Correlated two-pion exchange in peripheral *NN* scattering

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The correlated two-pion exchange in the mesonic σ channel as derived using chiral symmetry and large-N_c constraints is shown to improve the description of the proton-proton (pp) peripheral partial waves $(L \geqslant 3)$ below and above the inelastic threshold. With inclusion of one-pion exchange it, furthermore, can account for the energy dependence observed for uncoupled partial waves. The coupled waves are signaling the need for an additional long-range tensor force.

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Structure and behavior of the peripheral *NN* partial waves $(L \geqslant 3)$ are closely connected with the dynamics of the lightest QCD mode—the pion, which is a Goldstone boson of the spontaneously broken $SU_L \times SU_R$ chiral symmetry. Furthermore, peripheral *NN* scattering is largely perturbative $\mathcal{T}_{NN} \simeq \mathcal{V}_{NN}$ suggesting an unique way for directly accessing the *NN* force. The long-range part of the *NN* interaction is well established and is represented by one-pion exchange (OPE). The OPE accounts for the bulk of the energy dependence of the peripheral *NN* phase shifts up to $T_{\text{lab}} \simeq 100$ MeV and its presence in the *NN* force can be confirmed with high statistical significance. For lower partial waves and higher momentum transfer where the *NN* dynamics becomes complex, the physical transparency underlying the analysis in terms of OPE is lost and one has to rely on phenomenological models or take into account the multipion exchange explicitly. In this region the σ meson [1] effectively represents the correlated scalar-isoscalar two-pion exchange and is an important ingredient in microscopic and one-boson exchange (OBE) models.

Convincing evidence for the presence of the genuine two-pion exchange (TPE) in the long-range *NN* force has been presented in Ref. [2]. There based on OPE and chiral TPE a quantitative description of the elastic *pp* scattering data below the inelastic threshold has been obtained. In baryon chiral perturbation theory (BχPT), which is utilized in the analysis of Ref. [2], the TPE is understood mainly as uncorrelated exchange and all other higher order effects, such as *t*-channel singularities or Δ-isobar degrees of freedom are effectively included in the set of low-energy constants (LEC's). The important feature of the BχPT is that it can treat the TPE force in a model independent way [3]. The χPT machinery is usually employed below the pion production threshold, where a high-quality description of the peripheral *NN* scattering is already available [4]. For higher energies, where the σ-like $\pi\pi$ correlations are expected to be seen, the high-momentum components of the exchanged pions become important—a feature which cannot be properly treated in the dimensionally regulated BχPT [5]. The long-range behavior of the TPE force and the effect of correlated σ-like scalar-isoscalar $\pi\pi$ exchange are related problems. The success and dynamical content of OBE models suggest that meson-meson correlations are important. This is supported by dispersion theory, where the main effects of higher-order interactions can be accounted for by inclusion of experimentally known meson resonances. Note, that in dispersion theory σ exchange can be explained by correlated two-pion exchange (CrTPE) with a broad spectral distribution which leads to the isoscalar central attraction between two nucleons [6]. At the same time both the empirical evidence for the σ meson and the role played by the CrTPE in the *NN* interaction remain controversial. Interestingly, in BχPT the proper consideration of the tree level scalar-isoscalar $\pi\pi$ correlations (first diagram in Fig. 1) shows that these terms are small and, even more, lead to a weak repulsion [7].

Recently it was pointed out that—in the context of the unitary χPT [8]—the effective σ exchange of OBE models does not represent the actual correlated $\pi\pi$ exchange in the scalar-isoscalar channel. Recall that in χPT, with nonlinear realization of chiral symmetry, the σ state shows up as a resonance in the $\pi\pi$ system and can be generated dynamically by resummation of infinite series of chiral loops [9]. It was shown [8] that coupling of the unitary $\pi\pi$ correlation function, which contains a dynamical pole corresponding to the σ state, to the *NN* system, Fig. 1, generates a central scalar-isoscalar *NN* force which has an unconventional behavior. It is strongly repulsive at scales less than $\simeq 1$ fm and has only a moderate attraction at internucleon distances >1 fm. Note that a similar structure of the scalar-isoscalar central *NN* potential was previously discovered in the Skyrme model [10]. This feature of the dynamically generated σ meson differs from the effective σ meson in OBE models. The latter leads to a central potential which is always attractive.

In our previous work [11] we reconsidered the role played by the CrTPE in the *NN* interaction for the construction of

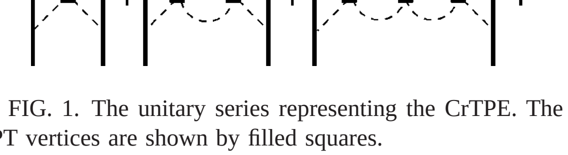

FIG. 1. The unitary series representing the CrTPE. The $\mathcal{O}(p^2)$ χPT vertices are shown by filled squares.

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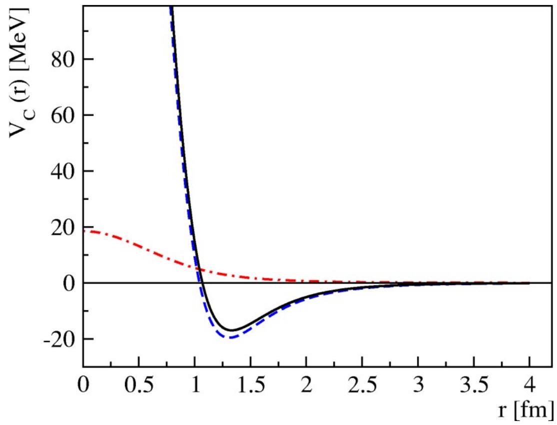

FIG. 2. (Color online) The central CrTPE potential in coordinate space (solid curve). The parts nonvanishing and vanishing in the chiral limit are shown by the dashed and dot-dashed curves, respectively.

the scalar-isoscalar central and spin-orbit *NN* potentials. We found that the abovementioned unconventional behavior of the central *NN* force is general. It mainly relies on the particular functional form of the scalar form factor of the nucleon and is not the effect of unitarization. We have considered the effect of the tree level and unitary two-pion correlations on the *NN* interaction in the limit of a large number of colors (large-N_c limit) using the hedgehog *Ansätze* of the quark-soliton model [12]. This way we confirmed the structure of the central *NN* force observed in the unitarized χPT [8]. We found (see Fig. 2) a strong repulsion at short distances and a moderate attraction at $r > 1$ fm—even with the tree level $\pi\pi$ interaction. Interestingly, the long-range tail of the central CrTPE potential is driven by the the pion-nucleon (πN) sigma term and saturates the LEC c_1 from BχPT. In this tail region it also is consistent with the effective σ exchange in OBE models. In addition, the structure of the *NN* force is consistent with the Kaplan-Manohar large-N_c power counting [13], with large-N_c behavior of phenomenological OBE models [14] and is dominated by terms which are nonvanishing in the chiral limit.

The goal of this work is to test the findings in Ref. [8] and in our recent analysis [11] against empirical data. Another point is to clarify the possible role played by the CrTPE and particularly by the dynamical σ state in the *NN* interaction. In the approach followed here to show the effect we use the analytic representation of the central scalar-isoscalar CrTPE force derived in Ref. [11]. The latter can represent rather well the full unitary series of chiral loops generating the dynamical σ state and is completely determined by the axial coupling constant g_A, pion mass M_π, and pion decay constant f_π. It is given by

$$\mathcal{V}_C(r) = \mathcal{V}_C^{(1)}(r) + \mathcal{V}_C^{(2)}(r), \tag{1}$$

where superscripts (1) and (2) refer to the parts nonvanishing and vanishing in the chiral limit, respectively,

$$\mathcal{V}_C^{(1)}(r) = 16\pi^2 f_\pi \sqrt{\frac{2\,g_A}{3\pi}} \times \left[\frac{80 + 8\tilde{\xi} + 3\tilde{\xi}^2 - 3\tilde{\xi}^3}{(2+\tilde{\xi})^2(4+\tilde{\xi}+2\sqrt{2\tilde{\xi}})^2(4+\tilde{\xi}-2\sqrt{2\tilde{\xi}})^2} \right], \tag{2}$$

$$\mathcal{V}_C^{(2)}(r) = \frac{M_\pi^2}{r}\left(\frac{3\,g_A^2}{64\,f_\pi^2}\right)\left[\arctan\left(\sqrt{\frac{\tilde{\xi}}{2}}\right) - \mathcal{D}(\tilde{\xi})\right], \tag{3}$$

where $\tilde{\xi} = 16\pi r^2 f_\pi^2/(3g_A)$ and

$$\mathcal{D}(\tilde{\xi}) = \frac{1}{2} \times \begin{cases} \arctan\left(\dfrac{2\sqrt{2\tilde{\xi}}}{4-\tilde{\xi}}\right), & \tilde{\xi} \leqslant 4, \\ \pi - \arctan\left(\dfrac{2\sqrt{2\tilde{\xi}}}{\tilde{\xi}-4}\right), & \tilde{\xi} > 4. \end{cases} \tag{4}$$

In Eqs. (2) and (3) we have related g_A in the soft pion limit, using $g_A \sim R_s^2 = 1/M_q^2$, to the soliton size R_s and to the dynamical quark mass M_q arising from spontaneous chiral symmetry breaking [11]. The r-space behavior of the central force for $g_A \simeq 1.27$, $f_\pi \simeq 0.1$ GeV and averaged pion mass $M_\pi = (M_{\pi^0} + 2M_{\pi^\pm})/3$ is shown in Fig. 2. As one can see, $\mathcal{V}_C^{(1)}$ (dashed curve) and $\mathcal{V}_C^{(2)}$ (dot-dashed curve) do not vanish at small separations, where both are repulsive. Furthermore, the symmetry breaking part $\mathcal{V}_C^{(2)}$ is small and the term $\mathcal{V}_C^{(1)}$, which is nonvanishing in the chiral limit, dominates the CrTPE. Driven by $\sim 1/M_q$, the total strength of the repulsive core at origin is $\simeq 600$ MeV. Remarkably, at distances $r > 1$ fm the CrTPE generates an attractive dip with strength $\simeq -17$ MeV in good agreement with Ref. [8]. In coordinate

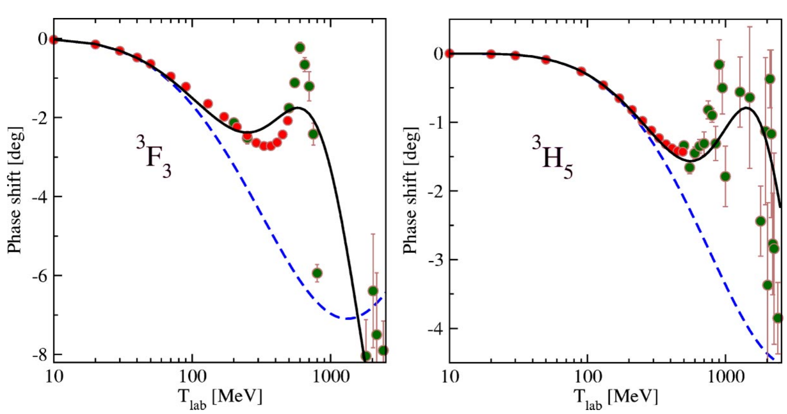

FIG. 3. (Color online) The *pp* phase shifts for the spin triplets $^3F_3$ (left) and $^3H_5$ (right). The dashed curve is the LO result, i.e., OPE, while the solid line refers to OPE+CrTPE. The filled circles are from the partial wave analysis (see text).

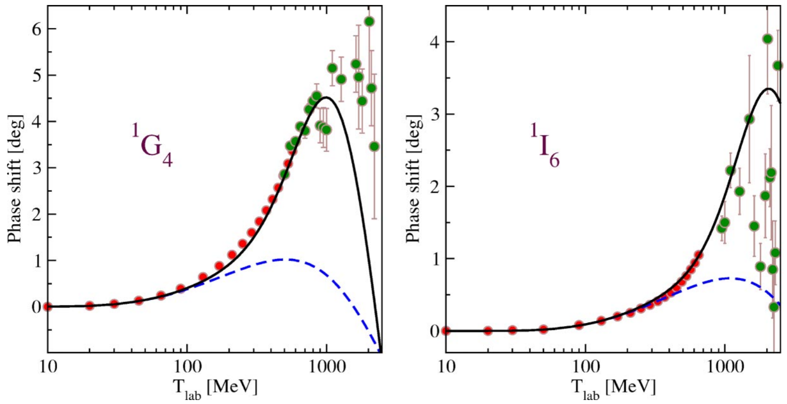

FIG. 4. (Color online) The same as in Fig. 3, but for the spin-singlets $^1G_4$ (left) and $^1I_6$ (right).

space the spin-orbit force can be obtained using the central potential by $\mathcal{V}_{LS}=-(\partial V_C/\partial r)/(2rM_N^2)$. Actually, the spin-orbit interaction is small and repulsive: it is suppressed relative to $\mathcal{V}_C$ by N_c^{-2}, where N_c is the number of colors [13]. For $N_c=3$ we have $\mathcal{V}_{LS}/\mathcal{V}_C\sim O(1/N_c^2)\simeq 1/9$ [11] and in the present analysis this small contribution can be neglected.

Indeed, the structure of the central force provided by Eq. (1) and based on the $\pi\pi$ dynamics only is of very long range and must be important for the peripheral *NN* scattering. The numerical iteration procedure employed here to obtain the scattering phase shifts follows the analysis given in Ref. [15]. To demonstrate the effect it is sufficient to consider the isotriplet *pp* scattering. Our results for the uncoupled partial waves with $L\geqslant 3$, i.e., spin triplets $^3F_3$ and $^3H_5$ and spin singlets $^1G_4$, $^1I_6$, are shown in Figs. 3 and 4. There the central CrTPE force, Eqs. (2) and (3), is supplemented by the leading order (LO) contribution generated by the one-pion exchange (OPE) $\mathcal{V}_{\rm OPE}$, i.e., the model *NN* potential consists of two terms: $\mathcal{V}_{NN}=\mathcal{V}_{\rm OPE}+\mathcal{V}_C$. $\mathcal{V}_{\rm OPE}$ is of standard type being truncated by the dipole form factor with cutoff $\Lambda=780$ MeV [15,16]. In addition, in Figs. 3 and 4 the results of the SAID/SM97 parametrizations [17] are shown up to about $T_{\rm lab}\simeq 500$ MeV, above this energy single-energy solutions are used. As one can see the *pp* phase shifts are fully described by the OPE (dashed curves) at low energies as it should be. For energies close to threshold the OPE is not sufficient and certainly fails describing the empirical data indicating the need for a portion of the long-range attraction. Here the role of the central *NN* force provided by Eqs. (2) and (3) becomes important and the combined effect of both OPE+CrTPE (solid curves) improves the behavior of phase shifts. It can reproduce the generic trends of the uncoupled partial waves up to 1 GeV. Note that the increase of the phase shifts relative to OPE is due to the attractive part of the CrTPE, whereas its further decrease is due to the repulsive core at distances less than $\simeq 1$ fm. These specific features of the CrTPE, where the strong repulsive interactions are generated by the $\pi\pi$ dynamics itself, predict that the spin triplets $^3F_3$ and $^3H_5$ should change the slope twice. This behavior is supported by the partial wave analysis (see Fig. 2). For spin singlets the behavior is smooth up to $\simeq 1$ GeV. It is instructive to consider the coupled channels, where we find that the effect of the CrTPE relative to OPE is common and contributes an additional attraction. As an example we show in Fig. 5 our results for the coupled $^3F_4$ and $^3H_4$ waves. Our results are consistent with the SAID data base below the inelastic threshold with less impressive agreement above the threshold. Considering the corresponding mixing angle ϵ_4, Fig. 5 (bottom), the need for an additional long-range tensor force is noted. Because no new tensor components with respect to OPE were introduced, we cannot expect any improvements relative to the OPE result here. For other peripheral coupled waves not shown here the situation is similar.

In summary, we have considered the effect of the scalar-isoscalar CrTPE on the very peripheral elastic *pp* scattering. We have tested its behavior—obtained in unitarized χPT [8] and supported by our recent analysis [11]—against empirical data. We have shown that a minimal model setup consisting of two parts—the standard OPE and the dominant central part of the CrTPE—can account for the energy dependence observed for very peripheral partial waves below and above

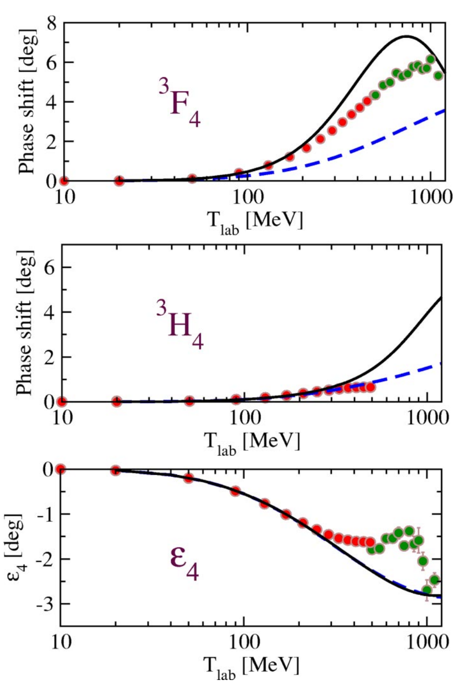

FIG. 5. (Color online) Phase shifts and mixing angle ϵ_4 for the coupled triplets $^3F_4$ and $^3H_4$. Notations for the curves and data points are the same as in Fig. 3.

the inelastic threshold. The model force results in a specific behavior for the uncoupled spin triplets, it further on describes spin singlets and appears to be important in the description of their high momentum transfer behavior. The coupled waves indicate the need for an additional long-range tensor force. Altogether, the CrTPE in the mesonic σ channel, where the moderate attraction and short-range repulsion are relevant ingredients, can provide an important mechanism, which explains peripheral *NN* scattering in a simple and transparent way.

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