Breakdown of vibrational structure in 98Ru

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The nucleus ⁹⁸Ru, the lightest collective Ru isotope with *N*>50, has been studied employing the $(\alpha, 2n)$ reaction using YRAST Ball detectors. The new data, as well as a systematic set of limits on the intensities of unobserved transitions, reveal a structure that resembles a near harmonic vibrator spectrum up through the two-phonon triplet but show an almost complete breakdown of vibrator structure thereafter. These results are not understood theoretically and are at variance with previous interpretations of this nucleus. A rare anomaly in yrast $B(E2)$ values is also pointed out.

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I. INTRODUCTION

The Ru isotopes with $N>50$ lie in a region of structural change that has long been a challenge to theoretical interpretations. The Zr and Sr isotopes near $A = 100$ undergo the most rapid spherical-deformed transition in heavy nuclei. The rate of change of structure with neutron number becomes more gradual with increasing proton number in Mo, Ru, Pd, and Cd. The Ru isotopes seem to show a smooth increase of collectivity with neutron number. This is illustrated in Fig. 1 which shows that the energy of the first excited state, $E(2_1^+)$, drops smoothly, $R_{4/2} = E(4^+_1)/E(2^+_1)$ increases, and the $B(E2, 2^+_1 \rightarrow 0^+_1)$ values increase as *N* increases from 52 to 64.

The nucleus 98 Ru (Z=44,*N*=54) is of particular interest. It is the lightest Ru isotope above $N=50$ with $R_{4/2} > 2$, that is, that should be amenable to a collective model approach. At first glance, its structure would seem to be that of a nearharmonic vibrator (HV). The 2^+_1 level is at 653 keV, a value typical of vibrational nuclei in this region (e.g., $E(2_1^+)$ in $108-116$ Cd is \sim 550 keV). The *R*_{4/2} ratio is 2.14 and there is an apparent two-phonon triplet of levels $(0^+, 4^+, 2^+)$ closely grouped together at 1321, 1398, and 1415 keV, respectively (see Ref. [1]). However, closer inspection reveals difficulties with this interpretation. One of the most puzzling and striking of these difficulties is shown in the lowest panel of Fig. 1, namely, the fact that the $B(E2; 4^+_1 \rightarrow 2^+_1)$ is 12 W.u. [2] while the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value is 32 W.u. It is very unusual and difficult to explain how $B_{4/2} = B(E2; 4^{+}_1 \rightarrow 2^{+}_1)/B(E2; 2^{+}_1)$ \rightarrow 0⁺₁) can be <1 in collective nuclei. *B*_{4/2} is 2 in a pure geometric vibrator and about 1.5 in the finite particle interacting boson approximation (IBA) model. It is 1.43 in a pure rotor. In 98 Ru, it is 0.38 (11) according to the most recent lifetime measurement [2]. We note here the equally surprising result from Ref. [2] that the $B(E2; 6^+_1 \rightarrow 4^+_1)$ value is also anomalously small $(12.9 W.u.)$ and much less than the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value.

A second difficulty with a vibrator interpretation is that the possible candidates for three-phonon states in ⁹⁸Ru are considerably spread out in energy and as we shall see, cannot be fit with the anharmonic vibrator model.

In the literature the structure of $98Ru$ has had various interpretations, ranging from shell model calculations [3] to a pure vibrator [4] to a strongly perturbed rotor [5]. The structure of this nucleus, which lies in one of the most important, complex, and challenging shape transition regions, is thus still significantly in doubt.

The purpose of this paper is to discuss new experiments on ⁹⁸Ru and their resulting interpretation. In order to do a

FIG. 1. Systematics of basic observables in the Ru isotopes showing $\mathcal{B}(E2; 2^+_1 \rightarrow 0^+_1)$, and $B(E2; 4^+_1 \rightarrow 2^+_1)$ $B(E2; 2^+_1 \rightarrow 0^+_1)$ values for even neutron numbers between 50 and 70.

thorough study of the low lying levels of $98Ru$ we exploited a nonselective population technique, the $(\alpha, 2n)$ reaction. Reactions of this type provide essentially complete level schemes in certain spin and energy ranges. In particular, with relatively low angular momentum in the formation region, this reaction populates sets of low and medium spin states up to rather high excitation energies. A partial level scheme was constructed up to 6591 keV, with spins up to $J=15$: special emphasis, however, was placed on a careful study of the low-lying low spin states and an assessment of vibrational structure in this nucleus.

Eighteen new levels and 31 new γ rays were observed, some γ rays were eliminated, others were moved to new placements on the basis of both our singles and γ - γ coincidence data, and a number of γ -ray intensities were altered. Several levels previously suggested were found not to exist, or to have specific spin limitations. Intensities were measured for each observed γ -ray from the coincidence data and upper intensity *limits* for *all* spin allowed (using *M*1, *E*2 multipolarities) unobserved transitions were extracted for levels below about 2600 keV. This latter aspect proved absolutely crucial to the interpretation.

The results show that the vibrational structure of 98 Ru disintegrates after the two-phonon triplet, with no evidence for states with three or four-phonon structure.

II. EXPERIMENTAL METHODS AND RESULTS

A 96 Mo (4.51 mg/cm²) target was bombarded by α particles at four different beam energies from the ESTU tandem accelerator at WNSL at Yale University. The experiment was done in two parts. In the first part, the beam energies were 26, 28, and 30 MeV and in the second 24 and 30 MeV. The beam intensities ranged from ~ 0.5 to 2.5 pnA. The total measuring time amounted to about 60 h. Surrounding the target was an array of 6 Compton suppressed clover detectors from the YRAST Ball [6]. Four were mounted in a 90° ring with respect to the beam direction, the others were mounted at 140°. The data were recorded with both single and double γ -ray triggers. A ¹⁵²Eu source was used for the energy and efficiency calibrations in both experiments. The Radware [7] software package was used to analyze the data. In order to obtain the best peak discrimination for low lying levels the γ -ray energy dispersion was set to record γ rays up to 2 MeV. Several beam energies were useful to distinguish γ rays in ⁹⁸Ru from contaminant lines.

The data are excellent, exhibiting about 30 times better statistics than earlier α -induced reactions [4] in both γ - γ coincidence and singles spectra. Examples of these spectra are shown in Figs. 2 and 3. Further spectra will be shown below in the discussion of specific levels and transitions.

The analysis was done using both the γ - γ coincidence and singles data. The coincidence data were crucial to the development of the level scheme, while the singles data were especially useful for obtaining upper intensity limits on some spin-allowed unobserved γ -ray transitions (see later). The observed γ rays in ⁹⁸Ru and their intensities (normalized to 1000 for the 652.6 keV line) are listed in Table I, along with their placements in the level scheme. The level scheme itself

FIG. 2. Total projection spectrum for ⁹⁸Ru. The insets (which, here and in Figs. 3 and 8, have the same axis labels and intensity scale as the main plot) show the regions around 480 and 1593 keV. Here and in Figs. 3 and 8, the vertical scale is expanded to show weaker peaks so that the strongest peaks extend above the limits of the plot.

is shown in Figs. 4–6. The decay intensities from each level below \sim 2600 keV are given in Table II.

One of the advantages of the $(\alpha, 2n)$ reaction is that the population intensity of different levels is a smooth function of their excitation energy and spin [8,9]. Level populations have a bell-shaped distribution with spin for a given excitation energy. They are small for 0^+ levels, rise up to $J \sim 8$ and then fall off again for higher spins. Our interest lies primarily in the lower spin levels and, hence, the populations tend to increase with spin in the low energy spectrum. Level populations also decrease with increasing excitation energy. The observation that the level populations are smooth as a function of spin and excitation energy implies that, for excitation energies and spins for which the populations should be above experimental sensitivities, the levels must be observed. Otherwise, the original premise of smooth dependence on E_{ex} and J^{π} is contradicted.

Therefore, the $(\alpha,2n)$ reaction can be considered spectroscopically complete—that is, all levels will be populated—in these E_{ex} and *J* ranges. In our case, this applies to levels with $2 \le J \le 8$ and $E_{ex} \le 2.6$ MeV. The $(\alpha, 2n)$ reaction shows population patterns reminiscent of (n, γ) spectra [10] where

FIG. 3. Example of gated spectrum for 98 Ru. The inset to the 653 keV-gated spectrum shows the region around 1301 keV.

TABLE I. Observed γ -ray transitions in ⁹⁸Ru, their relative intensities, and their placements.

$E_\gamma^{~{\rm a}}$	Rel. int. ^b	\mathcal{E}_i	$E_{\it f}$
189.5°	$1.2\,$	4824.1	4634.4
214.3°	1.9	4216.1	4002.1
229.8°	2.3	3476.4	3246.4
253.8	14.5	2267.2	2013.4
260.3°	$1.4\,$	4824.1	4563.8
272.8	$2.2\,$	3852.6	3579.9
280.5°	3.7	2547.8	2267.2
295.5	2.6	3579.9	3284.5
303.4°	3.1	5219.2	4915.5
312.7	1.9	3852.6	3539.8
317.3°	$6.7\,$	3857.1	3539.8
320.6	14.6	2868.4	2547.8
324.6	15.0	2547.8	2223.3
339.0°	15.0	4563.8	4224.7
382.7	24.8	1797.6	1414.9
399.1	$6.7\,$	1797.6	1398.2
406.8°	2.3	5626.0	5219.2
410.7°	$3.2\,$	4634.4	4224.7
412.4	21.9	3539.8	3127.4
469.6	55.0	2267.2	1797.6
522.5	2.4	3070.3	2547.8
542.8	6.6	2810.0	2267.2
563.3^c	1.3	2786.6	2223.3
589.6	2.4	2603.5	2013.4
591.3^c	3.9	3252.1	2660.8
594.7°	4.7	3252.1	2657.5
598.9	22.9	2013.4	1414.9
599.4 ^c	5.5	4824.1	4224.7
609.9	1.3	2427.9	1817.9
615.0	25.8	2013.4	1398.2
623.7°	$2.6\,$	4848.2	4224.7
627.1	14.0	3284.5	2657.5
630.3	$1.2\,$	2427.9	1797.6
632.6°	3.4	4634.4	4002.1
645.1	2.3	2868.4	2223.3
652.6	1000	652.6	$\boldsymbol{0}$
661.3	1.4	3852.6	3191.3
668.1^d	1.3	1320.7	652.6
676.3°	1.7	4216.1	3539.8
722.9	30.0	4007.4	3284.5
725.2	54.4	3852.6	3127.4
745.6	831	1398.2	652.6
754.5°	8.7	4006.6	3252.1
762.3	39.9	1414.9	652.6
810.7	24.5	4002.1	3191.3
816.1°	13.7	4007.4	3191.3
821.3	35.0	4673.9	3852.6
822.1°	10.8	4824.1	4002.1
825.1	491	2223.3	1398.2
835.4 ^c	$3.2\,$	3058.7	2223.3
840.7	16.5	4848.2	4007.4
848.0	20.4	5521.9	4673.9
863.2	18.0	2660.8	1797.6

$E_\gamma^{\ a}$	Rel. int. ^b	\mathcal{E}_i	\mathcal{E}_f
869.2	15.8	2267.2	1398.2
874.7	57.6	4002.1	3127.4
879.6°	9.5	4007.4	3127.4
	6.8	2278.0	1398.2
889.3°	2.3	4135.7	3246.4
904.1	216	3127.4	2223.3
913.4	16.3	4915.5	4002.1
940.4°	1.8	5614.3	4673.9
968.0	121	3191.3	2223.3
979.2	37.9	3246.4	2267.2
984.9	8.4	3252.1	2267.2
987.6	11.3	4989.7	4002.1
992.0	17.4	3539.8	2547.8
1012.1^f	9.3	4258.5	3246.4
1024.9°	4.3	4216.1	3191.3
1029.7^{g}	$<$ 4	2427.9	1398.2
1030.4°	4.1	4221.7	3191.3
1032.1^f	9.9	3579.9	2547.8
1033.3^{f}	26.9	4224.7	3191.3
1061.2	54.9	3284.5	2223.3
1069.3	0.8	6591.2	5521.9
1088.5°	2.6	4216.1	3127.4
1097.5	13.3	4224.7	3127.4
1124.8°	5.0	5349.5	4224.7
1145.1	95.5	1797.6	652.6
1149.7	69.1	2547.8	1398.2
1165.3	6.4	1817.9	652.6
1205.7	4.6	2603.5	1398.2
1217.3°	4.3	5219.2	4002.1
1225.4^c	3.1	4416.7	3191.3
1253.2^c	3.5	3476.4	2223.3
1259.3	46.6	2657.5	1398.2
1322.2	2.9	2720.4	1398.2
1360.9	2.6	2013.4	652.6
1415.0	19.4	1414.9	$\boldsymbol{0}$
1436.6^c	4.4	4563.8	3127.4
1625.4	7.4	2278.0	652.6
1668.5	6.7	3066.7	1398.2
1723.1^{f}	3.1	3946.4	2223.3
1776.4^h	19.9	2427.9	652.6
1818.4^{i}	0.9	1817.9	$\boldsymbol{0}$

TABLE I. (*Continued.*)

^aAll energies in the table are in keV. Relative uncertainties range from ± 0.2 keV for the strong and intermediate intensity transitions up to ±0.4 keV for the weakest transitions.

^bIntensities are normalized to 1000 keV for the 652.6 keV transition. The uncertainties on the relative intensities range from 1% for the strongest transitions ($I_{\gamma} \ge 100$) to 2–4% for transitions with $100 \le I_{\gamma} \le 10$. For I_{γ} values below 10 the uncertainties range upwards with decreasing intensity from 5% to 15% for the weakest lines. Exceptions are explicitly noted.

^cNew γ -ray transitions assigned to ⁹⁸Ru in this work.

 d Energy uncertainty ± 0.8 keV.

e Doubly placed. Intensities from coincidence spectra. Uncertainty on intensity about ±15%.

 f Gamma-ray placed differently than adopted in ref. [1].

^gTransition is tentative.

h Doublet. Only one of the γ rays belongs to ⁹⁸Ru. Energy uncertainty ± 0.6 keV.

ⁱTentative, appears at some beam energies. The summing contribution to this line is estimated at ≤ 0.3 units of intensity.

FIG. 4. Decay of levels in ⁹⁸Ru in this work up to \sim 2600 keV. Gamma-ray energies and intensities (normalized to 1000 for the 653 keV γ ray) are given above each transition arrow. Adopted J^{π} values (from Ref. [1] and the present work) and level energies (keV) are noted for each level. We do not observe the 2374 keV level. (see Ref. [1]). For completeness, we show it as a short horizontal line. If the 1818.4 keV transition exists, then the spin of the 1817.9 keV level would be 2⁺. The 1593 keV transition from the 2246 keV level [5], shown as dotted, was not observed.

the maximum population is centered around that of the capture state spin. The main difference in $(\alpha,2n)$ is a broader and higher entry angular momentum distribution and hence a broader distribution of accessible spin states.

The population systematics, that is, the total γ -ray feeding intensity to each level, obtained by summing the depopulating γ -ray intensities, is shown in Fig. 7. Since limits on the intensities of unobserved deexcitation transitions were also obtained and are generally quite small, the depopulation intensity is a good estimate of the populating intensity. Since the maximum detectable γ -ray energy in this study was 2 MeV, the decay of levels with $J \leq 4$ lying above 2653 keV to the 2^+_1 level at 653 keV would proceed with γ -ray transitions outside our range. Therefore, above 2653 keV the populations are incomplete for low spin states and Fig. 7 therefore only includes a few well known higher spin states above this energy.

Clearly there is a reasonable separation according to spin (higher spins, up to some limit, are more strongly fed) for well established levels, and a decreasing trend with excitation energy. We will use Fig. 7 to help extract J^{π} values, or limits, especially for weakly fed levels. For example, as in (n, γ) , if the upper limits on the γ -ray intensities of the deexcitations of a given level are well below a given value (estimated using the appropriate curve in Fig. 7), a level of that spin can be effectively ruled out. In some cases this leads us to conclude that the level itself does not exist. Several data points (labeled by level energy) are shown in Fig. 7 for levels that will be discussed in this context.

III. SPECIFIC LEVELS

Here we briefly remark on a few specific levels where comments may help to clarify changes made to the previously existing level scheme [1].

A. 1321 keV level

The 1321 keV level has been seen [11] in the (p, p') reaction and, quite tentatively, Ref. [5] also assigns a γ ray of 669.7 keV as depopulating this level to the first 2^+ state. The spin has been assigned as 0^+ . A weak peak in the 653 keV gate in the present data with an energy of 668.1 keV was found. It is not clear if this is the same transition as placed in Ref. [5]. Analysis of the energy region around 669 keV gives an intensity limit for a 669.7 keV transition of \leq 1.3. Our adapted level energy is 1320.7 keV, based on the 668.1 keV transition.

FIG. 5. Same as for Fig. 4 except for levels from 2657 to 3852 keV. J^{π} values from Ref. [1].

B. 1818 keV level

This level is populated $[12]$ in a (d,t) reaction with an $L=2$ angular distribution and has been assigned a spin of 0^+ or $2^{\text{+}}$. In Ref. [13] an 1817.4 keV γ ray was placed as a ground state transition. We tentatively observe a weak γ ray of 1817.9 keV which may be the same transition. If this γ ray is, in fact, a ground state transition from the 1818 keV level, a 0^+ spin parity would, of course, be ruled out. However, the total population of this state is well below the trend for 2^+ states (see Fig. 7) but yet seems high for 0^+ (given the weak population of the known 0^+ state at 1321 keV). The existing data from the literature and the present work are therefore inconclusive. What seems clear is that the level does exist and that its spin parity is either 0^+ or 2^+ , although neither assignment gives completely consistent results.

C. 1953 keV level

This level was tentatively proposed and assigned as a 3^+ state by Samudra *et al.* [5] on the basis of a 1301 keV γ -ray decay to the 653 keV level. However, we found no evidence for this γ -ray in our γ - γ coincidence or singles spectra despite the high statistics and the fact that a population of about 90 units would be expected from Fig. 7 for a 3^+ level at this energy. Figure 3 shows the relevant portion of a gate on the 653 keV $2^+_1 \rightarrow 0^+_1$ peak. There is no definite evidence for the

1301 keV γ ray and the upper limit on its intensity is <2.5. From Fig. 7, we see that this definitely rules out a $J^{\pi} \geq 2^+$ value. We conclude that there is no positive evidence for the level.

D. 2241 keV level

This level was tentatively observed in Ref. [5] with an 843 keV γ ray to the 1398 keV 4⁺ level, implying a spin of 2 or greater. However, there is no evidence for this γ ray in either our γ - γ coincidence data or singles spectra. The upper intensity limit is $<$ 2. Again, inspection of the intensity scale for $J \geq 2$ in Fig. 7 suggests that it is highly unlikely that this level exists.

E. 2246 keV level

This level was observed in Refs. [5,12]. Reference [5] tentatively placed a 1593 keV γ ray as deexciting the 2246 keV level to the 653 keV level. We did not observe this γ ray in either the γ - γ coincidence or singles data (see inset to Fig. 2). The upper intensity limit is ≤ 1.5 . The evidence [12] for the level in (d,t) , however, is convincing, and an *L*=2 angular distribution is observed, which would suggest a spin parity of $J^{\pi} \le 5^+$. However, from Fig. 7, most of these spins would seem to be ruled out. Therefore, the most likely assignment is $J^{\pi} = 0^+$.

FIG. 6. Same as for Fig. 4 except for levels above 3852 keV. J^{π} values from Ref. [1].

F. 2374 keV level

This level was proposed on the basis of a 1723 keV γ ray to the 2^+_1 level in an (α, xn) study [4] and also seen [12] weakly in (d,t) with a nondescript angular distribution. In our γ - γ data, a peak at 1723 keV is in coincidence with both the 745 and 825 keV γ rays from the yrast 4⁺ and 6⁺ levels, respectively (see Fig. 8). Gating on the 1723 keV line shows peaks at 652, 745, and 825 keV (see Fig. 8). Thus, we propose that it deexcites a new level at 3946 keV. There is therefore no longer any evidence for γ -ray decay from a possible level at 2374 keV. If that level exists, then, from Fig. 7, it would have to be a 0^+ level.

G. 2430 keV level

The only previous evidence for this level is a tentative 1032.4 keV γ ray to the 1398 level [5]. We do not observe this transition in the 745 keV gate. However, the present γ - γ data show that a γ ray of about this energy deexcites the 3580 keV level to the state at 2548 keV. We conclude that the existence of the 2430 keV level is doubtful. (Note that this level is not the same as the 2427.9 keV level which definitely exists and has three deexcitation transitions.)

H. 2467 keV level

A level at this energy was proposed in Ref. [13] and apparently observed [12] weakly in (d,t) with an $L=0$ triton angular distribution, which only allows spins of 2^+ or 3^+ . The strongest proposed decay is a 670 keV γ ray to the 1798 keV level. We see no evidence for this γ ray in our coincidence spectra, with an upper limit on its intensity of 0.3. From the

FIG. 7. Level populations (obtained from the sum of outgoing intensities) against excitation energy. The curves are drawn to guide the eye through points for each given spin. The downward arrows show upper limits on level populations for states with no observed transitions.

J. 2548 keV level

I. 2474 keV level

 $3⁺$ suggesting that the level may not exist.

population plot of Fig. 7 this likely rules out both $J^{\pi} = 2^{+}$ and

The only previous evidence for this level is a 197 keV γ ray to the 2278 keV level [5]. However, in the spectra of Ref. [5], the peak is dominated by a nearby contaminant. We see no evidence for the 197 keV γ ray in our coincidence data.

FIG. 8. Spectra gated on the 825 and 1723 keV transitions. The inset in the spectrum gated on the 825 keV line shows the region around 1723 keV.

This level was previously assigned a spin parity of J^{π} $=(5,6)^+$. We found a new transition deexciting this level of 280.5 keV to the 2267.2 keV 4^+ level. The population intensity (see Fig. 7) is consistent with $J=5^+, 6^+$.

K. 2603 keV level

This level is well established with 1205 and 589 keV deexcitation γ rays. Previously, spins of 2^{\pm} , 3^{\pm} , 4^{\pm} , or 5^{\pm} were adopted [1]. From the population plot in Fig. 7, the low feeding of this level argues against $J=4,5$ values of leaving more likely possibilities of 2 or 3. The transition to the 4^+ level at 1398 keV eliminates 2−. Population [12] with *L*=2 in (d, t) indicates positive parity. Hence, we conclude the most likely J^{π} values are 2^{+} , 3^{+} .

L. Levels above 2603 keV

The level scheme above 2603 keV was constructed on the basis of the coincidence data. We stress that, because of our effort to thoroughly study the low lying levels in 98 Ru, the portion of the scheme in Figs. 5 and 6 may well be incomplete for γ transitions above 2 MeV. Also, on account of this limitation some levels are based on a single γ -ray deexcitation, and must be viewed with some caution.

IV. DISCUSSION

⁹⁸Ru looks at first like an excellent vibrator but the energy spreading of the likely candidates for three-phonon levels and the ratio $B_{4/2}$ < 1 suggest a more complex structure. We will address these issues in the rest of this paper.

A. Yrast states

Often, yrast states are purer in phonon structure than others, since they are separated in energy from other states of the same spin. Therefore, it is useful to first compare the

FIG. 9. Comparison of experimental yrast energies in 98Ru with harmonic and anharmonic vibrator models.

yrast energies with the harmonic and anharmonic vibrator models (AHV) [14]. This is done in Fig. 9 where, for the model predictions, the phonon energy was normalized to the 2_1^+ level at 652.6 keV. For the AHV, the experimental twophonon 4^+_1 energy was used to fix the anharmonicity (ϵ_4) , and the higher levels predicted using the relation

$$
E(n - ph) = nE(2_1^+) + \frac{n(n - 1)}{2} \epsilon_4,
$$
 (1)

where, for the yrast states, the phonon number $n = J/2$. The value of ϵ_4 was rather small, namely ϵ_4 =93.0 keV. The results are shown on the right in Fig. 9.

Inspection of the figure shows that the harmonic vibrator gives a poor representation of the data, while the AHV gives an excellent fit up through the 8^+ level. Beyond that, deviations grow rapidly. This is due to a clear backbend at spin 10^{+} .

An alternate, and visually informative, way of testing for vibrational or rotational character is to use the recently proposed technique of E-GOS (E_v over spin) plots [15], in which $E_{\gamma}(J \rightarrow J-2)/J$ is plotted against spin *J* for the yrast states. To see the utility of this, consider a harmonic vibrator nucleus. This gives

$$
E_{\gamma}J = E(2^+)/J,\tag{2}
$$

which shows that E_y/J decreases rapidly (hyperbolically) with increasing spin.

In contrast, for a rotor

$$
E_{\gamma}J = [\hbar^2/2\Theta][4 - (2/J)] \tag{3}
$$

which shows that E_y/J increases gradually with spin from 3 (in units of $\hbar^2/2\Theta$) to 4.

An E-GOS plot for $98Ru$ is shown in Fig. 10. The HV, AHV, and rotor results can also be seen in the figure. Clearly, the rotor limit fails completely. As the spin increases, E_{γ}/J decreases as is appropriate for any nucleus with $R_{4/2}$ < 3.0. The harmonic vibrator deviates from the data significantly, even at low spins. The AHV, however, accounts extremely well for the data up through the 8^+ level. Therefore, again, 98 Ru looks like a vibrator nucleus with small anharmonicity at low spin. After the 8⁺ level, ⁹⁸Ru has smaller E_{γ}/J values because of the backbending. Although both the AHV and E-GOS plots support a vibrator multiphonon structure for the yrast levels, we will see that such an interpretation must be strongly modified when non-yrast levels are considered.

To consider these states, it is useful to look at other data, namely relative $B(E2)$ values, for a more accurate assessment of the phonon structure. We will see that such an analysis gives unexpected and puzzling results.

Before entering into this we discuss an additional crucial feature of the data analysis.

B. Importance of unobserved transitions

A particularly important aspect of this work was the extraction of upper limits on all unobserved transitions that are allowed by the angular momentum selection rules for dipole or quadrupole transitions. These spin-allowed unobserved transitions (SAUTs) have proved to be absolutely essential to understanding the phonon structure of this nucleus. Extensive tabulations of SAUTs in our recent study $[16]$ of 156 Dy also were highly useful. Figure 11 gives the level scheme of

FIG. 10. E-GOS plot of yrast energies in ⁹⁸Ru compared with harmonic vibrator, AHV, and rotor models. See text for details.

FIG. 11. Same as Fig. 4 but including SAUTs (see text). See caption to Fig. 4 concerning the 1818.4 keV transition.

Fig. 4 again, but extended to show (as dashed lines) the SAUTs and their intensity limits for levels up through \sim 2600 keV.

To obtain relative $B(E2)$ values for transitions from each level, the intensities in Table II were corrected for the E^5_γ dependence of *E*2 transitions and, where known, for *E*2/*M*1 admixtures. This gives relative experimental $B(E2)$ values for each level including limits for the spin-allowed transitions. These are given in Table III for levels up to \sim 2600 keV. {For a few low energy SAUTs the limits give no useful information on relative $B(E2)$ values $[I_{\gamma}(E2)/E_{\gamma}^{5}]$ and these are therefore omitted in Table III and Fig. 11}.

As we shall see, the use of these limits completely changes the interpretation of a number of levels in 98 Ru and, therefore, of the structure of this nucleus as a whole. The systematic extraction of limits on all SAUTs is, in fact, so useful that we recommend it as a general technique for γ -ray studies of low lying levels.

It is useful to give two specific examples of this, namely for the 1818 keV level and the 2278 keV level. For the 1818 keV level, which, for the present purpose, we take as having $J^{\pi} = 2^{+}$, Fig. 12 shows the *observed* deexcitations only. This level is a candidate for a three-phonon state. The only two observed transitions are an 1165 keV γ ray to the 2^+_1 level (one-phonon) and a (tentative) 1818 keV transition

to the ground state. Note that both of these transitions are forbidden in the vibrator model. The former changes the phonon number by 2 and the latter by 3, if the 1818 keV state is a three-phonon level. We note that the relative Δn_d $=$ 2 *B*(*E*2) value is about 50 times larger than the Δn_d = 3 value. The allowed transitions to the two-phonon levels are not observed, but, given the reduction in their intensities due to their lower γ -ray energies, this is not, *per se*, an argument for or against three-phonon character for the 1818 keV level. Thus, one would conclude that the data on the observed transitions are consistent with, but do not prove, that this level could be a three-phonon state. We will see later that the extraction of SAUTs for this level changes the situation.

For the 2278 keV level, there are again two observed transitions (see Fig. 13). If this level is thought to be a fourphonon state, both of these transitions would be forbidden and, again, the less forbidden one is stronger (by a factor of \sim 20). Therefore, these limited data show no inconsistencies with a four-phonon structure.

If, as an alternative, the 2278 keV level is considered to be a three-phonon state then the 879 keV transition to the $4⁺₁$ (two-phonon) level would be allowed and the 1625 keV transition to the 2^+_1 state forbidden. Experimentally, the relative $B(E2)$ values are in the ratio of $\sim 20:1$ in favor of the allowed transition. Thus one could conclude that these data support a three-phonon interpretation.

TABLE II. Deexcitation transitions from low lying levels. Levels up through \sim 2600 keV. All energies in the table are in keV. In the last column, the intensities are normalized to 1000 for the strongest transition for each level. Uncertainties on level energies are ± 0.2 keV unless otherwise specified. Uncertainties on intensities have the same relative values as in Table I.

E_i	J^{π}	E_f	E_{γ}	Rel. int.	Normalized int.
652.6	2^+	θ	652.6	1000	1000
1320.7 ^a	0^+	652.6	668.1 ^a	1.3	1000
1398.2	4^+	652.6	745.6	831	1000
1414.9	2^+	θ	1415.0	19.4	486
		652.6	762.3	39.9	1000
1797.6	3^+	652.6	1145.1	95.5	1000
		1398.2	399.1	6.7	70
		1414.9	382.7	24.8	260
1817.9	0^+ , 2^+	$\overline{0}$	1818.4^{b}	0.9	141
		652.6	1165.3	6.4	1000
2013.4	3^+	652.6	1360.9	2.6	101
		1398.2	615.0	25.8	1000
		1414.9	598.9	22.9	888
2223.3	$6+$	1398.2	825.1	491	1000
2267.2	4^+	1398.2	869.2	15.8	287
		1797.6	469.6	55.0	1000
		2013.4	253.8	14.5	264
2278.0	2^+	652.6	1625.4	7.4	1000
		1398.2	879.6	6.8	919
2427.9	2^+	652.6	1776.4	19.9	1000
		1398.2	1029.7°	$<$ 4	201
		1797.6	630.3	1.2	60
		1817.9	609.9	1.3	65
2547.8	5^+ , 6^+	1398.2	1149.7	69.1	1000
		2223.3	324.6	15.0	217
		2267.2	280.5	3.7	54
2603.5^{d}	$2^+, 3^+$	1398.2	1205.7	4.6	1000
		2013.4	589.6	2.4	522

^aLevel and γ -ray energy uncertainty $\sim \pm 0.8$ keV.

^bTentative. γ -ray transition observed in some spectra.

Tentative.

 d Energy uncertainty ± 0.5 keV.

As we shall see later, when the evidence from the SAUTs is included, exactly opposite conclusions emerge. Specifically, they clearly show that neither the 1818 or the 2278 keV level can be described in a phonon picture.

C. Candidates for three-phonon levels

With Fig. 11 and Table III we can now discuss possible candidates for multiphonon structure more sensitively.

The lowest candidate for a three-phonon state is the $3₁⁺$ level at 1798 keV. While the allowed transitions to the twophonon states $(4₁⁺$ at 1398 keV, $2₂⁺$ at 1415 keV) have much stronger $B(E2)$ values than the forbidden transition to the one-phonon 653 keV level, the relative $B(E2)$ values to the

two phonon states have a ratio $B(E2; 3^+_1 \rightarrow 2^+_2)/B(E2; 3^+_1)$ \rightarrow 4⁺₁ $=$ 0.64 compared to the harmonic vibrator value (see Table IV) of 2.5. However, there is a large uncertainty in the $E2/M1$ mixing ratio for the 383 keV $3^+_1 \rightarrow 2^+_2$ transition and a significantly larger *E*2 component is allowed within the experimental errors and therefore one cannot use this data alone to definitively rule out a three-phonon interpretation for the 1798 keV level.

However, there is another significant problem which is related to the energy of the 3^+_1 level. It is well known [14,18] that the anharmonicities in the two-phonon states allow us to uniquely predict the three-phonon energies within a vibrator model with up to two-body diagonal anharmonicities. This is illustrated in Fig. 14 where the observed anharmonicities, ϵ_l , of the two-phonon states $\left[\epsilon_j = E_j - 2E(2_1^+)\right]$ are used to predict the energies of the three-phonon states. This comparison shows that the predicted energy of the $3₁⁺$ three-phonon level is almost 500 keV higher than observed. If the 1798 keV $3₁⁺$ level is not a three-phonon-like level, its structure is difficult to understand. The simplest configuration would be the neutron excitation $(d_{5/2}, g_{7/2}, J=3)$. However, this leads to a multiplet of states with $J=1-6$, in which the 6^+ is expected to lie lowest for a short range residual interaction. We conclude that there is no satisfactory interpretation of this level, and that, specifically, there are serious difficulties with an interpretation as a three-phonon state.

Another possible candidate for the three-phonon 3^+ level is that at 2013 keV. Its energy is in better agreement with the predicted value of 2138 keV and the forbidden transition to the 2_1^+ level is also weak (as with the 1798 keV level). However, again, there is a discrepancy with the ratio of allowed *B*(*E*2) values: *B*(*E*2; 3^{+}_{2} \rightarrow 2^{+}_{2})/*B*(*E*2; 3^{+}_{2} \rightarrow 4^{+}_{1})=0.9(+0.06, −0.16) experimentally, compared to 2.5 in the vibrator.

Yrast multiphonon states are often in better agreement with vibrator predictions and, indeed, as noted earlier, the AHV predicts the $6₁⁺$ energy very well. However, there is a problem mentioned above that the $B(E2; 6^+_1 \rightarrow 4^+_1)$ value (relative to 32 W.u. for the $2^+_1 \rightarrow 0^+_1$ transition) is impossible to understand in a vibrator model. Therefore, even the yrast candidate for a three-phonon level exhibits strong disagreement with the AHV prediction.

For the other possible candidates for three-phonon states, the 2^+ level at 1818 keV, the 4^+ level at 2267 keV, and the 2^+ level at 2278 keV, the deviations of relative $B(E2)$ values from the vibrator model are also severe. In order to test if these deviations can be understood in terms of breaking of the vibrator symmetry we have therefore carried out calculations with the IBA (Interacting Boson Approximation) model [19].

The nucleus ⁹⁸Ru $(Z=44, N=54)$ has (relative to *Z* =40,*N*=50) eight valence nucleons or four bosons. The IBA can therefore attempt to describe states up to a spin of 8. This is adequate for $\frac{98}{8}$ Ru since the yrast states have a backbend at spin 10^+ which is beyond the scope of the simple IBA-1. We use a simple Hamiltonian

$$
H = \epsilon n_d + \kappa Q \cdot Q, \qquad (4)
$$

where

TABLE III. Relative $B(E2)$ values and limits for deexcitations of the positive parity levels below \sim 2600 keV. All energies in the table are in keV. The Table includes upper limits for essentially all spin allowed (*M*1 or *E*2) unobserved transitions (SAUTs) from each level, based on careful examinations of the singles and coincidence spectra. A few limits on SAUTs for low energy transitions where no useful $B(E2)$ limit is obtainable are omitted. These limits give important structural information (see text). For each level, the sixth column gives the γ -ray intensity or limits relative to 1000 for the 652.6 keV line. Where known from the literature, the intensities have been corrected to include only the *E*2 contribution in the last two columns. All other transitions are assumed to be pure *E*2. The seventh column gives the relative $B(E2)$ values from each level for each γ ray (or limit) by dividing the intensities by E^5_γ and normalizing the largest observed value to 1000 for each level.

E_i	E_f	J_i^{π}	J_f^π	E_{γ}	I_{γ} (E2)	Rel. $B(E2)$ value
652.6	$\boldsymbol{0}$	2^+	0^+	652.6	1000	1000
1320.7	652.6	0^+	2^+	668.1	1.3	1000
1398.2	652.6	4^+	2^+	745.6	831	1000
1414.9	$\boldsymbol{0}$	2^+	0^+	1415.0	19.4	22.2
	652.6		2^+	762.3	39.7	1000
1797.6	652.6	3^+	2^+	1145.1	95.5^{a}	73
	1398.2		4^+	399.1	6.7	$1000\,$
	1414.9		2^+	382.7	$3.5^{\rm b}$	644
1817.9	$\boldsymbol{0}$	$0^+, 2^+$	0^+	1818.4	$0.9\,$	15.2
	652.6		2^+	1165.3	6.4	1000
	1320.7		0^+	497.2	< 0.30	$<$ 3400
	1398.2		4^+	419.7	< 0.40	$<$ 10400
	1414.9		2^+	403.0	< 0.55	$<$ 17400
2013.4	652.6	3^+	2^+	1360.9	$2.6\,$	1.9
	1398.2		4^+	615.0	25.8 ^a	$1000\,$
	1414.9		2^+	598.9	20.4^c	903
	1797.6		3^+	215.8	< 8.3	< 60500
	1817.9		$0^+, 2^+$	195.5	< 0.05	< 600
2223.3	1398.2	$6+$	4^+	825.1	491	1000
2267.2	652.6	4^+	2^+	1614.6	< 0.30	$<\!\!0.1$
	1398.2		4^+	869.2	15.8	$2.5\,$
	1414.9		2^+	852.3	< 0.55	$<\!\!0.1$
	1797.6		3^+	469.6	14.3^d	49.6
	1817.9		$0^{\rm +}$, $2^{\rm +}$	449.3	< 0.22	< 0.9
	2013.4		3^+	253.8	13.3	1000
2278.0	652.6	2^+	2^+	1625.4	7.4	50.4
	1320.7		0^+	957.3	< 0.33	$< \negthinspace 32$
	1398.2		4^+	879.6	6.8	1000
	1414.9		2^+	863.1	< 1.00	< 162
	1797.6		3^+	480.4	< 0.30	< 910
	1817.9		$0^{\rm +}$, 2 ⁺	460.1	< 1.10	$<$ 4130
	2013.4		3^+	264.6	< 0.16	< 9550
2427.9	652.6	2^+	2^+	1776.4	19.9	73.0
	1320.7		0^+	1107.2	< 0.66	$<$ 26 $\,$
	1398.2		4^+	1029.7	$<$ 4 $^{\circ}$	$<$ 225
	1414.9		2^+	1013.0	< 1.10	$<\!67$
	1797.6		3^+	630.3	1.2	783
	1817.9		$0^+, 2^+$	609.9	1.3	1000
	2013.4		3^+	414.5	< 0.66	$<$ 3500
	2246		0^+	282	$<\!\!0.1$	$<$ 32600
	2267.2		4^+	160.7	< 0.06	$<$ 36500

E_i	E_f	J_i^{π}	J_f^{π}	E_{γ}	I_{ν} (E2)	Rel. $B(E2)$ value
	2278.0		2^+	149.9	< 0.06	$<$ 51500
2547.8	1398.2	5^+ , 6^+	4^+	1149.7	69.1	8.3
	1797.6		3^+	750.2	< 0.70	< 0.7
	2013.4		3^+	534.4	< 2.10	<12
	2223.3		$6+$	324.6	15.0	1000
	2267.2		4^+	280.5	3.7	512
2603.5	652.6	2^+ , 3^+	2^+	1950.9	< 0.06	< 0.1
	1398.2		4^+	1205.7	4.6	54
	1414.9		2^+	1188.6	< 0.6	< 8
	1797.6		3^+	805.9	< 0.2	< 18
	1817.9		$0^+, 2^+$	785.6	< 0.3	$<$ 30
	2013.4		3^+	589.6	2.4	1000
	2223.3		$6+$	380.2	$<$ 4.0	< 15000
	2246		0^+	357	< 0.06	$<$ 307
	2267.2		4^+	336.3	< 0.11	< 765
	2278.0		2^+	325.5	< 1.5	< 12300
	2427.9		2^+	175.6	$<$ 4.5	< 812000

TABLE III. (*Continued.*)

^a Possibly some *M*1 content. Full observed intensity assumed to be *E*2.

^bLarge uncertainty on δ (see Ref. [1]). *E*2 component ranges from 1% to 82%. The value from Ref. [1] of 14% adopted.

 ${}^{c}B(E2)$ obtained with an *E*2 component of 89% +6%, −17% (see Ref. [1]).

^dLarge uncertainty on δ . *E*2 component taken from Ref. [1] as 26% +40%, −6%.

^eComplex multiplet. Limit estimated from comparison of singles and coincidence spectra.

$$
Q = (s^{\dagger} \widetilde{d} + d^{\dagger} s) + \chi (d^{\dagger} \widetilde{d})^{(2)}
$$

with parameters ϵ , κ , and χ . Given the near-harmonicity evident in the energies of the two-phonon states, the resulting parameters and wave functions must be close to the vibrator. That is, ϵ *must* be large and $\sim E(2_1^+)$. With only four bosons, this implies that the effects of the $O \cdot O$ terms are small and it

FIG. 12. Decay of the 1818 keV level showing the observed transitions, with relative $B(E2)$ values. Of course, if the 1818.4 keV transition is correctly placed, the level spin is 2^+ . This is not directly relevant to the point being illustrated in this figure.

FIG. 13. Decay of the 2278 keV level showing the observed transitions, with relative $B(E2)$ values.

cussed later since small energy shifts would have little if any significant effects on the transition rates.

The calculated energies from the IBA are compared to the data in Fig. 14, where we also show a comparison of the known absolute $B(E2)$ values up through the two-phonon triplet and the 6^+_1 level. [The calculated $B(E2; 2^+_1 \rightarrow 0^+_1)$ value is normalized to the measured value of 32 W.u.].

As can be seen, the energy agreement for the two phonon states is reasonably good. The $B(E2)$ values for the 2^{+}_{2} state are also in excellent agreement with the data. However, as we have already pointed out, the small $B(E2; 4^+_1 \rightarrow 2^+_1)$ [and $B(E2; 6^+_1 \rightarrow 4^+_1)$] values from the most recent measurements [2] are impossible to understand with any known collective model using reasonable parameters. This is a major issue that must be addressed by future experiments or understood with new theoretical approaches. For our present purposes, we allow the possibility that the existing $B(E2)$ values may be altered by such experiments and focus on an analysis of

TABLE IV. Relative $B(E2)$ values from three-phonon to twophonon states in the harmonic vibrator model. Normalized to 35 for the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value (see Refs. [17,18]). This normalization gives results in convenient integer form. Also, since the $B(E2; 2₁⁺)$ \rightarrow 0⁺₁) in ⁹⁸Ru is 32 W.u., the entries in the table are approximately (in Weisskopf units) what would be expected for the decay of threephonon states in ⁹⁸Ru.

Two-phonon	Three-phonon				
	0^+	2^+	3^+	4^+	
0^+		49			
2^+	105	20	75	55	
4^+		36	30	50	105

FIG. 14. Comparison of the energies of candidates for threephonon states with the AHV (left), and with the IBA (right). Absolute $B(E2)$ values are shown up through the two-phonon triplet levels and the $6₁⁺$ level. The theoretical $B(E2)$ values (in Weisskopf units) are normalized to 32 W.u. for the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value.

three- and four-phonon candidates. We now use the IBA calculations to assess the structure of the next levels above the two-phonon triplet.

1. 1798 keV level

The IBA calculations for this level are very similar to the vibrator for a three-phonon level since with only four bosons there is no other 3^+ state with which to mix. Both the predicted energy of this level (see Fig. 14, right) and the ratio of allowed $B(E2)$ values to the 4_1^+ and 2_2^+ states (see earlier discussion) disagree with the data.

2. 1818 keV level

Figure 15 shows the comparison of the measured relative $B(E2)$ values (or limits) for the decay of the 1818 keV level, which we take for this discussion as having a spin of 2^+ , with the IBA calculation. To highlight the relation between allowed and forbidden vibrator transitions, it is convenient to normalize the relative $B(E2)$ values from the 1818 keV level to the largest relative $B(E2)$ value $(2^+_3 \rightarrow 2^+_2)$ (even though this is just an upper limit). The strict limits on the unobserved transitions allow us to conclude that the 1818 keV level can not be well described as a three-phonon state. The strongest evidence comes from the limit on the allowed $B(E2; 2^+_3 \rightarrow 0^+_2)$ which is less than four times the forbidden $B(E2; 2_3^{\pm} \rightarrow 2_1^{\pm})$ value. In contrast, the IBA calculation, which has a slightly perturbed vibrator structure, predicts that the allowed $B(E2)$ value is more than 3000 times larger. Also, as seen in Fig. 15, the predicted energy is several hundred keV higher the observed level energy. Hence, a three-phonon description of this level fails. A similar, albeit slightly weaker, argument applies if the 1818 keV level is 0^+ .

FIG. 15. Comparison of experimental relative $B(E2)$ values with the IBA for the decay of the 1818 keV level (taken as 2^+ for this comparison), *including* limits on all SAUTs. Here the normalization is to 100 for the strongest relative $B(E2)$ value, including the upper limits. For each transition the upper number on the transition arrow is the experimental value or limit and the lower value is the IBA prediction. Predicted $B(E2)$ values less than 0.1 are written as 0. In Figs. 15–17 any transitions with only a single number are the relative experimental $B(E2)$ values for cases where the final state is not within the IBA model space.

3. 2013 keV level

The situation is basically unchanged from the earlier analysis of this level with the vibrator model $B(E2)$ values since the limits on the SAUTs (which in this case would both be forbidden or noncollective transitions) are not stringent enough to provide additional arguments.

4. 2267 keV level

As is seen from Fig. 16, the strongest evidence concerning the structure of this level comes from the small upper limit on the $B(E2; 4^+_3 \rightarrow 2^+_2)$ value (852 keV transition). Although this transition is allowed in the vibrator it is unobserved and, in fact, the $B(E2)$ to the 2^+_2 state is about $500-10000$ times less than the forbidden $B(E2)$ values to

FIG. 17. Same as Fig. 15 for the 2278 keV level as a candidate for a four-phonon level.

either of the 3^+ states. In contrast, the IBA calculation predicts that the $B(E2; 4^+_2 \rightarrow 2^+_2)$ value is about 110 times greater than the forbidden transition. In addition, the allowed 852 and 869 keV transitions to the 2^+_2 and 4^+_1 two-phonon states, experimentally give a ratio $B(E_2^2; 4^+_2 \rightarrow 2^+_2)/B(E_2^2; 4^+_2)$ \rightarrow 4⁺₁ $>$ 0.04 while the predicted *B*(*E*2) ratio is 1.1 in both the vibrator model and in the IBA calculation. Therefore, this level cannot be described as a three-phonon state.

5. 2278 keV level

As the fourth 2^+ level, this state might be considered a candidate for a four-phonon state. Of course, from our previous discussion, we do not have good candidates for threephonon states but let us assume for the moment that the 3⁺ levels at 1798 keV and/or 2013 keV may have some threephonon amplitude. An interpretation of the 2278 keV level as a possible four-phonon level is tested in Fig. 17. If the relative $B(E2; 2_4^+ \rightarrow 4_1^+)$ and $B(E2; 2_4^+ \rightarrow 3_1^+)$ values are compared, the forbidden $B(E2; 2^+_4 \rightarrow 4^+_1)$ value is at least 1.1 times larger than the experimental limit (480.4 keV transition: see Fig. 2) on the allowed $B(E2; 2^+_4 \rightarrow 3^+_1)$ value. In contrast, in the IBA calculation, the latter $B(E2)$ value is larger by a factor of about 2.5×10^4 . If the nearby 2013 keV level is taken (for the sake of this test) as the three-phonon 3⁺ state, there is still a factor of $10³$ or more discrepancy with the IBA calculations. Finally, the energy of the 2278 keV level is half a MeV or more lower than predicted for a fourphonon state. Such a description is therefore untenable.

Another possibility is that the 2^+_4 level could be a threephonon state because we have already shown that the 1818 keV level is not the three-phonon state. Moreover, the three-phonon energy predicted by the anharmonic vibrator (see Fig. 14) is closer to the energy of the 2^+_4 level than the 2^+_3 level. However, as shown in Fig. 18, in the IBA calculations, the allowed $B(E2; 2^+_4 \rightarrow 0^+_2)$ is 6000 times larger than the forbidden $B(E2; 2^+_4 \rightarrow 2^+_1)$ value, while it is less than the forbidden $B(E2; 2^+_4 \rightarrow 2^+_1)$ value in the data. Moreover, the $B(E2; 2^+_4 \rightarrow 0^+_2)/B(E2; 2^+_4 \rightarrow 4^+_1)$ ratio is 1.36 in the vibrator and about 1.86 in the IBA calculation while it is ≤ 0.032

FIG. 18. Same as Fig. 15 for the 2278 keV level as a candidate for a three-phonon level.

experimentally. Therefore, the 2^+_4 level cannot be described at all in a phonon picture.

V. CONCLUSIONS

⁹⁸Ru is an intriguing nucleus. At first glance it appears to be a near harmonic vibrator. However, above the two-phonon levels there are no reasonable candidates for three-phonon levels. That is, none of the possible three-phonon states behaves as a vibrational level. The only non-yrast levels that even approximately could have significant three-phonon structure are either the 1798 or 2013 3^+ levels. Both exhibit deviations in branching ratios of allowed transitions from the vibrator model of factors of about 3, although the larger uncertainty in the *E*2/*M*1 mixture for the 382 keV transition allows a branching ratio within the vibrator range for the 1798 keV level. However, even in that case, the 1798 keV level energy is in serious disagreement with the vibrator predictions, including the allowance for anharmonicities.

It is not easy to understand why the vibrator model breaks down so suddenly (at or below the three-phonon states) in 98 Ru while, in nuclei like $^{108-114}$ Cd, at least some states (even non-yrast states) with up to five phonons have intact structure $[20,21]$. Since collectivity in the proton sector in ⁹⁸Ru is likely small (only the 1*g*9/2 level is available) the explanation presumably depends on the number of valence neutrons [4 in 98 Ru, 12–16 in $^{110-114}$ Cd]. However, further theoretical work is clearly needed.

The yrast states present another especially puzzling feature that has apparently gone unnoticed. In collective nuclei the nearly universal behavior is that yrast $B(E2)$ values *increase* with spin, at least up to the region of the first backbend (here, through the 8^+ level). The latest measurements for ⁹⁸Ru [2], however, give $B(E2; 4^{+}_{1} \rightarrow 2^{+}_{1})$ and $B(E2; 6^{+}_{1})$ \rightarrow 4⁺₁) values *significantly* less than the *B*(*E*2; 2⁺₁ → 0⁺₁) value. The $B_{4/2}$ value in ⁹⁸Ru is extremely small (0.38 ± 0.11) and cannot be ascribed to an intruder structure since there are no extra levels below 1800 keV. This phenomenon can occur in near magic nuclei where the seniority scheme applies [22], but is extremely rare for collective nuclei in the mass 90–200 region [23]. We note, however, that, with the previously measured (see Ref. [1]) $B(E2; 4_1^+ \rightarrow 2_1^+)$ value (40 W.u.), $B_{4/2}$ would be >1 . Thus, there is an important need for a remeasurement of this transition. If the latest value persists, this would represent a very serious breakdown of the vibrator picture even at the two-phonon level and a puzzle beyond the ability of current collective models to interpret. Such measurements are planned.

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