PHYSICAL REVIEW C **70**, 051303(R) (2004)

Parametrization of SU(3) spectroscopic factors for light nuclei within an algebraic model

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(Received 30 June 2004; published 18 November 2004)

A parametrization of SU(3) shell-model spectroscopic factors for light nuclei is proposed. It is shown that spectroscopic factors, as calculated from first principles, can be reproduced nearly perfectly, including taking full account of the Pauli principle, without recurring to sophisticated microscopic procedures. The results show that microscopic spectroscopic factors follow a surprisingly simple pattern.

DOI: 10.1103/PhysRevC.70.051303 PACS number(s): 21.10.Jx, 21.60.Fw, 21.60.Gx

The computation of spectroscopic factors for diverse cluster systems plays an important role in the description of α and heavy cluster decay (cluster radioactivity) [1,2]. Much effort was been invested in the 1970s and 1980s to derive the spectroscopic factors in various cluster channels [3–9] within the framework of the SU(3) shell model. All of them evoke rather sophisticated procedures in order to assure the antisymmetry of the many-particle system. When more complicated systems, or systems not yet addressed, are considered, such methods turn out to be impractical and/or very unattractive. This has inhibited the use of such methods for those not accustomed to the use of such shell-model methods.

In this Rapid Communication we propose a practical method for the parametrization of the spectroscopic factor within the SU(3) shell model for light nuclei. The parameters are adjusted to reproduce some *calculated* spectroscopic factors and then compare them to the rest. The results show that the spectroscopic factors obtained by using SU(3) shell-model procedures can be almost perfectly reproduced using a rather simple procedure, and raise the expectation that the method can be applied to other systems where shell-model results are either not available or too difficult to consider calculating. The results show that the calculations of Refs. [4] and [8] yield a relatively simple pattern.

The basis of the procedure is the semimicroscopic algebraic cluster model (SACM) [10,11]. Each cluster is represented by an irreducible representation (irrep) of the $\mathrm{SU}_i(3)$ ($i=1,2$) group. The relative motion is described by a $\mathrm{SU}_R(3)$ group. The relevant group chain is

$$\begin{array}{l}\mathrm{SU}_1(3)\otimes\mathrm{SU}_2(3)\otimes\ \mathrm{SU}_R(3)\supset\mathrm{SU}_C(3)\otimes\mathrm{SU}_R(3)\supset\\ (\lambda_1,\mu_1)\quad(\lambda_2,\mu_2)\quad(n_\pi,0)\quad(\lambda_C,\mu_C)\end{array}$$

$$\mathrm{SU}(3)\supset\mathrm{SO}(3)\supset\mathrm{SO}(2)$$

$$(\lambda,\mu)\quad \kappa L\quad M, \tag{1}$$

where (λ_i,μ_i) refer to the SU(3) irrep of the individual clusters, which are coupled to intermediate irrep (λ_C,μ_C); n_π is the number of relative oscillator quanta (vibron number), limited from below by the Wildermuth condition [12] and from above by the total number of π plus s bosons (see Refs. [10] and [11] for details); (λ,μ) is the total SU(3) irrep; and L and M are the angular momentum and its projection.

In the SACM, the set of allowed (λ,μ) values is obtained by multiplying the cluster irreps with that of the relative motion, and then checking against the list of allowed SU(3) irreps in the shell model. Only those irreps which also appear in the shell model are retained. In this way the Pauli exclusion principle is taken into account. The SACM is called *semimicroscopic* because the model space has a shell-model equivalent, but as noted in Refs. [10,11], the operators themselves contain parameters.

An attempt to parametrize the spectroscopic factor was done successfully in [13]. However, in that case it was applied only to a limited data set and the parameters were changed from one system to another. Furthermore, a linear dependence on scalar operators in the SACM was proposed in [13]. In [14,15] the spectroscopic factor for the cluster radioactivity [2] was investigated and the need for an exponential ansatz was demonstrated. The form proposed here for the spectroscopic factor is based on the experience in [14] and it is given by

$$\begin{aligned}S=\exp[&A+Bn_\pi+C\mathcal{C}_2(\lambda_1,\mu_1)+D\mathcal{C}_2(\lambda_2,\mu_2)+E\mathcal{C}_2(\lambda_c,\mu_c)\\&+F\mathcal{C}_2(\lambda,\mu)+G\mathcal{C}_3(\lambda,\mu)+H\Delta n_\pi]\end{aligned}$$

$$\begin{aligned}&|\langle(\lambda_1,\mu_1)\kappa_1L_1,(\lambda_2,\mu_2)\kappa_2L_2\|(\lambda_C,\mu_C)\kappa_CL_C\rangle_{\rho_C}\\&\times\langle(\lambda_C,\mu_C)\kappa_CL_C,(n_\pi,0)1l\|(\lambda,\mu)\kappa L\rangle_1|^2,\end{aligned} \tag{2}$$

where $\mathcal{C}_3(\lambda,\mu)$ is the third-order Casimir operator of SU(3) with eigenvalue $(\lambda-\mu)(2\lambda+\mu+3)(\lambda+2\mu+3)$, which is important in order to distinguish excited states like (λ,μ) and (μ,λ) that are conjugates of one another. The $\mathcal{C}_2(\lambda_k,\mu_k)$ ($k=1,2,C$) is the second-order Casimir operator with eigenvalue $(\lambda_k^2+\lambda_k\mu_k+\mu_k^2+3\lambda_k+3\mu_k)$. Finally, Δn_π gives the difference in the number of relative quanta in excited states to

TABLE I. Spectroscopic factors used in the fitting procedure. Γ_i ($i=1,2$) represents (λ_i,μ_i), Γ_C (λ_C,μ_C), and Γ (λ,μ). Data from Ref. [4] are used, except for the last system. For $^{12}C+^{12}C$ we used Ref. [8]. All spectroscopic factors are normalized relative to the ground state spectroscopic factor of $^{16}O+\alpha$. The "Alg" refers to the algebraic model used in this contribution and "Data" to data against which the results are compared.

System	n_π	Γ_1	Γ_2	Γ_C	Γ	L	Alg	Data
$^{16}O+\alpha$	8	(0,0)	(0,0)	(0,0)	(8,0)	0	1.00	1.00
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(8,2)	0	0.29	0.31
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(8,2)	6	0.010	0.012
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(8,4)	0	0.22	0.21
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(8,4)	6	0.032	0.032
$^{20}Ne+\alpha$	8	(4,2)	(0,0)	(4,2)	(8,4)	0	0.15	0.19
$^{20}Ne+\alpha$	10	(8,0)	(0,0)	(8,0)	(14,2)	0	1.68	1.68
$^{22}Ne+\alpha$	8	(8,2)	(0,0)	(8,2)	(4,8)	0	0.13	0.12
$^{22}Ne+\alpha$	8	(8,2)	(0,0)	(8,2)	(4,8)	6	0.0085	0.0072
$^{22}Ne+\alpha$	8	(4,4)	(0,0)	(4,4)	(4,8)	0	0.17	0.20
$^{24}Mg+\alpha$	8	(8,4)	(0,0)	(8,4)	(0,12)	0	0.075	0.08
$^{24}Mg+\alpha$	8	(8,4)	(0,0)	(8,4)	(0,12)	6	0.0033	0.0032
$^{24}Mg+\alpha$	8	(6,2)	(0,0)	(6,2)	(12,0)	0	0.058	0.057
$^{24}Mg+\alpha$	8	(8,4)	(0,0)	(8,4)	(12,0)	0	0.048	0.042
$^{12}C+^{12}C$	14	(0,4)	(0,4)	(4,0)	(14,2)	0	0.0043	0.0032
$^{12}C+^{12}C$	14	(0,4)	(0,4)	(0,8)	(10,4)	0	0.17	0.15
$^{12}C+^{12}C$	14	(0,4)	(0,4)	(2,4)	(12,0)	0	0.0071	0.010

the number in the ground state, which is determined by the Wildermuth condition, and l is the angular momentum of the relative motion. Here we calculate the spectroscopic factor of the cluster system, when each cluster is in its ground state, i.e., in the weak-coupling limit. In this paper we treat only clusters with even proton and neutron numbers. We focus on even-even nuclei for the sake of simplicity. For odd-even, even-odd, and odd-odd nuclei, an additional complication arises due to the appearance of spin (S) dependencies. This requires more parameters but it is an interesting question for a future investigation. The projection on the weak coupling basis states is required by the data and is sufficient for the light nuclei we consider.

The factor in the last two lines of (2) describes the coupling of the two clusters in their ground state ($L_1=0=L_2$) to a cluster irrep (λ_C,μ_C) in its ground state ($L_C=0$) and the latter to a relative motion term $(n_\pi,0)$ with angular momentum l to a total irrep (λ,μ) with angular momentum L. Coefficients of the type $\langle....||..\rangle_\rho$ are isoscalar factors of the SU(3) group [16–19] with ρ_C a multiplicity index in the direct product coupling $(\lambda_1,\mu_1)\otimes(\lambda_2,\mu_2)\rightarrow(\lambda_C,\mu_C)$. The symbol κ_i is the multiplicity of the angular momentum L_i in the irrep (λ_i,μ_i) of the ith cluster, $i=1,2$. A similar explanation applies for κ_C and L_C.

The dependence on n_π can be understood as follows: The probability of finding the two clusters at a distance R with

TABLE II. Spectroscopic factors for various core-plus-α-particle cases. The data are from [4]; an asterisk refers to the data used in the fit. Different systems are separated by two blank rows while a change from one SU (3) irrep to another (in the clusters or in the total irrep) within the same system is indicated by a single blank row. The values of the spectroscopic factors are normalized to the ground state spectroscopic factor of $^{16}O+\alpha$.

System	n_π	Γ_1	Γ_2	Γ_C	Γ	L	Alg	Data
$^{16}O+\alpha$	8	(0,0)	(0,0)	(0,0)	(8,0)	0	1.00	1.00*
$^{16}O+\alpha$	8	(0,0)	(0,0)	(0,0)	(8,0)	2	1.00	1.00
$^{16}O+\alpha$	8	(0,0)	(0,0)	(0,0)	(8,0)	4	1.00	1.00
$^{16}O+\alpha$	8	(0,0)	(0,0)	(0,0)	(8,0)	6	1.00	1.00
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(8,2)	0	0.29	0.31*
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(8,2)	2	0.22	0.24
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(8,2)	4	0.10	0.11
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(8,2)	6	0.010	0.012*
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(4,4)	0	0.54	0.41
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(4,4)	2	0.47	0.36
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(4,4)	4	0.34	0.26
$^{18}O+\alpha$	8	(4,0)	(0,0)	(4,0)	(4,4)	6	0.18	—
$^{18}O+\alpha$	8	(0,2)	(0,0)	(0,2)	(8,2)	0	0.63	0.75
$^{18}O+\alpha$	8	(0,2)	(0,0)	(0,2)	(8,2)	2	0.61	0.73
$^{18}O+\alpha$	8	(0,2)	(0,0)	(0,2)	(8,2)	4	0.68	0.68
$^{18}O+\alpha$	8	(0,2)	(0,0)	(0,2)	(8,2)	6	0.58	0.58
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(8,4)	0	0.22	0.21*
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(8,4)	2	0.081	0.078
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(8,4)	4	0.0003	0.00
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(8,4)	6	0.032	0.032*
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(4,6)	0	0.29	0.21
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(4,6)	2	0.075	0.053
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(4,6)	4	0.044	0.032
$^{20}Ne+\alpha$	8	(8,0)	(0,0)	(8,0)	(4,6)	6	0.031	0.023
$^{20}Ne+\alpha$	8	(4,2)	(0,0)	(4,2)	(8,4)	0	0.15	0.19*
$^{20}Ne+\alpha$	8	(4,2)	(0,0)	(4,2)	(8,4)	2	0.11	0.14
$^{20}Ne+\alpha$	8	(4,2)	(0,0)	(4,2)	(8,4)	4	0.050	0.063
$^{20}Ne+\alpha$	8	(4,2)	(0,0)	(4,2)	(8,4)	6	0.0045	0.0063
$^{20}Ne+\alpha$	8	(0,4)	(0,0)	(0,4)	(8,4)	0	0.45	0.64
$^{20}Ne+\alpha$	8	(0,4)	(0,0)	(0,4)	(8,4)	2	0.43	0.61
$^{20}Ne+\alpha$	8	(0,4)	(0,0)	(0,4)	(8,4)	4	0.38	0.54
$^{20}Ne+\alpha$	8	(0,4)	(0,0)	(0,4)	(8,4)	6	0.30	0.42
$^{22}Ne+\alpha$	8	(8,2)	(0,0)	(8,2)	(4,8)	0	0.13	0.12*
$^{22}Ne+\alpha$	8	(8,2)	(0,0)	(8,2)	(4,8)	2	0.032	0.029
$^{22}Ne+\alpha$	8	(8,2)	(0,0)	(8,2)	(4,8)	4	0.016	0.014
$^{22}Ne+\alpha$	8	(8,2)	(0,0)	(8,2)	(4,8)	6	0.0085	0.0072*

TABLE II. (*Continued.*)

System	n_π	Γ_1	Γ_2	Γ_C	Γ	L	Alg	Data
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(10,2)	0	0.091	0.083
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(10,2)	2	0.026	0.023
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(10,2)	4	0.0031	0.0024
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(10,2)	6	0.018	0.016
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(6,4)	0	0.149	0.089
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(6,4)	2	0.012	0.0072
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(6,4)	4	0.026	0.016
^{22}Ne+α	8	(8,2)	(0,0)	(8,2)	(6,4)	6	0.0008	0.00
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(4,8)	0	0.17	0.20*
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(4,8)	2	0.040	0.047
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(4,8)	4	0.020	0.024
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(4,8)	6	0.011	0.013
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(10,2)	0	0.09	0.10
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(10,2)	2	0.05	0.06
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(10,2)	4	0.0073	0.0084
^{22}Ne+α	8	(4,4)	(0,0)	(4,4)	(10,2)	6	0.0072	0.0084
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(0,12)	0	0.075	0.08*
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(0,12)	2	0.017	0.018
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(0,12)	4	0.008	0.008
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(0,12)	6	0.0033	0.0032*
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(12,0)	0	0.048	0.042*
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(12,0)	2	0.0109	0.0096
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(12,0)	4	0.0036	0.0032
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(12,0)	6	0.0091	0.0080
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(6,6)	0	0.081	0.057
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(6,6)	2	0.0058	0.0040
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(6,6)	4	0.015	0.010
^{24}Mg+α	8	(8,4)	(0,0)	(8,4)	(6,6)	6	0.0005	0.00
^{24}Mg+α	8	(6,2)	(0,0)	(6,2)	(12,0)	0	0.058	0.078*
^{24}Mg+α	8	(6,2)	(0,0)	(6,2)	(12,0)	2	0.033	0.046
^{24}Mg+α	8	(6,2)	(0,0)	(6,2)	(12,0)	4	0.0045	0.0064
^{24}Mg+α	8	(6,2)	(0,0)	(6,2)	(12,0)	6	0.0042	0.0056

respect to one another is proportional to $|F(R)|^2$ [12], where $F(R) \sim \exp(-aR^2)$ is the relative motion wave function. On the other hand the expectation value of R satisfies $\langle R \rangle \sim \sqrt{n_\pi}$ when the SACM is mapped onto a geometrical picture [20] and this results in the n_π dependence in the exponent. From this consideration one expects a negative value for the parameter $B=-a$.

The parameters were adjusted to specific spectroscopic factors from Refs. [4,8] listed in Table I as "Data." For the system ^{12}C+^{12}C at two excitation quanta, the quality of the fit depends on the cluster irreps (λ_C, μ_C). The (0,4) can be coupled with (0,4) to the irreps (4,0), (3,2), (2,4), (1,6), and (0,8). Because the individual clusters are assumed to be in

TABLE III. The same as in Table II but for various cluster systems In this case the data is from [8] rather than [4].

System	n_π	Γ_1	Γ_2	Γ_C	Γ	L	Alg.	Data
^{12}C+^{12}C	14	(0,4)	(0,4)	(4,0)	(14,2)	0	0.0064	0.0032*
^{12}C+^{12}C	14	(0,4)	(0,4)	(4,0)	(14,2)	2	0.0055	0.0028
^{12}C+^{12}C	14	(0,4)	(0,4)	(4,0)	(14,2)	4	0.0037	0.0002
^{12}C+^{12}C	14	(0,4)	(0,4)	(0,8)	(10,4)	0	0.17	0.15*
^{12}C+^{12}C	14	(0,4)	(0,4)	(0,8)	(10,4)	2	0.13	0.13
^{12}C+^{12}C	14	(0,4)	(0,4)	(0,8)	(10,4)	4	0.051	0.072
^{12}C+^{12}C	14	(0,4)	(0,4)	(2,4)	(12,0)	0	0.0071	0.010*
^{12}C+^{12}C	14	(0,4)	(0,4)	(2,4)	(12,0)	2	0.0057	0.0085
^{12}C+^{12}C	14	(0,4)	(0,4)	(2,4)	(12,0)	4	0.0032	0.0042
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(14,2)	0	1.68	1.68*
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(14,2)	2	1.17	—
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(14,2)	4	0.40	—
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(10,4)	0	2.99	0.60
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(10,4)	2	1.63	—
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(10,4)	4	0.18	—
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(12,3)	0	0.14	0.34
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(12,3)	2	0.89	—
^{20}Ne+α	10	(8,0)	(0,0)	(8,0)	(12,3)	4	1.73	—
^{16}O+^{8}Be	14	(0,0)	(4,0)	(4,0)	(14,2)	0	0.069	0.065
^{16}O+^{8}Be	14	(0,0)	(4,0)	(4,0)	(14,2)	2	0.063	—
^{16}O+^{8}Be	14	(0,0)	(4,0)	(4,0)	(14,2)	4	0.050	—

their ground state with total angular momentum zero, the (3,2) and (1,6) cluster irreps can be excluded because they do not contain a zero angular momentum state. One also has to ensure that the cluster irreps (λ_C, μ_C) can be coupled with the relative motion factor to the corresponding final irrep. The values of (λ_C, μ_C) as given in Table III correspond to the best fit. The data, given in [8] where divided by 0.23 [21] in order to normalize them to the ground state spectroscopic factor of ^{16}O+α. For the fitting procedure we used the MINUIT routine from the CERN library [22]. The resulting parameter values are: $A=3.616\,3$, $B=-0.361\,13$, $C=-0.054\,389$, $D=-0.11764$, $E=0.060\,728$, $F=-0.008\,665\,4$, $G=0.000\,021\,097$, and $H=1.9090$. As can be seen, the dominant part is given by the constant term in the exponent and the term proportional to n_π. Though the parameter G appears to be small, one must recognize that the eigenvalues of $\mathcal{C}_3(\lambda, \mu)$ are generally very large. For example, for (12,0) the eigenvalue is 4860 and with the factor it gives a contribution on the order of 0.01. The terms depending on the SU(3) irreps are related to the deformation of the clusters and the total system [23,24]. In Table I we show the data to which the parameters had been adjusted. The χ^2 value was defined as $\Sigma_i\{\ln[S(i)]-\ln[S_{dat}(i)]\}^2$, where the sum runs over all data points, $S(i)$ is the spectroscopic factor as obtained by our

parametrization and $S_{dat}(i)$ denote the data used. In Tables II and III we show all 93 data to which we compared the fit. The χ^2 value obtained in our fit is 0.52. As can be seen, the agreement is very good, save for the (10,4) irrep of the system $^{20}\mathrm{Ne}+\alpha$.

In summary, we note that a parametrization of spectroscopic factors from SU(3) shell-model calculations has been introduced. The agreement with the data obtained from first principles is very good and gives rise to an expectation that it might be possible to extend this approach into areas not yet considered and/or too difficult to determine from first principles. For example, there is a renewed interest in describing the fusion cross section of $^{12}\mathrm{C}+^{12}\mathrm{C}$ and additional experimental data will be available soon [25]. Extrapolating the spectroscopic factor to highly excited states would be of great use, but first it has to describe the spectroscopic factors at low energy. The fact that such a simple parametrization can reproduce the exact result so well suggests that the parametrization carries a deeper meaning. The dependence on the relative oscillation quanta n_π is easily understood. The dependence on the isoscalar factors of $\mathrm{SU}(3)\supset\mathrm{SO}(3)$ can be interpreted as representing the overlaps of the total SU(3) state with the product of the SU(3) cluster states. The dependence on the other terms in the exponential, which are related to the deformation of the system, is more difficult to understand. The negative sign of C and D indicate that the spectroscopic factor increases when the deformation is lowest for the clusters, using the result of Refs. [23,24] that the eigenvalue of the second-order Casimir operator is proportional to the deformation squared. The positive sign of E means that the two clusters have to be joined in the most elongated form [20] in order to increase the decay probability. The negative sign of F reflects the fact that the cluster decay probability decreases with the deformation of the parent nucleus. The positive H shows that with increasing intershell excitations the spectroscopic factor increases, which is normally observed because with increasing Δn_π the overlap of the initial nucleus with the cluster configuration increases. The positive G requires a prolate elongation of the total system [$\mathcal{C}_3(\lambda,\mu)$ is proportional to $(\lambda-\mu)$] for increasing the spectroscopic factor, which is consistent with the positive sign of E.

Since the parametrization gives very valuable information on the dependence of the spectroscopic factor on its microscopic structure and quantum numbers, it will be important to find a deeper understanding of this simple formulation.

Financial support from DGAPA (IN119002), the CONACyT-MTA collaboration, CONACyT, OTKA (T37502) and the U.S. National Science Foundation (0140300) is acknowledged.

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