Lepton interferometry in relativistic heavy ion collisions: A case study

Jan-e Alam,[1] Bedangadas Mohanty,[1] A. Rahaman,[2] Sourav Sarkar,[1,*] and Bikash Sinha[1,3]
[1]*Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700064, India*
[2]*Durgapur Government College, Durgapur, West Bengal, India*
[3]*Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India*
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We propose intensity interferometry with identical lepton pairs as an efficient tool for the estimation of the source size of the expanding hot zone produced in relativistic heavy ion collisions (RHIC). This can act as a complementary method to two-photon interferometry. The correlation function of two electrons with the same helicity has been evaluated for RHIC energies. The thermal shift of the ρ meson mass has negligible effects on the Hanbury Brown–Twiss radii.

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I. INTRODUCTION

In high-energy heavy ion collisions, two-particle correlations have been extensively studied both experimentally [1–3] and theoretically [4–6] to obtain direct information about the size, shape, and dynamics of the source at freeze-out. This is usually done via selection of transverse momentum and rapidity of the correlated particles. Such calculations are based on the fact that identical particles occurring nearby in phase-space experience quantum statistical effects resulting from the (anti)symmetrization of the multiparticle (fermion) boson wave function. For bosons, the two-particle coincidence rate shows an enhancement at small momentum difference between the particles; the opposite behavior is observed for fermions. The momentum range of this enhancement or depletion can be related to the size of the particle source in coordinate space.

Recently, it has been argued that Hanbury Brown and Twiss (HBT) interferometry [7] (see Ref. [8] for application of HBT interferometry in particle physics) is a sensitive probe of the QCD equation of state and hence formation of QGP [9–11]. Further, it has been argued that in contrast to hadrons, two-particle intensity interferometry of photons [12–15] which are produced throughout the space-time evolution of the reaction and which suffer almost no interactions with the surrounding medium can provide information on the history of the evolution of the hot matter created in heavy ion collisions [16].

The two-fermion interferometry is a well known method used in nuclear physics to estimate the size of the source formed after nuclear collisions [17] (see Ref. [18] for a review). In case of fermion interferometry, the symmetric (antisymmetric) space part of the wave function is coupled with the antisymmetric (symmetric) spin part of the wave function. In this work, we will evaluate the correlation function for the symmetric spin part and hence antisymmetric space wave functions. Statistical spin mixture has been neglected here [18].

*Present address: IFIC, University of Valencia, Aptdo. Correos 22085, 46071 Valencia, Spain.

In our previous work on photon interferometry [12], the spin-averaged source function was used to study the two-photon correlation functions. Here we use spin-dependent source functions to evaluate the two-lepton correlation function. The main aim of this study is to extract the HBT radii of the source from the spin-dependent electron correlation function and show that the HBT radii extracted here are similar to those obtained from the two-photon interferometry. This indicates that the results from photon interferometry are not very sensitive to the spin of the photon. In addition to this, it will be useful to see the effects of the shift of vector meson masses with temperature on the correlation function of two electrons.

The paper is organized as follows. In the next section, we give a general discussion on the correlation function for fermions and the associated kinematics. This is followed by the section which deals with the space-time evolution. Section IV deals with the results on two-lepton correlations at relativistic heavy ion collision (RHIC) energies, the results with mass variation of vector mesons in a hot medium, and a brief discussion on the results of two-electron interferometry vis-à-vis two photon interferometry. We summarize our findings in the last section.

II. CORRELATION FUNCTION

The two-particle correlation function in momentum space is defined as

$$C_2(\vec{k}_1,\vec{k}_2) = \frac{P_2(\vec{k}_1,\vec{k}_2)}{P_1(\vec{k}_1)P_1(\vec{k}_2)}, \tag{1}$$

where $\vec{k}_1$ and $\vec{k}_2$ are the three-momenta of the two particles. $P_1(\vec{k}_i)$ and $P_2(\vec{k}_1,\vec{k}_2)$ represent the one- and two-particle inclusive electron spectra, respectively. These are defined as

$$P_1(\vec{k}) = \int d^4x S(x,k) \tag{2}$$

and

$$P_2(\vec{k}_1,\vec{k}_2) = P_1(\vec{k}_1)P_1(\vec{k}_2) - \int d^4x_1 d^4x_2 S(x_1,K)S(x_2,K) \times \cos(\Delta x^\mu \Delta k_\mu), \tag{3}$$

where $K=(k_1+k_2)/2$, $\Delta k_\mu = k_{1\mu} - k_{2\mu} = q_\mu$, x_i and k_i are the four-coordinates of position and momentum, respectively, and $S(x,k)$ is the source function, which defines the average number of electrons with four-momentum k emitted from a source element centered at the space-time point x. In the present case, $S(x,k)$ is the thermal emission rate of electrons per unit four volume. For processes of the form $\alpha(p_1)+\beta(p_2)\to e^+(k_1)+e^-(k)$, we have [19]

$$S(x,k) = \frac{\mathcal{N}}{16(2\pi)^7 k}\int_{(m_1+m_2)^2}^{\infty} ds \int_{t_{\min}}^{t_{\max}} dt |\mathcal{M}|^2 \times \int dE_1 \int dE_2 \frac{f(E_1)f(E_2)}{\sqrt{aE_2^2 + 2bE_2 + c}}, \tag{4}$$

where α and β are either quarks or pions, and $k_i=(E_i,\vec{k}_i)$ is the four-vector for the particle i. The masses of quarks and electrons are neglected here. $\mathcal{N}$ is the overall degeneracy of the particles α and β, $f(E_i)$ denotes the thermal distribution functions, and s,t,u are the usual Mandelstam variables. The expressions for a,b,c and the integration limits, $E_{1\min}$, $E_{2\min}$, and $E_{2\max}$, are given in Ref. [19].

The spin-dependent invariant amplitude, $\mathcal{M}$, for the processes $q_R^- q_L^+ \to e_R^- e_L^+ (e_L^- e_R^+)$, $q_L^- q_R^+ \to e_R^- e_L^+ (e_L^- e_R^+)$, and $\pi^-\pi^+ \to e_R^- e_L^+ (e_L^- e_R^+)$ have been calculated using standard field theoretic techniques [20].

We shall be presenting the results as a function of longitudinal ($q_{\rm long}$), outward ($q_{\rm out}$), sideward ($q_{\rm side}$), and invariant momentum differences ($q_{\rm inv}$) of the two leptons. These are defined as

$$q_{\rm long} = k_{1z} - k_{2z} = k_{1T}\sinh(y_1) - k_{2T}\sinh(y_2), \tag{5}$$

$$q_{\rm out} = \frac{\vec{q}_T\cdot\vec{K}_T}{|K_T|} = \frac{(k_{1T}^2 - k_{2T}^2)}{\sqrt{k_{1T}^2 + k_{2T}^2 + 2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}}, \tag{6}$$

$$q_{\rm side} = \left|\vec{q}_T - q_{\rm out}\frac{\vec{K}_T}{K_T}\right| = \frac{2k_{1T}k_{2T}\sqrt{1-\cos^2(\psi_1-\psi_2)}}{\sqrt{k_{1T}^2 + k_{2T}^2 + 2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}}, \tag{7}$$

$$q_{\rm inv}^2 = -2k_{1T}k_{2T}[\cosh(y_1-y_2) - \cos(\psi_1-\psi_2)], \tag{8}$$

where $\vec{q}_T = \vec{k}_{1T} - \vec{k}_{2T}$, $\vec{K}_T = (\vec{k}_{1T}+\vec{k}_{2T})/2$, with the subscript T indicating the transverse component, y_i is the rapidity, and ψ_i's are the angles made by k_{iT} with the x axis.

The HBT radii can be extracted by parametrizing the calculated correlation function in the following Gaussian form:

$$C_2^{\rm asym} = 1 - \lambda\exp(-R_{\rm out}^2 q_{\rm out}^2 - R_{\rm side}^2 q_{\rm side}^2 - R_{\rm long}^2 q_{\rm long}^2). \tag{9}$$

The superscript "asym" indicates that antisymmetric space wave functions have been used. Also note that a negative sign appears before the exponential (as opposed to a positive sign in the case of the two-boson correlation function). The λ in Eq. (9) is called the chaoticity parameter, $\lambda=0$ for a coherence source and $\lambda=1$ for a completely chaotic source. We have set $\lambda=1$ as we are concerned here with a completely thermalized system from where the leptons are emitted. If the source function has more than one maxima, then Gaussian parametrization fails to describe the structure of the correlation function at large values of outward, sideward, or invariant momenta defined in Sec. II. For "binary" source functions like mixed phase in the present context, more than one maxima in correlation functions are possible [6]. The values of $C_2^{\rm asym}$ will lie between 0 and 1 for a chaotic source.

A gross idea of the source size can also be obtained from $R_{\rm inv}$, which is defined as

$$C_2^{\rm asym} = 1 - \exp(-R_{\rm inv}^2 q_{\rm inv}^2). \tag{10}$$

III. SPACE-TIME EVOLUTION

For the evaluation of two-electron correlations at RHIC energies, we will consider a scenario where QGP is formed in the initial state which evolves with time into a hadronic phase via an intermediate mixed phase in a first-order phase transition scenario. The mixed phase is a mixture of both quark matter and hadronic matter, with the fraction of quark matter decreasing with time to zero when the phase transition is complete. The hot hadronic gas then expands until the system freezes-out. We also evaluate the correlation function when the vector meson masses (ρ in this case) vary with temperature according to the universal scaling scenario proposed by Brown and Rho (BR) [21].

The initial condition for RHIC energies in terms of the initial temperature (T_i) is calculated from the number of particles per unit rapidity at the midrapidity region for those energies according to the following equation:

$$T_i^3 = \frac{2\pi^4}{45\zeta(3)}\frac{1}{\pi R_A^2 \tau_i 4a_k}\frac{dN}{dy} = \frac{1}{\pi R_A^2\tau_i 4a_k}\frac{dS}{dy}, \tag{11}$$

where $dS(dN)$ is the entropy (number) contained within a volume element $\Delta V = \pi R^2 \tau_i dy$ (R is the radius of the colliding nuclei). $g_{\rm eff}$ is the effective statistical degeneracy, $\zeta(3)$ denotes the Riemann zeta function, and y is the rapidity. For massless bosons (fermions), the ratio of dS to dN is given by $2\pi^4/[45\zeta(3)]\sim 3.6$ (4.2), which is a crude approximation for heavy particles. For example, the above ratio is 3.6 (7.5) for 140 MeV pions (938 MeV protons) at a temperature of 200 MeV. $a_k = \pi^2 g_k/90$ is determined by the statistical degeneracy (g_k) of the system formed after the collision. Taking the particle multiplicity per unit rapidity to be 1100 for Au+Au collisions at RHIC, we get $T_i = 264$ MeV for an initial time of 0.6 fm/c [22]. The critical temperature (T_c) is taken to be 170 MeV [23] here. The freeze-out temperature (T_f) is taken to be 120 MeV, which describes the transverse momentum distributions of produced hadrons [24] with the same space-time evolution model used here. For the equation of state (EOS) which plays a central role in the space-time evolution, we have considered a hadronic gas with particles of mass up to 2.5 GeV and including the effects of nonzero widths of various mesonic and hadronic degrees of freedom

[25]. The velocity of sound (c_s) corresponding to this EOS at freeze-out is about 0.18 [25]. The bag model EOS has been used for the QGP phase. Using the above inputs and assuming boost invariance along the longitudinal direction [26], the (3+1)-dimensional hydrodynamic equations have been solved to study the space-time evolution [27] from the initial QGP phase to freeze-out with an intermediate mixed phase of QGP and hadrons. The initial energy density profile used here is the same as the one used in Ref. [28] and the initial radial velocity is taken as zero.

IV. RESULTS

In this section, we present the results of two-electron interferometry at RHIC energies. In Fig. 1, the transverse momentum distribution of the electrons for Au+Au collisions is depicted for various phases. The initial temperature is taken as $T_i = 264$ MeV. We have assumed that the mass of the intermediary ρ in pion annihilation varies according to Brown-Rho scaling [21].

The results on the interferometry can be presented for several combinations of the variables y, ψ, and k_T. For simplicity, we will present all two-lepton correlation functions for single electrons with momentum 1 GeV/c, with the assumption that around this value of transverse momentum most of the electrons will have a thermal origin. We take $\psi_2 = 0$ and $y_2 = 0$ for all cases, varying ψ_1 and y_1 wherever necessary.

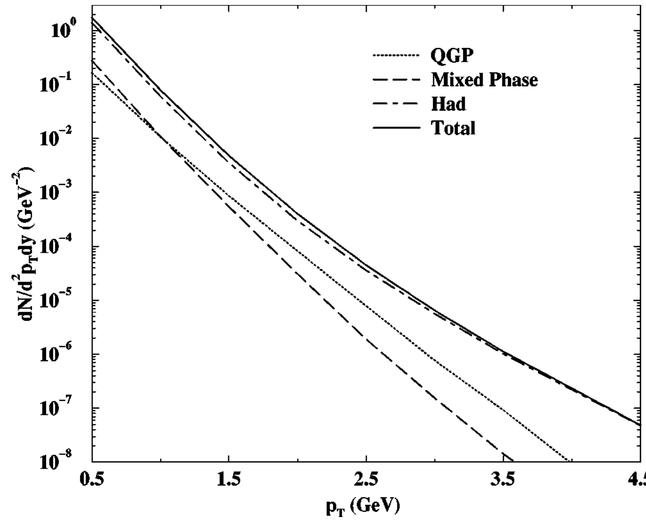

FIG. 1. Transverse momentum distribution of a single electron for Au+Au collisions at 200 A GeV at RHIC. Dotted (dashed) line indicates results for QGP (mixed) phase and dot-dashed (solid) line represents p_T spectra for hadronic (total) phase.

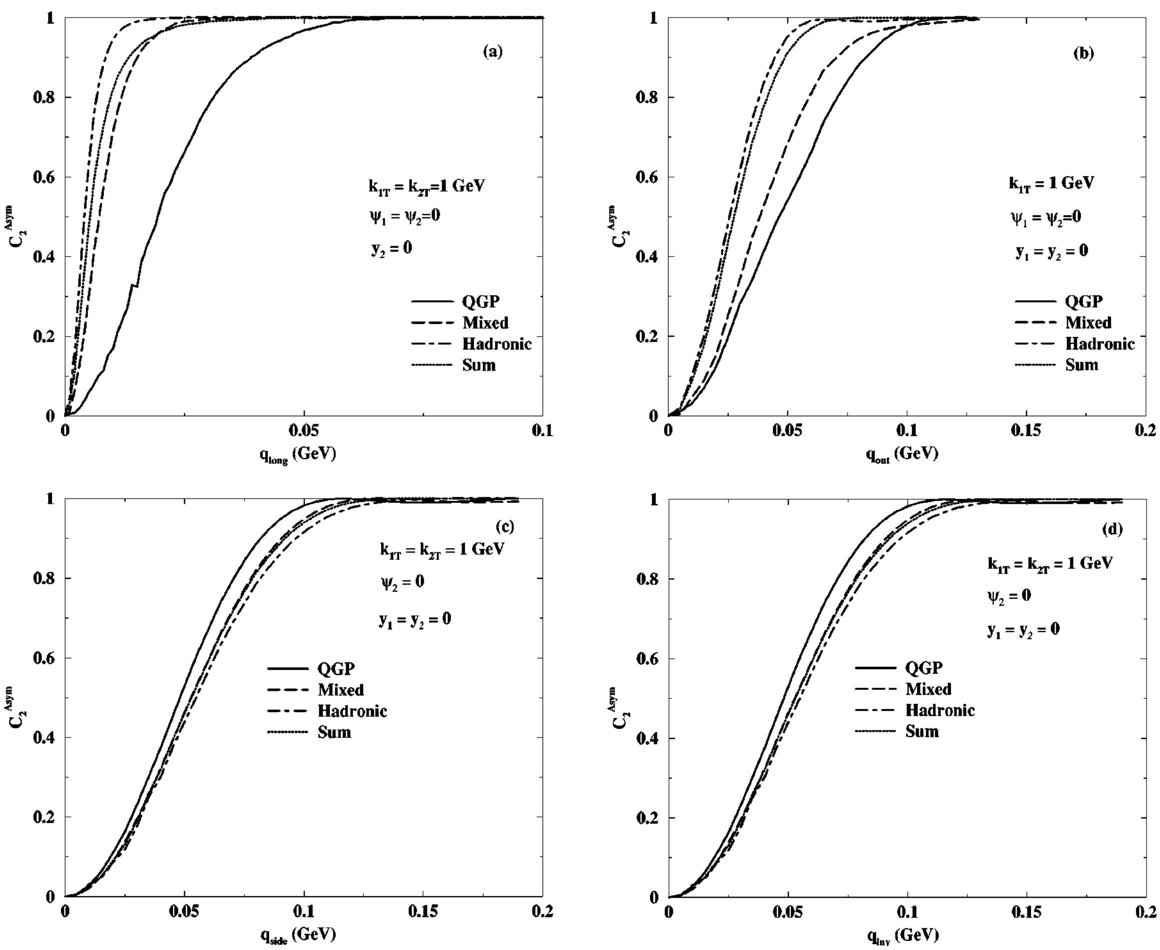

FIG. 2. Correlation function, C_2^{asym} as a function of q_{long}, q_{out}, q_{side}, and q_{inv} for Au+Au collisions at 200 A GeV at RHIC. Solid (dashed) line indicates results for QGP (mixed) phase and dot-dashed (dotted) line represents correlation function for hadronic (sum) phase.

TABLE I. Values of the various HBT radii in fm.

		R_{inv}	R_{out}	R_{side}	R_{long}
RHIC	QGP	3.5	3.5	3.5	0.6
	Mixed	3.2	4.3	3.2	1.8
	Hadron	3.0	6.5	3.0	3.0
	Sum	3.2	6.0	3.2	3.3

Figures 2(a)–2(d) show the variation of the correlation strength (C_2^{asym}) as a function of q_{long}, q_{out}, q_{side}, and q_{inv} for various phases (sum$\equiv$QGP+mixed+hadrons). The HBT dimensions extracted from these correlation functions are shown in Table I. To obtain the longitudinal dimension R_{long} in the longitudinally comoving system (LCMS) of reference, one should multiply the numbers given in Table I by the Lorentz factor $\gamma_K(=\cosh y_K)$, where y_K is the rapidity corresponding to K defined in Sec. II. The width of the correlation function for the hadronic phase is the largest as compared to the other phases along q_{out}, followed by that for the mixed phase and the QGP phase. For q_{side}, the values are comparable for QGP and mixed phases, while they are slightly lower for the hadronic phase. The HBT dimensions satisfy the relation $R_{\mathrm{out}}/R_{\mathrm{side}}\sim 2$ for the correlation functions denoted by "sum" in Table I. This is consistent with the earlier calculations on pion interferometry [29] that the ratio $R_{\mathrm{out}}/R_{\mathrm{side}}$ will be larger than unity in a first-order phase transition scenario due to the appearance of a mixed phase and hence time delay due to the slow down of the expansion rate. We would like to emphasize here that the HBT radii give the length of homogeneity of the source [30], and this is equal to the geometric size if the source is static. However, for a dynamic source, e.g., the system formed after ultrarelativistic heavy ion collisions, the HBT radii are smaller than the corresponding geometric sizes (see Refs. [31–35]).

As indicated earlier, we also consider a scenario where the ρ meson mass varies with the temperature of the medium. The obvious motivation is to comment on this very important issue based on two-electron interferometry. In Figs. 3(a) and 3(b), we show the variation of the correlation strength (C_2) as a function of q_{out} and q_{side} for various phases with the masses of the ρ modified in the medium according to BR scaling. Unfortunately, these results are not very different from the corresponding ones without medium effects shown in Fig. 2. We do not present the results for q_{long} and q_{inv} as they are very similar to the case without mass variation. All these indicate that lepton interferometry is not, probably, a suitable probe to detect the in-medium modification of vector mesons.

We also note that the HBT radii of various phases extracted from photon interferometry [12] with the spin-averaged source function and those obtained here from electron interferometry with the spin-dependent source functions are similar. This indicates that the results from photon interferometry are not very sensitive to the spin of the photon.

V. SUMMARY AND OUTLOOK

The two-electron correlation functions have been evaluated for RHIC energies. The spin dependence of the source functions of the electrons originating from the QGP phase and hadronic phase have been considered explicitly through the invariant amplitude. (3+1)-dimensional relativistic hydrodynamics has been used for the evolution in space and time from the initial QGP phase to the final hadronic phase with intermediate mixed phase of QGP and hadrons in a first-order phase transition scenario. In contrast to the invariant mass distribution of the dileptons [19,36], the HBT radii obtained from the two-electron interferometry is seen to be insensitive to the in-medium modifications of the intermediary vector mesons, apparently because of the cancellation of such effects between the numerator and the denominator of Eq. (1). The results obtained from the electron interferometry here are similar to those obtained from the photon interferometry [12] (where spin averaged amplitudes were used for photon production). The values of HBT radii extracted from the two-lepton correlation functions show that $R_{\mathrm{out}}/R_{\mathrm{side}}\sim 2$, consistent with the assumption of a first-order phase transition scenario [29]. It has been checked that the HBT radii extracted from the correlation function evaluated by using symmetric space and antisymmetric spin wave function is similar to the values given in Table I. Experimentally it is

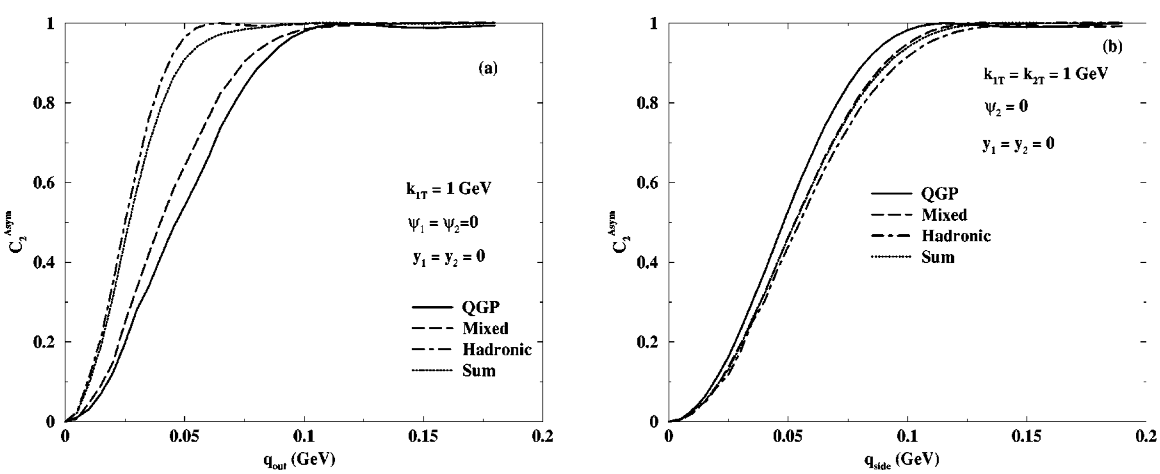

FIG. 3. Same as Figs. 2(b) and 2(c) when the mass of the ρ varies with temperature according to BR scaling.

difficult to verify the predictions made here, however a first attempt to measure two-photon correlation functions in heavy ion collisions has already been made by WA98 Collaboration [15].

Unlike hadrons which are dominantly emitted from the freeze-out surface of the fireball, the leptons and photons are produced and emitted from all the evolution stages of the matter formed after the nuclear collisions. Therefore, lepton and photon interferometry can give, in principle, "the length of homogeneity" of the system at any stage of the evolution. We have not come across any theoretical work on lepton interferometry with spin-dependent invariant amplitude and $(3+1)$-dimensional hydrodynamical expansion. However, there are scopes of improvements of the present work. A first-order phase transition is assumed here in the absence of satisfactory understanding of the order of the phase transition from lattice QCD [23,37]. It will be interesting to look into the two-particle correlation functions with a continuous transition. Near the phase transition, the quarks and gluons may not behave as massless particles due to their interactions with other particles in the thermal bath, although they are treated here as massless. The bag model equation of state has been used here for simplicity; work with more realistic EOS [23] is necessary. We have assumed that e^- from annihilation of thermal quarks and pions dominates at $p_T=1$ GeV. To justify this, a detailed analysis of the p_T distribution of e^-'s from other sources, e.g., Dalitz decays, open charm decays, Drell-Yan processes, etc., is required. A detailed calculation taking into account some of these factors will be published elsewhere.

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