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Six-Body variational Monte Carlo study of ${}^{6}_{\Lambda\Lambda}$He

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Variational Monte Carlo calculations are carried out for ${}^{6}_{\Lambda\Lambda}$He using realistic *NN*, *NNN*, and phenomenological ΛN and ΛNN interactions. For the $\Lambda\Lambda$ interaction we employ the various phase equivalent Nijmegen interactions. By incorporating the various components of Λ-nuclear interactions in stages, and keeping $B_\Lambda({}^{5}_{\Lambda}\text{He})$ around 3.12 MeV, it is demonstrated that the incremental energy $\Delta B_{\Lambda\Lambda}$ for ${}^{6}_{\Lambda\Lambda}$He is sensitive to the three-body ΛNN force and the exchange part of the ΛN interaction. The $\Lambda\Lambda$ interaction obtained is only somewhat weaker than the ΛN interaction. We also report the results for the rearrangement energy of the α core. We discuss the implications of our results.

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Recently, a number of $\Lambda\Lambda$ hypernuclei calculations have been performed in the *s*- and *p*-shell regions. These studies have been sparked by the recent identification of a ${}^{6}_{\Lambda\Lambda}$He event [1] at the High Energy Accelerator Research Organization (KEK), which gives a considerably lower value for the incremental $\Lambda\Lambda$ energy $\Delta B_{\Lambda\Lambda}$ ($1.01\pm0.20^{+0.18}_{-0.11}$MeV) compared to the older [2] ($\approx$4.7 MeV) but doubtful [3] emulsion event. The $\Lambda\Lambda$ interaction is one of the basic *BB* forces and is fundamental for understanding the interaction between strange baryons. The information on the multiply strange system is of wide physics interest in the realm of QCD and nuclear physics. In order to reliably obtain the $\Lambda\Lambda$ interaction we perform complete *six*-body variational Monte Carlo VMC calculations for ${}^{6}_{\Lambda\Lambda}$He using realistic interactions with highly flexible correlations. Earlier many-body calculations of $\Lambda\Lambda$ hypernuclei have been made either using central *NN* and ΛN interactions [4–6] or in three- or four-body (for *p*-shell $\Lambda\Lambda$ hypernuclei) cluster models in which the nuclear clusters have been treated as inert [7,8]. We demonstrate that *six*-body calculations of ${}^{6}_{\Lambda\Lambda}$He with realistic interactions present a different dynamics, which in turn requires a stronger $\Lambda\Lambda$ interaction than the one deduced from $\alpha-\Lambda\Lambda$ model calculations. We show that the three-body ΛNN interaction and the exchange two-body ΛN interaction have a large effect on the binding energy of ${}^{6}_{\Lambda\Lambda}$He. For the $\Lambda\Lambda$ scattering length we obtain $a_{\Lambda\Lambda}\approx-1.24\pm0.5$ fm, whereas from the $\alpha-\Lambda\Lambda$ model calculations one gets only $a_{\Lambda\Lambda}\approx-0.8$ fm [7,8]. Experimentally, the ΛN scattering length is $a_{\Lambda N}=-1.5\pm0.15\pm0.3$ fm [9,10]. Thus, our conclusion that $|V_{\Lambda\Lambda}|\leqslant|V_{\Lambda N}|$ violates the flavor SU(3) requirement $|V_{\Lambda\Lambda}|\ll|V_{\Lambda N}|$ implying that SU(3) symmetry is broken. A relatively stronger $\Lambda\Lambda$ force will perhaps lead to a bound ${}^{4}_{\Lambda\Lambda}$H [11–14]. The results for this hypernucleus shall be reported elsewhere. Within the variational framework [15], we also report the results of rearrangement energy calculations and discuss their implications on the results of the cluster $\alpha-\Lambda\Lambda$ model calculations.

For the nuclear part of the Hamiltonian, corresponding to strangeness $S=0$, we use Argonne V_{18} two-body *NN* [16] and Urbana IX three-body [17] *NNN* potentials Argonne V_{18} accounts well for the *NN* scattering data up to 350 MeV and when combined with Urbana IX *NNN*, gives an excellent account for all the *s*-shell nuclei [18].

For the sector $S=-1$, we use the phenomenological potentials of Ref. [19] which consist of central, Majorana space exchange, and spin-spin ΛN components

$$V_{\Lambda N}=[V_c(r)-\overline{V}T_\pi^2(r)](1-\epsilon+\epsilon P_x)+\frac{1}{4}V_\sigma T_\pi^2(r)\sigma_\Lambda\cdot\sigma_N \tag{1}$$

where P_x is the Majorana space-exchange operator and ϵ is the exchange parameter. $V_c(r)$ is a Woods-Saxon core, $\overline{V}$ and V_σ are, respectively, the spin-average and spin-dependent strength, and T_π is a one-pion tensor shape factor.

The ΛNN interaction consists of a two-pion exchange and a dispersive part [19,20]. These arise mainly from the elimination of the Σ degrees of freedom. The $V_{\Lambda NN}$ is found to be essential for a consistent phenomenology of *s*-shell hypernuclei, when use is made of $V_{\Lambda N}$ Eq. (1), which accounts for the low-energy Λp scattering data. This is also borne out by the recent calculations of Nemura *et al.* and Nogga *et al.* [21], who explicitly consider the Σ degrees of freedom in their calculations of the *s*-shell hypernuclei.

Following [19], we use $\epsilon=0.20$, which is consistent, though slightly on the low side, with the forward to backward ratio of the low-energy Λp scattering data [22]. Usmani and Bodmer [20], and also Millener [23], however, consider a value $\epsilon=0.25$, which is consistent with the hypernuclear spectroscopic data of *p*-shell and higher mass hypernuclei. The effects which arise for $\epsilon=0.20$ would be somewhat larger if higher values are used.

In Table I we give sets of ΛN and ΛNN potential parameters which we use in the present study. C_p and W_0 are the strength parameters of the two pion and the dispersive parts of the ΛNN potential. The first interaction $\Lambda N1$ is our full interaction with all components, namely, space exchange and the ΛNN parts.

In Table II, we present results for *s*-shell hypernuclei for our preferred model $\Lambda N1$. These results are slightly different,

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TABLE I. ΛN and ΛNN interaction parameters. $a_{\Lambda N}$ and r_o are the spin-averaged scattering length and effective range in fm. Except for ϵ, other quantities are in MeV.

ΛN	$\overline{V}$	V_σ	ϵ	C_p	W_0	$B_\Lambda(^5_\Lambda\text{He})$	$-a_{\Lambda N}$	r_o
$\Lambda N1$	6.15	0.176	0.2	1.50	0.028	3.17(3)	1.69	5.65
$\Lambda N2$	6.11	0.0	0.0	1.50	0.028	3.19(3)	1.56	3.83
$\Lambda N3$	6.025	0.0	0.0	0.0	0.0	3.13(2)	1.32	4.25

but consistent, with those of [19] due to use of more flexible correlations and better statistics [24]. The charge symmetry breaking in the ΛN interaction is unimportant for the present study. We therefore take the average of B_Λ of $^4_\Lambda$H and $^4_\Lambda$He. We also give the space-exchange contribution, SEC, which arises due to an approximately equivalent weak ΛN interaction in the relative p state.

The rearrangement energy E_R arises because of the modification of the α core due to the presence of the Λs. This modification is through the change in the nucleonic part of the wave function from minimizing the energy of the hypernucleus. E_R is obtained from

$$E_R \approx \frac{\langle \Psi_m | H_N | \Psi_m \rangle}{\langle \Psi_m | \Psi_m \rangle} - \frac{\langle \Psi | H_N | \Psi \rangle}{\langle \Psi | \Psi \rangle}, \tag{2}$$

where Ψ_m represents the core nucleus wave function as modified due to the presence of one or two Λs in a hypernucleus, and Ψ is the optimized wave function of the isolated core nucleus. H_N is the nuclear Hamiltonian. The present estimate of E_R is an approximation to the rigorous definition given in [15], but is much simpler to implement.

For the $\Lambda\Lambda$ potential, we use low-energy phase equivalent Nijmegen interactions represented by a sum of three Gaussians [7,25,26],

$$V_{\Lambda\Lambda} = v^{(1)} \exp(-r^2/\beta_{(1)}^2) + \gamma v^{(2)} \exp(-r^2/\beta_{(2)}^2) + v^{(3)} \exp(-r^2/\beta_{(3)}^2), \tag{3}$$

where the strength parameter v^i and the range parameter β_i are taken from [7]. The values of γ=0.5463, 1.0, and 1.2044 correspond to Nijmegen interactions NSC97e, ND, and NEC00, respectively.

Our variational wave function is of the form

TABLE II. Λ separation energy B_Λ, space-exchange contribution (SEC), three body contribution $\langle V_{\Lambda NN}\rangle$, and the rearrangement energy, E_R for s-shell hypernuclei. All values are in MeV.

Pot.		$^4_\Lambda$H	$^4_\Lambda$H*	$^5_\Lambda$He
$\Lambda N1$	B_Λ	2.15(2)	1.06(2)	3.17(3)
	SEC	0.22(1)	0.18(1)	0.49(3)
	$-\langle V_{\Lambda NN}\rangle$	1.39(2)	0.61(2)	0.87(2)
	E_R			0.39(6)
Experiment	B_Λ	2.22(4)	1.12(4)	3.12(2)

$$|\Psi_v(\Lambda\Lambda)\rangle = \left[1 + \sum_{i<j<k} (U_{ijk} + U_{ijk}^{TNI}) + \sum_{i<j,\Lambda} U_{ij,\Lambda} + \sum_{i<j} U_{ij}^{LS}\right] \times |\Psi_p(\Lambda\Lambda)\rangle, \tag{4}$$

where the pair wave function $|\Psi_p(\Lambda\Lambda)\rangle$ is

$$|\Psi_p(\Lambda\Lambda)\rangle = S\prod_{i<j}(1+U_{ij})S\prod_{i,\Lambda}(1+U_{i\Lambda})\left|\Psi_J\left(^A_{\Lambda\Lambda}Z\right)\right\rangle. \tag{5}$$

The operator S symmetrizes the various noncommuting operators which occur in U. The Jastrow wave function $|\Psi_J(^A_{\Lambda\Lambda}Z)\rangle$ for the s-shell $\Lambda\Lambda$ hypernucleus consists of two- and three-body central correlations represented by various fs,

$$\left|\Psi_J\left(^A_{\Lambda\Lambda}Z\right)\right\rangle = f_c^{\Lambda\Lambda}\prod_{i,\Lambda} f_c^{i\Lambda} \prod_{i<j<k} f_c^{ijk} \prod_{i<j} f_c^{ij} \left|\Psi_{JT}\left(^{A-2}Z\right)\right\rangle A|\downarrow\Lambda\uparrow\Lambda\rangle, \tag{6}$$

where $|\Psi_{JT}(^{A-2}Z)\rangle$ represents the spin and isospin wave function of the s-shell nucleus with definite total angular momentum J and isospin T, and $A|\downarrow\Lambda\uparrow\Lambda\rangle$ is the antisymmetric wave function of the two Λ particles coupled to total angular momentum zero. The correlation $f_c^{\Lambda\Lambda}$ between the two Λs is obtained through a solution of a Schrödinger-type two-body equation with an effective potential containing a number of variational parameters. All the correlation components of the wave function were determined using techniques described in [18,19,24]. We also incorporate additional flexibility in the correlations by adding to each link f a correction through cosine polynomials

$$f \to f + \sum_{n=1}^{4} a_n \cos\left(\frac{n\pi r}{r_d}\right) \quad \text{for } r \leqslant r_d. \tag{7}$$

The a_n are variational parameters. We vary three of them; the remaining one is fixed by the condition that the correction term in Eq. (8) becomes zero at $r=r_d$, the healing distance which is also a variational parameter. The cosine series has the property that its first derivative at the boundaries is zero, which is essential for applying the correction.

In Table III and Fig. 1, we present results for the incremental energy $\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}-2B_\Lambda$ for $^6_{\Lambda\Lambda}$He, where $B_{\Lambda\Lambda}$ is the separation of the two Λs from the core nucleus. The quantity $\Delta B_{\Lambda\Lambda}$ is closely related to the interaction energy of the two Λ's. We have made calculations for γ=0.773 an intermediate value between NSC97e and ND, and have called it NM. This is done to facilitate better interpolation. We also give expectation values of $V_{\Lambda NN}$ and SEC to emphasize their importance. For the interaction $\Lambda N3$, which consists of a two-body central ΛN potential only, our results for $\Delta B_{\Lambda\Lambda}$ are in close

TABLE III. $\Delta B_{\Lambda\Lambda}$ for ${}^{6}_{\Lambda\Lambda}$He with various $\Lambda N+\Lambda NN$ and $\Lambda\Lambda$ Nijmegen interactions. For NM see text. Except for γ, other quantities are in MeV. The * refers to the s-wave Faddeev calculations of [7].

Potential		NSC97e	NM	ND	NEC00
	γ	0.5463	0.773	1.0	1.2044
$\Lambda N1$	$\mathbf{\Delta B_{\Lambda\Lambda}}$	**0.24(8)**	**0.84(7)**	**1.90(8)**	**3.90(8)**
	$-\langle V_{\Lambda NN}\rangle$	2.33(2)	2.44(4)	2.40(2)	2.78(4)
	SEC	1.27(2)	1.59(1)	2.12(1)	2.72(2)
	E_R	1.70(6)	1.81(6)	2.01(6)	2.27(6)
$\Lambda N2$	$\mathbf{\Delta B_{\Lambda\Lambda}}$	**0.42(7)**	**1.05(8)**	**2.44(7)**	**4.20(7)**
	$-\langle V_{\Lambda NN}\rangle$	2.36(4)	2.45(4)	2.82(4)	3.03(5)
	E_R	1.42(6)	1.73(6)	1.82(6)	2.09(6)
$\Lambda N3$	$\mathbf{\Delta B_{\Lambda\Lambda}}$	**0.58(6)**	**1.42(6)**	**2.95(6)**	**4.75(9)**
	E_R	1.03(6)	1.06(6)	1.14(6)	1.28(6)
$\alpha-\Lambda\Lambda$ with Isle Pot.					
VMC Present	$\mathbf{\Delta B_{\Lambda\Lambda}}$	**0.69**		**2.96**	**4.84**
Faddeev (Ref. [7,8])	$\mathbf{\Delta B_{\Lambda\Lambda}}$	**0.71**		**2.99**	**4.51***

agreement with the three-body Faddeev calculations of Filikhin and Gal [7,8], who use an $\alpha-\Lambda\Lambda$ model for ${}^{6}_{\Lambda\Lambda}$He in which the α core is treated as inert. However, for $\Lambda N2$ and $\Lambda N1$, significant differences arise because of the importance of ΛNN and exchange ΛN contributions.

In an $\alpha-\Lambda\Lambda$ model the $\alpha-\Lambda$ potential plays a crucial role. By using different $\alpha-\Lambda$ potentials, fitted to $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})=3.12$ MeV, one gets quite different values of $\Delta B_{\Lambda\Lambda}$. This was demonstrated quite some time ago by Bodmer and Usmani [27], who performed accurate variational calculations for ${}^{6}_{\Lambda\Lambda}$He in an $\alpha-\Lambda\Lambda$ model. They obtained values for $\Delta B_{\Lambda\Lambda}$ from 2.71 to 4.65 MeV for a given $\Lambda\Lambda$ potential, but using mostly different Isle type $\alpha-\Lambda$ potentials fitted to $B_\Lambda({}_{\Lambda}^{5}\mathrm{He})=3.12$ MeV. Their study was partly based on extracting the $\alpha-\Lambda$ potential from VMC calculations of ${}^{5}_{\Lambda}$He using simplified $NN,\Lambda N$, and ΛNN potentials. In particular, with ΛNN potentials, they found a decrease in $\Delta B_{\Lambda\Lambda}$ by $\sim$0.4 MeV. We notice similar trends in the present complete *six*-body calculations also. We demonstrate the accuracy of the variational calculations of Ref. [27] by using the same type of product wave function in $\alpha-\Lambda\Lambda$ model, namely

$$\left|\Psi_v\left({}^{6}_{\Lambda\Lambda}\mathrm{He}\right)\right\rangle = f_{\alpha\Lambda}(r_{\alpha\Lambda_1})f_{\alpha\Lambda}(r_{\alpha\Lambda_2})f_{\Lambda\Lambda}(r_{\Lambda_1\Lambda_2}) \tag{8}$$

with the local Isle potential of [7]. The results of the present VMC calculations are in excellent agreement with the *exact* Faddeev calculations of [7,8] as evident from the last two rows of Table III. For NSC97e and ND interactions, Faddeev calculations are for the angular momentum states up to $\ell_{\alpha\Lambda}=6$ and $\ell_{\Lambda\Lambda}=6$ [8], whereas for model NEC00 $\ell_{\alpha\Lambda}=0$ and $\ell_{\Lambda\Lambda}=0$, and is not strictly comparable. In the VMC calculations all angular momentum states are included.

It therefore follows from above that both in the complete *six*-body calculations as well as in the $\alpha-\Lambda\Lambda$ cluster model the various components of ΛN interaction play significant roles in ${}^{6}_{\Lambda\Lambda}$He, in the latter through the $\alpha-\Lambda$ potential.

The effect of the exchange potential is much more pronounced for $\Lambda N1$ in decreasing $\Delta B_{\Lambda\Lambda}$, particularly for larger values of γ. Large positive values of the space-exchange contributions arise, because of the differences in the $\Lambda N+\Lambda NN+\Lambda\Lambda$ and $NN+NNN$ correlations, since corresponding interactions are very different. With simplified NN and ΛN interactions the space-exchange contribution will be much smaller [4,28].

It is evident from Table III that the rearrangement energies for ${}^{6}_{\Lambda\Lambda}$He are substantial. For $\Lambda N1$ the rearrangement energy for ${}^{5}_{\Lambda}$He is $\sim$0.4 MeV, whereas for ${}^{6}_{\Lambda\Lambda}$He it is $\sim$2.0 MeV, a fourfold to fivefold increase. In an $\alpha-\Lambda\Lambda$ cluster model where the α is treated as inert, it may be hard

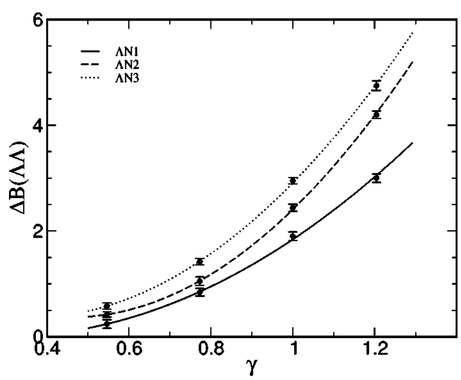

FIG. 1. $\Delta B_{\Lambda\Lambda}$ vs γ. The full circles are the values from Table III. The lines are the best fit results as described in the text.

to simulate this large change in the E_R for ${}^{5}_{\Lambda}$He and ${}^{6}_{\Lambda\Lambda}$He, even if one has reliable knowledge of the $\alpha-\Lambda$ potential. Thus the comments in the preceding two paragraphs, combined with the rearrangement energy effects, limit the usefulness of the cluster model calculations especially for more complex interactions.

For the three interactions, we fitted the calculated $\Delta B_{\Lambda\Lambda}$ values by $\Delta B_{\Lambda\Lambda}=a+b\gamma+c\gamma^2$ (Fig. 1). From these fits, we obtain the values of γ and the scattering lengths $a_{\Lambda\Lambda}$ corresponding to $\Delta B_{\Lambda\Lambda}=1.01\pm0.20$,

$$\Lambda N1:\gamma = 0.816^{+0.050}_{-0.055}, \quad a_{\Lambda\Lambda} = -1.36^{+0.28}_{-0.24}\ \text{fm}$$

$$\Lambda N2:\gamma = 0.762^{+0.044}_{-0.052}, \quad a_{\Lambda\Lambda} = -1.12^{+0.20}_{-0.18}\ \text{fm}$$

$$\Lambda N3:\gamma = 0.681^{+0.047}_{-0.056}, \quad a_{\Lambda\Lambda} = -0.84^{+0.13}_{-0.15}\ \text{fm}. \tag{9}$$

For s-shell nuclei, the VMC energies are generally higher compared to Green's function Monte Carlo [18] calculations by less than 3%. The hyperonic part of the wave function has a much simpler correlation structure than the nuclear wave function, since the corresponding interaction is much simpler. In addition, the ΛN correlations are weaker compared to NN correlations. It is therefore reasonable to assume that the calculated hypernuclear energies will also be higher, roughly by 3% as compared to the exact values. Thus the errors being systematic and small, because of the variational nature of the problem and the reasons mentioned above, one may expect that the values of B_Λ and $B_{\Lambda\Lambda}$ (and hence of $\Delta B_{\Lambda\Lambda}$) would be greater than, but close to, 3% of the exact values of these quantities.

If we combine the uncertainties associated with ΛN interaction, MC errors, and the experimental result, we obtain

$$a_{\Lambda\Lambda} = -1.24^{+0.50}_{-0.40}\ \text{fm}. \tag{10}$$

In (10), we have excluded the results for the purely central ΛN potential $\Lambda N3$ as this interaction considerably underbinds the mass four hypernuclei [24]. The value $a_{\Lambda\Lambda}=-1.24$ fm represents the average of $\Lambda N1$ and $\Lambda N2$. This value is considerably closer to $a_{\Lambda N}\approx-1.5$ fm and in absolute value larger than ≈-0.80 fm [7,8], the value deduced from $\alpha-\Lambda\Lambda$ cluster calculations. It should be noted from Eq. (9) that $\Lambda N3$ gives $a_{\Lambda\Lambda}$ very close to that obtained from the $\alpha-\Lambda\Lambda$ model [7,8]. This is perhaps not surprising since a folding model for the $\alpha-\Lambda$ potential works very well in this case [27].

We have ignored the effects of $N\Xi$ and $\Sigma\Sigma$ channels. Their contribution in ${}^{6}_{\Lambda\Lambda}$He is presumably repulsive [29], implying a more attractive $\Lambda\Lambda$ interaction in free space. We believe that these and other many-body effects are covered in the uncertainties of (10).

In summary, we have made complete *six*-body calculations for ${}^{6}_{\Lambda\Lambda}$He using realistic NN and NNN, and phenomenological ΛN and ΛNN interactions, and have demonstrated the limitations of the cluster $\alpha-\Lambda\Lambda$ model calculations. A more attractive $\Lambda\Lambda$ interaction has implications for the stability of ${}^{4}_{\Lambda\Lambda}$H [12,13]; the role of coupling between $\Lambda\Lambda, N\Xi$, and $\Sigma\Sigma$ channels [29], properties of strange hadronic matter in bulk [30], and the breaking of SU(3) symmetry in baryonic interactions.

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