},\mathbf{s})]_{J_iM_i}\frac{f^{(\alpha J_i)}_{\alpha' J_f lj}(r)}{r}, \quad (7)$$

where by $\Psi^{(\alpha')}_{J_fM_f}$ we denoted the wave function of the even-even core, which is similar to (2), but of course without the proton wave function $\Phi^{(\alpha\rho)}$ [29]. The identification of Eqs. (5) and (7), together with the orthonormality of the coefficients $A(\alpha',J_f,K_f)$, entering the wave function of the even-even core, leads to the following relation for the radial components in the laboratory system:

$$f^{(\alpha J_i)}_{\alpha' J_f lj}(r) = \sum_{K_iKK_f}A(\alpha',J_f,K_f)\overline{A}(\alpha;J_i,j,J_f;K_i,K,K_f)g^{(\alpha\rho=1)}_{ljK}(r). \quad (8)$$

We will investigate transitions to the ground state, with $J_f=0$, and therefore the coefficient $A(\alpha',J_f,K_f)$ will be unity. Moreover, the angular momentum of the emitted proton is given by the initial spin $J_i=j$ and the summation in (5) is restricted to equal intrinsic projections $K_i-K=K_f=0$. We will consider that ρ is a good quantum number and consequently (5) contains only one term.

By using the expression of the wave function in the laboratory system given by Eqs. (5) and (7) one obtains in a standard fashion the coupled system of differential equations describing the radial motion of the proton in the field induced by the core in the intrinsic system. We want to stress that in our calculations the spin-orbit potential is deformed and, therefore, our system of equations contains also first-order derivatives of the wave function [19]. In order to integrate this system of equations in the intrinsic system for positive energies we calculate a set of N linear independent vector functions $\{\mathcal{R}\}$ which are regular at the origin, and another set $\{\mathcal{H}\}$ which is outgoing at large distance, i.e.,

$$\mathcal{R}_{ljK;l'j'K'}(r)\to_{r\to 0}\delta_{ll'}\delta_{jj'}\delta_{KK'}r^{l+1},$$
$$\mathcal{H}^{(+)}_{ljK;l'j'K'}(r)\to_{r\to\infty}\delta_{ll'}\delta_{jj'}\delta_{KK'}H^{(+)}_l(kr) = \delta_{ll'}\delta_{jj'}\delta_{K'K}[G_l(kr)+iF_l(kr)], \quad (9)$$

where $F_l(kr)$ and $G_l(kr)$ are the regular and irregular Coulomb functions.

The internal and external solutions can be written as

$$g^{(int)}_{ljK}(r) = \sum_{l'j'K'}\mathcal{R}_{ljK;l'j'K'}(r)B_{l'j'K'},$$
$$g^{(ext)}_{ljK}(r) = \sum_{l'j'K'}\mathcal{H}^{(+)}_{ljK;l'j'K'}(r)C_{l'j'K'}\to_{r\to\infty}C_{ljK}H^{(+)}_l(kr), \quad (10)$$

where for simplicity we dropped the upper indices α, ρ. We determine the matching constants as usual, i.e., by using the continuity of the functions and their derivatives at some radius r_0. In order to find a nontrivial solution one obtains the secular equation

$$det\begin{bmatrix} \mathcal{R}(r_0) & \mathcal{H}^{(+)}(r_0) \\ \frac{d}{dr}\mathcal{R}(r_0) & \frac{d}{dr}\mathcal{H}^{(+)}(r_0) \end{bmatrix} = 0, \tag{11}$$

which may have real (bound) as well as complex (resonance) solutions. These correspond to the poles of the S matrix in the complex energy plane. These solutions are the energies of the deformed Gamow states. However, if the Coulomb barrier is high, as it should be for the resonance to be measurable (a mean life of a picosecond corresponds to a width of 6.6×10^{-10} MeV), the regular Coulomb functions are negligible and the solutions are virtually real functions.

Notice that the coefficients B and C in Eq. (10) are fully determined by using the normalization of the wave function in the internal region. Moreover, since the proton state corresponds to a very narrow resonance, one can evaluate the internal wave function by a diagonalization procedure using a harmonic-oscillator basis. The matching constant of the external part can be found by inverting (10), i.e.,

$$C_{ljK} = \sum_{l'j'K'} [\mathcal{H}^{(+)}_{ljK;l'j'K'}(r_0)]^{-1} g^{(int)}_{l'j'K'}(r_0). \tag{12}$$

The "exact" results, i.e., those corresponding to the S-matrix poles (Gamow functions), are close to the approximated ones when, as in this case, the resonances are narrow. This well-known feature is shown, e.g., in Ref. [30].

For bound states the outgoing solution is replaced in (9) by an exponentialy decreasing state. We label the solutions of Eq. (11) (bound states and narrow resonances) by the eigenvalue index n.

For large distances the radial function $f(r)$ in Eq. (7) defines the usual scattering amplitude. Due to the orthonormality of the angular functions the decay width in the $J_f=0$ channel is given by

$$\Gamma_{0nlj} = \hbar v u_n^2 \lim_{r\to\infty} \int \Psi^{\dagger}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s})\Psi_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) r^2 d\hat{r} d\omega$$
$$= \hbar v u_n^2 \lim_{r\to\infty} \left| \sum_{K_i} f_{0nljK_i-K_i}(r)\right|^2 = \hbar v u_n^2 |\overline{f_{0nlj}}|^2, \tag{13}$$

where we considered the scalar product over the spin variable and we omitted the upper indices α, J_i in the radial wave function. Here we assumed that the transition proceeds to the final ground state $J_f=0$. The number u_n is the BCS amplitude for the deformed level n and

$$\overline{f_{0nlj}} = \sum_{K_i} (-)^{j-K_i} A(j,K_i,)\langle j,K_i;j,-K_i|0,0\rangle C_{nljK_i}. \tag{14}$$

Notice that in the above relation one has a coherent summation over intrinsic projections K_i because all these terms have a common angular function with $K_i - K=0$ in (5).

To evaluate the angular distribution of the decaying proton one has to integrate over the rotational variables ω for each direction of decay $\hat{r}$, i.e.,

$$\Gamma_n(\hat{r}) = \hbar v u_n^2 \lim_{r\to\infty} \int \Psi^{\dagger}_{J_iM_i}(\omega,\mathbf{r},\mathbf{s})\Psi_{J_iM_i}(\omega,\mathbf{r},\mathbf{s}) r^2 d\omega$$
$$= \frac{\Gamma_n}{4\pi} W(\theta),$$
$$W(\theta) = 1 + \sum_{L\geq 1} a_L P_L(\cos\theta). \tag{15}$$

For transitions to the ground state, with $J_f=0$, $(J_i=j)$ the angular momentum of the emitted proton is given by the initial spin $J_i=j$ and the coefficient a_L has a simple expression, namely,

$$a_L = (-)^{M_i+1/2}(2J_i+1)\langle J_i,-M_i;J_i,M_i|L,0\rangle$$
$$\times\left\langle J_i,\tfrac{1}{2};J_i,-\tfrac{1}{2}|L,0\right\rangle. \tag{16}$$

This relation is simpler than the one corresponding to the α-particle angular distribution for transitions in odd-mass nuclei [31]. It does not depend upon nuclear structure details; in particular, it is independent of the deformation parameters. The main reason for this is that in the case of favored α decay the spin J_i of the odd nucleon is not changed during the decay process, i.e., $J_i=J_f$. Therefore, the angular momentum of the emitted particle takes several values, namely $|J_i - J_f| \leq l \leq J_i + J_f$. In our case the odd proton is emitted with the same spin as the initial state $j=J_i$ and the coefficients a_L do not depend upon the details of the nuclear structure, given by the amplitudes $\overline{f_{J_fnlj}}$. The angular distribution in this case provides only information about the initial distribution of the spin projection M_i. A similar conclusion was reached for axially deformed nuclei in Ref. [32].

We applied the formalism described above to analyze the decay of the proton emitters ^{161}Re, with a Q value of $E_p = 1192(2)$ keV and a half-life $T_p=0.37(4)$ ms [33], and ^{185}Bi with $E_p=1585(9)$ keV and $T_p=44(16)$ μs [34]. Thus, the energy of the decaying resonance has been measured but the corresponding spin J_i is not experimentally known in this nuclei yet.

The triaxially deformed mean field corresponding to the even-even core was chosen to be of a Woods-Saxon-type with the radius given by

$$R(\theta,\phi) = R_0\left[1 + \beta_2 \cos\gamma Y_{20}(\theta) + \frac{\beta_2}{\sqrt{2}}\sin\gamma(Y_{22}(\theta,\phi) + Y_{2-2}(\theta,\phi))\right]. \tag{17}$$

Our potential-energy surface calculations reveal the interesting feature that the ground state of ^{161}Re has negative parity and the rather weak quadrupole deformation given by $\beta_2=0.110$, $\gamma=0.7°$. Instead, the first excited state has positive parity and a strong quadrupole deformation, i.e., $\beta_2 = 0.100$, $\gamma=-26.3°$. We thus deal with a case of strong polarizing effect of the single particle upon the core, where the negative-parity ground state is axially symmetric and the excited state of positive parity shows strong triaxial deformation. A similar case emerges in ^{185}Bi, where the first excited state has positive parity and a strong triaxial shape with $\beta_2 = 0.156$, $\gamma=-22.1°$, whereas the ground state with negative

TABLE I. Ratio T/T_{exp} between the calculated and experimental half-lives corresponding to proton decay from the Fermi level in ^{161}Re. The deformation parameter γ is given in degrees while β_2 =0.1, 0.2, and 0.3 for the cases (a), (b), and (c), respectively. The quantity b^2_{nlj} is the wave-function amplitude squared corresponding to the single-particle angular momentum (l,j) in our basis.

β_2	γ	lj	b^2_{nlj}	T/T_{exp}
		(a)		
0.100	0.0	$h_{9/2}$	0.003	1.9×10^8
		$h_{11/2}$	0.994	3.4×10^5
0.100	−26.3	$f_{7/2}$	0.016	1.1×10^5
		$h_{9/2}$	0.002	1.7×10^8
		$h_{11/2}$	0.979	3.2×10^5
		(b)		
0.200	0.0	$h_{9/2}$	0.010	5.5×10^7
		$h_{11/2}$	0.974	3.2×10^5
0.200	−26.3	$p_{3/2}$	0.001	4.6×10^6
		$f_{5/2}$	0.001	7.4×10^6
		$f_{7/2}$	0.049	3.7×10^4
		$h_{9/2}$	0.001	4.9×10^7
		$h_{11/2}$	0.922	4.0×10^5
		(c)		
0.300	0.0	$h_{9/2}$	0.019	3.2×10^7
		$h_{11/2}$	0.941	3.3×10^5
0.300	−26.3	$s_{1/2}$	0.048	1.8×10^2
		$d_{3/2}$	0.288	2.2×10^2
		$d_{5/2}$	0.172	1.4×10^3
		$g_{7/2}$	0.202	1.1×10^4
		$d_{9/2}$	0.028	5.0×10^8
		$i_{11/2}$	0.012	5.1×10^{10}

parity is close to axial symmetry, having a quadrupole deformation of β_2=0.100, γ=−7°. For these deformations we diagonalized the triaxial rotator in order to compute the $A(j,K_i)$ coefficients entering (14). It turns out that the major component is given by the maximal intrinsic angular projection for all values of the total spin $j=J_i$.

We reproduced the energy of the proton resonant state by adjusting the depth of the central Woods-Saxon potential while the other parameters which are not determined by our minimal-energy procedure correspond to the universal set of parameters [35].

We will now present the calculation of the half-lives corresponding to proton emission from ^{161}Re.

One would (naively) expect that the most likely state in which the proton would move while decaying is the one corresponding to the largest component of the wave function. However, our results, shown in Table I contradict strongly this expectation. One sees that, in general, the largest wave-function component $b^2_{nlj}=\Sigma_K B^2_{nljK}$, with B_{nljK} given by (10), does not correspond to the lowest half-life. Instead, there is a subtle interplay between the centrifugal barrier and the amplitude of the considered component. For β_2=0.1 and 0.2 one obtains a strong dependence upon the angular momentum but the calculated values of T are still too large. For β_2=0.3, γ=−26.3° the $s_{1/2}$ and $d_{3/2}$ components become lowest in energy and provide comparable half-lives, but still overestimating the experimental value by one order of magnitude.

TABLE II. Ratio T/T_{exp} between the calculated and experimental half-lives for proton decay from the lowest excited states in ^{161}Re. Only the calculated values corresponding to the configuration $s_{1/2}$, with amplitude squared $b^2_{ns_{1/2}}$, are given. The energy of the state is ε_n (in MeV). The deformation parameter γ is given in degrees.

ϵ_n	β_2	γ	$b^2_{ns_{1/2}}$	T/T_{exp}
0.916	0.100	0.0	0.670	39.7
0.491	0.100	−26.3	0.327	4.6
1.487	0.200	0.0	0.621	39.0
1.042	0.200	−26.3	0.232	7.0
2.197	0.300	0.0	0.583	40.2
2.051	0.300	−26.3	0.156	15.4

Since the negative-parity ground state by no means can account for the decay width, we consider the first excited state having positive parity. Its structure is dominated by the components $s_{1/2}$ and $d_{3/2}$. Notice that the largest component, given in Table II is always $s_{1/2}$ for all considered cases.

Again, we adjust the energy E_n of the states to the experimental value corresponding to the decaying resonance. This is now an excited state with excitation energy $\epsilon_n=E_n-E_F$, where E_F is the Fermi energy provided by our calculation.

The remarkable feature in Table II is that one finds that the best value for T is obtained by using the deformation parameters corresponding to the minimal energy. In addition, this is the first excited state in the mother nucleus. One also sees that between the state corresponding to the triaxial case and the axial one there is a difference of one order of magnitude. One thus finds that proton decay indeed is a powerful tool to probe deformations in nuclei.

For the proton decay from ^{185}Bi we proceeded as before, i.e., we first evaluated the half-lives among the (lj) components of the deformed proton wave function at the Fermi level having negative parity. We found, again, that the centrifugal barrier hinders the decay too much and, therefore, the experimental value is best reproduced by the state with the lowest angular momentum. This can be seen in Table III, where we only show the case of β_2=0.156, because the general tendency of T as a function of β_2 is as before.

One sees that there is not any (l,j) combination for which the calculated half-life from the ground state agrees with experiment. We therefore tried to evaluate T starting from the lowest excited states. The results are shown in Table IV. Once again we obtained the best agreement for the case of $s_{1/2}$ corresponding to the first excited state and for the deformation parameters predicted by the minimal energy.

In summary, we have analyzed in this paper the influence of triaxiality upon the half-life corresponding to proton decay. For this we solved the Schrödinger equation by using a coupled-channel approach. Our analysis showed that the angular distribution corresponding to transitions to the ground state is not sensitive to nuclear structure details, a feature which is at variance with α decay from odd-mass nuclei, where the anisotropy is proportional to the quadrupole deformation.

TABLE III. The same as in Table I, but for ^{185}Bi and β_2 =0.156.

γ	lj	b^2_{nlj}	T/T_{exp}
0.0	$p_{1/2}$	0.034	1.5×10^2
	$p_{3/2}$	0.047	1.1×10^4
	$f_{5/2}$	0.202	1.0×10^5
	$f_{7/2}$	0.058	2.0×10^5
	$h_{9/2}$	0.616	1.2×10^9
−22.1	$p_{1/2}$	0.015	3.4×10^2
	$p_{3/2}$	0.015	3.5×10^4
	$f_{5/2}$	0.134	3.8×10^4
	$f_{7/2}$	0.028	3.2×10^5
	$h_{9/2}$	0.661	2.3×10^7

TABLE IV. The same as in Table II, but for ^{185}Bi and β_2 =0.156.

γ	ϵ_n	lj	b^2_{nlj}	T/T_{exp}
0.0	1.028	$s_{1/2}$	0.677	11.4
		$d_{3/2}$	0.226	1.0×10^4
		$d_{5/2}$	0.075	7.9×10^4
		$g_{7/2}$	0.020	9.8×10^5
		$g_{9/2}$	0.002	1.7×10^9
−22.1	0.020	$s_{1/2}$	0.374	3.8
		$d_{3/2}$	0.158	7.1
		$d_{5/2}$	0.046	1.3×10^2
		$g_{7/2}$	0.095	3.5×10^3
		$g_{9/2}$	0.004	2.3×10^5

We calculated the equilibrium deformation parameters by the procedure of minimal energy [28] in the proton emitter cases of ^{161}Re and ^{185}Bi. We found large triaxial deformations in these nuclei, depending on the particular structure of the odd particle. We adjusted the depth of the nuclear interaction to obtain the experimental proton energy. It turns out that the decay width is very sensitive to the triaxial deformation. This becomes especially clear for the $s_{1/2}$ state. But the most important feature of our calculation is that the deformation predicted by the minimal energy provides a half-life that reproduces well the corresponding experimental value assuming that the ground state of the mother nucleus is a resonance $s_{1/2}$. This is an impressive achievement of the model, particularly considering that the width can vary by many orders of magnitude by changing the deformation parameters as well as the configurations involved in the decay, as seen in the tables.

The high sensitivity of the calculated half-life upon angular momenta and deformation allows one to assert that proton decay is a powerful tool to determine spin, as well as to uncover triaxial shapes in nuclei.

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