Bounds on the bound η-^{3}He system

A. Sibirtsev,[1] J. Haidenbauer,[1] J. A. Niskanen,[2] and Ulf-G. Meißner[1,3]

[1]*Institut für Kernphysik (Theorie), Forschungszentrum Jülich, D-52425 Jülich, Germany*
[2]*Department of Physical Sciences, PO Box 64, FIN-00014 University of Helsinki, Finland*
[3]*Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Universität Bonn, Nußallee 14-16, D-53115 Bonn, Germany*

We investigate the relation between the η-^{3}He binding energy and width and the (complex) η-^{3}He scattering length. Following our systematic analysis of the η-^{3}He scattering length we set limits on the η-^{3}He binding. If bound states exist the binding energy (width) should not exceed 5 MeV (10 MeV). In addition, we comment on a recently claimed observation of an η mesic ^{3}He quasibound state by the TAPS collaboration based on η photoproduction data. Although our limits are in reasonable agreement with the values reported by this collaboration, our analysis of these data does not lead to a solid conclusion concerning the existence of an η-mesic bound state. More dedicated experiments are necessary for further clarification.

The formation of η-nucleus quasibound states has been investigated for a long time. While no such states have been directly observed, a quantity closely related to the existence of bound states, namely, the η-nucleus scattering length, has been intensively studied both experimentally and theoretically. By factorizing the strongly energy-dependent final state interaction part from the slowly energy-dependent production mechanism [1,2], one may hope to extract the scattering length from production reactions [3]. However, that information is necessarily indirect, since the quantity actually related to binding, namely, the sign of the real part Re $a_{\eta\text{-He}}$ of the complex scattering length, cannot be determined from the cross section of the reactions $pd \to \eta$-^{3}He and $\pi^+ t \to \eta$-^{3}He [4,5]. One may note that there is a presumably quite difficult experimental possibility of using charge symmetry breaking in pion production in the neighborhood of the η threshold to do this [6]. On the other hand, several theoretical studies attempt to calculate η-nucleus scattering starting from the elementary ηN interaction using for example optical models based on the impulse approximation [7], sophisticated versions of multiple scattering expansions [8,9], or even Faddeev-Yakubovsky equations [10]. Although these calculations are quite contradictory in numerical details, some of them still indicate a possibility of a large negative Re $a_{\eta\text{-He}}$—the latter being considered as a smoking gun evidence for the existence of a η-^{3}He bound state. In contrast, the optical model calculations of Refs. [11–14] exclude the possibility of the η binding in the three-nucleon system and proposed [14] that the lightest bound system would be with ^{4}He. Also a Skyrme model calculation suggests ^{4}He to give weak binding, while the case of ^{3}He is inconclusive [15]. Recently our knowledge of the η-^{3}He scattering length was revisited [5] via a systematic analysis of presently available data on the $pd \to \eta$-^{3}He reaction. It is natural to extend this study and to investigate in how far the limits deduced for the scattering length in Ref. [5] provide also constraints on the binding energy for the η-^{3}He system. In particular, a phenomenological understanding of the relation between the scattering length and the depth of the binding and the width of the state would be valuable in planning possible experi-

ments aimed for the direct observation of bound states. In this paper we present a calculation where both the depths and widths of the bound states together with the corresponding scattering length are interconnected in terms of complex potentials.

The η-nucleus optical potential is taken to be proportional to the density of the ^{3}He nucleus

$$V = -\frac{4\pi}{2\mu_{\eta\text{-He}}}(V_R + iV_I)\rho(r), \tag{1}$$

for which the Gaussian form

$$\rho(r) = \frac{1}{(\sqrt{\pi}\alpha)^3}e^{-r^2/\alpha^2}, \quad \alpha = \sqrt{\frac{2}{3}\langle r^2\rangle} \tag{2}$$

corresponding to a root-mean-square radius 1.9 fm is adopted, and $\mu_{\eta\text{-He}}$ is the reduced mass. Within the standard optical model calculations the strength parameters V_R and V_I were taken as thrice the elementary ηN scattering length adjusted with the ratio of the reduced masses of the η-nucleus system and the ηN system as

$$V_R + iV_I = 3a_{\eta N}\frac{\mu_{\eta\text{-He}}}{\mu_{\eta N}}. \tag{3}$$

More precisely, this factor of 3 stems from the impulse approximation underlying such an approach. Here the sign definition of the scattering length a is given by the standard effective range convention in meson physics

$$q \cot \delta = \frac{1}{a} + \frac{1}{2}r_0q^2 + O(q^4), \tag{4}$$

where δ is the phase shift and r_0 is the effective range. We should emphasize, however, that the present study is not an optical model calculation in the above narrow sense. Instead, here the strength parameters V_R and V_I are freely varied to study for which scattering lengths one might expect bound states to exist and with which energies. As a numerical check, the values of $V_R = 2.235$ fm and $V_I = 1.219$ fm yield the η-^{3}He scattering length $a_{\eta\text{-He}} = -2.31 + i2.57$ fm in agree-

ment with the result of Ref. [7]. For the scattering calculations the solution of the Schrödinger equation with the asymptotic boundary condition

$$\psi_{0k}(r) \to \cos \delta \, j_0(kr) - \sin \delta \, n_0(kr) \qquad (5)$$

is standard and the numerics is accurate, i.e., better than 0.01 fm in the relevant calculated quantities. The bound state is obtained from the corresponding homogeneous integral equation

$$\psi_E(r) = \int_0^\infty G_0(r,r') V_{\text{opt}}(r') \psi_E(r') r'^2 dr' \qquad (6)$$

by iterating to a self-consistent energy eigensolution. Here the numerics is most difficult in the real and barely bound region. Therefore, no results with less than 0.1 MeV binding are actually used. In this worst situation still we would consider the accuracy of our calculations to be better than 20 keV. The above potential with varying real and imaginary strengths is applied to calculate both the complex binding energies and scattering lengths. Figure 1 shows our results for the η-^{3}He binding energy B and width Γ as functions of the imaginary and real parts of the η-^{3}He scattering length. The different symbols indicate our results for B and Γ of 1, 2, 4, 5, 10, and 20 MeV. The solid and dashed contours show our $\chi^2 + 1$ solutions for the η-^{3}He scattering length [5]. The solid line is the solution obtained by a simultaneous fit of the $pd \to \eta$-^{3}He data from Mayer *et al.* [3] and Berger *et al.* [18], while the dashed line is our result evaluated from the data of Mayer *et al.* [3] alone. The calculations shown in Fig. 1 indicate that, for example, the V_R and V_I strength of Ref. [7] resulting in $a_{\eta\text{-He}} = -2.31 + i2.57$ fm could not provide binding of the η-^{3}He system. Furthermore, it is immediately clear that the predictions for the η-^{3}He scattering length from Refs. [7–10] do not lie in the region of binding, even though some of these studies indicate support for bound states [16] since they yield a negative Re $a_{\eta\text{-He}}$. Looking at the results in Fig. 1 in more detail one detects some strong dependencies. It seems impossible to get binding under the condition Im $a_{\eta\text{-He}} \gtrsim |\text{Re } a_{\eta\text{-He}}|$. This finding is in line with earlier expectations as conditions of quasibound states for the scattering lengths formulated [14] as $-\text{Re } a_{\eta\text{-He}} > \text{Im } a_{\eta\text{-He}} \gtrsim 0$ or the more restrictive condition $\text{Re}[a_{\eta\text{-He}}^3(a_{\eta\text{-He}}^* - r_{0\eta\text{-He}}^*)] > 0$ involving also the effective range, given in Ref. [5]. Often these conditions are overlooked as criteria of bound states. However, it should be also noted that neither of these conditions is *sufficient* for the existence of a bound state. As an interesting feature it was found that a very small imaginary strength V_I could produce a large imaginary part in the energy $E = -B - i\Gamma/2$ and scattering length $a_{\eta\text{-He}}$ in the case of barely bound systems. The effect is relatively larger in the former by a factor of 2. For example, for $V_R = 2.1$ fm giving only 0.25 MeV binding, a change of V_I from zero to 0.1 fm changes B to 0.21 MeV and Γ to 0.38 MeV, thus a 5% imaginary strength produces essentially as large an imaginary part in the energy as the real part. This may be understood, if one considers that close to threshold most of the (real) potential contribution is cancelled by the kinetic energy. In the scattering length the above change is "only"
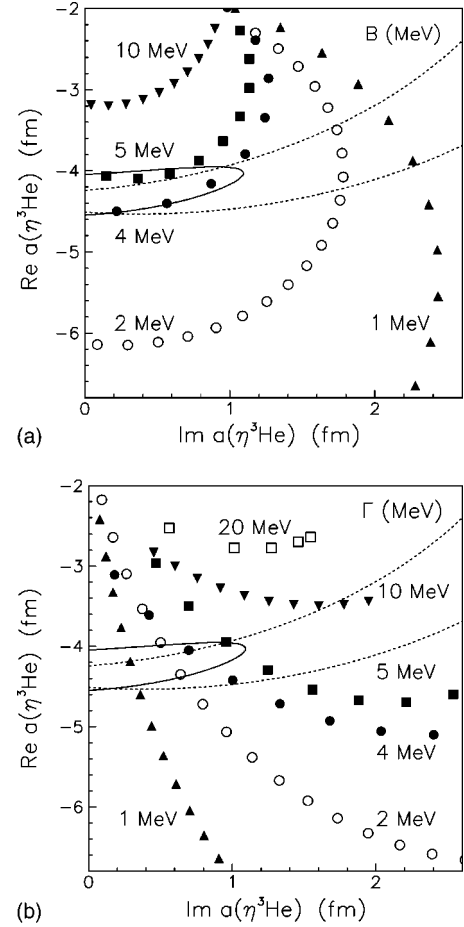

(a)

(b)

FIG. 1. The η-^{3}He binding energy B (upper panel) and width Γ (lower panel) as functions of the imaginary and real parts of the η-^{3}He scattering length. Results are presented for B and Γ of 1 MeV (triangles), 2 MeV (open circles), 4 MeV (closed circles), 5 MeV (closed squares), 10 MeV (inverse triangles), and 20 MeV (open squares). The solid and dashed contours show our $\chi^2 + 1$ solutions for the η-^{3}He scattering length [5] for the various data sets as explained in the text.

$a_{\eta\text{-He}} = -17.1$ fm $\to -14.3 + i6.0$ fm. However, close to the binding threshold also changes in the scattering length are drastic, both in the real and imaginary parts, a fact related to the loss of binding. With deeper binding the imaginary part of the scattering length becomes smaller. However, with an increasing imaginary strength or Im $a_{\eta\text{-He}}$ the binding vanishes even for strongly attractive potentials and at the same time the state may become really very wide. There is an accumulation point at about $-2.0 + i0.9$ fm to which all equal value contours converge with strengthening potentials. An even more important numerical finding was that, while E and $a_{\eta\text{-He}}$ are individually dependent on the potential shape, the relation between them was relatively model independent. For two very different density profiles $\rho(r)$ the differences between the complex energies were less than 10% for the *same* complex scattering lengths in the region of interest. However, while a well-known relation between the binding energy, scattering length and effective range [17] holds quite well for real binding energies of even 10 MeV, we found that it fails for the complex case. Therefore, the numerically es-
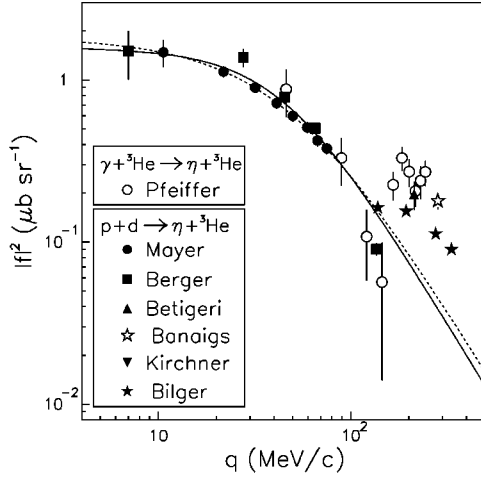
FIG. 2. Spin and angle averaged transition amplitudes $|f|^2$ extracted from γ-^{3}He$\rightarrow\eta$-^{3}He [19] and $pd\rightarrow\eta$-^{3}He [3,18,20–23] data as functions of the final momentum q in the c.m. system. The $|f|^2$ from coherent photoproduction was arbitrarily multiplied by a factor of 6. The solid line is our overall fit [5] to low-energy data from Mayer *et al.* [3] and Berger *et al.* [18], while the dashed line shows our fit to the data from Mayer *et al.* [3] alone.

FIG. 3. The γ-^{3}He$\rightarrow\eta$-^{3}He cross section (open circles) and the difference between the γ-^{3}He$\rightarrow\pi^0 pX$ reaction for $170°\leqslant\theta_{\pi^0 p}\leqslant 180°$ and that for $150°\leqslant\theta_{\pi^0 p}\leqslant 170°$ (solid squares) as a function of the invariant collision energy $\sqrt{s}$. The arrow indicates the η production threshold. The dashed curve is a fit to the data based on Eq. (9) and the solid curve is the result based on Eq. (7), employing the amplitude shown in Fig. 2 (solid line).

tablished only weakly model-dependent relation cannot be expressed by a simple analytic formula. Presently there are no data or any solid arguments to prove that the real part of the η-^{3}He scattering length actually is negative and that the η-^{3}He system should be bound. But, to estimate the possible binding energy B and width Γ we select our solution for $a_{\eta\text{-He}}$ with a negative real part. It may be noted that the result of our analysis [5] while taking Re $a_{\eta\text{-He}}<0$ would be within the binding region as shown by the contour lines in Fig. 1. In fact, the standard rectangular error limits [5] $a_{\eta\text{-}^3\text{He}}=(-4.3\pm0.3)+i(0.5\pm0.5)$ fm would suggest a bound state with the binding energy $B=4.3\pm1.2$ and width $\Gamma=2.8\pm2.8$ MeV. Taking our solution shown by the dashed lines in Fig. 1 and obtained from the data of Mayer *et al.* [3] alone we deduce the limits for η-^{3}He binding energy $B\leqslant5$ MeV and the width $\Gamma\leqslant10$ MeV.

There are some new data on η and π^0 photoproduction on ^{3}He from the TAPS Collaboration [19] at MAMI. Figure 2 shows the spin and angle averaged squared transition amplitude $|f|^2$ extracted from data on $pd\rightarrow\eta$-^{3}He [3,18,20–23] cross sections σ [5] as

$$|f|^2 = \frac{k}{q}\frac{\sigma}{4\pi}, \tag{7}$$

where k and q are the initial and final particle momenta in center-of-mass (c.m.) system, together with corresponding results obtained from the new TAPS data on γ-^{3}He$\rightarrow\eta$-^{3}He [19]. The lines in Fig. 2 show the squared reaction amplitude given by [1,2]

$$|f|^2 = \left|\frac{f_p}{1-iaq}\right|^2, \tag{8}$$

where f_p is the s-wave production operator, assumed to be independent of the final momentum q near the reaction

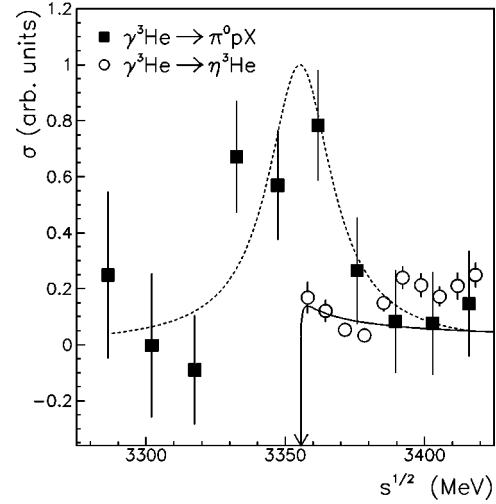threshold and a is the complex η-^{3}He scattering length. The solid line in Fig. 2 shows our overall fit [5] to low-energy data from Mayer *et al.* [3] and Berger *et al.* [18], while the dashed line shows our fit to the data from Mayer *et al.* [3] alone. For $q\leqslant100$ MeV, where the η-^{3}He final state interaction dominates, the coherent photoproduction data are in good agreement with $pd\rightarrow\eta$-^{3}He measurements and can be reasonably reproduced by our s-wave analysis [5]. In this sense the γ-^{3}He$\rightarrow\eta$-^{3}He data could not provide new information about the sign of Re $a_{\eta\text{-He}}$ and from these data alone it is not possible to draw conclusions about the existence of the η-^{3}He bound state. However, Ref. [19] also reports a small enhancement in the γ-^{3}He$\rightarrow\pi^0 pX$ reaction for the $\pi^0 p$ 180° opening angle spectra as compared with other opening angles for energies near the η threshold. The enhancement is assumed to arise in particular from the pionic decay of an $N^*(1535)$ resonance at rest in the ^{3}He nucleus so that its decay products should have opposite momenta in the c.m. system. The $N^*(1535)$ resonance in turn is thought of being formed by absorption of a bound η meson on a proton. Accordingly, the enhancement is seen as a signature for an η-^{3}He bound state and a combined analysis including also γ-^{3}He$\rightarrow\eta$-^{3}He data yielded a binding energy $B=4.4\pm4.2$ MeV and width $\Gamma=25.6\pm6.1$ MeV. Curiously enough our prediction for B presented above—derived under the assumption that a bound state exists—is in line with this result and we would like to discuss it a little further. The solid squares in Fig. 3 show the difference between the cross section of the reaction γ^3He$\rightarrow\pi^0 pX$ for $170°\leqslant\theta_{\pi^0 p}\leqslant180°$ and that for $150°\leqslant\theta_{\pi^0 p}\leqslant170°$ as a function of the invariant collision energy $\sqrt{s}$. This difference exhibits an enhancement in the vicinity of the γ^3He$\rightarrow\eta$ ^{3}He threshold, the latter being indicated by the arrow. Considering this enhancement to be entirely due to the formation of a η^3He

bound state we adopt the same strategy as in Ref. [19] and fit this resonant cross section, i.e., solid squares in Fig. 3, using the nonrelativistic Breit-Wigner form

$$\sigma = \frac{\Gamma^2/4}{(\sqrt{s} - m_\eta - m_{^3\text{He}} + B)^2 + \Gamma^2/4}, \qquad (9)$$

with $m_\eta = 547.3$ MeV, $m_{^3\text{He}} = 2808.398$ MeV. The fit is shown by the dashed line in Fig. 3. We obtain $B = 0.43 \pm 2.9$ MeV and $\Gamma = 27.9 \pm 7.2$ MeV with a $\chi^2/N_{\text{DF}} = 0.89$ based on the data points (squares) shown in Fig. 3. Obviously our result is smaller than the values quoted in Ref. [19] and, as a matter of fact, so close to the threshold that it is compatible with zero. Thus, it is impossible to conclude from these data alone whether the structure is a signature of a bound state or not. To improve the analysis one might combine the γ-^{3}He $\to \pi^0 pX$ and γ-^{3}He $\to \eta$-^{3}He data as was done in Ref. [19]. The latter data are in line with our analysis of the η-^{3}He scattering length, see Fig. 2, and accordingly with the limits we set for the binding energy. However, as already mentioned above, these data do not provide any constraints on the sign of Re $a_{\eta\text{-He}}$. The sign is solely inferred from the data on γ-^{3}He $\to \pi^0 pX$ and therefore subject to the ambiguity that is reflected in the large uncertainty of the values for B and Γ that we (but also the authors of Ref. [19]) deduced. In this context we want to point out that with the opening of the η-^{3}He channel one would anyway expect a cusp structure at the threshold which would give rise to a similar enhancement as exhibited by the dashed curve in Fig. 3, but corresponds to a positive sign of Re $a_{\eta\text{-He}}$. Due to these reasons we do not consider the results of Ref. [19] as an unambiguous signature of an η-^{3}He bound state.

In summary, we have studied the relation between the η-^{3}He binding energy and width and the η-^{3}He scattering length. While, based on final state analyses of η production reactions, one cannot obtain direct information about the existence of η-^{3}He bound states, it is, nevertheless, possible to find constraints regarding the range of energies and widths where such a bound state could be possible. Thus, assuming that the real part of the η-^{3}He scattering length is negative and following our systematic analysis [5] of $a_{\eta\text{-He}}$ evaluated from available data for the $pd \to \eta$-^{3}He reaction, we set the limits for the binding energy to $B \leq 5$ MeV and for the width to $\Gamma \leq 10$ MeV. However, whether or not a bound state indeed exists cannot be deduced from our analysis, simply because we have no reliable empirical information on the sign of Re $a_{\eta\text{-He}}$.

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