Patterns of the ground states in the presence of random interactions: Nucleon systems

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We present our results on properties of ground states for nucleonic systems in the presence of random two-body interactions. In particular, we calculate probability distributions for parity, seniority, spectroscopic (i.e., in the laboratory frame) quadrupole moments, and discuss α clustering in the ground states. We find that the probability distribution for the parity of the ground states obtained by a two-body random ensemble simulates that of realistic nuclei with $A \geqslant 70$: positive parity is dominant in the ground states of even-even nuclei, while for odd-odd nuclei and odd-mass nuclei we obtain with almost equal probability ground states with positive and negative parity. In addition, assuming pure random interactions, we find that, for the ground states, low seniority is not favored, no dominance of positive values of spectroscopic quadrupole deformation is observed, and there is no sign of α-clustering correlation, all in sharp contrast to realistic nuclei. Considering a mixture of a random and a realistic interaction, we observe a second-order phase transition for the α-clustering correlation probability.

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I. INTRODUCTION

It was discovered in Ref. [1] that the dominance of spin-zero ground states (0 g.s.) can be obtained by diagonalizing a scalar two-body Hamiltonian with random valued matrix elements, a so-called two-body random ensemble (TBRE) Hamiltonian. The 0 g.s. dominance was soon confirmed in Ref. [2] for *sd*-boson systems. This feature was found to be robust and insensitive to the detailed statistical properties of the random Hamiltonian, suggesting that the 0 g.s. dominance holds for a very large ensemble of two-body interactions other than a simple monopole paring interaction. An understanding of this discovery is very important, because this observation seems to be contrary to what is traditionally assumed in nuclear physics, where the 0 g.s. dominance in even-even nuclei is usually explained as a reflection of attractive pairing interaction between like nucleons.

There have been many efforts to understand this observation, but a fundamental understanding is still out of reach [3]. There are also many works [4] studying other robust phenomena of many-body systems in the presence of the TBRE, for example the studies of odd-even staggering of binding energies, generic collectivity, the behavior of energy centroids of fixed spin states, correlations, etc.

The purpose of the present paper is to focus our attention on some physical quantities in the ground states which have not been studied yet, specifically parity, seniority, spectroscopic quadrupole moments (i.e., measured in the laboratory frame), and α-clustering probability. For realistic nuclei, these quantities show a very regular pattern. In this paper, we shall discuss whether these regular patterns are robust in the presence of random interactions.

As is well known, all even-even nuclei have positive-parity ground states (i.e., 100%), whereas the ground states of nuclei with odd mass numbers have only a slightly higher probability for positive parity than for negative parity. Odd-odd nuclei have almost equal probabilities for positive- and negative-parity ground states ($\sim$50%). The statistics for the ground-state parity of nuclei with mass number $A \geqslant 70$ are summarized in Table I. As the first subject, we will study the ground-state parity distribution using random interactions.

TABLE I. The positive parity distribution of the ground states of atomic nuclei. We included all ground-state parities of nuclei with mass number $A \geqslant 70$. The data are taken from Ref. [5]. We have not taken into account those nuclei for which the ground-state parity was not measured.

Counts	Even-even	Odd-*A*	Odd-odd
verified (+)	487	281	118
verified (−)	0	215	104
tentative (+)	0	159	70
tentative (−)	0	126	60

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The next subject that we shall discuss in this paper is the distribution of seniority in the ground states. Seniority [6] has been proven to be a very relevant concept in nuclear physics, in particular for spherical or transitional nuclei. Seniority (v) is uniquely defined for a single-j shell; it was generalized to the case of many-j shells by Talmi in Ref. [7]. In Refs. [1,8] it was reported that the pairing phenomenon seems to be favored simply as a consequence of the two-body nature of the interaction. The "pairing" of Refs. [1,8] was characterized by a large matrix element of the S pair annihilation operator between the ground states of an n fermion system and an $n-2$, $n-4,\ldots$ system, where the S pair structure is determined by using the procedure of Talmi's generalized seniority scheme. This indicated that the S-pair correlation is dominant for the spin-0 g.s. of these systems. However, an examination of this "pairing" correlation of fermions in a single-j shell in Ref. [9] showed that an enhanced probability for low seniority in the spin-0 g.s. is not observed in most of the calculations using a TBRE Hamiltonian. For many-j shells, there have been only a few discussions to clarify this point so far.

Another subject that we shall discuss is the α-clustering correlation in the presence of random interactions. The importance of the α-clustering correlation in light and medium nuclei has been emphasized by many authors [10]. The α-clustering correlation also plays an important role in astrophysical processes, such as the Salpeter process in the formation of ^{12}C. Many calculations of low-lying states, using the antisymmetrized molecular-dynamics model, have been done in recent years [11] to study the α-clustering and other clustering correlations for both stable and unstable light nuclei. α-cluster condensation was suggested by Horiuchi, Schuck, and collaborators in Ref. [12]. As a function of the admixture of a realistic to the TBRE interaction, a phase transition is observed for the α-clustering probability in the ground state.

In this paper, we also discuss the spectroscopic quadrupole moments (i.e., measured in the laboratory frame) of the ground states. A positive value of spectroscopic quadrupole deformation is dominant in the low-lying states of atomic nuclei. Recently, it has been argued in Ref. [13] that this is due to the interference of spin-orbit and l^2 terms of the Nilsson potential.

Our calculations are based on the use of TBRE interactions. The single-particle energies are set to be zero. The Hamiltonian that we use conserves the total angular momentum and isospin,

$$H=\sum_{j_1j_2j_3j_4}^{JT}\sqrt{2J+1}\sqrt{2T+1}G^{JT}_{j_1j_2j_3j_4}\frac{1}{\sqrt{1+\delta_{j_1j_2}}\sqrt{1+\delta_{j_3j_4}}}\times[(a^\dagger_{j_1t}\times a^\dagger_{j_2t})^{(JT)}\times(\tilde{a}_{j_3t}\times\tilde{a}_{j_4t})^{(JT)}]^{(00)},\quad(1)$$

where the $G^{JT}_{j_1j_2j_3j_4}$ are defined as $\langle j_1j_2JT|V|j_3j_4JT\rangle$ and follow the following distribution:

$$\rho(G^{JT}_{j_1j_2j_3j_4})=\frac{1}{\sqrt{2\pi x}}\exp\left(-\frac{(G^{JT}_{j_1j_2j_3j_4})^2}{2x}\right)\quad(2)$$

with

$$x=\begin{cases}1 & \text{if } |(j_1j_2)JT\rangle=|(j_3j_4)JT\rangle\\ \frac{1}{2} & \text{otherwise}.\end{cases}\quad(3)$$

The Hamiltonian so defined is called a TBRE Hamiltonian. Here j_1, j_2, j_3, and j_4 denote the respective single-particle orbits, and $J(T)$ denotes the total angular momentum (isospin) of two nucleons. For each system, 1000 runs of calculations are performed in order to accumulate stable statistics.

This paper is organized as follows. In Sec. II, we present our results for parity distributions for a variety of systems. In Sec. III, we discuss the distribution of seniority in the ground states using random interactions. In Sec. IV, we show our results for spectroscopic quadrupole moments of the ground states which suggest prolate or oblate shapes. In Sec. V, we discuss the α-clustering correlation in the ground states. A summary will be given in Sec. VI.

II. PARITY

We select four model spaces for studying the parity distribution in the ground states obtained by random interactions.

(A) Both protons and neutrons are in the $f_{5/2}p_{1/2}g_{9/2}$ shell, which corresponds to nuclei with both proton number Z and neutron number $N\sim 40$.

(B) Protons in the $f_{5/2}p_{1/2}g_{9/2}$ shell and neutrons in the $g_{7/2}d_{5/2}$ shell, which corresponds to nuclei with $Z\sim 40$ and $N\sim 50$.

(C) Both protons and neutrons are in the $h_{11/2}s_{1/2}d_{3/2}$ shell, which corresponds to nuclei with Z and $N\sim 82$.

(D) Protons in the $g_{7/2}d_{5/2}$ shell and neutrons in the $h_{11/2}s_{1/2}d_{3/2}$ shell, which corresponds to nuclei with $Z\sim 50$ and $N\sim 82$.

These four model spaces do not correspond to a complete major shell but have been truncated in order to make the calculations feasible. These truncations are based on the subshell structures of the involved single-particle levels. We study the dependence on valence-proton number N_p and valence-neutron number N_n in these four model spaces. It is noted that the numbers of states [denoted as $D(I)$] for positive and negative parity are very close to each other for all these examples. The $D(I)$'s for a few examples are shown in Fig. 1. One thus expects that the probability of the ground states with positive parity is around 50%, if one assumes that each state of the full shell model space is equally probable in the ground state.

The calculated statistics for the parity of the ground states, using a TBRE Hamiltonian, is given in Table II. This clearly shows that positive parity is favored, and dominant for most examples, for the ground states of even-even nuclei in the presence of random interactions.

The statistics for nuclei with odd mass numbers and nuclei with odd values of both N_p and N_n (the number of protons and the number of neutrons, respectively) is also given in Table II. These statistics show that the probabilities to have positive or negative parity in the ground states are almost equal to each other with some exceptions. In general, there is no favoring for either positive parity or negative

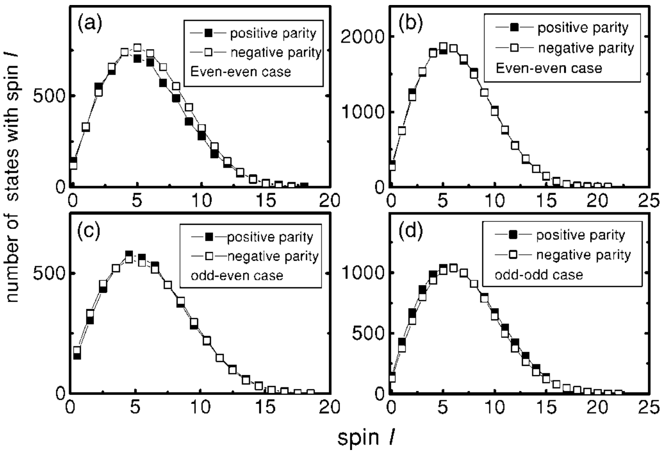

FIG. 1. Number of states with total angular momentum I [denoted as $D(I)$] vs I. One sees that the $D(I)$ of positive parity levels and that of negative parity levels are very close to each other. (a) Two protons in the $1g_{9/2}2p_{1/2}1f_{5/2}$ shell and four neutrons in the $2d_{5/2}1g_{7/2}$ shell; (b) two protons and two neutrons in the $1g_{9/2}2p_{1/2}1f_{5/2}$ shell; (c) two protons and three neutrons in the $1g_{9/2}2p_{1/2}1f_{5/2}$ shell; (d) three protons in the $1h_{11/2}3s_{1/2}2d_{5/2}$ shell and three neutrons in the $2d_{5/2}1g_{7/2}$ shell.

parity in the ground states of odd mass nuclei and doubly odd nuclei in the presence of random interactions. It is noted that these calculations are done for the beginning of the shell. For the end of the shell, the results show a similar trend.

TABLE II. The positive parity probability for the ground states (in %). Numbers of neutrons and protons (N_p, N_n) are given in parentheses for each configuration.

Basis A							
(0,4)	(0,6)	(2,2)	(2,4)	(2,6)			
86.8%	86.2%	93.1%	81.8%	88.8%			
(2,3)	(1,4)	(1,3)	(0,5)	(1,5)	(6,1)	(2,1)	
42.8%	38.6%	77.1%	45.0%	69.8%	38.4%	31.2%	
Basis B							
(2,2)	(2,4)	(4,2)					
72.7%	80.5%	81.0%					
(3,4)	(3,3)	(2,3)	(5,1)	(3,2)	(4,1)	(1,4)	(5,0)
42.5%	74.9%	72.4%	42.9%	39.1%	75.1%	26.4%	44.1%
Basis C							
(2,2)	(2,4)	(4,0)	(6,0)				
92.2%	81.1%	80.9%	82.4%				
(1,3)	(1,5)	(2,3)	(5,0)	(4,1)			
73.0%	64.4%	52.0%	42.6%	56.5%			
Basis D							
(2,2)	(4,2)	(2,4)	(0,6)				
67.2%	76.1%	74.6%	83.0%				
(3,3)	(3,2)	(2,3)	(0,5)				
54.5%	54.2%	54.0%	45.9%				

We also find that the above regularities for parity distributions also hold for very simple cases, namely single-closed two-j shells with one positive and one negative parity. We have checked this explicitly in the cases $(2j_1, 2j_2) = (9,7)$, (11,9), (13,9), (11,3), (13,5), (19,15), (7,5), and (15,1). The statistics is very similar to the above results: The probability of ground states with positive parity is about 85% for an even number of nucleons, and about 50% for an odd number of nucleons.

It is interesting to note that for all even-even nuclei, the $P(0^+)$ is usually two orders of magnitude larger than $P(0^-)$. It would be very interesting to investigate the origin of this large difference, i.e., why the 0^- is not favored in the ground states. As is the case for an odd number of bosons with spin l [14], spin $I=0$ is *not* a sufficient condition to be favored in the ground states of a many-body system in the presence of the random interactions. It should be noted that for a realistic g.s., not only is $I=0$ required but also positive parity.

One simple and schematic system to study the parity distribution of the ground states in the presence of random interactions is the *sp*-boson system. First, we note that an *sp*-boson system with an odd number of particles (denoted as n) has the same number of states with positive and negative parity, and for an even value of n there are *slightly* more states with positive parity (the difference is only $n+1$). The calculated results of Ref. [15] showed that when the number n of *sp* bosons is even, the dominant I of the ground states is 0 or n (about 99%), with positive-parity dominance [the parity for *sp* bosons is given by $(-)^I$]. When n is odd, only about 50% of the ground states in the ensemble have spin 0, and about 50% have $I=1$ or $I=n$. This leads to about equal percentages for positive- and negative-parity ground states. This pattern is very similar to that observed for fermion systems.

III. SENIORITY

In this section, we discuss the distribution of the seniority, the number of particles not pairwise coupled to angular momentum 0, of the ground states of nuclei in the *sd* shell in the presence of random interactions. Because seniority is used in classifying the states in our basis, we define the expectation value for seniority in the ground states as follows [16]:

$$\langle v \rangle = \sum_i f_i^2 v_i, \tag{4}$$

where f_i is the amplitude of the ith component in the ground-state wave function, and v_i is the seniority number of the corresponding component.

For even-even nuclei, we consider the spin-0 g.s. because previous discussions [8,9] were focused on spin-0 ground states. For odd-mass nuclei, we consider the $I=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ ground states, because these spin I's are equal to the angular momenta of the single-particle levels in the *sd* shell and are favored as the ground states in the presence of random interactions. For odd-odd nuclei in this section we consider the ground states with $I=1$ (most favored) and $I=0$ states. The examples that we have calculated include $(N_p, N_n) = (0,4)$, (0,6), (2,2), (2,4), (2,6), (4,6), (0,5), (2,3), (2,5), (4,3), (4,5), (3,3), (1,5), and (3,5).

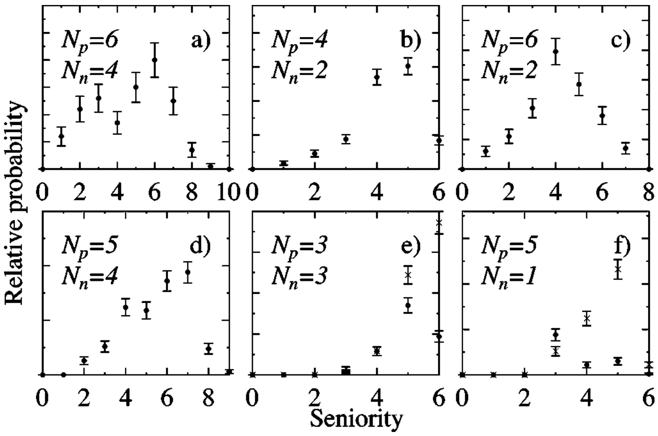

FIG. 2. Distribution of seniority in the ground states with spin zero for even-even nuclei [refer to panels (a), (b), and (c)], spin $I = j_1, j_2, j_3$ for the odd-A case [refer to panel (d)], or spin $I=1,0$ for odd-odd nuclei [refer to panels (e) and (f)]. The error bar is defined by the square root of the count (statistics) for each seniority bin (step width is 1). The dominance of seniority zero components of ground states is not observed.

Typical examples of the distribution of average seniority ($\langle v \rangle$) in the ground states are shown in Fig. 2 in arbitrary units (i.e., relative probability). The figure shows that for none of the cases is a small value for $\langle v \rangle$ preferred. These distributions of seniority in the ground states show that the large matrix elements of the S-pair operator between the spin-0 g.s. of an n-nucleon system and that of an $(n+2)$-nucleon system, as observed in Ref. [8], should not be understood as an indication of a large S pair condensate in the spin-0 g.s. of TBRE Hamiltonians. Further studies are necessary to understand the implications of Ref. [8].

IV. SPECTROSCOPIC QUADRUPOLE MOMENT

In this section, we study the quadrupole moments of the ground or low-lying states. If the ground-state spin I is smaller than 1 (i.e., 0 or $\frac{1}{2}$), the spectroscopic quadrupole moment necessarily vanishes (even though there could be a finite intrinsic moment) because the triangle relationship of angular momentum coupling cannot be obeyed by the two I's $\left(I \leqslant \frac{1}{2}\right)$ and the angular momentum for the quadrupole operator (=2). For these cases, one can use an alternative, namely the quadrupole moment of the next lowest state with $I > \frac{1}{2}$. For all cases that we have checked, it is found that the essential statistics for positive and negative quadrupole moments obtained by this alternative is very close to that obtained by neglecting cases with ground state $I < \frac{1}{2}$. In this paper, we show the statistics which does not include the runs of spin-0 and spin-$\frac{1}{2}$ ground states. The total number of calculated spectroscopic quadrupole moments is thus much less than 1000. We note that a negative spectroscopic quadrupole moment implies a positive quadrupole moment in the intrinsic frame and thereby a prolate deformation.

The spectroscopic quadrupole moment is defined by

$$Q = \langle \beta I | r^2 Y_{2M} | \beta I \rangle \tag{5}$$

for both proton and neutron degrees of freedom. In Eq. (5), $|\beta I\rangle$ is the wave function of the ground state. In this paper, Q will be used to refer to the spectroscopic quadrupole moment following from Eq. (5).

We have calculated Q for a number of cases in the *sd* shell and for several fillings of the four single-particle bases mentioned in Sec. II. The results are given in Table III. One sees that negative values for Q (corresponding to prolate deformations) are dominant with two exceptions, $(N_p, N_n) = (4,3), (6,5)$ in the *sd* shell. In general we observe that for the *sd* shell, the statistics for positive and negative values for Q is comparable if N_p and/or N_n are close to their midshell values.

From Table III, we conclude that at the beginning of the shell, negative values for Q are dominant, while at the end of the shell, positive value dominate. This is similar to the result for a harmonic-oscillator potential, for which prolate deformation occurs at the beginning and oblate deformation at the end of the shell [17].

V. α CLUSTERING

It was shown in Ref. [18] that the essential parts of the $I=0, T=0$ ground state for ^{20}Ne with two protons and two neutrons in the *sd* shell are dominated by components with the highest orbital symmetry [4]; 91.8% of the ground state is given by components with orbital symmetry [4] which corresponds to a pure α-clustering configuration. One may use the expectation value of the Majorana interaction, P_M, as the fingerprint for the α-cluster wave function. Another similar example is the $I=0, T=0$ ground state for ^{8}Be with two protons and two neutrons in the p shell. If one uses the Cohen-Kurath interaction, one sees that the expectation value of P_M is −5.76, close to −6, which is the eigenvalue of Majorana force. The overlap between the g.s. wave function obtained by diagonalizing the Cohen-Kurath interaction for ^{8}Be and that for exact SU(4) symmetry (namely, full symmetry [4] for the ground state) is 0.97.[1] This dominance of the full symmetry [4] with respect to the permutation of orbital degrees of freedom in the $I=0$ and $T=0$ ground states of these nuclei is an indication of α-clustering correlation from the perspective of the shell model. In this paper, we concentrate on these two examples using random interactions.

To set the scale, we can calculate the matrix element of P_M in the $I=T=0$ (spin-isospin singlet) ground state by assuming that all the possible $I=T=0$ states with different symmetries with respect to the exchange of the orbital degrees of freedom appear at an equal probability. We call the P_M so obtained the geometric P_M. To do so, one needs the number of $(I=0, T=0)$ states for each orbital symmetry.

The procedure to construct the states with particular spin-isospin symmetry is given in Ref. [19], while that for constructing wave functions with certain orbital symmetry is given in Ref. [20] for the Elliott model [21], with tables for the *sd*, *pf*, and *sdg* shells. Finally, the spin-isospin functions should be coupled to the orbital functions with their conju-

[1] In this paper we set the single particle energies to zero. If we take the single particle energies of Cohen-Kurath interaction, this overlap becomes 0.99.

TABLE III. The number of cases with positive (negative) spectroscopic quadrupole moments are given in bold (*italic*) font, respectively. We omitted the cases for which the spin of the ground state is less than 1; see the text for further details.

Both protons and neutrons in the *sd* shell									
(N_p,N_n)		(2, 1)			(2, 3)			(2, 5)	
	280		*418*	**338**		*430*	**306**		*402*
(N_p,N_n)		(2, 1)			(4, 3)			(4, 5)	
	287		*425*	**434**		*374*	**320**		*370*
(N_p,N_n)		(6, 1)			(6, 3)			(6, 5)	
	201		*530*	**400**		*444*	**420**		*348*
Basis (A): protons and neutrons in $f_{5/2}p_{1/2}g_{9/2}$									
(N_p,N_n)		(1, 2)			(1, 3)			(1, 4)	
	267		*469*	**283**		*481*	**246**		*535*
(N_p,N_n)		(1, 6)			(2, 3)			(0, 5)	
	207		*566*	**284**		*564*	**447**		*459*
Basis (B): protons $(f_{5/2}p_{1/2}g_{9/2})$, neutrons $(g_{7/2}d_{5/2})$									
(N_p,N_n)		(1, 4)			(4, 1)			(2, 4)	
	374		*507*	**278**		*632*	**253**		*428*
(N_p,N_n)		(3, 4)			(4, 3)			(6, 1)	
	278		*620*	**330**		*560*	**233**		*660*
Basis (C): protons and neutrons in $s_{1/2}d_{3/2}h_{11/2}$									
(N_p,N_n)		(2, 3)			(2, 5)			(4, 3)	
	231		*657*	**238**		*472*	**392**		*498*
(N_p,N_n)		(5, 1)			(5, 0)			(3, 3)	
	213		*628*	**212**		*659*	**349**		*449*
Basis (D): protons $g_{7/2}d_{5/2}$, neutrons $s_{1/2}d_{3/2}h_{11/2}$									
(N_p,N_n)		(14, 13)			(15, 12)				
	781		*183*	**610**		*333*			

gate symmetry to obtain the fully antisymmetric wave functions with respect to an exchange of two particles. The angular momentum for each state is given by coupling S and L.

Table IV presents the number of $I=0$ states for two protons and two neutrons in the p shell and the sd shell with all possible orbital symmetries. From Table II, one obtains the geometric P_M for the $I=T=0$ states: P_M is $-\frac{6}{5}$ for the p shell and $-\frac{22}{21}$ for the sd shell.

Using a TBRE Hamiltonian, we obtain the following probabilities for spin-I ground states: For 1000 runs, one obtains 485 and 365 runs with $(I,T)=(0,0)$ ground states for ^{8}Be and ^{20}Ne, respectively. This is consistent with the result [1,8] of the $I=T=0$ g.s. dominance in the presence of random interactions. The average value of P_M for the $(I,T)=(0,0)$ g.s. that we obtain is −1.26 (the geometric value is $-\frac{6}{5}=-1.20$) and −1.66 (the geometric value is $-\frac{22}{21}=-1.05$) for the p shell and the sd shell, respectively. The average value of P_M for a TBRE Hamiltonian and the corresponding geometric value are very close to each other for the p shell, indicating that α-clustering correlation is not favored by random interactions. For the case of the sd shell, the average value of P_M for a TBRE Hamiltonian deviates sizably from its geometric value.

To check whether this deviation becomes larger for larger shells, we calculate the case of two protons and two neutrons in the sdg shell, for which we obtained 385 cases with $(I,T)=(0,0)$ ground states among 1000 sets of TBRE Hamiltonians. The average P_M value for these states is −0.629, while that obtained by assuming a random orbital symmetry is $-\frac{2}{5}$, which is close to the above value.

It is also interesting to study the distribution of overlaps between the $I=T=0$ ground state obtained from the realistic interactions and those obtained by pure random interactions or by a combination of realistic and random interactions. As an example, we discuss here the case of two protons and two neutrons in the p shell where the realistic interaction is chosen as the Cohen-Kurath interaction. We thus define a Hamiltonian

$$H = (1-\lambda)H_{\rm TBRE} + \lambda H_{\rm real}. \quad (6)$$

Here $\lambda=0$ corresponds to the pure TBRE Hamiltonian and $\lambda=1$ corresponds to the realistic Cohen-Kurath interaction.

TABLE IV. The number of $I=0$ states for two valence protons and two valence neutrons in the *p* shell and the *sd* shell with definite symmetry with respect to exchange orbital degree of freedom of two particles and the corresponding conjugate symmetry with respect to exchange spin-isospin $(S-T)$ degrees of freedom. L is the total orbital angular momentum and S is the total spin. The last column gives number of the $I=0$ states with $T=0$.

L	S	$I=0$	$I=T=0$
	The *p* shell		
[4] $0,2,4$	0	0	0
[31] $1,2,3$	$0,1^2$	0^2	0
[22] $0,2$	$0^2,1,2$	0^3	0^2
[211] 1	$0,1^3,2$	0^3	0
	The *sd* shell		
[4] $0^4,2^5,3,4^4,5,6^2,8$	0	0^4	0^4
[31] $0^2,1^4,2^7,3^6,4^5,5^3,6^2,7$	$0,1^2$	0^{10}	0^4
[22] $0^3,1,2^5,3^2,4^3,5,6$	$0^2,1,2$	0^{12}	0^8
[211] $1^5,2^3,3^5,4^2,5^2$	$0,1^3,2$	0^{18}	0^5
[1111] $1,2,3$	$0,1,2$	0^2	

We will vary λ in the range from 0 to 1, corresponding to the situation of nuclear forces with different mixtures of random noise.

The results for $\lambda=0$, 0.3, 0.5, 0.7, and 0.9 are shown in Figs. 3(a)–3(e). The error bars indicate the statistical errors in determining the numbers, defined by the square root of the number of counts for each bin. For case (a) with $\lambda=0$ one sees that the overlaps distribute "randomly" from 0 to 1. This suggests that pure random interactions produce "random" overlaps of the $I=T=0$ ground states with the realistic ground state. However, for $\lambda > 0.5$, the $I=T=0$ ground states are close to that of the realistic interactions for most of the cases. This is especially clear from Fig. 4, where the overlap, averaged over the different Hamiltonians in the ensemble Eq. (6), is plotted versus λ. The statistical inaccuracies are indicated by the error bars in this figure. For values of λ exceeding 0.6, the overlap is very close to unity, while for larger admixtures of the random component in the interaction, the overlap decreases approximately linearly with λ. This trend has all the signatures of a second-order phase transition. Only for limited magnitude of the random interaction does the g.s. have a realistic structure, which breaks down when a critical value is exceeded.

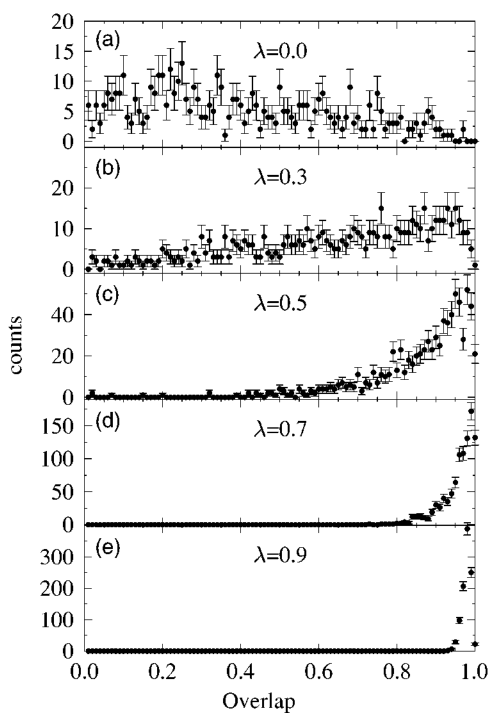

FIG. 3. The overlaps between the $I=T=0$ ground states for two protons and two neutrons in the *p* shell obtained by Cohen-Kurath interactions and those obtained by the Hamiltonian Eq. (6). (a)–(e) correspond to $\lambda=0$, 0.3, 0.5, 0.7, and 0.9, respectively.

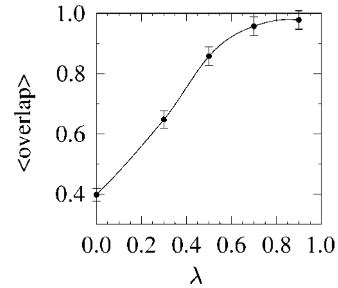

FIG. 4. Average overlap of the g.s. of the Hamiltonian of Eq. (6) with that of the realistic Hamiltonian as a function of the mixing parameter λ. The line is plotted to guide the eyes.

VI. DISCUSSION AND SUMMARY

The present paper was stimulated by the discovery of the spin-zero ground-state dominance (0 g.s.) of even fermion systems [1] and boson systems [2] in the presence of the random two-body ensemble (TBRE). This discovery sparked off a sudden interest in many-body systems under the TBRE. It also led to extensive studies of other physical quantities [4], such as energy centroid of fixed spin states, collectivity, etc. The purpose of this paper was to study the robustness of some features which are well known in nuclear physics but have not been studied under the TBRE.

First, we calculated in Sec. II the parity distribution of the g.s. for a TBRE Hamiltonian. It was found that positive parity is dominant for the g.s. of systems with even numbers of valence protons and neutrons. For odd-*A* and doubly odd systems, the TBRE Hamiltonian leads to ground states with comparable probability for both positive and negative parity. This is similar to the global statistics for realistic nuclei with $A \geqslant 70$ (refer to Table I). Unlike the spin-0 g.s. dominance in the presence of random interactions, the dominance of positive parity in the ground states of even-even nuclei has not been pointed out explicitly so far. Since parity is a much simpler quantity than angular momentum, an understanding of the parity dominance of even-even systems may be helpful in understanding the spin-0 g.s. dominance of even-even nuclei in the presence of random interactions.

Second, our investigation showed that the seniority distribution for the g.s. of *sd*-shell nuclei is not dominated by low seniority components, contrary to the situation for realistic nuclei. Our investigation also suggests that the correlation between the wave function of the spin-0 g.s. for A nucleons and that for $A+2$ nucleons discussed in Ref. [8] should not be understood as an indication of the dominance of the seniority zero component.

Third, the dominance of negative spectroscopic quadrupole moments at the beginning of the shell and positive quadrupole moments at the end of the shell is also observed in the g.s. obtained by using the TBRE interactions. This situation is similar to the prediction obtained from a simple harmonic-oscillator potential. This means that the TBRE Hamiltonians do not lead to an overall dominance of the prolate deformation. However, also in realistic nuclei a dominance of prolate deformation is observed when both valence protons and neutrons are in the first half of a major shell.

Last, we studied the α-clustering correlation by calculating the expectation value of the Majorana operator in the $I=0$, $T=0$ g.s. of TBRE interactions. We also calculated the overlaps between the $I=0$, $T=0$ ground states of the TBRE Hamiltonian and the ground state obtained from realistic interactions. Our calculations on ^{8}Be and ^{20}Ne showed that the α-clustering structure is not favored by a pure TBRE Hamiltonian. It is interesting to note that, as a function of the admixture of a realistic Hamiltonian to a TBRE Hamiltonian, a second-order phase transition is observed. For Hamiltonians that contain less than ~ 0.4 admixture of random interactions, the structure of the g.s. is close to the realistic case, but for higher admixtures the overlap with a realistic wave function becomes progressively worse.

In conclusion, we have observed in this paper the dominance of positive parity in the ground states of even-even nuclei in the presence of pure random two-body interactions. Because parity is intrinsically a simpler quantum number than angular momentum, it will be interesting to understand the mechanism for this. In addition, it has been shown that, even though the quantum numbers for the g.s. are realistic, the dynamical properties of the ground states under the TBRE Hamiltonian, such as seniority, which is a signature of pairing correlation, the α-clustering probabilities, and the sign of quadrupole moments, are in sharp disagreement with those of realistic nuclei.

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