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Low-momentum interaction in few-nucleon systems

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The low-momentum nucleon-nucleon interaction $V_{\text{low }k}$ is applied to three- and four-nucleon systems. We investigate the ^{3}H, ^{3}He, and ^{4}He binding energies for a wide range of the momentum cutoffs. By construction, all low-energy two-body observables are cutoff independent, and therefore, any cutoff dependence is due to missing three-body or higher-body forces. We argue that for reasonable cutoffs $V_{\text{low }k}$ is similar to high-order interactions derived from chiral effective field theory. This motivates augmenting $V_{\text{low }k}$ by corresponding three-nucleon forces. The set of low-momentum two- and three-nucleon forces can be used in calculations of nuclear structure and reactions. We find that three-nucleon force contributions are perturbative for small cutoffs.

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Microscopic nuclear many-body calculations are complicated by the short-distance repulsion in nuclear forces, which leads to strong high-momentum components in nuclear wave functions. Usually, one solves this problem by introducing an effective interaction, the Brueckner G matrix, which resums in-medium particle-particle scattering. The G matrix is a soft interaction, which is both energy and nucleus dependent and typically requires approximations in practice.

An alternative strategy to construct a soft interaction by integrating out the high-momentum components in free space has been formulated in [1]. Using a renormalization-group (RG) approach, phenomenological two-body potential models can be evolved to an effective low-momentum interaction, called $V_{\text{low }k}$, which is energy independent, Hermitian, and preserves the on-shell T matrix below a cutoff Λ in momentum space as well as the deuteron binding energy. For $\Lambda \lesssim 2$ fm^{-1}, the matrix elements of $V_{\text{low }k}$ are practically independent of the potential model it is derived from and thus unifies all nuclear forces used in microscopic nuclear structure calculations [2]. By construction, $V_{\text{low }k}$ is much softer than the modern potential models, and thus can be used directly for microscopic nuclear calculations in different mass regions [3,4] or for different densities [5,6]. This is clearly important to theoretically extrapolate to the drip lines.

Over the last few years, there has also been an immense progress in our understanding of nuclear interactions from chiral effective field theory (EFT). The spontaneous breaking of the approximate chiral symmetry leads to the appearance of light Goldstone bosons, the pions. Their masses are well below any other hadronic excitation and they drive the long-range nuclear interaction. Due to their derivative coupling one can formulate a power counting that restricts the diagrams contributing to the nuclear interaction at low energy. This approach qualitatively explains the hierarchy of two-nucleon (2N), three-nucleon (3N), and higher-body forces [7], which is observed using phenomenological models. On a quantitative level, it was shown that the resulting 2N and consistent higher-body interactions lead to a quite good description of 2N as well as 3N observables [8–12]. In the pion-full EFT approach, the Lippmann-Schwinger equation is regularized by imposing a cutoff $\Lambda \approx 2.5-3.0$ fm^{-1}. Thus, the chiral potentials are also low-momentum interactions, and with the universal property of $V_{\text{low }k}$, this suggests that $V_{\text{low }k}$ effectively parametrizes higher-order chiral 2N interactions. While EFT offers the only known systematic approach to consistent 2N and higher-body forces, $V_{\text{low }k}$ can be evolved to arbitrary cutoffs with cutoff-independent 2N observables.

Since $V_{\text{low }k}$ is constructed within the 2N system, one neglects many-body forces due to degrees of freedom missing in the effective theory (contributions from the Δ) as well as due to the truncation to low momenta (contributions from high-momentum nucleons). In any effective theory, these effects are inseparable. In this Communication, we use cutoff dependence as a tool to assess the effects of many-body forces. Motivated by the similarities between $V_{\text{low }k}$ and chiral low-momentum interactions, we combine $V_{\text{low }k}$ with the leading chiral 3N force to absorb the cutoff dependence in $A \leqslant 4$ binding energies. Finally, we examine the expectation values of the various force components to check that the hierarchy of nuclear two- and three-body forces is maintained.

We first calculate 3N and 4N binding energies by solving the Faddeev-Yakubovsky equations with only the two-body $V_{\text{low }k}$. We include electromagnetic and isospin-breaking effects and vary the cutoff over a wide range. Our results are numerically stable for the studied cutoff values, which require a careful treatment of the necessary interpolations in the vicinity of the sharp cutoff. We also checked the convergence with respect to the included partial waves. We estimate an accuracy of 2 keV for the ^{3}H and ^{3}He and 50 keV for the ^{4}He calculations. More details about the numerical method can be found in [13].

In Fig. 1, we give results for binding energies of the 3N system. We show results for the $V_{\text{low }k}$ derived from the CD-

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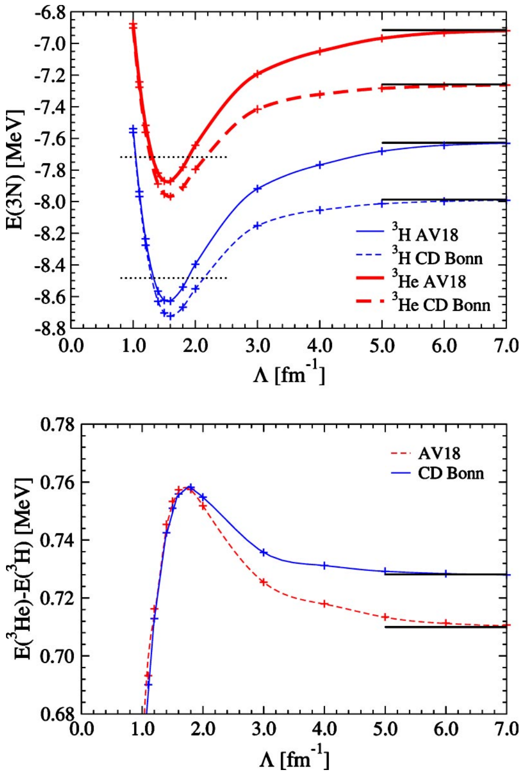

FIG. 1. (Color online) Cutoff dependence of the 3N binding energies and the binding-energy difference of ^{3}H and ^{3}He. Results are shown for the Argonne v_{18} and the CD-Bonn 2000 potential. The horizontal solid lines represent results for the bare two-body interactions and the dotted lines denote the experimental binding energies.

Bonn 2000 [14] and Argonne v_{18} [15] interactions. In both cases electromagnetic interactions were included. The cutoff dependence is due to missing three-body forces. For large cutoffs, we reproduce the known binding energies obtained with the bare interactions only. For intermediate cutoffs, we find a stronger binding with $V_{\mathrm{low}\ k}$. This could be expected, because softer interactions generally lead to stronger binding. It is also consistent with the correlation between the triton binding energy and the deuteron *D*-state probability observed for phenomenological potentials [16]. For $V_{\mathrm{low}\ k}$ the *D*-state probability decreases monotonically with a decreasing cutoff. Therefore, this correlation evidently breaks down for cutoffs below $\Lambda \approx 1.6\ \mathrm{fm}^{-1}$. The binding then decreases, as attractive parts of the bare interactions are integrated out.

For cutoffs $\Lambda \lesssim 2m_\pi$, truly model-independent results are obtained and the binding-energy curves for the CD-Bonn 2000 and Argonne v_{18} $V_{\mathrm{low}\ k}$ interactions collapse. In Fig. 1, we also show the cutoff dependence of the difference in ^{3}He and triton binding energies, which is due to electromagnetic and isospin-breaking contributions. The difference varies by 60 keV and correlates with the binding energy, since the latter is related to the charge radius [17]. For special choices of the cutoff, both experimental binding energies can be reproduced simultaneously without a 3N interaction. We emphasize that 3N forces will contribute to other observables.

Our results indicate that 3N forces due to the truncation to low momenta are of the same order as adjusted 3N forces due to missing excitations of nucleons, although these effects cannot be separated. The bare 3N forces provide about 0.7–1 MeV of binding in conventional models, whereas the binding energies given by $V_{\mathrm{low}\ k}$ change by 1 MeV over the large cutoff range. In this sense the truncation to low momenta does not induce strong three-body forces in low-energy observables, such as nuclear binding energies. We note that this is in contrast to the interpretation given in [18]. There, the size of 3N forces was assessed by comparing the $V_{\mathrm{low}\ k}$ binding energies to the results of the bare 2N potential model. This neglects the uncertainty in the binding-energy predictions of traditional 2N forces and misses that, in effective theory approaches, the effects of the truncation to small cutoffs are inseparable from those of missing degrees of freedom like the Δ. Because these two contributions to higher-body forces cannot be disentangled at low energies, we will absorb both by augmenting $V_{\mathrm{low}\ k}$ with a chiral 3N force below.

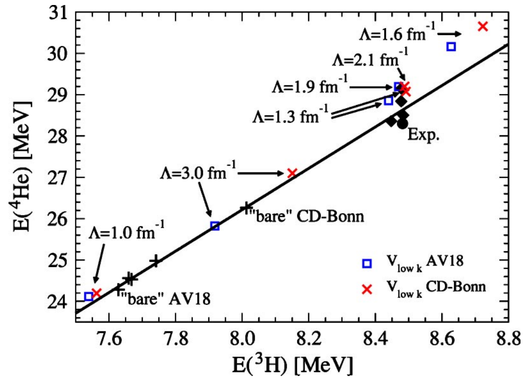

FIG. 2. (Color online) Correlation of the ^{3}H and ^{4}He binding energies. The results are shown for several modern potential models alone (pluses) and with adjusted 3N forces (diamonds) [20]. The $V_{\mathrm{low}\ k}$ results are for the Argonne v_{18} (squares) and the CD-Bonn 2000 potential (crosses). The solid line is a linear fit to the 2N force model results only.

For further insight, we have calculated the α-particle binding energy. To obtain an overview, calculations are performed for the smallest cutoff considered, $\Lambda = 1.0\ \mathrm{fm}^{-1}$, in the maximum of the triton binding energy at $\Lambda = 1.6\ \mathrm{fm}^{-1}$, for two cutoffs which lead to 3N binding energies close to the experimental one, $\Lambda = 1.3\ \mathrm{fm}^{-1}$ and $\Lambda = 1.9\ \mathrm{fm}^{-1}$ (Argonne v_{18}), or $\Lambda = 2.1\ \mathrm{fm}^{-1}$ (CD-Bonn 2000), and for a cutoff in the tail at $\Lambda = 3.0\ \mathrm{fm}^{-1}$. The focus of our studies is whether the cutoff dependence of $V_{\mathrm{low}\ k}$ can be related to correlations observed when traditional two-body interactions are used. From [13,19,20], it is well known that there is an almost linear relation between 3N and 4N binding energies, known as the Tjon line. This correlation holds with very good accuracy for all modern interactions, but is slightly broken by the action of 3N forces. As can be seen in Fig. 2, the various $V_{\mathrm{low}\ k}$ results do not differ significantly more from the phenomenological Tjon line than calculations with adjusted 3N forces. We see that as a further indication that 3N and 4N contributions are not unexpectedly large due to the low-momentum truncation, at least for the triton and α particle. Already at $\Lambda = 3.0\ \mathrm{fm}^{-1}$ the $V_{\mathrm{low}\ k}$ prediction is almost ex-

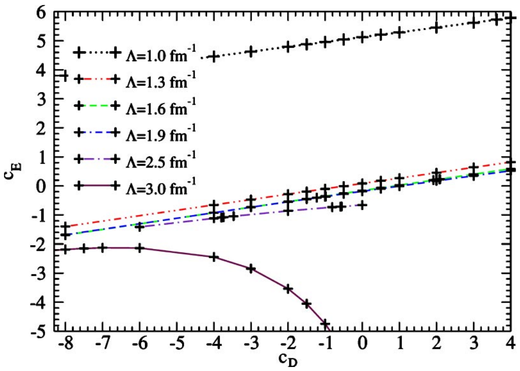

FIG. 3. (Color online) Relation between c_D and c_E obtained by requiring that $V_{\mathrm{low}\ k}$ augmented by the 3N force predicts the ^{3}H binding energy correctly.

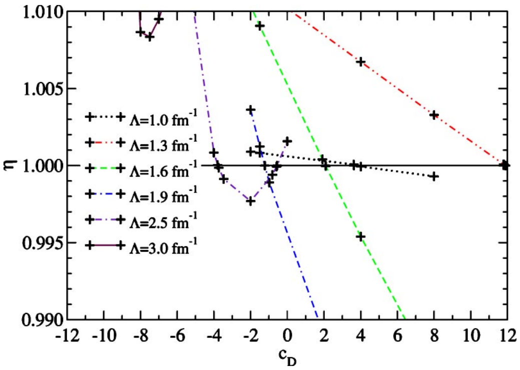

FIG. 4. (Color online) Dependence of the eigenvalue η of the Yakubovksy equation on c_D for various cutoffs. A deviation of $\eta-1=0.01$ corresponds to a deviation of approximately 600 keV from the experimental value.

actly on the Tjon line given by the phenomenological models.

As also seen in Fig. 2, even if a cutoff is chosen that leads to a good description of the 3N binding energies, the 4N binding energy deviates from experiment. Clearly, 3N or higher-body forces must act for these values of the cutoff. In the following, we construct a low-momentum 3N interaction by fitting the leading chiral 3N force to $V_{\mathrm{low}\ k}$. For simplicity we restrict ourselves to the $V_{\mathrm{low}\ k}$ derived from the Argonne v_{18} potential. The chiral 3N force to leading order contains a long-range 2π exchange part, an intermediate range 1π exchange (D term), and a zero-range contact interaction (E term), (see [11,12]). For the operator form and the definition of the strength constants, we refer the reader to Eqs. (2) and (10) in [12]. The interaction is regularized by exponential cutoff functions of the form $\exp[-(p/\Lambda)^8]$ with the cutoff taken from $V_{\mathrm{low}\ k}$. The very high exponent guarantees a very sharp drop to zero at $p=\Lambda$. The 2π exchange part is determined by strength constants c_i, which we take from [21], where they were obtained by a fit to NN data.The dimensionless strength constants c_D and c_E were obtained from a fit to the ^{3}H and ^{4}He binding energies. First, a relation between c_D and c_E was established by requiring that the ^{3}H binding energy of −8.482 MeV is described accurately. The resulting dependence for various cutoffs is shown in Fig. 3. For small cutoffs we obtain a linear relationship, which suggests that the D and E terms are perturbative in this region. We have checked explicitly and also for the c terms that these are perturbative for $\Lambda \lesssim 2\ \mathrm{fm}^{-1}$. This could be useful for applications, where it is practically impossible to include the 3N force into the dynamical equations, but a perturbative treatment is feasible.

In Fig. 4, we show the eigenvalue η of the Yakubovsky equation for ^{4}He versus c_D. In all cases c_E was chosen according to Fig. 3. The binding energy of ^{4}He agrees with the experimental one of −28.3 MeV for $\eta=1$. In the considered range for c_D, we find a unique solution for the cutoff choices up to $\Lambda=1.9\ \mathrm{fm}^{-1}$. For $\Lambda=2.5\ \mathrm{fm}^{-1}$, the relation of η and c_D is strongly nonlinear and we find two solutions. We observed a very similar behavior, when the N3LO chiral interaction of [10] was augmented by the same 3N force. For $\Lambda=3.0\ \mathrm{fm}^{-1}$, we cannot describe the ^{3}H and ^{4}He binding energies simultaneously. For this cutoff, we choose $c_D=7.5$, for which η is minimal and the binding energy is best described. The resulting c_D/c_E pairs are compiled in Table I, where the (∗) indicates that the ^{4}He binding energy is reproduced only approximately as −28.8 MeV for $\Lambda=3.0\ \mathrm{fm}^{-1}$, and (a) and (b) label the two possible solutions for $\Lambda=2.5\ \mathrm{fm}^{-1}$.

A very important task is to estimate the size of 3N forces in a systematic way. We decided to calculate the expectation values of the 2N and the different parts of the 3N interactions and compare their magnitude. The results are summarized in Table II. As a worst case scenario, we compare the maximum of the individual 3N force terms to the 2N interaction for ^{4}He. As expected from Fig. 3, for $\Lambda \lesssim 2\ \mathrm{fm}^{-1}$, all 3N parts are perturbative. For these cutoffs, we obtain contributions of 4 %–10 %, which are comparable to 3N forces for phenomenological models [13,22]. For larger cutoffs, the 2π exchange contribution (c terms) grows rapidly, which is canceled by the E term. We take this as an indication that, in this range, our ansatz for the 3N force is not reliable.

In summary, we have thoroughly assessed the size of 3N forces in the $V_{\mathrm{low}\ k}$ approach. Based on the $V_{\mathrm{low}\ k}$ results for the ^{3}H and ^{4}He binding energies, we found that the dependence on the cutoff is not unnaturally large for $\Lambda \geqslant 1.0\ \mathrm{fm}^{-1}$. This suggests that higher-body interactions are small. We emphasize that the large cutoff range, for which $V_{\mathrm{low}\ k}$ is available, will enable similar studies for other low-energy observables, e.g., all binding and excitation energies,

TABLE I. Fit results for c_D and c_E for various cutoffs of the $V_{\mathrm{low}\ k}$ derived from the Argonne v_{18} potential [for (a), (b), and (∗) see text]. The strength of the 2π exchange part is determined by $c_1=-0.76\ \mathrm{GeV}^{-1}$, $c_3=-4.78\ \mathrm{GeV}^{-1}$, and $c_4=3.96\ \mathrm{GeV}^{-1}$ [21].

Λ (fm^{-1})	c_D	c_E
1.0	3.621	5.724
1.3	11.889	2.265
1.6	2.080	0.230
1.9	−1.225	−0.405
2.5(a)	−0.560	−0.707
2.5(b)	−3.794	−1.085
3.0(∗)	−7.500	−2.151

TABLE II. Expectation values of the kinetic energy (T), 2N interaction ($V_{\text{low }k}$), 2π exchange part of the 3N force (c terms), and D and E term for ^{3}H and ^{4}He [for (a), (b), and (*) see text]. All energies are in MeV.

	^{3}H					^{4}He				
Λ (fm^{-1})	T	$V_{\text{low }k}$	c terms	D term	E term	T	$V_{\text{low }k}$	c terms	D term	E term
1.0	21.06	−28.62	0.02	0.11	−1.06	38.11	−62.18	0.10	0.54	−4.87
1.3	25.71	−34.14	0.01	1.39	−1.46	50.14	−78.86	0.19	8.08	−7.83
1.6	28.45	−37.04	−0.11	0.55	−0.32	57.01	−86.82	−0.14	3.61	−1.94
1.9	30.25	−38.66	−0.48	−0.50	0.90	60.84	−89.50	−1.83	−3.48	5.68
2.5(a)	33.30	−40.94	−2.22	−0.11	1.49	67.56	−90.97	−11.06	−0.41	6.62
2.5(b)	33.51	−41.29	−2.26	−1.42	2.97	68.03	−92.86	−11.22	−8.67	16.45
3.0(*)	36.98	−43.91	−4.49	−0.73	3.67	78.77	−99.03	−22.82	−2.63	16.95

and that this is a powerful tool to isolate missing parts in effective interactions. Furthermore, we have extended $V_{\text{low }k}$ by the leading chiral 3N force and fitted the two unknown parameters to the ^{3}H and ^{4}He binding energies. We assessed the strength of the 3N force by calculating expectation values of its individual parts. By requiring that not only the sum, but also the individual parts are of natural size, we found that our ansatz for the 3N force is reliable for cutoffs $\Lambda \lesssim 2$ fm^{-1}. It turned out that the 3N force contribution can be treated perturbatively for this range of cutoffs. This completes a soft nuclear interaction model, which will be important for many-body calculations. Applications to symmetric nuclear matter are in preparation.

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