Exponential enhancement of nuclear reactions in a condensed matter environment

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A mechanism that uses the environment to increase the probability of the nuclear reaction when a beam of accelerated nuclei collides with a target nucleus implanted in condensed matter is suggested. The effect considered is exponentially large for low collision energies. For $t+p$ collision the mechanism becomes effective when the energy of the projectile tritium is below 1 KeV per nucleon. The gain in probability of the nuclear reaction is due to a redistribution of energy and momentum of the projectile in several "preliminary" elastic collisions with the target nucleus and the environmental nuclei in such a way that the final inelastic projectile-target collision takes place with larger relative velocity, which is accompanied by the corresponding decrease of the center of mass energy. This increase of the relative velocity exponentially increases the penetration through the Coulomb barrier.

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It is well known that nuclear reactions at low energies are suppressed by the Coulomb repulsion between the nuclei. Recent experimental papers [1,2] indicate that the solid state environment of the target nucleus can, possibly, significantly boost the probability of the DD fusion. The previously suggested mechanisms of [3,5] are efficient only when the energy is too low to make the fusion observable in modern experiments. Here we examine the nonsymmetrical collisions when the projectile nucleus is heavier than the target. We show that in this case the environment produces an exponential enhancement of the cross section of a nuclear reaction that can be experimentally observable.

We consider a nuclear reaction that is due to a collisions of a beam of the projectile nuclei with the target nucleus that is implanted in the condensed matter environment. The projectile energy is presumed to be below the Coulomb barrier, where the probability of the nuclear reaction is proportional to the Coulomb suppression factor

$$P(v) = \frac{2\pi Z_{\text{proj}} Z_{\text{tar}} e^2}{\hbar v} \exp\left(-\frac{2\pi Z_{\text{proj}} Z_{\text{tar}} e^2}{\hbar v}\right). \qquad (1)$$

Here Z_{proj} and Z_{tar} are the charges of the projectile and target nuclei and v is their relative velocity. For the collision in the vacuum this velocity equals the initial velocity of the projectile V, $v=V$. However, a condensed matter environment alters the situation because the velocity of the collision can be changed due to redistribution of the momentum and energy of the projectile in collisions with the environment nuclei and the target nucleus. We show below that a chain of (quasi) elastic collisions with the environmental nuclei can, in fact, enlarge the collision velocity. This, according to Eq. (1), gives an exponential gain of the probability of the nuclear reaction. There is a price for this increase. The probability for the projectile and the target to remain on the collision course after collisions with the environment nuclei is small. The more elastic collisions take place, the smaller it is. However, for sufficiently low collision energies the exponential gain due to the increase of the velocity inevitably prevails.

The interest in studying the role of the condensed matter environment in nuclear reactions is inspired by a few mentioned publications (see Refs. [1,2], and references therein), that claim an increase of the DD fusion cross section in solids. Several possible mechanisms that allow an increase in collision velocity in the environment were considered previously [3–5]. Two of them [3,4] are related to the motion of the target nuclei due to the vibration of atoms in solids. Another one is related to a sequence of three elastic collisions [5]. This sophisticated chain of events, called a "carambole" collision in Ref. [5], produces a gain for the nuclear reaction, but this happens for very low energy of the projectile D (below 0.5 KeV) which were not tested in the mentioned experiments. In the present paper we examine collisions of the projectile nucleus that is heavier than the target nucleus. In this case there exists a *rescattering* mechanism to increase the collision velocity that relies on only two preliminary elastic collisions. This more simple chain of events prove to be more effective than the carambole mechanism of Ref. [5].

There exists the long standing discrepancy between by the experimental data on astrophysical fusion reactions at low energies [6–10] and calculations; the latter include effects of the electron screening, vacuum polarization, bremsstrahlung and atomic polarization, see Ref. [11], and references therein. The theoretical data slightly, but systematically underestimate the probability of the fusion.

Let the projectile and the target nuclei have masses M and m correspondingly and $M>m$. The masses of the environment nuclei will be considered large $M_{\text{env}} \gg m$. Consider the following sequence of events shown in Fig. 1. The projectile

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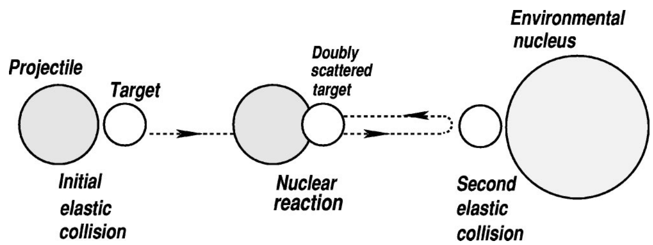

FIG. 1. Collision of the projectile nucleus with the target nucleus in the solid state environment. The rescattering mechanism involves two "preliminary" elastic collisions. First the elastic collision between the projectile and the target nuclei takes place. Then, the recoiled target nucleus collides with the heavy nucleus of the environment. After that the final collision of the projectile and the (doubly recoiled) target results in the nuclear reaction. This chain of events increases the collision velocity making the nuclear reaction more probable.

with the original velocity **V** first collides elastically with the target nucleus initiating its motion in the direction of **V**. The final velocities V' and v' of the projectile and the target, which are parallel to **V**, are given by

$$V' = \frac{1 - m/M}{1 + m/M}V, \quad v' = \frac{2V}{1 + m/M}. \tag{2}$$

Suppose now that there exists an atom of environment located on the path of the recoiled target nucleus, as is shown in Fig. 1. Then there is an opportunity for the target nucleus to be scattered backward due to its collision with this heavy nucleus. The velocity of the doubly scattered target $v'' = -v' = -2V/(1+m/M)$ becomes opposite to the projectile velocity. After these two preliminary collisions the projectile and the target nuclei find themselves on the collision course for the second time. Let us presume that their second encounter results in the nuclear reaction. Note that the relative velocity w of the target and the projectile in this collision is

$$w = V' - v'' = \frac{3 - m/M}{1 + m/M}V, \tag{3}$$

which is larger than the initial collision velocity V, $w > V$. For a heavy projectile $w \simeq 3V$. Thus the two preliminary elastic collisions produce a substantial increase of the relative velocity that results in the exponential increase of the probability of the nuclear reaction Eq. (1).

However, there is a dumping factor that arises due to a necessity for the projectile and the target to remain on the path that leads to their final inelastic collisions. In order to calculate this dumping factor let us introduce the cross sections for the elastic collisions: $d\sigma_{\text{proj,tar}}/d\Omega$ will be the differential cross section that governs the first elastic collision between the projectile and target ($d\Omega$ is the solid angle of the recoiled target; we need this cross section for the situation when the direction of the velocity of the recoiled target coincides with the velocity of the incoming projectile. In the center of mass frame (cmf) this corresponds to the backward scattering). The flux J of the projectile and target during their final inelastic collision equals

$$J = \frac{1}{b^4}\frac{d\sigma_{\text{proj,tar}}}{d\Omega}\frac{d\sigma_{\text{tar,env}}}{d\Omega}, \tag{4}$$

where b is a path of the target nucleus between the environmental nucleus and the point of the final collision with the projectile, and $d\sigma_{\text{tar,env}}/d\Omega$ is the differential cross section for the backward scattering of the light target on the heavy environment nucleus. From simple kinematics it follows that

$$b = \frac{v' - V'}{v' + V'}a = \frac{1 + m/M}{3 - m/M}a, \tag{5}$$

where a is the distance between the initial position of the target nucleus and the environmental nucleus. For a heavy projectile $b \simeq a/3$. Equation (4) can be explained without calculations. For the considered energy range the nuclear wavelengths are much smaller than typical distances between atoms in condensed matter. This implies that the elastic nuclear collisions happen at separations that are much smaller than typical atomic separations. In other words, the scattering amplitudes for the two preliminary elastic collisions are much smaller than separations between atoms. This allows one to approximate the wave functions that govern the two elastic collisions by their asymptotes that have the conventional form $\psi_{\text{elast}} \simeq (f/r)\exp(ikr)$, where f is the elastic scattering amplitude in the cmf. Within this approximation one can factorize the amplitude of the complicated process into the product of elastic scattering amplitudes. Correspondingly, the probability is presented as a product of the cross sections in Eq. (4). Alternatively, one can validate Eq. (4) on the purely classical grounds. The first cross section $d\sigma_{\text{proj,tar}}/d\Omega$ specifies the initial elastic collision, while the quantities $(d\sigma_{\text{proj,tar}}/d\Omega)/b^2$ and $(\sigma_{\text{nuc}})/b^2$, where σ_{nuc} is the nuclear cross section [that is not presented explicitly in Eq. (4), but will be taken into account later on, see Eq. (6)], can be considered as two spherical angles, i.e., the two probabilities that define the necessary kinematic conditions that allow the final inelastic collision to take place.

It follows from the above discussion that the ratio F of the probability of the nuclear reaction in the environment due to the rescattering mechanism to the probability of the nuclear reaction in the vacuum equals

$$F = J\frac{P(w)}{P(V)} = \frac{1}{b^4}\frac{d\sigma_{\text{proj,tar}}}{d\Omega}\frac{d\sigma_{\text{tar,env}}}{d\Omega} \times \frac{V}{w}\exp\left[\frac{2\pi Z_{\text{proj}} Z_{\text{tar}} e^2}{\hbar}\left(\frac{1}{V} - \frac{1}{w}\right)\right]. \tag{6}$$

Here Z_{proj} and Z_{tar} are the charges of the projectile and the target nuclei, V is the velocity of the projectile, w is the velocity of the final projectile-target collision Eq. (3) (which is preceded by the two preliminary elastic collisions). The Coulomb factors $P(V)$, $P(w)$ arise from the nuclear cross sections for the collisions with velocities V and w, respectively. We presume here that the velocity-dependence of the nuclear cross section is due entirely to the Coulomb factor, which is usually a very good approximation for low-energy nuclear reaction. Equation (6) shows that the discussed mechanism provides the exponential enhancement of the nuclear reaction, which is moderated by the power-type damping J-factor related to the two elastic collisions. For sufficiently low projectile energy the increase always prevails.

In order to evaluate the factor F in Eq. (6) one needs to find the differential cross sections. They can be calculated in the classical approximation (because the wavelengths of all colliding nuclei are small). In examples discussed below we consider the proton as a target. The potential that describes the interaction of the proton with some other nucleus should include the nuclear Coulomb repulsion that is partly compensated by the electron screening. For light nuclei the screening is insignificant since for the considered energy range the scattering takes place due to those events that happen at very small separations between nuclei. For heavier nuclei the screening is more important. Having this in mind, we consider a model in which the internuclear potential is approximated by an interaction of the bare proton with the Thomas-Fermi potential of the heavier nucleus. Calculating the relative trajectory of the colliding nuclei in this potential one finds the elastic cross section [12]

$$\frac{d\sigma_{\rm el}}{d\Omega} = \frac{\rho(\chi)}{\sin\chi}\left|\frac{d\rho}{d\chi}\right|, \tag{7}$$

where ρ is impact parameter, χ is the scattering angle, $\chi=180°$ for our case, both for the projectile-target collision and collision of the recoiled-target with the environmental nucleus.

Consider two numerical examples. The first one is the collision of the projectile tritium with the proton as a target, with the reaction $t+p\to{}^3\mathrm{He}+n$, or $t+p\to{}^4\mathrm{He}+\gamma$. The rescattering increases the collision velocity by a factor of 2, $w=2V$. Another one is the ^{7}Li as the projectile and the proton as a target that leads to the reaction $^7\mathrm{Li}+p\to{}^4\mathrm{He}+{}^4\mathrm{He}$ with the collision velocity increase due to the rescattering by a factor of 2.5, $w=2.5V$. In both cases we assume that the environmental nucleus is Pd. (This assumption is not crucial since the elastic cross section very smoothly depends on the atomic charge of the heavy nucleus.) We estimate a magnitude of the rescattering effect for the two values of the parameter b, taking $b=1$ and $b=3$ in Bohr radius. Figure 2 shows the results of calculations of the enhancement factor F that describes the effectiveness of the rescattering mechanism comparing it with the reaction in the vacuum (sometimes in literature the enhancement factor is defined as $f=1+F$). It is shown versus the factor $V_{\rm proj}/Z_{\rm proj}$ that, according to Eq. (1), is a natural measure for the probability of the reaction.

Figure 2 shows that, indeed, the rescattering mechanism becomes very efficient for sufficiently low projectile energy. According to Fig. 2 the rescattering is effective for the $t+p$ collision when $V_{\rm proj}/Z_{\rm proj}\leqslant 0.17-0.2$ a.u., which corresponds to the projectile energy $\varepsilon\leqslant 0.7$–1 KeV per nucleon. The energy range down to $\varepsilon\simeq 1$ KeV per nucleon was probed for the DD synthesis in Ref. [1]. This result gives a hope that the rescattering mechanism can be studied experimentally in the $t+p$ case in the very near future. Figure 2 demonstrates also that the rescattering for the $t+p$ collision

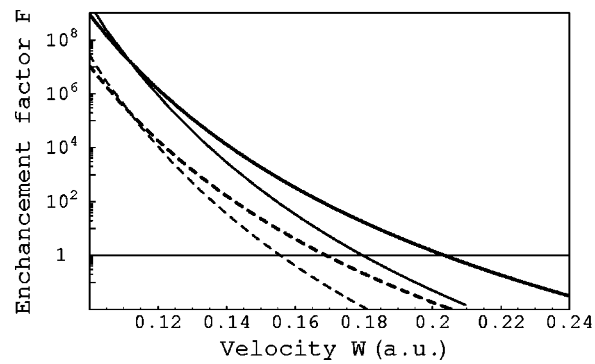

FIG. 2. Collision of the projectile nucleus with the target proton implanted in the condensed matter. The enhancement factor F defined in Eq. (6) is shown versus the velocity per the projectile charge $W=V/(Z_{\rm proj}e^2/\hbar)$. Thick lines, tritium as a projectile; solid line, $b=1$; dotted line, $b=3$ in Bohr radii, where b is the distance that separates the final nuclear event from the environmental nucleus. Thin lines, ^{7}Li as a projectile; solid line, $b=1$; dotted line, $b=3$.

proves be more efficient than for the $^7\mathrm{Li}+p$ case. (This happens because for the given ratio of $V_{\rm proj}/Z_{\rm proj}$ the elastic cross sections in the $t+p$ case are larger than for $^7\mathrm{Li}+p$ case.)

Among several factors that were left outside the scope of our analyses, probably the most significant one is related to a dependence of the results on the geometrical structure of the condensed matter. To make the rescattering mechanism effective the projectile, the target and the environmental nucleus were assumed to be located on one and the same line. If this condition is slightly violated, then the rescattering remains possible, but the relative velocity of the final nuclear event becomes smaller. The stronger is the deviation from the rectilinear geometry, the smaller is the collision velocity and less effective is the rescattering mechanism. Having this fact in mind we specifically presented data for sufficiently large parameter b, $b=3$ a.u. For the $t+p$ reaction this corresponds to a sufficiently large separation between the proton and the environmental atom in the condensed matter $a=6$ a.u. One can hope that for this separation possible deviations from the rectilinear configuration can be made insignificant. This point should be kept in mind and verified more accurately in the future when some particular, condensed matter environment is chosen for experimental studies.

We verified above that the nuclear reaction can be boosted by two preliminary elastic collisions. Similarly, one can consider the more sophisticated scenario when the target nucleus is elastically scattered several, $2n$, $n=1,2,\ldots$ times by the target nucleus and the nucleus of the environment. In this "game" the target nucleus plays a role of a "ball" that bounces forward and backward in between the projectile and the environmental nuclei n times acquiring with each bounce larger and larger velocity. One can find a similarity of this mechanism with a moving billiard wall [12] that is also related to the known Fermi mechanism of acceleration [13]. Carraro *et al.* [14] proposed a similar idea (calling it the knock-on mechanism) discussing a possible enhancement of cluster-impact fusion yields.

There are several reasons that restrict the number of bounces. During the bouncing "game" the projectile should keep its velocity in the initial direction. To satisfy this condition the projectile must be sufficiently heavy, for $n=2$ case the projectile must be at least six times heavier than the target, for larger n the mass ratio must be even greater. This restriction rules out sophisticated $n>1$ cases for $t+p$ collision. A ratio of the yield of the nuclear reaction after n cycles of bouncing to its yield after $n-1$ cycles is proportional to $\propto \exp[(1/w_{n-1}-1/w_n)S]$, where $S=2\pi Z_{\rm proj}Z_{\rm tar}e^2/\hbar$ and w_n is the collision velocity between the projectile and the target during their nuclear reaction after n cycles of elastic rescattering. This estimate shows that the effectiveness of the multiple elastic collisions diminishes with the increase of the number n of cycles. For sufficiently large velocity of the target the two more additional elastic collisions make this velocity only slightly larger, while the price for additional collisions represented by the dumping factor J (which roughly can be estimated as n-independent) remains the same. Thus the multiple collisions are effective only if $F \gg 1$. Therefore they can give a contribution to the magnitude of the rescattering effect, but the mere fact of the exponential enhancement of the probability of the nuclear reaction follows from the simplest case of one cycles $n=1$, when only two preliminary elastic collisions take place.

In summary, the considered rescattering mechanism proves be effective. Our estimations for the $t+p$ collision show that when the energy of the projectile tritium is in the region of $\sim 0.7-1$ KeV per nucleon then the probability of the nuclear reaction induced by this mechanism exceeds the probability of the direct event. For lower energies the discussed mechanism provides an exponential boost for the reaction.

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