

Accurate approximation for the Coulomb potential between deformed nuclei

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We present a useful procedure for calculating the Coulomb potential between deformed nuclei. The corresponding results are compared with those obtained from the numerical resolution of the six-dimensional integral involved in double-folding calculations.

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I. INTRODUCTION

The forces between extended objects is an important subject in heavy-ion collisions, where the double-folding potential plays a fundamental role in the description of the corresponding interaction. The folded Coulomb potential between heavy ions involves a six-dimensional integral which can easily be solved for two spherically symmetric charge distributions. However, for deformed densities the corresponding numerical resolution requires lengthy calculations. Therefore, faster methods for obtaining good approximations to the deformed potential are quite convenient. In a recent work [1], we presented a procedure for calculating in an approximate form the Coulomb and nuclear potentials between deformed nuclei. That procedure provides precise results for the Coulomb interaction at large distances but only reasonable estimates in the inner region. In this work, we have improved the method with the aim of obtaining good accuracy also at small distances.

II. THE COULOMB POTENTIAL

Denoting the projectile and target densities by ρ_1 and ρ_2 , the Coulomb interaction between them is given by

$$V(\vec{R}, \alpha) = \int d\vec{r}_1 d\vec{r}_2 \frac{e^2}{|\vec{R} + \vec{r}_2 - \vec{r}_1|} \rho_1(\vec{r}_1) \rho_2(\vec{r}_2), \quad (1)$$

where \vec{R} is the position vector of the center of mass of nucleus 2 measured from that of nucleus 1, and α represents the ensemble of intrinsic coordinates, implicit in ρ_1 and ρ_2 . The densities are appropriately normalized as

$$\int \rho_i(\vec{r}) d\vec{r} = Z_i. \quad (2)$$

We express the interaction in terms of the Fourier transform representation [2],

$$V(\vec{R}, \alpha) = \frac{e^2}{2\pi} \int d\vec{k} \int d\vec{r}_1 d\vec{r}_2 \rho_1(\vec{r}_1) \rho_2(\vec{r}_2) \frac{e^{i\vec{k} \cdot (\vec{R} + \vec{r}_2 - \vec{r}_1)}}{k^2}, \quad (3)$$

and, for simplicity, we assume the projectile and target to be uniformly charged objects with a sharp surface,

$$\rho_i(\vec{r}) = \rho_{oi} \Theta(R_i(\Omega) - r). \quad (4)$$

The angle-dependent radius is given in terms of the deformation parameters as

$$R_i(\Omega) = R_i \left(1 + \sum_{\lambda\mu} a_{\lambda\mu}^{(i)} Y_{\lambda\mu}^*(\Omega) \right), \quad (5)$$

where the sum is performed for $\lambda \geq 2$. To second-order terms, the normalization can be expressed by

$$\frac{4\pi}{3} R_i^3 \rho_{oi} \left(1 + \frac{3}{4\pi} \sum_{\lambda\mu} |a_{\lambda\mu}^{(i)}|^2 \right) = Z_i. \quad (6)$$

We use the spherical decomposition of the plane wave,

$$e^{i\vec{k} \cdot \vec{r}} = 4\pi \sum_{\lambda\mu} i^\lambda j_\lambda(kr) Y_{\lambda\mu}^*(\Omega_k) Y_{\lambda\mu}(\Omega_r), \quad (7)$$

to obtain a multipole expansion of the interaction,

$$\begin{aligned} V(\vec{R}, \alpha) &= 32\pi e^2 \sum_{\substack{\lambda\lambda_1\lambda_2 \\ \mu\mu_1\mu_2}} i^{\lambda+\lambda_2-\lambda_1} \int_0^\infty dk \int d\vec{r}_1 d\vec{r}_2 \\ &\times \rho_1(\vec{r}_1) \rho_2(\vec{r}_2) j_\lambda(kR) j_{\lambda_1}(kr_1) j_{\lambda_2}(kr_2) \\ &\times Y_{\lambda\mu}^*(\Omega_R) Y_{\lambda_1\mu_1}(\Omega_{r_1}) Y_{\lambda_2\mu_2}(\Omega_{r_2}) \\ &\times \int d\Omega_k Y_{\lambda\mu}(\Omega_k) Y_{\lambda_1\mu_1}^*(\Omega_k) Y_{\lambda_2\mu_2}^*(\Omega_k). \end{aligned} \quad (8)$$

Defining an angular momentum coupling coefficient,

$$\begin{aligned} \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \mu_1 & \mu_2 & \mu_3 \end{bmatrix} &= \int d\Omega_k Y_{\lambda\mu}(\Omega_k) Y_{\lambda_1\mu_1}^*(\Omega_k) Y_{\lambda_2\mu_2}^*(\Omega_k) \\ &= (-1)^{\mu_3} \sqrt{\frac{(2\lambda_1+1)(2\lambda_2+1)(2\lambda_2+1)}{4\pi}} \\ &\quad \times \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \mu_1 & \mu_2 & -\mu_3 \end{pmatrix} \end{aligned} \quad (9)$$

and the multipoles of the densities,

$$M_{\lambda\mu}^{(i)}(k) = \int d\vec{r} \rho_i(\vec{r}) j_\lambda(kr) Y_{\lambda\mu}(\Omega_r), \quad (10)$$

we can rewrite the expression for the interaction as

$$\begin{aligned} V(\vec{R}, \alpha) &= 32\pi e^2 \sum_{\lambda\lambda_1\lambda_2} Y_{\lambda\mu}^*(\Omega_R) i^{\lambda+\lambda_2-\lambda_1} \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda \\ \mu_1 & \mu_2 & \mu \end{bmatrix} \\ &\quad \times \int_0^\infty dk j_\lambda(kR) M_{\lambda_1\mu_1}^{(1)}(k) M_{\lambda_2\mu_2}^{(2)}(k). \end{aligned} \quad (11)$$

Substituting the sharp-radius approximation to the density in the expression for the multipoles, we can rewrite the latter as

$$M_{\lambda\mu}^{(i)}(k) = \frac{\rho_{oi}}{k^3} \int d\Omega_r Y_{\lambda\mu}(\Omega_r) \int_0^{kR_i(\Omega_r)} x^2 j_\lambda(x) dx. \quad (12)$$

To evaluate these, we expand them to second order in the deformations,

$$\begin{aligned} M_{\lambda\mu}^{(i)}(k) &= \frac{\rho_{oi}}{k^3} \left\{ \delta_{\lambda 0} \sqrt{4\pi} \int_0^{x_i} y^2 j_0(y) dy + x_i^3 j_\lambda(x_i) a_{\lambda\mu}^{(i)} \right. \\ &\quad + x_i^2 \left(x_i j_\lambda(x_i) + \frac{x_i^2}{2} \frac{dj_\lambda}{dx_i} \right) \sum_{\lambda_a\lambda_b} a_{\lambda_a\mu_a}^{(i)} a_{\lambda_b\mu_b}^{(i)} \\ &\quad \left. \times \int d\Omega_r Y_{\lambda\mu}(\Omega_r) Y_{\lambda_a\mu_a}^*(\Omega_r) Y_{\lambda_b\mu_b}^*(\Omega_r) \right\}, \end{aligned} \quad (13)$$

where $x_i = kR_i$. Using the angular coupling coefficients and the normalization condition, Eq. (6), we obtain

$$\begin{aligned} M_{\lambda\mu}^{(i)}(k) &= \frac{3Z_i}{4\pi} \left(1 - \frac{3}{4\pi} \sum_{\lambda\mu} |a_{\lambda\mu}^{(i)}|^2 \right) \left\{ \delta_{\lambda 0} \sqrt{4\pi} \frac{j_\lambda(x_i)}{x_i} + j_\lambda(x_i) a_{\lambda\mu}^{(i)} \right. \\ &\quad + \left(j_\lambda(x_i) + \frac{x_i}{2} \frac{dj_\lambda}{dx_i} \right) \\ &\quad \left. \times \sum_{\lambda_a\lambda_b} \begin{bmatrix} \lambda_a & \lambda_b & \lambda \\ \mu_a & \mu_b & \mu \end{bmatrix} a_{\lambda_a\mu_a}^{(i)} a_{\lambda_b\mu_b}^{(i)} \right\}. \end{aligned} \quad (14)$$

We now substitute $M_{\lambda\mu}^{(i)}(k)$ in Eq. (11) and retain only terms to second order in the deformations. We can systematize the resulting expression by defining the following integrals:

$$F_0^{(0)}(R, X, Y) = \frac{18}{\pi} \int_0^\infty dk j_0(kR) \frac{j_1(kX)}{kX} \frac{j_1(kY)}{kY}, \quad (15)$$

$$F_\lambda^{(1)}(R, X, Y) = \frac{18}{\pi} \int_0^\infty dk j_\lambda(kR) j_\lambda(kX) \frac{j_1(kY)}{kY}, \quad (16)$$

$$F_\lambda^{(2)}(R, X, Y) = \frac{18}{\pi} \int_0^\infty dk j_\lambda(kR) \left[j_\lambda(kX) + \frac{kX}{2} \frac{dj_\lambda}{d(kX)} \right] \frac{j_1(kY)}{kY}, \quad (17)$$

$$F_{\lambda\lambda_a\lambda_b}^{(3)}(R, X, Y) = \frac{18}{\pi} \int_0^\infty dk j_\lambda(kR) j_{\lambda_a}(kX) j_{\lambda_b}(kY). \quad (18)$$

Properties and methods for the efficient numerical evaluation of these integrals can be found in Refs. [3,4].

Furthermore, to simplify the angular dependence, we take as the reference axis the one defined by \vec{R} . We then assume that the deformation parameters are defined through the independent rotations of a set of axially symmetric deformations determined by the parameters $\beta_{\lambda i}$. In this scenario, $\theta_{\lambda i}$ and $\phi_{\lambda i}$ represent the orientation of the symmetry axis of the λ deformation of nucleus i relative to \vec{R} . Thus, the Coulomb potential can be obtained from

$$V(\vec{R}, \alpha) = V_0(R) + V_{Cor}(\vec{R}, \alpha), \quad (19)$$

$$V_{Cor}(\vec{R}, \alpha) = V_1(\vec{R}, \alpha) + V_2^{(0)}(\vec{R}, \alpha) + V_2(\vec{R}, \alpha), \quad (20)$$

where

$$V_0(R) = Z_1 Z_2 e^2 F_0^{(0)}(R, R_1, R_2), \quad (21)$$

$$\begin{aligned} V_1(\vec{R}, \alpha) &= Z_1 Z_2 e^2 \sum_\lambda [F_\lambda^{(1)}(R, R_1, R_2) Y_{\lambda 0}^*(\theta_{\lambda 1}, 0) \beta_{\lambda 1} \\ &\quad + (-1)^\lambda F_\lambda^{(1)}(R, R_2, R_1) Y_{\lambda 0}^*(\theta_{\lambda 2}, 0) \beta_{\lambda 2}], \end{aligned} \quad (22)$$

$$\begin{aligned} V_2^{(0)}(\vec{R}, \alpha) &= -\frac{Z_1 Z_2 e^2}{4\pi} \sum_\lambda [3(\beta_{\lambda 1}^2 + \beta_{\lambda 2}^2) F_0^{(0)}(R, R_1, R_2) \\ &\quad - F_0^{(2)}(R, R_1, R_2) \beta_{\lambda 1}^2 - F_0^{(2)}(R, R_2, R_1) \beta_{\lambda 2}^2 \\ &\quad - F_{0\lambda\lambda}^{(3)}(R, R_1, R_2) \beta_{\lambda 1} \beta_{\lambda 2}], \end{aligned} \quad (23)$$

$$\begin{aligned} V_2(\vec{R}, \alpha) &= Z_1 Z_2 e^2 \left\{ \sum_{\lambda \neq 0} (2\lambda + 1) \begin{pmatrix} \lambda_a & \lambda_b & \lambda \\ 0 & 0 & 0 \end{pmatrix} \right. \\ &\quad \times [F_\lambda^{(2)}(R, R_1, R_2) \beta_{\lambda_a 1} \beta_{\lambda_b 1} \\ &\quad \times H_{\lambda\lambda_a\lambda_b}(\theta_{\lambda_a 1}, \phi_{\lambda_a 1}; \theta_{\lambda_b 1}, \phi_{\lambda_b 1}) \\ &\quad \left. + (-1)^\lambda F_\lambda^{(2)}(R, R_2, R_1) \beta_{\lambda_a 2} \beta_{\lambda_b 2} \right\} \end{aligned}$$

$$\left. \begin{aligned} & \times H_{\lambda\lambda_a\lambda_b}(\theta_{\lambda_a 2}, \phi_{\lambda_a 2}; \theta_{\lambda_b 2}, \phi_{\lambda_b 2}) \\ & + i^{\lambda+\lambda_b-\lambda_a} F_{\lambda\lambda_a\lambda_b}^{(3)}(R, R_1, R_2) \beta_{\lambda_a 1} \beta_{\lambda_b 2} \\ & \times H_{\lambda\lambda_a\lambda_b}(\theta_{\lambda_a 1}, \phi_{\lambda_a 1}; \theta_{\lambda_b 2}, \phi_{\lambda_b 2}) \end{aligned} \right\}, \quad (24)$$

$$H_{\lambda\lambda_a\lambda_b}(\theta_1, \phi_1; \theta_2, \phi_2) = \sum_{\mu_a \mu_b} \begin{pmatrix} \lambda_a & \lambda_b & \lambda \\ \mu_a & \mu_b & 0 \end{pmatrix} Y_{\lambda_a \mu_a}^*(\theta_1, \phi_1) \times Y_{\lambda_b \mu_b}^*(\theta_2, \phi_2). \quad (25)$$

The term V_0 is just the Coulomb potential between two uniformly charged spheres. The potential V_1 does not depend on the azimuthal angles ϕ_{λ_i} , since it is the contribution of the deformation perturbation of one nucleus with the spherical distribution of the other. The monopole term $V_2^{(0)}$ arises from the normalization of the densities, Eq. (6). This term was not present in Ref. [1], since it was assumed there that $4\pi R_i^3 \rho_{oi}/3 \approx Z_i$.

An analysis of the asymptotic behavior ($R > X+Y=R_1+R_2$) of our expressions shows that

$$F_0(R, X, Y) = \frac{1}{R}, \quad (26)$$

$$F_\lambda^{(1)}(R, X, Y) = \frac{3}{2\lambda+1} \frac{X^\lambda}{R^{\lambda+1}}, \quad (27)$$

$$F_\lambda^{(2)}(R, X, Y) = \frac{3}{2} \frac{(\lambda+2)}{(2\lambda+1)} \frac{X^\lambda}{R^{\lambda+1}}, \quad (28)$$

$$F_{\lambda\lambda_a\lambda_b}^{(3)}(R, X, Y) = 0, \quad \lambda \neq \lambda_a + \lambda_b. \quad (29)$$

In particular, we have $F_0^{(2)}(R, X, Y) = 3/R$ and $F_{0\lambda\lambda}^{(3)}(R, X, Y) = 0$ so that the term $V_2^{(0)}$ vanishes and the monopole potential reduces to $V_0(R) = Z_1 Z_2 e^2 / R$ as it should. The first-order correction becomes

$$V_1(\vec{R}, \alpha) = 3Z_1 Z_2 e^2 \sum_{\lambda} \frac{1}{(2\lambda+1)R^{\lambda+1}} [R_1^\lambda \beta_{\lambda 1} Y_{\lambda 0}^*(\theta_{\lambda 1}, 0) + R_2^\lambda \beta_{\lambda 2} Y_{\lambda 0}^*(\pi - \theta_{\lambda 2}, 0)]. \quad (30)$$

The angular momentum sums in V_2 cannot be reduced, in general, so we do not attempt to write a simplified form for it.

We now generalize our expressions for nonvanishing diffuseness. We represent the density by a deformed two-parameter Fermi distribution,

$$\rho_i(\vec{r}) = \frac{\rho_{oi}}{1 + \exp\left(\frac{r - R_i(\Omega)}{a}\right)}. \quad (31)$$

We define the corresponding nondeformed density by

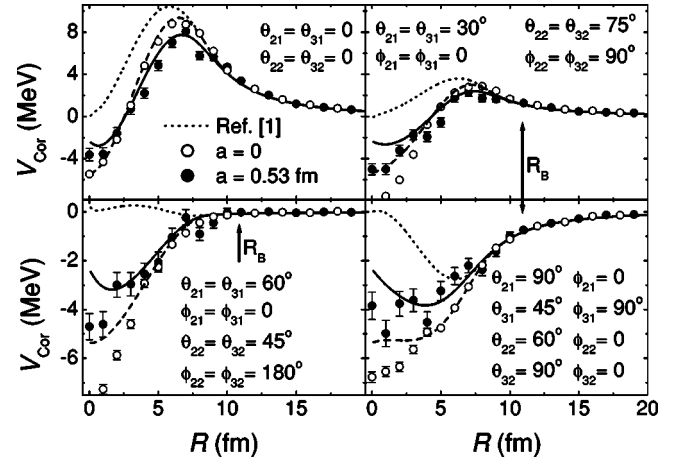


FIG. 1. The figure presents the correction of the Coulomb potential obtained through the numerical resolution of the six-dimensional integral, Eq. (1), for $a=0$ (open circles) and $a=0.53$ fm (closed circles). The calculations were performed for the $^{58}\text{Ni}+^{58}\text{Ni}$ system, considering only quadrupole and octupole deformations: $\beta_2=0.205$ and $\beta_3=0.235$ for both nuclei. The angles of the deformation axes are indicated in the figure. The lines correspond to the approximate expressions for V_{Cor} presented in Ref. [1] and in the present work. The arrows indicate the position of the s -wave barrier radius.

$$\rho_i^{(0)}(r) = \frac{\rho_{oi}^{(0)}}{1 + \exp\left(\frac{r - R_i}{a}\right)}. \quad (32)$$

One should observe that, due to the normalization given in Eq. (2), $\rho_{oi} \neq \rho_{oi}^{(0)}$. The Coulomb potential can be obtained using the same set of equations (19)–(25), but substituting the terms $j_1(kR_i)/kR_i$ with $\hat{\rho}_i(k)/3$ in the form factors of Eqs. (15)–(17), where

$$\hat{\rho}_i(k) = \frac{4\pi}{Z_i} \int_0^\infty J_0(kr) r^2 \rho_i^{(0)}(r) dr. \quad (33)$$

In Ref. [1] this procedure was applied only to the V_0 term,

$$V_0(R) = \frac{2Z_1 Z_2 e^2}{\pi} \int_0^\infty J_0(kR) \hat{\rho}_1(k) \hat{\rho}_2(k) dk. \quad (34)$$

The present method implies that V_{Cor} also depends on the value of the diffuseness parameter.

We point out that, in Refs. [1,5], we provided an extensive systematics for the radius, diffuseness, and deformation parameters of the densities. We found that the radii of the charge distributions are well represented by

$$R_i = 1.76Z_i^{1/3} - 0.96 \text{ fm}, \quad (35)$$

with an average diffuseness value of $a=0.53$ fm.

III. DISCUSSION AND CONCLUSION

We have calculated Eq. (1) using a Monte Carlo method and obtained the corresponding values for $V_{Cor}(\vec{R}, \alpha)$

$=V(\vec{R}, \alpha) - V_0(R)$. These results thus have statistical uncertainties. Figure 1 shows the V_{Cor} values obtained for $a=0$ (open circles) and $a=0.53$ fm (closed circles) for four different sets of deformation angles in the case of the $^{58}\text{Ni} + ^{58}\text{Ni}$ system. The dotted lines represent the approximate results for V_{Cor} obtained with the method presented in Ref. [1], while the dashed ($a=0$) and solid ($a=0.53$ fm) lines correspond to the results of the present work. Clearly, the new method provides much better approximations than those obtained in Ref. [1], and reproduces with very good accuracy the exact results from the surface region to quite internal distances. However, the major differences appear at distances smaller than the barrier radius (see Fig. 1). This inner region can be probed through scattering experiments only at high bombarding energies. Even so, the method presented in this work is superior to that of Ref. [1], because it provides better

precision with approximately the same amount of numerical calculation. In fact, the calculation of V_{Cor} through the six-dimensional integral of Eq. (1) required about five days of CPU time for each configuration of the deformation angles, while the corresponding calculation with the approximate expressions presented in this work (and also with those of Ref. [1]) took less than a minute.

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