Calculation of screening masses in a chiral quark model

Xiangdong Li
Department of Computer System Technology, New York City College of Technology of the City University of New York, Brooklyn, New York 11201, USA

Hu Li, C. M. Shakin,* and Qing Sun
Department of Physics and Center for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210, USA
(Received 12 May 2004; published 4 October 2004)

We consider a simple model for the coordinate-space vacuum polarization function which is often parametrized in terms of a screening mass. We discuss the circumstances in which the value $m_{sc}=\pi T$ is obtained for the screening mass. In the model considered here, that result is obtained when the momenta in the relevant vacuum polarization integral are small with respect to the first Matsubara frequency.

DOI: 10.1103/PhysRevC.70.048201 PACS number(s): 12.39.Fe, 12.38.Aw, 14.65.Bt

In early work Eletskii and Ioffe [1] considered the leading QCD diagram in the calculation of the screening mass and suggested that for $T\to\infty$, $m_{sc}=2\pi T$. On the other hand, Florkowski and Friman [2] obtained values closer to $m_{sc}=\pi T$ for $T\simeq 400$ GeV in the context of an analytic study of the Nambu-Jona-Lasinio (NJL) model at a finite temperature.

In our studies of hadronic current correlation functions we did not obtain exponential behavior for the Euclidean-space correlator and were not able to define a screening mass. We have traced this problem to our use of a quite large cutoff for the momentum circulating in the vacuum polarization diagram. In the present work we wish to show that exponential behavior is found for all values of the distance z between the two points defining the correlation function if there is a relatively small cutoff for the momentum in the polarization diagram. If that momentum is less than the screening mass we obtained the expected exponential behavior with a screening mass that agrees with that found in Ref. [2].

In a number of recent works [3–5] we have calculated various hadronic correlation functions and compared our results to results obtained in lattice simulations of QCD [6–8]. The lattice results for the correlators, $G(\tau,T)$, may be used to obtain the corresponding spectral functions, $\sigma(\omega,T)$, by making use of the relation

$$G(\tau,T)=\int_0^\infty d\omega\ \sigma(\omega,T)K(\tau,\omega,T), \tag{1}$$

where

$$K(\tau,\omega,T)=\frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)}. \tag{2}$$

The procedure to obtain $\sigma(\omega,T)$ from the knowledge of $G(\tau,T)$ makes use of the maximum entropy method (MEM) [9–11], since $G(\tau,T)$ is only known at a limited number of points.

*Electronic address: casbc@cunyvm.cuny.edu

In our studies of meson spectra at $T=0$ and at $T<T_c$ we have made use of the Nambu-Jona-Lasinio (NJL) model. The Lagrangian of the generalized NJL model we have used in our studies is

$$\begin{aligned}L &= \bar{q}(i\partial\!\!\!/ - m^0)q + \frac{G_S}{2}\sum_{i=0}^{8}[(\bar{q}\lambda^i q)^2 + (\bar{q}i\gamma_5\lambda^i q)^2] \\ &\quad - \frac{G_V}{2}\sum_{i=0}^{8}[(\bar{q}\lambda^i\gamma_\mu q)^2 + (\bar{q}\lambda^i\gamma_5\gamma_\mu q)^2] \\ &\quad + \frac{G_D}{2}\{\det[\bar{q}(1+\lambda_5)q] + \det[\bar{q}(1-\lambda_5)q]\} + \mathcal{L}_{conf}. \end{aligned} \tag{3}$$

Here, m^0 is a current quark mass matrix, $m^0=\mathrm{diag}(m_u^0,m_d^0,m_s^0)$. The λ_i are the Gell-Mann (flavor) matrices and $\lambda^0=\sqrt{2/3}\mathbf{1}$, with $\mathbf{1}$ being the unit matrix. The fourth term is the 't Hooft interaction and $\mathcal{L}_{conf}$ represents the model of confinement used in our studies of meson properties.

In the study of hadronic current correlators it is important to use a model which respects chiral symmetry, when $m^0=0$. Therefore, we make use of the Lagrangian of Eq. (3), while neglecting the 't Hooft interaction and $\mathcal{L}_{conf}$. In order to make contact with the results of lattice simulations we use the model with the number of flavors, $N_f=1$. Therefore, the λ^i matrices in Eq. (3) may be replaced by unity. We then have used

$$\begin{aligned}\mathcal{L} &= \bar{q}(i\partial\!\!\!/ - m^0)q + \frac{G_S}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5 q)^2] \\ &\quad - \frac{G_V}{2}[(\bar{q}\gamma_\mu q)^2 + (\bar{q}\gamma_5\gamma_\mu q)^2], \end{aligned} \tag{4}$$

in order to calculate the hadronic current correlation functions in earlier work [3–5].

In order to present our results in the simplest form, we consider only the scalar interaction proportional to $(\bar{q}q)^2$. We also extend the definition of $\sigma(\omega,T)$ of Eq. (1) to include a dependence upon the total moment of the quark and anti-

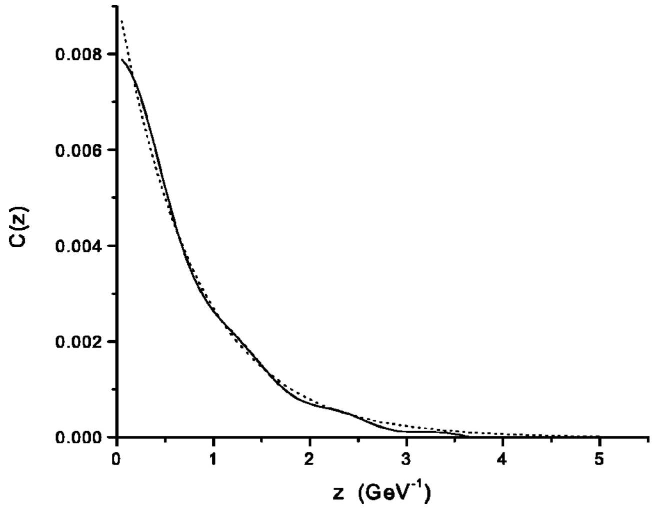

FIG. 1. The function $C(z)$ of Eq. (6) is shown for a sharp cutoff of $k_{max}=0.1$ GeV. The dotted line represents an exponential fit to the curve using $m_{sc}=1.23$ GeV. (We recall that πT is equal to 1.27 GeV.)

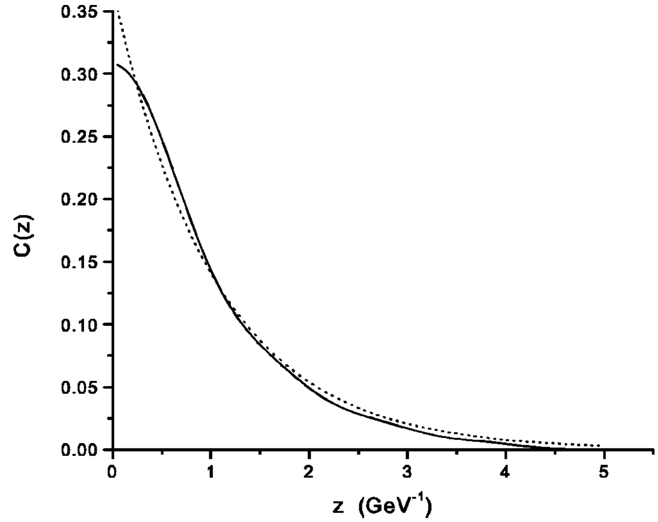

FIG. 2. The function $C(z)$ of Eq. (6) is shown for a sharp cutoff of $k_{max}=0.4$ GeV. The dotted line represents an exponential fit to the curve using $m_{sc}=0.961$ GeV. (We recall that πT is equal to 1.27 GeV.

quark appearing in the polarization integral. Thus we consider the imaginary part of the correlator, $\sigma(\omega,\vec{P})$. Since we place $\vec{P}$ along the z-axis this quantity may be written as $\sigma(\omega,0,0,P_z)$ in accord with the notation of Ref. [12]. In this work we will present our results for the coordinate-dependent correlator $C(z)$ which is proportional to the correlator defined in Eq. (1) of Ref. [12],

$$C(z) = \frac{1}{2}\int_{-\infty}^{\infty} dP_z\, e^{iP_z z}\int_0^{\infty} d\omega \frac{\sigma(\omega,0,0,P_z)}{\omega}. \tag{5}$$

We may also use the form

$$C(z) = \frac{1}{4}\int_{-\infty}^{\infty} dP_z\, e^{iP_z z}\int_0^{\infty} dP^2 \frac{\sigma(P^2,0,0,P_z)}{P^2}. \tag{6}$$

We have made a study of the screening mass in a simple model in order to understand the origin of exponential behavior for the correlator. To that end we make use of Ref. [13]. We consider the Matsubara formalism and note that the quark propagator may be written, with $\beta=1/T$, as

$$S_\beta(\vec{k},\omega_n) = \frac{\gamma^0(2n+1)\pi/\beta + \vec{\gamma}\cdot\vec{k} - M}{(2n+1)^2\pi^2/\beta^2 + \vec{k}^2 + M^2}. \tag{7}$$

For bosons the vacuum polarization function is given as Eq. (1.51) of Ref. [13],

$$\Pi(\vec{p},p^0) = \frac{g^2}{2\beta}\sum_n \frac{d^3k}{(2\pi)^3} \times \frac{1}{\frac{4n^2\pi^2}{\beta^2} + \vec{k}^2 + M^2}\cdot\frac{1}{\left(\frac{2n\pi}{\beta}+p^0\right)^2 + (\vec{k}+\vec{p})^2 + M^2}. \tag{8}$$

We modify Eq. (8) to refer to fermions. In this case the Matsubara frequencies are

$$\omega_n = \frac{(2n+1)\pi}{\beta}, \tag{9}$$

and we have

$$\Pi(\vec{p},p^0) = \frac{g^2}{2\beta}\mathrm{Tr}\int \frac{d^3k}{(2\pi)^3} \times \frac{[(\gamma^0\pi/\beta + \vec{\gamma}\cdot\vec{k})(\gamma^0(p^0+\pi/\beta) + \vec{\gamma}\cdot(\vec{k}+\vec{p}))]}{\left(\frac{\pi^2}{\beta^2}+\vec{k}^2\right)\left[\left(\frac{\pi}{\beta}+p^0\right)^2 + (\vec{k}+\vec{p})^2\right]}, \tag{10}$$

if we keep only the first term in the sum, where $\omega_0=\pi/\beta$. As a next step we drop p^0, so that we have

$$\Pi(\vec{p},0) = \frac{g^2}{2\beta}\mathrm{Tr}\int \frac{d^3k}{(2\pi)^3} \times \frac{[(\gamma^0\pi/\beta + \vec{\gamma}\cdot\vec{k})(\gamma^0\pi/\beta + \vec{\gamma}\cdot(\vec{k}+\vec{p}))]}{\left[\left(\frac{\pi}{\beta}\right)^2+\vec{k}^2\right]\left[\left(\frac{\pi}{\beta}\right)^2 + (\vec{k}+\vec{p})^2\right]}. \tag{11}$$

We then take $\vec{p}$ along the z axis and write $\Pi(p_z)=\Pi(\vec{p},0)$. We define

$$C(z) = \int dp_z\, e^{ip_z z}\Pi(p_z). \tag{12}$$

In our calculation we replace $g^2/2\beta$ by unity and use a sharp cutoff so that $|\vec{k}|<k_{max}$.

The results of our calculation of $C(z)$ of Eq. (6) are given in Figs. 1 and 2. In Fig. 1 we use $k_{max}=0.1$ GeV and in Fig.

2we put $k_{max}=0.4$ GeV. For our calculations, we have $m_{sc}=\pi T=1.27$ GeV when $T=1.5\ T_c$ and $T_c=0.27$ GeV. Thus, the k_{max} values considered here are less than m_{sc} and that feature leads to the exponential behavior seen in Figs. 1 and 2. If k_{max} is made larger than 0.4 GeV we begin to see deviations from exponential behavior for $C(z)$. (Since in our calculations reported in Refs. [3–5], the integrals were regulated with a Gaussian regulator $\exp[-\vec{k}^2/\alpha^2]$ with $\alpha \simeq 4$ GeV, we can see that the $\vec{k}$ values in those calculations are so large as to preclude obtaining exponential behavior for our coordinate-space correlator.)

Our goal in this work was to consider a simple quark model for the calculation of a hadronic current correlation function and to determine the conditions under which the coordinate-space correlator is dominated by the screening mass which is given by the first Matsubara frequency. We have found that the standard result is obtained if the quark and antiquark momenta in the vacuum polarization calculation are small compared to that frequency.

[1] V. L. Eletskii and B. L. Ioffe, Sov. J. Nucl. Phys. **48**, 384 (1988).
[2] W. Florkowski and B. L. Friman, Acta Phys. Pol. B **25**, 49 (1994).
[3] B. He, H. Li, C. M. Shakin, and Q. Sun, Phys. Rev. D **67**, 014022 (2003).
[4] B. He, H. Li, C. M. Shakin, and Q. Sun, Phys. Rev. D **67**, 114012 (2003).
[5] B. He, H. Li, C. M. Shakin, and Q. Sun, Phys. Rev. C **67**, 065203 (2003).
[6] I. Wetzorke, F. Karsch, E. Laermann, P. Petreczky, and S. Stickan, Nucl. Phys. B, Proc. Suppl. **106**, 510 (2002).
[7] F. Karsch, S. Datta, E. Laermann, P. Petreczky, S. Stickan, and I. Wetzorke, Nucl. Phys. **A715**, 701c (2003).
[8] F. Karsch, E. Laermann, P. Petreczky, S. Stickan, and I. Wetzorke, Phys. Lett. B **530**, 147 (2002).
[9] M. Asakawa, T. Hatsuda, and Y. Nakahara, Nucl. Phys. **A715**, 863 (2003).
[10] T. Umeda, K. Nomura, and H. Matsufuru, hep-ph/0211003.
[11] I. Wetzorke, Invited talk at the 7th Workshop on Quantum Chromodunamics, Villefranche-sur-mer, France, 2003.
[12] P. Petreczky, J. Phys. G **30**, S431 (2004).
[13] A. Das, *Finite Temperature Field Theory* (World Scientific, Singapore, 1997).