

**Simple functional form for the  $n + {}^{208}\text{Pb}$  total cross section between 5 and 600 MeV**

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The total cross section for neutron scattering from  ${}^{208}\text{Pb}$  with energies between 5 and 600 MeV has been analyzed, extending a previously defined simple function of three parameters to reveal a Ramsauer-like effect throughout the whole energy range. This effect can be parametrized in a simple way so that it may be anticipated that the complete function prescription will apply for total cross sections from other nuclei.

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Total cross sections from the scattering of nucleons by nuclei and for energies to 600 MeV or more are required in a number of fields of study in basic science as well as many of applied nature. Often, those cross sections have been evaluated using phenomenological optical potentials and much effort has gone into defining global sets of parameter values for optical potentials with which to estimate cross sections as yet unmeasured. In a recent study, Koning and Delaroche [1] gave a detailed specification of such. However, it would be useful and convenient if total cross sections could be well approximated by a simple function form. We show herein, in the case of neutron scattering from  ${}^{208}\text{Pb}$ , that there is a simple three-parameter function form one can use to form estimates without recourse to optical potential calculations. With data from other nuclei having so similar an appearance, the same form can be used for any target. Further, the required values of the three parameters of that function form themselves trend sufficiently smoothly with energy that they may be interpolated to estimate any cross section value at energies that have not been measured.

We also investigate whether a simple functional form for energy variation of the parameters themselves may exist. But like the cross section at relatively low incident energies, one parameter required to fit data exhibits a noticeably large scale structure. With actual data, that effect, defined as the Ramsauer effect, varies smoothly with target mass so that in the past it has been attributed to characteristics of the nuclear geometry and was interpreted semiclassically [2] as due to the interference between parts of the scattering wave function passing through the nuclear medium and parts that do not. The focus in optical model wave functions [3] is another result of such interference. The variation of the total cross sections then was formulated as the Ramsauer model. Herein we use that Ramsauer model in conjunction with the simple three-parameter function form and find that the measured total cross-section data can be well fitted.

The utility of the Ramsauer model has been demonstrated recently with extended versions used to study the isospin

effect noted in comparison of total neutron cross sections from select medium mass nuclei [4] and to interpret zero angle ( $p, n$ ) cross-section data [5]. Also, by using Wick's limit, estimations of neutron reaction cross sections were made [6]. This portends an interesting development relevant to our approach. If the functional form scheme gives results satisfying Wick's limit to within a few percent, then the approach [6] to specify the total reaction cross section for the same wide span of energy may be used with some confidence. Inverse scattering theory [7–9] may then be used to define a complex local optical potential that reproduces those data and which is essentially free of any model prescription.

While the total cross sections and their large scale structures should be the result of a more sophisticated specification of the optical potential describing neutron-nucleus scattering, and indeed aspects of the Ramsauer effect have been elicited from a  $g$ -folding optical potential [10,11], the predicted results never quite match satisfactorily the observed Ramsauer structure especially at energies below 50 MeV.

The total cross sections for nucleon scattering from nuclei can be expressed in terms of partial wave scattering matrices specified at energies  $E \propto k^2$ , namely,  $S_l^\pm(k) = \eta_l^\pm(k) e^{2i \text{Re}[\delta_l^\pm(k)]}$ , where  $\delta_l^\pm(k)$  are the (complex) scattering phase shifts and  $\eta_l^\pm(k)$  are the moduli of the  $S$  matrices. The superscript designates  $j = l \pm 1/2$ . In terms of these quantities, the total cross section is

$$\begin{aligned} \sigma_{\text{tot}}(E) &= \frac{2\pi}{k^2} \sum_l \sigma_{\text{tot}}^{(l)}(E) \\ &= \frac{2\pi}{k^2} \sum_l [(l+1)(1 - \eta_l^+(k) \cos\{2 \text{Re}[\delta_l^+(k)]\}) \\ &\quad + l(1 - \eta_l^-(k) \cos\{2 \text{Re}[\delta_l^-(k)]\})]. \end{aligned} \quad (1)$$

There are equivalent forms for the total reaction and total elastic cross sections and a study of such cross-section data [12,13] established that the partial total cross sections may be described by a simple function form

$$\begin{aligned} \sigma_{\text{tot}}^{(l)}(E) &\equiv \sigma_{\text{th}}^{(l)}(E) = (2l+1)(1 + e^{(l-l_0)/a})^{-1} \\ &\quad + \epsilon(2l_0+1)e^{(l-l_0)/a}(1 + e^{(l-l_0)/a})^{-2}. \end{aligned} \quad (2)$$

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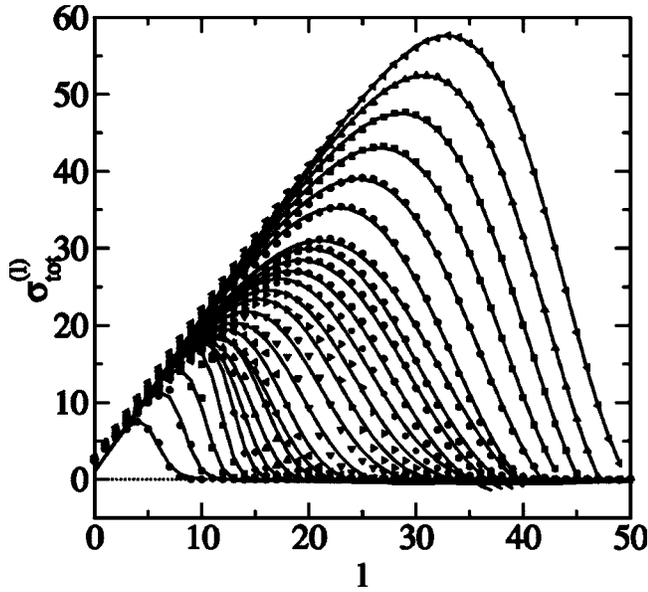


FIG. 1. The partial total cross sections for scattering of neutrons from  $^{208}\text{Pb}$  for energies between 10 and 600 MeV. The largest energy has the broadest spread of values. The “data” were obtained from  $g$ -folding optical potential calculations.

As with our previous studies, this form for the total cross section is suggested by the values of the partial total cross sections found from energy-dependent, optical potentials generated from a  $g$ -folding formalism [10]. With that form excellent reproduction of the proton total reaction cross sections for many targets and over a wide range of energies was found with parameter values that varied smoothly with energy and mass. For the case of scattering from  $^{208}\text{Pb}$ , Skyrme-Hartree-Fock model (SKM\*) densities [14] have been used to form the  $g$ -folding optical potentials to give the initiating parameter values. That structure when used to analyze proton and neutron scattering differential cross sections at 65 and 200 MeV gave quite excellent results [15]. Indeed those analyses were able to show selectivity for that SKM\* model of structure and for the neutron skin thickness of 0.17 fm that it proposed. That same structure model has been used in forming  $g$ -folding optical potentials from which our initial guess at the structure of the partial total cross section values at all energies to 600 MeV were obtained.

The partial total cross sections from  $g$ -folding optical potential calculations for neutron scattering from  $^{208}\text{Pb}$  are shown in Fig. 1. Those values are shown as diverse open and closed symbols in Fig. 1 and we take them as “data” against which to find the first guess for the energy variation of the three parameters in Eq. (2). Each curve shown in that figure is the result of a search for the best fit values of the three parameters  $l_0$ ,  $a$ , and  $\epsilon$ . From the sets of values that result from that fitting process, the two parameters  $a$  and  $\epsilon$  can themselves be expressed by the parabolic functions

$$\begin{aligned} a &= 1.29 + 0.00250E - 1.76 \times 10^{-6}E^2, \\ \epsilon &= -1.47 - 0.00234E + 4.16 \times 10^{-6}E^2. \end{aligned} \quad (3)$$

With  $a$  and  $\epsilon$  so fixed, we then adjusted the values of  $l_0$  in each case so that actual measured neutron total cross-section

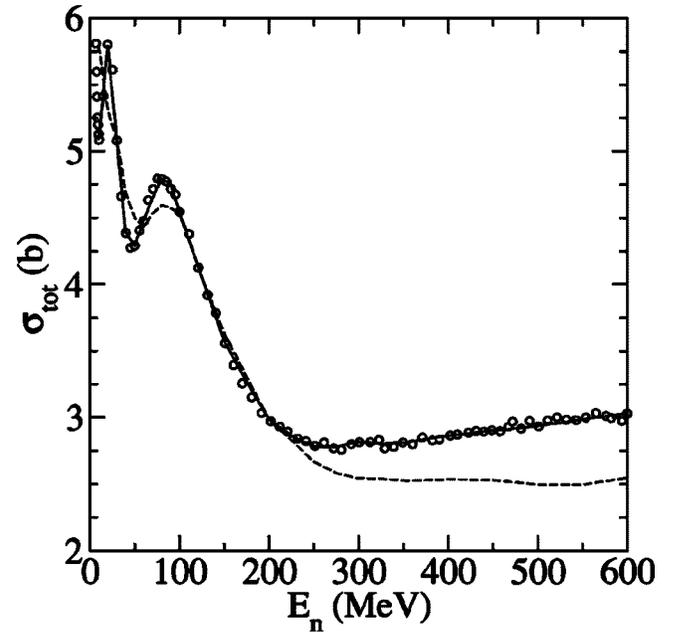


FIG. 2. Total cross sections for  $n$ - $^{208}\text{Pb}$  scattering. The curves are as defined in the text.

data were fitted using Eq. (2). The quality of fits to measured total cross-section data are shown in Fig. 2. Using the SKM\* model structure, the  $g$ -folding optical potentials gave the total cross sections shown by the dashed curve in Fig. 2. Clearly there is a need to improve this model for energies at and above pion threshold. Nonetheless, it does do quite well for lower energies, most notably giving a reasonable account of the Ramsauer resonance [1] near 100 MeV. However, recall that these  $g$ -folding values serve only to provide a set of partial cross sections to define an initial set of the three parameters of the function form. With  $a$  and  $\epsilon$  set by Eq. (3), adjustment of  $l_0(E)$  produces the solid curve shown in Fig. 2; an excellent reproduction of the data, as it was designed to be. There are obvious oscillations in that tabulation of values of  $l_0(E)$  for energies below 100 MeV reflecting the Ramsauer effects in the cross-section data. But for energies above 250 MeV, the  $l_0$  values are well approximated as a straight line. Without retaining the excellent fit to values below 100 MeV, a simple representation of the  $l_0$  values that is useful is the energy-dependent function

$$l_0^{\text{th}}(E) = 0.0384E + 16.28 - 11.22 \left( 1 - \frac{E}{132.9} \right) e^{-0.0164E}. \quad (4)$$

With  $a$  and  $\epsilon$  specified by Eq. (3), this function form leads to an average background total cross section upon which a regular oscillatory contribution from the Ramsauer effect is found. Of course, phenomenological optical potentials by appropriate parameter adjustments [1] will define the relevant cross sections well for all energies. But if a global (smooth) variation of those parameter values is made then the quality of reproduction deteriorates.

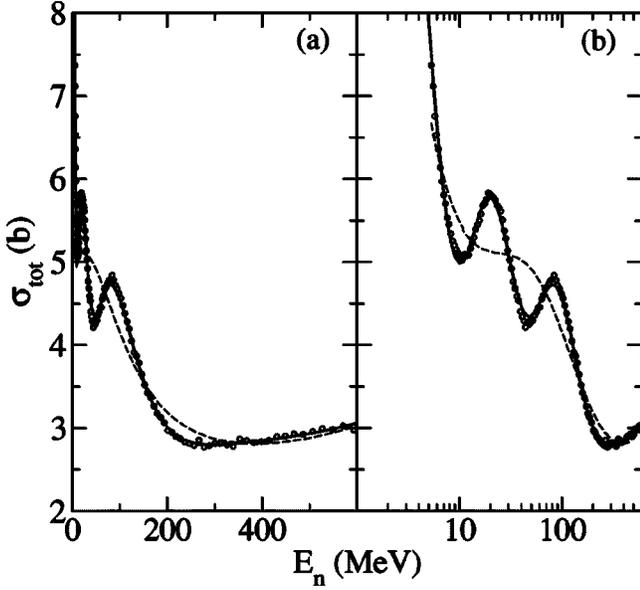


FIG. 3. The optimal background cross section with the data from  $n$ - $^{208}\text{Pb}$  scattering. The curves are as for Fig. 2.

A Ramsauer-like effect has been included in past data analyses to describe the large scale variations of total cross sections from an otherwise smooth monotonic background [6]. Under approximation, this correction is a coherent scaling of a theoretical model (diffraction, global optical, or functional form) of the background, namely,

$$\sigma_{\text{tot}}(E) \equiv \sigma_{\text{th}}(E)\{1 - \alpha(E)\cos[\beta(E)]\}. \quad (5)$$

Dietrich *et al.* [6] also linked this by Wick's limit to extract reaction cross sections. Wick's limit is that of an inequality involving the zero degree cross section and which arises from the optical theorem, namely, since

$$\text{Im}[f(0^\circ)] = \frac{k}{4\pi}\sigma_{\text{tot}}(E),$$

$$\sigma(0^\circ) = |f(0^\circ)|^2 \geq \{\text{Im}[f(0^\circ)]\}^2 \geq \left(\frac{k}{4\pi}\sigma_{\text{tot}}(E)\right)^2. \quad (6)$$

The equality specifies the cross section at the Wick limit,  $\sigma^W(0^\circ)$ . Then as the Ramsauer model is based upon a form for the scattering amplitude of

$$f(\theta) = i\frac{k}{4\pi}\sigma_{\text{th}}(E)[1 - \alpha(E)e^{i\beta(E)}], \quad (7)$$

the zero degree cross section is

$$\begin{aligned} \sigma(0^\circ) &= \left(\frac{k}{4\pi}\sigma_{\text{th}}(E)\right)^2 \{[1 - \alpha\cos(\beta)]^2 + \alpha^2\sin^2(\beta)\} \\ &= \sigma^W(0^\circ) \left(1 + \frac{\alpha^2\sin^2(\beta)}{[1 - \alpha\cos(\beta)]^2}\right). \end{aligned} \quad (8)$$

In this and the next equation the energy variable has been omitted for convenience. Thus the validity of Wick's limit in

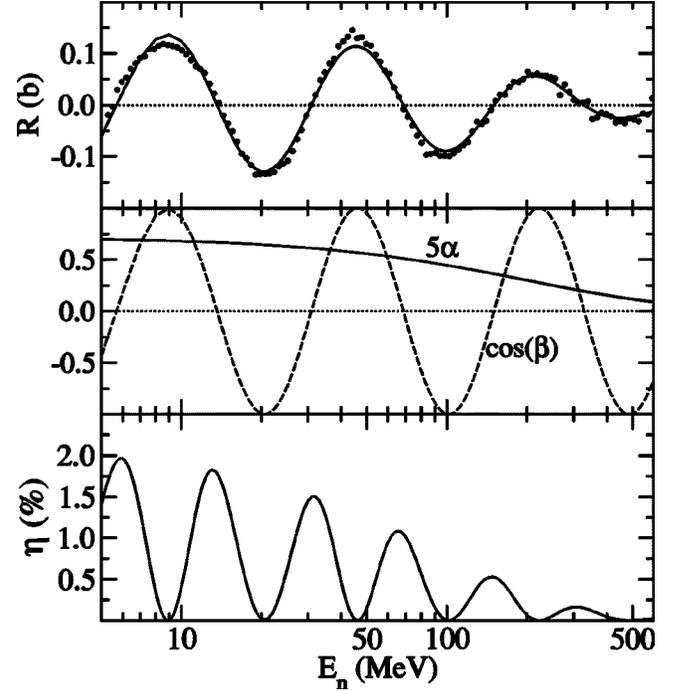


FIG. 4. The Ramsauer corrections in data (top), parameters (middle), and fractional deviation from Wick's limit (bottom) based upon the three-parameter functional form model.

this model is measured by the fractional deviation found for the zero degree cross-section expectation,

$$\eta(\%) = 100 \frac{[\sigma(0^\circ) - \sigma^W(0^\circ)]}{\sigma^W(0^\circ)} = 100 \left( \frac{\alpha \sin(\beta)}{1 - \alpha \cos(\beta)} \right)^2. \quad (9)$$

Results for  $n$ - $^{208}\text{Pb}$  scattering are displayed in Figs. 3 and 4. With the particular choice for the energy dependence of  $l_0(E)$  as given in Eq. (4), we note a Ramsauer effect that continues to high energies but which has a very regular character. The total cross sections are shown linearly with energy in the (a) segment in Fig. 3 while they are displayed on a logarithmic-linear plot in the (b) segment. The (a) segment illustrates the overall smoothness of the data variation and suggests that the functional form results are very good for energies above 150 MeV. Indeed adjustment of the values in Eq. (4) can give a much better fit to the high energy data, but the chosen set give a more regular behavior to the Ramsauer effect contributions. The logarithmic-linear plot emphasizes the low energy regime and illustrates more clearly the Ramsauer effect. The (b) panel also reveals that the Ramsauer effect has a sinusoidal variation with  $\log_{10}(E)$  and that the wavelength is  $\sim 0.7 \log_{10}(E)$ .

In Fig. 4, details of the Ramsauer correction given the chosen background form are displayed. All three graphs are shown in linear-logarithmic form to emphasize the scale in which the corrections are nearest to sinusoidal. In the top panel we show the factor  $R(E) = \alpha(E)\cos[\beta(E)]$ . Clearly the Ramsauer variation has a regular behavior with energy with the wavelength indicated above. Also in this scale, the two

parameters  $\alpha(E)$  and  $\beta(E)$  vary smoothly with energy. The former is bounded by  $\pm 0.2$  and monotonically decreases with energy. Finally in the bottom panel we show the fractional deviation from Wick's limit as defined by Eq. (9). Never does this analysis exceed a 2% violation of that limit and so the approach of Dietrich *et al.* [13] to extract neutron reaction cross sections from the total cross sections, and with the reasonable accuracy that those authors require, may be used then with the functional form for the background cross section.

Of course the actual effect displayed depends upon what is used as the smooth background. However, the indicated maxima and minima should be a good first order estimate. These results also show that the Ramsauer effect is not necessarily constrained to low energies as one may have assumed. Indeed, save for very low energies where channel coupling and discrete resonance features in scattering are expected to be important, the effect has been and should be a feature of the most appropriate optical potential [4,6] and which should be complex, energy dependent and nonlocal [10]. Such if formed microscopically [10] require, at the very least, an appropriate complex medium and energy-dependent effective interaction between the incident nucleon and each and every bound target nucleon as well as a significant large basis, many nucleon prescription of that target. That is a not

insignificant endeavor though for  $^{208}\text{Pb}$  just such an effort led to aspects of the Ramsauer structure in the total cross section [11] as well as showing that analyses of proton scattering data provide a very reasonable criterion for the skin thickness [15].

We suggest a simple function form for partial total cross sections whose sum, when scaled by the Ramsauer effect, will give neutron- $^{208}\text{Pb}$  total cross sections for any energy without recourse to phenomenological optical potential parameter searches. That basic function form also reproduces proton reaction cross sections. The parameters that fit actual data show smooth trends with energy. Our results suggest that the Ramsauer effect, visually obvious for energies below 100 MeV, persists at all energies. Further, with our functional form, Wick's limit remains valid to within 2% and so may be used to find the total reaction cross section by using the method of Dietrich *et al.* [6]. With both data, inverse scattering theory may be used to specify an optical potential without recourse to defined radial forms. Finally, given the similarity in the shape of total cross section data from many nuclei, the method should be universal.

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