# Neutrino neutral reaction on $^4$He: Effects of final state interaction and realistic *NN* force

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The inelastic neutral reaction of neutrino on $^4$He is calculated microscopically, including full final state interaction among the four nucleons. The calculation is performed using the Lorentz integral transform method and the hyperspherical-harmonic effective interaction approach, with a realistic nucleon-nucleon interaction. A detailed energy dependent calculation is given in the impulse approximation. With respect to previous calculations, this work predicts an increased reaction cross section by 10–30% for neutrino temperature up to 15 MeV.

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The interest in neutrino reactions with nuclear targets stems from the role they play in major questions of contemporary physics. Such reactions are of central importance in various astrophysical phenomena, such as supernova explosion and the nucleosynthesis of the elements. In this Brief Report, we present a microscopic *ab initio* calculation of the neutral inelastic reactions of $^4$He with $\nu_x(\bar{\nu}_x)$ $(x=e,\mu,\tau)$.

Core collapse supernovae are widely accepted to be a neutrino driven explosion of a massive star. When the iron core of a massive star becomes gravitationally unstable it collapses until short-range nuclear forces halt the collapse and drive an outgoing shock through the outer layers of the core and the inner envelope. However, the shock loses energy through dissociation of iron nuclei and neutrino radiation, and gradually stalls; it becomes an accretion shock. It is believed, but to date not proven, that the shock is then revived as neutrinos emitted from the collapsed core (the proto-neutron star) deposit energy in the collapsing layers to overcome the infall and eventually reverse the flow to an outgoing shock which explodes the star. Hydrodynamic simulations of a collapsing star, which are restricted to spherical symmetry, fail in reviving the shock [1]. Lately it was shown [2] that even in full two-dimensional (2D) calculations the shock is not revived. In order to revive the shock, the neutrinos must deposit about 1% of their energy in the matter behind the shock. The latter, which is assumed to be in thermodynamic equilibrium, is composed mainly of protons, neutrons, electrons, and $^4$He nuclei. In contrast to the fairly known cross sections of neutrinos with electrons and nucleons, the interaction of neutrinos with $^4$He is not accurately known, and to date there is no realistic microscopic calculation of the inelastic $^4$He-neutrino cross section. The effect of neutrino-$^4$He interaction on the delayed shock mechanism was investigated by Bruenn and Haxton [3], through a presupernova 1D model of a 15 $M_\odot$ star. In that model, they found only a small reheating of the matter behind the shock, which can be attributed to the low mean energy of the neutrinos in comparison to the high threshold energy of the Alpha nucleus. This conclusion may change with different progenitor, or with enlarged inelastic neutrino-$^4$He cross sections.

The neutrinos migrating out of the proto-neutron star are in flavor equilibrium for most of their migration. The electron neutrinos remain in equilibrium with matter for a longer period than their heavy-flavor counterparts, due to the larger cross sections for scattering of electrons and because of charge current reactions. Thus the heavy-flavor neutrinos decouple from deeper within the star, where temperatures are higher. Typical calculations yield temperatures of ~10 MeV for $\mu$ and $\tau$ neutrinos [4], which is approximately twice the temperature of electron neutrinos. Consequently, there is a considerable amount of $\nu_{\mu,\tau}$ with energies above 20 MeV that can dissociate the $^4$He through neutral reaction.

The flux of neutrinos emitted in the collapse process is sufficiently large to initiate nucleosynthesis in the overlaying shells of heavy elements. Neutral reactions of Alpha and neutrino in the inner helium shell are part of reaction sequences leading to the production of the rare $A=7$ lithium and beryllium isotopes [5,6]. Thus better understanding of the $\nu-\alpha$ reaction can lead to better prediction for the abundances of these elements.

Theoretical understanding of the neutrino-nucleus scattering process is achieved through perturbation theory of the weak interaction model. The nuclear electroweak transition operator consists of one- and many-body components. The many-body currents are a result of meson exchange between the nucleons, and usually contribute up to 10% of the cross section in the supernova energy regime. However, when leading one-body terms are suppressed their contribution can be even larger. The current work is done in the impulse approximation, thus taking into account only one-body terms. The one-body currents connect the $^4$He ground state and final state wave functions. In order to calculate the cross section in a percentage level accuracy, one needs a solid estimate of these wave functions. Alas, for nuclear systems with more than three constituents, where particle correlation plays a decisive role, the computation of intermediate-energy continuum wave function is currently out of reach.

To facilitate the calculation of the neutral reaction of neutrino and alpha particle we introduce several modern methods. The calculation of the nuclear dynamics is carried out by combining two powerful tools: the Lorentz integral transform (LIT) method [7] and the effective interaction hyperspherical harmonics (EIHH) method [8]. First we use the LIT

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method in order to convert the scattering problem into a bound-state-like problem, and then the EIHH method is used to solve the resulting equations. Using this procedure we solve the final state interaction problem avoiding continuum wave functions. This method was used successfully to calculate the photoabsorption cross sections of up to six body nuclei [9–11]. To this end we use nuclear Hamiltonian consists of the realistic Argonne nucleon-nucleon potential model AV8′ [12].

In the limit of small momentum transfer (compared to the $Z$ particle rest mass), the effective Hamiltonian can be written as

$$\hat{H}_W = \frac{G}{\sqrt{2}}\int d^3x j_\mu(\vec{x})J^\mu(\vec{x}), \tag{1}$$

where $G$ is the Fermi weak coupling constant, $j_\mu(\vec{x})$ is the leptonic current, and $J^\mu$ is the hadronic current. The matrix element of the leptonic current is $\langle f|j_\mu|i\rangle = l_\mu e^{-i\vec{q}\cdot\vec{x}}$, where $l_\mu = \overline{u}(k_{\nu'})\gamma_\mu(1-\gamma_5)u(k_\nu)$. The nuclear current,

$$J_\mu^{hadronic} = (1-2\sin^2\theta_W)\frac{\tau_0}{2}J_\mu + \frac{\tau_0}{2}\vec{J}_\mu^5 - 2\cdot\sin^2\theta_W\tfrac{1}{2}J_\mu, \tag{2}$$

consists of one body weak currents, but also many body corrections due to meson exchange. In this work we use the impulse approximation. Since the momentum transfer relevant to our calculation are small compared to the nucleon mass, we ignore relativistic corrections. The differential cross section is given by Fermi's golden rule. Thus in order to consider recoil effects, and with unoriented and unobserved targets, the differential cross section takes the form

$$d\sigma = \int d\epsilon\,\delta\left(\epsilon - \omega + \frac{q^2}{2M_{^4\mathrm{He}}}\right)2\pi\frac{d^3\vec{k}_f}{(2\pi)^3} \times \sum_f \frac{\sum_{M_i=-J_i}^{J_i}}{2J_i+1}\sum_{helicities}|\langle f|\hat{H}_W|i\rangle|^2\delta(E_f - E_i + \epsilon), \tag{3}$$

where $\vec{k}_f$ is the momentum of the outgoing neutrino, $\omega$ is the energy transfer, and $\vec{q}$ is the momentum transfer.

Choosing the $\hat{z}$ direction to be parallel to the momentum transfer, and $\theta$ to be the angle between the incoming neutrino direction and outgoing neutrino direction, the cross section can be written as [13]

$$\begin{aligned}\frac{d\sigma}{dk_f} = \int d\epsilon\,\delta\left(\epsilon - \omega + \frac{q^2}{2M_{^4\mathrm{He}}}\right)\frac{4G^2}{2J_i+1}k_f^2\int_0^\pi \sin\theta d\theta \\ \times\Bigg\{\left[\sin^2\frac{\theta}{2} - \frac{q^\mu q_\mu}{2q^2}\cos^2\frac{\theta}{2}\right]\sum_{J\geqslant 1}[R_{\hat{M}_J}(\epsilon) + R_{\hat{E}_J}(\epsilon)] \\ \mp \sin\frac{\theta}{2}\sqrt{\sin^2\frac{\theta}{2} - \frac{q^\mu q_\mu}{2q^2}\cos^2\frac{\theta}{2}}\sum_{J\geqslant 1}2R_{\hat{E}_J\hat{M}_J}(\epsilon) \\ + \cos^2\frac{\theta}{2}\sum_{J\geqslant 0}R_{\hat{C}_J-(\omega/q)\hat{L}_J}(\epsilon)\Bigg\}\end{aligned} \tag{4}$$

the $-(+)$ is for neutrino (antineutrino). The functions

$$R_{\hat{O}_1\hat{O}_2}(\omega) = \int d\Psi_f\langle\Psi_0\|\hat{O}_1\|\Psi_f\rangle\langle\Psi_f\|\hat{O}_2\|\Psi_0\rangle\delta(E_f - E_0 - \omega) \tag{5}$$

are the response functions with respect to the transition operators $\hat{O}_1$ and $\hat{O}_2$ (when $\hat{O}_1 = \hat{O}_2$ we use the notation $R_{\hat{O}} = R_{\hat{O}\hat{O}}$). $|\Psi_{0,f}\rangle$ and $E_{0,f}$ are the wave function and energy of the ground and final state, respectively. The transition operators $C_J(q), L_J(q), E_J(q), M_J(q)$ are the reduced Coulomb, longitudinal, transverse electric, and transverse magnetic multipole operators. Since the relevant energy regime is up to $\approx$60 MeV, the main operators contributing to the inelastic cross section are the axial vector operators $E_2^5, L_2^5, M_1^5, L_0^5$ and the vector $C_1, E_1, L_1$. Usually, the main contribution comes from the Gamow-Teller $E_1^5$ operator but due to the closed shell character of the $^4$He nucleus, it is highly suppressed. In this energy range the long wavelength limit [13],

$$\begin{aligned} C_{1M}(q) &= F_V\frac{qr}{3}Y_{1M}(\hat{r}), \\ E_{1M}(q) &= -\sqrt{2}\frac{\omega}{q}C_{1M}(q), \\ L_{1M}(q) &= -\frac{\omega}{q}C_{1M}(q), \\ M_{1M}^5(q) &= F_A\frac{qr}{3}\vec{\sigma}\cdot\vec{Y}_{11M}(\hat{r}), \\ E_{2M}^5(q) &= -i\sqrt{\frac{3}{5}}F_A\frac{qr}{3}\vec{\sigma}\cdot\vec{Y}_{21M}(\hat{r}), \\ L_{2M}^5(q) &= \sqrt{\frac{2}{3}}E_{2M}^5(q), \\ L_{00}^5(q) &= -iF_A\frac{qr}{3}\vec{\sigma}\cdot\vec{Y}_{010}(\hat{r}) \end{aligned} \tag{6}$$

is a rather good approximation. (Here $\vec{r}$ is the nucleon's location relative to the system's center of mass.) However, in our calculations we have used the exact form of the multipole operators, and we may comment that in this case the long wavelength approximation is accurate to percentage level. The same holds for the contribution of higher multipoles.

The response functions are calculated by inverting the Lorentz integral transforms

$$L_{\hat{O}_1\hat{O}_2}(\sigma) = \int d\omega\frac{R_{\hat{O}_1\hat{O}_2}(\omega)}{(\omega-\sigma_R)^2 + \sigma_I^2} = \langle\tilde{\Psi}_1|\tilde{\Psi}_2\rangle,$$

where $\sigma = \sigma_R + i\sigma_I$, and $|\tilde{\Psi}_i\rangle$ $(i=1,2)$ are solutions of the Schrödinger-like equations

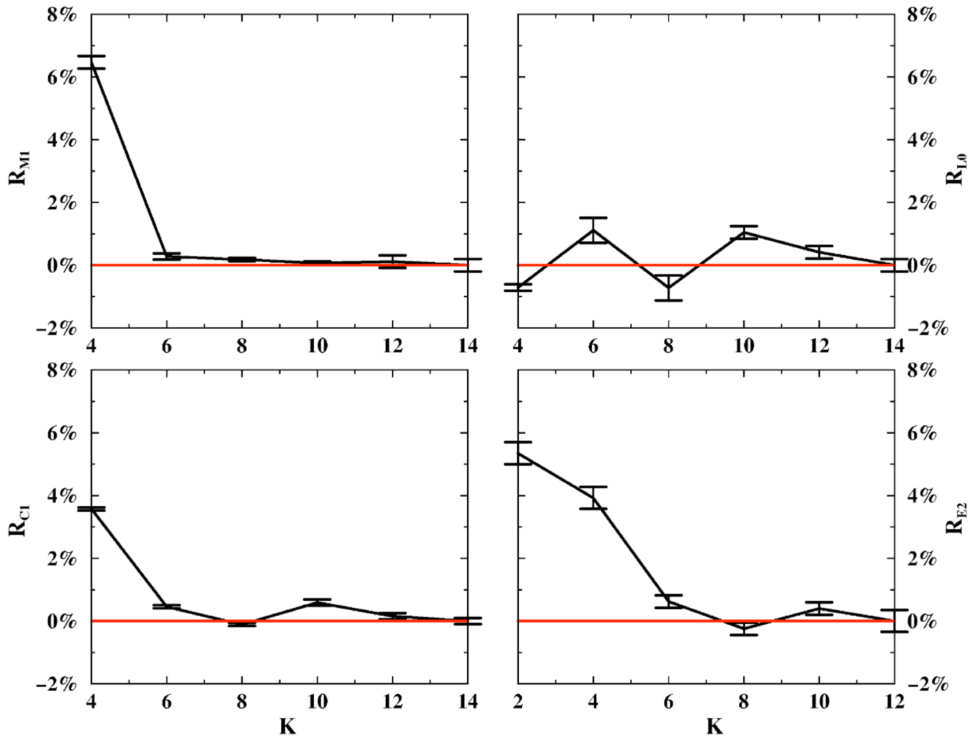


FIG. 1. (Color online) Relative error in the sum rule of the leading response functions with respect to the hyper-angular momentum quantum number $K$. The error bars reflect the uncertainty in inverting the LIT.

$$(H - E_0 - \sigma)|\tilde{\Psi}_i(\sigma)\rangle = \hat{O}_i|\Psi_0\rangle.$$

The localized character of the ground state, and the imaginary part of $\sigma$, give these equations an asymptotic boundary condition similar to a bound state. As a result, one can solve these equations using the EIHH [8,14] method. In this approach, the wave function is expanded in the hyperspherical-harmonics (HH) series. The expansion parameter is the hyperspherical (grand) angular momentum quantum number ($K$), and the expansion is truncated at some value $K=K_{max}$ which defines the model space. For this model space the bare potential is replaced by an Hermitian effective interaction constructed via the Lee-Suzuki method [15]. The resulting effective equations are solved by expanding $\Psi_0$ and $\tilde{\Psi}_i$ in four-body antisymmetrized HH basis functions [16,17]. We calculate the matrix element $\langle\tilde{\Psi}_1|\tilde{\Psi}_2\rangle$ using the Lanczos algorithm [18].

The combination of the EIHH and LIT methods brings to a rapid convergence in the Response functions. In Fig. 1, one can see the relative error in the sum rule of the main response functions with respect to the hyperangular momentum quantum number $K$. It can be seen that upon convergence the relative error is well below 1%. The error bars presented reflect the error in inverting the LIT. Bearing in mind that the cross section, up to kinematical factors, is the sum of the response functions, this is a measure of the accuracy in the calculation of the cross section.

It is well known that realistic two-body NN potentials lead to an underbinding of about 0.5–1 MeV for the $^3$He and the triton nuclei and an underbinding of about 3–4 MeV for $^4$He. For the AV8′ force with a simple Coulomb interaction we obtained a binding energy of 25.19 MeV for $^4$He, and 7.76 MeV for the triton. Thus our model has a discrepancy, $\Delta \approx 2.4$ MeV, with respect to the experimental inelastic reaction threshold. In order to correct for this difference we shifted the response function to the true threshold, i.e., $R(\omega) \to R(\omega - \Delta)$.

It is assumed that the neutrinos are in thermal equilibrium, thus their spectrum can be approximated by the Fermi-Dirac distribution with characteristic temperature $T$. As a result, the interesting quantities are the temperature averaged cross section and energy transfer cross section:

$$\frac{d\langle\sigma\rangle_T}{d\omega} = \int dk_i f(T,k_i)\frac{d\sigma}{dk_f} \quad (7)$$

$$\frac{d\langle\sigma\omega\rangle_T}{d\omega} = \omega\frac{d\langle\sigma\rangle_T}{d\omega}, \quad (8)$$

where $f(T,k)$ is the normalized Fermi-Dirac spectrum with zero chemical potential, temperature $T$, and energy $k$, i.e.,

$$f(T,k) = \frac{0.5546}{T^3}\frac{k^2}{e^{k/T}+1}. \quad (9)$$

As a typical example we present in Fig. 2 the calculated cross section for $T=10$ MeV. In Table I we present the calculated total temperature averaged cross section, $\langle\sigma\rangle_T = \frac{1}{2}(1/A)\langle\sigma_\nu + \sigma_{\bar{\nu}}\rangle_T$, and energy transfer cross section, $\langle\sigma\omega\rangle_T = \frac{1}{2}(1/A)\langle\omega\sigma_\nu + \omega\sigma_{\bar{\nu}}\rangle_T$, as a function of the neutrinos' temperature. Also presented are earlier results by Woosley *et al.* [5]. It can be seen that the current work predicts an enhancement of about 10–30% in the cross section.

The energy transfer cross section was fitted by Haxton to the formula [19]

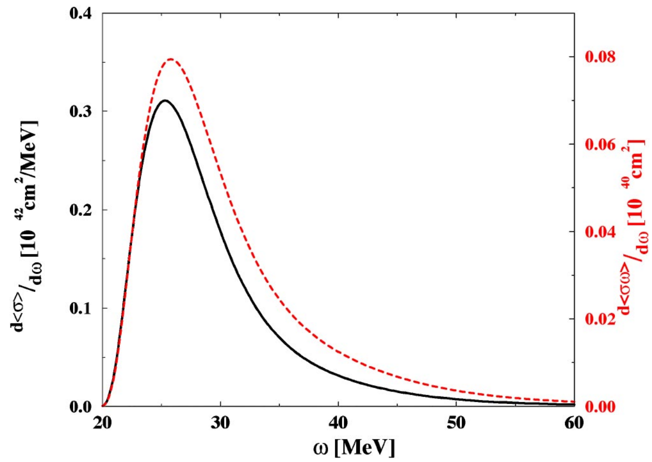


FIG. 2. (Color online) Temperature averaged inelastic cross sections at temperature $T=10$ MeV. The solid line is the differential cross section, $\langle d\sigma/d\omega\rangle_T=\frac{1}{2}(1/A)\langle d\sigma_\nu/d\omega+d\sigma_{\bar{\nu}}/d\omega\rangle_T$ (left scale). The dashed line is the differential energy transfer cross section, $\langle\omega(d\sigma/d\omega)\rangle_T=\frac{1}{2}(1/A)\langle\omega(d\sigma_\nu/d\omega)+\omega(d\sigma_{\bar{\nu}}/d\omega)\rangle_T$ (right scale).

$$\langle\sigma\omega\rangle_T=\alpha\left(\frac{T-T_0}{10\text{ MeV}}\right)^\beta \tag{10}$$

with the parameters $\alpha=0.62\times10^{-40}$ cm$^2$ MeV, $T_0=2.54$ MeV, $\beta=3.82$. A similar fit to our results yields $\alpha=0.64\times10^{-40}$ cm$^2$ MeV, $T_0=2.05$ MeV, $\beta=4.46$. It can be seen that the current work predicts a stronger temperature dependence of the cross sections. For example, a 15% difference between these calculations at $T=10$ MeV grows to a 50% difference at $T=16$ MeV.

In conclusion, a detailed realistic calculation of the inelastic neutrino-$^4$He neutral scattering cross section is given. The calculation was done in the impulse approximation with numerical accuracy of about 1%. The different approximations used here should result in about 10% error, mainly due to many-body currents, which were not considered in the current work. In order to estimate the effect of two-body currents, we present in Fig. 3 the contribution of the various operators to the total cross section at $T=10$ MeV. It can be seen that the axial vector part contributes more than 90% of the cross section given in Fig. 2. It is known from studies of inclusive electron scattering off $^4$He [20] that isovector electromagnetic two-body currents, which are proportional to the electroweak vector currents, produce a strong enhancement of the transverse response at low and intermediate energies. In the current case, the vector part is almost negligible with respect to the axial part, and the two-body axial currents are expected to give small contributions [21]. Thus two body currents should result in a percentage level error in our estimate for the cross section.

TABLE I. Flavor and temperature averaged inclusive inelastic cross section and energy transfer cross section calculated. The temperatures are given in MeV, the cross sections in $10^{-42}$ cm$^2$, and the energy transfer cross sections in $10^{-40}$ cm$^2$ MeV.

| $T$ (MeV) | $\langle\sigma\rangle_T[10^{-42}$ cm$^2]$ This work | Ref. [5] | $\langle\sigma\omega\rangle_T$ ($10^{-40}$ cm$^2$ MeV) |
|---|---|---|---|
| 4 | 2.09(−3) | | 5.27(−4) |
| 6 | 3.84(−2) | 3.87(−2) | 1.03(−2) |
| 8 | 2.25(−1) | 2.14(−1) | 6.30(−2) |
| 10 | 7.85(−1) | 6.78(−1) | 2.30(−1) |
| 12 | 2.05 | 1.63 | 6.27(−1) |
| 14 | 4.45 | | 1.42 |
| 16 | 8.52 | | 2.84 |

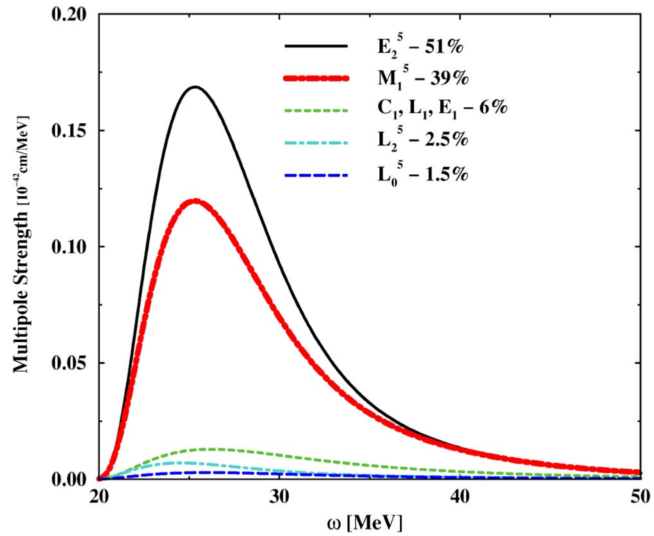


FIG. 3. (Color online) Temperature averaged inelastic multipole strength at temperature $T=10$ MeV. The different lines refer to different multipoles. The percent in the legends indicates the relative part of the specific multipole in the inelastic cross section.

The effect of these results on the supernova explosion mechanism should be checked through hydrodynamic simulations, of various progenitors. Nonetheless, it is clear that our results facilitate a stronger neutrino-matter coupling in the supernova environment. First, our calculations predict an enhanced cross section by 10–30% with respect to previous estimates. Second, we obtained steeper dependence of the energy transfer cross section on the neutrino's temperature, thus supporting the observation that the core temperature is a critical parameter in the explosion process. It is important to notice that the energy transfer due to inelastic reactions are 1–2 orders of magnitude larger than the elastic reactions, ergo the inelastic cross sections are important to an accurate description of the helium shell temperature.

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