# **Resonating group method study of kaon-nucleon elastic scattering in the chiral SU(3) quark model**

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The chiral SU(3) quark model is extended to include an antiquark in order to study the kaon-nucleon system. The model input parameters  $b_u$ ,  $m_u$ ,  $m_s$  are taken to be the same as in our previous work which focused on the nucleon-nucleon and nucleon-hyperon interactions. The mass of the scalar meson  $\sigma$  is chosen to be 675 MeV and the mixing of  $\sigma_0$  and  $\sigma_8$  is considered. Using this model the kaon-nucleon *S* and *P* partial waves phase

shifts of isospin  $I=0$  and  $I=1$  have been studied by solving a resonating group method equation. The numerical results of  $S_{01}$ ,  $S_{11}$ ,  $P_{01}$ ,  $P_{03}$ , and  $P_{11}$  partial waves are in good agreement with the experimental data while the phase shifts of *P*<sup>13</sup> partial wave are a little bit too repulsive when the laboratory momentum of the kaon meson is greater than 500 MeV in this present calculation.

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# **I. INTRODUCTION**

As is well known, the nonperturbative quantum chromodynamics (QCD) effect is very important in the light quark system, but up to now there is no serious practical approach to really solve the nonperturbative QCD problem. People still need QCD-inspired models to help. Among these models, the chiral SU(3) quark model [1] has been successful in reproducing the energies of the baryon ground states, the binding energy of deuteron, the nucleon-nucleon (*NN*) scattering phase shifts of different partial waves, and the hyperon-nucleon (*YN*) cross sections by the resonating group method (RGM) calculations [1,2]. In the study of the dibaryon structure, the binding energy of the *H* particle obtained from this model is around the threshold of two  $\Lambda$  [3], consistent with the recent experimental estimation from the binding energy of the double  $\Lambda$  hypernucleus [4]. Inspired by these achievements, we try to extend this model to the systems with antiquarks to study the baryon-meson interactions. With the antiquark  $(\bar{q})$  in the meson brought in, the complexity of the annihilation part in the interactions will appear. As a first step we start with the study of *KN* elastic scattering processes because in the *KN* system the annihilation to gluons and vacuum is forbidden and the  $u\bar{s}$  ( $d\bar{s}$ ) can only annihilate to kaon mesons.

Another motivation of the present work came from the discovery of the  $\Theta^+(1540)$  pentaquark state, an exotic  $K^+n$  or  $K^{0}p$  resonance reported by some laboratories recently [5–12]. The strangeness quantum number of this  $\Theta$  particle is  $S = +1$  and the upper limit of the width is about  $\Gamma_{\odot}$  < 25 MeV. It may be the first exotic hadron observed and has triggered great interest and heated discussions. However, the nature of this particle, its isospin, parity, and angular momentum, is still going to be determined. In order to obtain a reasonable interpretation of the data of the *uudds¯* system, a prior understanding of the kaon-nucleon interaction on a quark level is important and necessary.

Actually, the *KN* scattering had aroused particular interest in the past due to the kaon meson's high penetrating power [13,14], which makes the kaon one of the deepest probes of the nuclear medium in the energy range between 0 and 1 GeV/*c*. The model based on hadronic degrees of freedom [15] can give a good description of *KN* interaction, but Buettgen *et al.* had to add the exchange of a short range  $(\sim 0.2$  fm) repulsive scalar meson in order to reproduce the *S*-wave phase shifts in the isospin *I*=0 channel. The range of this repulsion is smaller than the nucleon radius, which clearly shows that the quark substructure of the kaon mesons and nucleons cannot be neglected. In Ref. [16], the *KN* phase shifts are calculated within a constituent quark model by numerically solving the RGM equation. In that calculation, the quark-quark potential includes gluon, pion, and sigma exchanges and the ground state energies of mesons can be reproduced, but the agreement of the obtained results with the experimental phase shifts is quite poor. Recently, Wang *et al.* [17] gave a study on the *KN* elastic scattering in a quark potential model. Their results are consistent with the experimental data, but in their model, a factor of color octet component is added arbitrarily and the size parameter of harmonic oscillator is chosen to be  $b<sub>u</sub> = 0.255$  fm, which is too small compared with the radius of nucleon.

The goal of the present work aims at studying the *KN* elastic scattering phase shifts of *S* and *P* partial waves of isospin  $I=0$  and  $I=1$  in the framework of the chiral SU(3) quark model by carrying on a resonating group method calculation. We take the same input model parameters  $b<sub>u</sub>$ ,  $m<sub>u</sub>$ ,  $m<sub>s</sub>$  as in our previous work [1,2], which successfully explained the existing *NN* and *YN* experimental data. The difference is that in the present work the mass of the scalar meson  $\sigma$  is chosen to be 675 MeV (in our previous work  $m_{\sigma}$ =595 or 625 MeV) and the mixing between  $\sigma_0$  and  $\sigma_8$  is considered. By this means the attraction of  $\sigma$  meson in *KN S*<sup>01</sup> partial wave can be reduced a lot. Except for the case of *P*<sub>13</sub>, the numerical results of different partial waves are in agreement with the experimental data. In comparison with the previous results [16,18], our calculation achieves a considerable improvement on the theoretical phase shifts. In this sense, it means that our model also works well when an antiquark is added in the system (at least for the *KN* system), so that one can regard that the interactions between two quarks obtained from this model is almost reasonable, which is useful for studying the structure of the  $\Theta^+(1540)$  pentaquark state from the constituent quark model point of view.

The paper is organized as follows. In the next section the framework of the chiral SU(3) quark model and the RGM approach applying to the *KN* system are briefly introduced. The calculated results of the isospin *I*=0 and *I*=1 *KN* phase shifts of *S* and *P* partial waves are shown in Sec. III, as well as some discussions are made in this section. Finally, conclusions are drawn in Sec. IV.

#### **II. FORMULATION**

## **A. The model**

Following Georgi's idea [19], the interaction Lagrangian of the quark-chiral SU(3) field can be written as

$$
\mathcal{L}_I = -\ g_{ch}(\bar{\psi}_L \Sigma \psi_R + \bar{\psi}_R \Sigma^+ \psi_L), \tag{1}
$$

with  $g_{ch}$  being the quark–chiral-field coupling constant,  $\psi_L$ and  $\psi_R$  being the quark-left and quark-right spinors, respectively, and

$$
\Sigma = \exp[i\pi_a \lambda_a/f], \quad a = 1, 2, \dots, 8,
$$
 (2)

where  $\pi_a$  is the Goldstone boson field and  $\lambda_a$  the Gell-Mann matrix of the flavor SU(3) group. Generalizing the linear realization of  $\Sigma$  from the SU(2) case to the SU(3) case, one obtains

$$
\Sigma = \sum_{a=0}^{8} \sigma_a \lambda_a + i \sum_{a=0}^{8} \pi_a \lambda_a,
$$
 (3)

and the interaction Lagrangian

$$
\mathcal{L}_I = -g_{ch}\overline{\psi}\left(\sum_{a=0}^8 \sigma_a \lambda_a + i \sum_{a=0}^8 \pi_a \lambda_a \gamma_5\right) \psi, \tag{4}
$$

where  $\lambda_0$  is a unitary matrix,  $\sigma_0, \ldots, \sigma_8$  are the scalar nonet fields, and  $\pi_0$ ,...,  $\pi_8$  the pseudoscalar nonet fields. Clearly,  $\mathcal{L}_I$  is invariant under the infinitesimal chiral  $SU(3)_L$  $\times$ SU(3)<sub>R</sub> transformation. Consequently, one obtains the interactive Hamiltonian as

$$
H_{ch} = g_{ch} F(q^2) \overline{\psi} \left( \sum_{a=0}^{8} \sigma_a \lambda_a + i \sum_{a=0}^{8} \pi_a \lambda_a \gamma_5 \right) \psi.
$$
 (5)

Here we insert a form factor  $F(q^2)$  to describe the chiral-field structure [20,21]. As usual,  $F(q^2)$  is taken as

$$
F(q^2) = \left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right)^{1/2},\tag{6}
$$

and the cutoff mass  $\Lambda$  indicates the chiral symmetry breaking scale [20–23].

From Eqs. (5) and (6) the SU(3) chiral-field-induced quark-quark potentials can be derived, and their expressions are given in the following:

$$
V_{\sigma_a}(\mathbf{r}_{ij}) = - C(g_{ch}, m_{\sigma_a}, \Lambda) X_1(m_{\sigma_a}, \Lambda, r_{ij}) [\lambda_a(i)\lambda_a(j)]
$$
  
+  $V_{\sigma_a}^{Is}(\mathbf{r}_{ij}),$  (7)

$$
V_{\sigma_a}^{I_s}(\mathbf{r}_{ij}) = -C(g_{ch}, m_{\sigma_a}, \Lambda) \frac{m_{\sigma_a}^2}{4m_{q_i}m_{q_j}} \left\{ G(m_{\sigma_a}r_{ij}) - \left(\frac{\Lambda}{m_{\sigma_a}}\right)^3 G(\Lambda r_{ij}) \right\} [L \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)][\lambda_a(i)\lambda_a(j)],
$$
\n(8)

and

$$
V_{\pi_a}(\mathbf{r}_{ij}) = C(g_{ch}, m_{\pi_a}, \Lambda) \frac{m_{\pi_a}^2}{12m_{q_i}m_{q_j}} X_2(m_{\pi_a}, \Lambda, r_{ij}) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)
$$
  
×[ $\lambda_a(i)\lambda_a(j)$ ], (9)

with

$$
C(g_{ch}, m, \Lambda) = \frac{g_{ch}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} m,
$$
 (10)

$$
X_1(m,\Lambda,r) = Y(mr) - \frac{\Lambda}{m}Y(\Lambda r),\tag{11}
$$

$$
X_2(m,\Lambda,r) = Y(mr) - \left(\frac{\Lambda}{m}\right)^3 Y(\Lambda r),\tag{12}
$$

$$
Y(x) = \frac{1}{x}e^{-x},
$$
 (13)

$$
G(x) = \frac{1}{x} \left( 1 + \frac{1}{x} \right) Y(x),
$$
 (14)

and  $m_{\sigma_a}$  being the mass of the scalar meson and  $m_{\pi_a}$  the mass of the pseudoscalar meson.

As mentioned in Ref. [24], in the chiral SU(3) quark model the interaction induced by the coupling of chiral field describes the nonperturbative QCD effect of the lowmomentum medium-distance range. To study the hadron structure and hadron-hadron dynamics, one still needs an effective one-gluon-exchange interaction  $V_{ij}^{OGE}$  which governs the short-range perturbative QCD behavior,

$$
\gamma_{ij}^{OGE} = \frac{1}{4} g_{i} g_{j} (\lambda_i^c \cdot \lambda_j^c) \left\{ \frac{1}{r_{ij}} - \frac{\pi}{2} \delta(r_{ij}) \left( \frac{1}{m_{q_i}^2} + \frac{1}{m_{q_j}^2} + \frac{4}{3} \frac{1}{m_{q_i} m_{q_j}} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \right) \right\} + V_{OGE}^{Js},
$$
(15)

with

*Vij*

$$
V_{OGE}^{l s} = -\frac{1}{16} g_i g_j (\lambda_i^c \cdot \lambda_j^c) \frac{3}{m_{q_i} m_{q_j}} \frac{1}{r_{ij}^3} L \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j), \quad (16)
$$

and a confinement potential  $V_{ij}^{conf}$  which provides the nonperpurbative QCD effect in the long distance,

$$
V_{ij}^{conf} = -a_{ij}^c (\lambda_i^c \cdot \lambda_j^c) r_{ij}^2 - a_{ij}^{c0} (\lambda_i^c \cdot \lambda_j^c).
$$
 (17)

For the *KN* system, we have to extend our chiral SU(3) quark model to the case with an antiquark. Now, the total Hamiltonian of *KN* system is written as

$$
H = \sum_{i=1}^{5} T_i - T_G + \sum_{i < j=1}^{4} V_{ij} + \sum_{i=1}^{4} V_{i5},\tag{18}
$$

where  $T_G$  is the kinetic energy operator of the center of mass motion, and  $V_{ij}$  and  $V_{i5}$  represent the interactions between quark-quark  $(q-q)$  and quark-antiquark  $(q-\bar{q})$ , respectively,

$$
V_{ij} = V_{ij}^{OGE} + V_{ij}^{conf} + V_{ij}^{ch},\tag{19}
$$

$$
V_{ij}^{ch} = \sum_{a=0}^{8} V_{\sigma_a}(\mathbf{r}_{ij}) + \sum_{a=0}^{8} V_{\pi_a}(\mathbf{r}_{ij}).
$$
 (20)

The interaction between  $u(d)$  and  $\overline{s}$  includes two parts [25]: direct interaction and annihilation parts,

$$
V_{i\bar{5}} = V_{i\bar{5}}^{dir} + V_{i\bar{5}}^{ann},\tag{21}
$$

with

$$
V_{i\overline{5}}^{dir} = V_{i\overline{5}}^{conf} + V_{i\overline{5}}^{OGE} + V_{i\overline{5}}^{ch}, \tag{22}
$$

and

$$
V_{i\bar{5}}^{conf} = -a_{i5}^c(-\lambda_i^c \cdot \lambda_5^{c^*})r_{i5}^2 - a_{i5}^{c0}(-\lambda_i^c \cdot \lambda_5^{c^*}),
$$
 (23)

$$
V_{i\overline{s}}^{OGE} = \frac{1}{4}g_i g_s(-\lambda_i^c \cdot \lambda_5^{c^*}) \left\{ \frac{1}{r_{i5}} - \frac{\pi}{2} \delta(r_{i5}) \left( \frac{1}{m_{q_i}^2} + \frac{1}{m_s^2} + \frac{4}{3} \frac{1}{m_q m_s} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_5) \right) \right\} - \frac{1}{16} g_i g_s(-\lambda_i^c \cdot \lambda_5^{c^*})
$$
  
 
$$
\times \frac{3}{m_{q_i} m_{q_5}} \frac{1}{r_{i5}^3} \boldsymbol{L} \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_5), \qquad (24)
$$

$$
V_{i\bar{5}}^{ch} = \sum_{j} (-1)^{G_j} V_{i5}^{ch,j}.
$$
 (25)

Here  $(-1)^{G_j}$  describes the *G* parity of the *j*th meson. For the *KN* system,  $u(d)\overline{s}$  can only annihilate into a *K* meson, i.e.,

$$
V_{i\overline{5}}^{ann} = V_{ann}^K,
$$
\n<sup>(26)</sup>

with

$$
V_{ann}^{K} = C_{ann}^{K} \left( \frac{1 - \sigma_q \cdot \sigma_{\bar{q}}}{2} \right)_{spin} \left( \frac{2 + 3\lambda_q \cdot \lambda_{\bar{q}}^*}{6} \right)_{color}
$$

$$
\times \left( \frac{38 + 3\lambda_q \cdot \lambda_{\bar{q}}^*}{18} \right)_{flavor} \frac{\Lambda^2}{r} e^{-\Lambda r},
$$
(27)

TABLE I. Model parameters. The meson masses and the cutoff masses:  $m_{\sigma'} = 980 \text{ MeV}$ ,  $m_{\kappa} = 1430 \text{ MeV}$ ,  $m_{\epsilon} = 980 \text{ MeV}$ ,  $m_{\sigma} = 675 \text{ MeV}$ ,  $m_{\pi} = 138 \text{ MeV}$ ,  $m_{\kappa} = 495 \text{ MeV}$ ,  $m_{\eta'} = 549 \text{ MeV}$ ,  $m_{\eta'}$ =957 MeV,  $\Lambda$ =1500 MeV for  $\kappa$  and 1100 MeV for other mesons.

$m_{\mu}$ (MeV)	313
$m_s$ (MeV)	470
$b_u$ (fm)	0.5
$g_u$	0.886
$g_s$	0.755
$a_{uu}^c$ (MeV/fm <sup>2</sup> )	52.40
$a^c_{us}$ (MeV/fm <sup>2</sup> )	75.30
$a_{uu}^{c0}$ (MeV)	$-50.37$
$a_{us}^{c0}$ (MeV)	$-66.80$
$C_{\text{ann}}^K$ (fm <sup>2</sup> )	$-0.137$

$$
C_{ann}^{K} = -\frac{\tilde{g}_{ch}^{2}}{4\pi m_{K}^{2} - (\tilde{m} + \tilde{m}_{s})^{2}},
$$
\n(28)

where  $\tilde{g}_{ch}$  is the effective coupling constant of chiral field in the annihilation case and  $\tilde{m}$  represents the effective quark mass. Actually,  $\tilde{m}$  is quark momentum dependent; here we treat it as an effective mass. In the present form of the annihilation interaction  $V_{ann}^K$ , a form factor  $F(q^2)$  [Eq. (6)], which is also used in the vertex of the quark–chiral-field coupling, is inserted to flatten the sharp behavior of the  $\delta$  function. In this work we treat  $C_{\text{ann}}^K$  as a parameter and adjust it to fit the mass of kaon meson.

#### **B. Determination of parameters**

We have three initial input parameters: the harmonicoscillator width parameter  $b_{\mu}$ , the up (down) quark mass  $m_{u(d)}$ , and the strange quark mass  $m_s$ . These three parameters are taken to be the same as in our previous work [1,2], i.e.,  $b_u = 0.5$  fm,  $m_{u(d)} = 313$  MeV, and  $m_s = 470$  MeV. By some special constraints, the other model parameters are fixed in the following way: the chiral coupling constant  $g<sub>ch</sub>$  is fixed by

$$
\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{NN\pi}^2}{4\pi} \frac{m_u^2}{M_N^2},\tag{29}
$$

with  $g_{NN\pi}^2/4\pi = 13.67$  taken as the experimental value. The masses of the mesons are also adopted to the experimental values, except for the  $\sigma$  meson, where its mass is treated as an adjustable parameter; in this work, it is adopted to be 675 MeV. The cutoff radius  $\Lambda^{-1}$  is taken to be the value close to the chiral symmetry breaking scale [20–23]. After the parameters of chiral fields are fixed, the one-gluon-exchange coupling constants  $g_u$  and  $g_s$  can be determined by the mass splits between *N*,  $\Delta$  and  $\Sigma$ ,  $\Lambda$ , respectively. The confinement strengths  $a_{uu}^c$ ,  $a_{us}^c$ , and  $a_{ss}^c$  are fixed by the stability conditions of *N*,  $\Lambda$ , and  $\Xi$ , and the zero point energies  $a_{uu}^{c0}$ ,  $a_{us}^{c0}$ , and  $a_{ss}^{c0}$ by fitting the masses of *N*,  $\Sigma$ , and  $\overline{\Xi + \Omega}$ , respectively. About  $C_{\text{ann}}^K$ , we adjust it to fit the mass of kaon meson. The resultant model parameters are tabulated in Table I and the masses of

TABLE II. The masses of octet and decuplet baryons.

			$N$ $\Sigma$ $\Xi$ $\Lambda$ $\Delta$ $\Sigma^*$ $\Xi^*$ $\Omega$		
Theor. 939 1194 1334 1116 1237 1375 1515 1657					
Expt. 939 1194 1319 1116 1237 1385 1530 1672					

octet and decuplet baryons obtained from this set of parameters are listed in Table II.

In our calculation, the meson mixing between the flavor singlet and octet mesons is considered, i.e.,  $\eta$ ,  $\eta'$  mesons are mixed by  $\eta_0$ ,  $\eta_8$ :

$$
\eta' = \eta_{8} \sin \theta^{PS} + \eta_{0} \cos \theta^{PS},
$$
  

$$
\eta = \eta_{8} \cos \theta^{PS} - \eta_{0} \sin \theta^{PS},
$$
 (30)

with the mixing angle  $\theta^{PS}$  taken to be the usual value  $-23^{\circ}$ and  $\sigma$ ,  $\epsilon$  mesons are ideally mixed by  $\sigma_0$ ,  $\sigma_8$ :

$$
\sigma = \sigma_8 \sin \theta^S + \sigma_0 \cos \theta^S,
$$
  

$$
\epsilon = \sigma_8 \cos \theta^S - \sigma_0 \sin \theta^S,
$$
 (31)

with  $\theta^{\mathsf{S}} = 35.264^{\circ}$ , which means that  $\sigma$  only acts on the *u(d)* quark, and  $\epsilon$  on the *s* quark, respectively. Under this ideal mixing, the scalar meson exchange interactions between  $u(d)$ and  $\bar{s}$  are totally vanished, so that the attraction force of scalar meson between *K* and *N* can be reduced a lot.

## **C. The RGM approach applying to the** *KN* **system**

In this section, we present the applying of the resonating group method (RGM) to the *KN* system. We take the following choice of the coordinates to construct the total wave function of the system:

$$
\xi_1 = r_2 - r_1,\tag{32}
$$

$$
\xi_2 = r_3 - \frac{r_1 + r_2}{2},\tag{33}
$$

$$
\xi_3 = r_5 - r_4,\tag{34}
$$

$$
R_{KN} = \frac{r_1 + r_2 + r_3}{3} - \frac{m_u r_4 + m_s r_5}{m_u + m_s},
$$
 (35)

$$
R_{c.m.} = \frac{m_u(r_1 + r_2 + r_3 + r_4) + m_s r_5}{4m_u + m_s}.
$$
 (36)

Here,  $\mathbf{r}_i$  is the coordinate of the *i*th quark,  $\xi_1$  and  $\xi_2$  are the internal coordinates for the cluster *N*, and  $\xi_3$  the internal coordinate for *K*.  $\mathbf{R}_{KN}$  is the relative coordinate between *K* and *N*, and  $\mathbf{R}_{c.m.}$  is the center of mass coordinate of the total system.

Following the cluster model calculation [26–28], the RGM wave function is written as

$$
\Psi = \mathcal{A}[\hat{\phi}_N(\xi_1, \xi_2) \hat{\phi}_K(\xi_3) \chi_{rel}(\mathbf{R}_{KN}) Z(\mathbf{R}_{c.m.})]_{ST},\qquad(37)
$$

$$
\phi_N(\xi_1, \xi_2) = \left(\frac{m_u \omega}{2\pi}\right)^{3/4} \left(\frac{2m_u \omega}{3\pi}\right)^{3/4} \exp\left[-m_u \omega\left(\frac{\xi_1^2}{4} + \frac{\xi_2^2}{3}\right)\right],\tag{38}
$$

$$
\phi_K(\xi_3) = \left(\frac{\omega}{\pi} \frac{m_u m_s}{m_u + m_s}\right)^{3/4} \exp\left[-\frac{\omega}{2} \frac{m_u m_s}{m_u + m_s} \xi_3^2\right], \quad (39)
$$

$$
Z(\boldsymbol{R}_{c.m.}) = \left(\frac{\omega}{\pi}(4m_u + m_s)\right)^{3/4} \exp\bigg[-\frac{\omega}{2}(4m_u + m_s)\boldsymbol{R}_{c.m.}^2\bigg].
$$
\n(40)

Here  $\phi_N(\xi_1, \xi_2)$  and  $\phi_K(\xi_3)$  denote the internal wave function in coordinate space of cluster *N* and *K*, respectively.  $\phi_N(\xi_1, \xi_2)$  represents the antisymmetrized wave function of cluster *N* and  $\hat{\phi}_K(\xi_3)$ , the wave function of cluster *K* with *N* and *K* further specifying all the quantum numbers of the relevant cluster.  $\chi_{rel}(\mathbf{R}_{KN})$  is the trial wave function of the relative motion between interacting clusters *K* and *N*, and  $Z(\mathbf{R}_{c.m.})$  is the wave function of the motion of the total center of mass. The oscillator frequency  $\omega$  is associated with the width parameter  $b_u$  by the constituent quark mass  $m_u$ .

$$
\frac{1}{b_i^2} = m_i \omega.
$$
 (41)

The symbol  $A$  is the antisymmetrizing operator defined as

$$
\mathcal{A} \equiv 1 - \sum_{i \in N} P_{i4} \equiv 1 - 3P_{34}.
$$
 (42)

*S* and *T* denote the total spin and isospin of the *KN* system, respectively. Substituting  $\Psi$  into the projection equation

$$
\langle \delta \Psi | (H - E) | \Psi \rangle = 0, \tag{43}
$$

where

$$
E = E_K + E_N + E_{rel},\tag{44}
$$

with  $E, E_K, E_N$ , and  $E_{rel}$  being the total energy, the inner energies of clusters *K* and *N*, and the relative energy between clusters *K* and *N*, respectively, we obtain RGM equation

$$
\int \mathcal{L}(\mathbf{R}', \mathbf{R}) \chi_{rel}(\mathbf{R}) d\mathbf{R} = 0, \qquad (45)
$$

with

$$
\mathcal{L}(\mathbf{R}',\mathbf{R}) = \mathcal{H}(\mathbf{R}',\mathbf{R}) - E\mathcal{N}(\mathbf{R}',\mathbf{R}),\tag{46}
$$

where the Hamiltonian kernel  $H$  and normalization kernel  $N$ can, respectively, be calculated by

with

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$$
\begin{cases}\n\mathcal{H}(\mathbf{R}',\mathbf{R}) \\
\mathcal{N}(\mathbf{R}',\mathbf{R})\n\end{cases} = \left\langle \begin{bmatrix} \hat{\phi}_N(\xi_1,\xi_2) \hat{\phi}_K(\xi_3) \delta(\mathbf{R}' - \mathbf{R}_{KN}) Z(\mathbf{R}_{c.m.}) \end{bmatrix}_{ST} \middle| \begin{Bmatrix} H \\ 1 \\ 1 \end{Bmatrix} \right| \\
\times \mathcal{A}[\hat{\phi}_N(\xi_1,\xi_2) \hat{\phi}_K(\xi_3) \delta(\mathbf{R} - \mathbf{R}_{KN}) Z(\mathbf{R}_{c.m.})]_{ST} \right\rangle. (47)
$$

In the actual calculation, the unknown  $\chi_{rel}$  is determined in the following way: First, we perform a partial wave expansion,

$$
\chi_{rel}(\boldsymbol{R}_{KN}) = \sum_{L} \chi_{rel}^{L}(\boldsymbol{R}_{KN}),
$$
\n(48)

and then, for a bound-state problem,  $\chi_{rel}^L(\boldsymbol{R}_{KN})$  is expanded as

$$
\chi_{rel}^L(\mathbf{R}_{KN}) = \sum_{i=1}^n c_i \int \left(\frac{\omega \mu_{KN}}{\pi}\right)^{3/4}
$$

$$
\times \exp\left[-\frac{\omega \mu_{KN}}{2}(\mathbf{R}_{KN} - \mathbf{S}_i)^2\right] Y_{LM}(\hat{\mathbf{S}}_i) \mathrm{d}\hat{\mathbf{S}}_i
$$

$$
= \sum_{i=1}^n c_i \frac{1}{R_{KN}} u^L(R_{KN}, S_i) Y_{LM}(\hat{\mathbf{R}}_{KN}), \qquad (49)
$$

with

$$
u^{L}(R_{KN}, S_i) \equiv 4\pi R_{KN} \left(\frac{\omega\mu_{KN}}{\pi}\right)^{3/4} \exp\left[-\frac{1}{2}\omega\mu_{KN}(R_{KN}^2 + S_i^2)\right] \times i_L(\omega\mu_{KN}R_{KN}S_i),
$$
\n(50)

where  $S_i$  is called the generate coordinate,  $\mu_{KN}$  is the reduced mass of  $KN$  system, and  $i<sub>L</sub>$  the *L*th modified spherical Bessel function. Usually  $\chi_{rel}(\boldsymbol{R}_{KN})$  is also expanded as

$$
\chi_{rel}(\boldsymbol{R}_{KN}) = \sum_{L} \frac{1}{R_{KN}} \chi_{rel}^{L}(R_{KN}) Y_{LM}(\hat{\boldsymbol{R}}_{KN}), \qquad (51)
$$

so equivalently, Eq. (49) can be written in a compact form

$$
\chi_{rel}^L(R_{KN}) = \sum_{i=1}^n c_i u^L(R_{KN}, S_i).
$$
 (52)

Now all the information about the relative wave function is contained in the coefficients  $c_i$ 's which are left to be solved. Performing variational procedure, one can deduce a *L*th partial-wave equation for the bound-state problem,

$$
\sum_{j=1}^{n} \mathcal{L}_{ij}^{L} c_j = 0 \quad (i = 1, ..., n), \tag{53}
$$

with

$$
\mathcal{L}_{ij}^L = \int u^L(R', S_i) \mathcal{L}^L(R', R) u^L(R, S_j) R' R dR' dR, \quad (54)
$$

$$
\mathcal{L}^{L}(R',R) = \int Y_{LM}^{*}(\hat{R}') \mathcal{L}(R',R) Y_{LM}(\hat{R}) d\hat{R}' d\hat{R}.
$$
 (55)

Solving Eq. (53), we can get the binding energy and the corresponding wave function of the two-cluster system.

For a scattering problem, the relative wave function is expanded as

$$
\chi_{rel}^L(R_{KN}) = \sum_{i=1}^n c_i \tilde{u}^L(R_{KN}, S_i),
$$
\n(56)

$$
\tilde{u}^{L}(R_{KN}, S_{i}) = \begin{cases} p_{i}u^{L}(R_{KN}, S_{i}), & R_{KN} \le R_{C} \\ \left[ h_{L}^{-}(k_{KN}R_{KN}) - s_{i}h_{L}^{+}(k_{KN}R_{KN}) \right] R_{KN}, & R_{KN} \ge R_{C} \end{cases}
$$
\n(57)

with  $h_L^{\pm}$  being *L*th spherical Hankel functions,  $k_{KN}$  $=\sqrt{2\mu_{KN}E_{rel}}$  the momentum of relative motion, and  $R_C$  a cutoff radius beyond which all the strong interactions can be disregarded. The complex parameters  $p_i$  and  $s_i$  are determined by the smoothness condition at  $R_{KN} = R_C$  and  $c_i$ '*s* satisfy  $\sum_{i=1}^{n} c_i = 1$ . Performing variational procedure, a *Lth* partial-wave equation for the scattering problem can be deduced as

$$
\sum_{j=1}^{n-1} \tilde{\mathcal{L}}_{ij}^{L} c_j = \tilde{\mathcal{M}}_i^{L} \quad (i = 1, ..., n),
$$
 (58)

with

$$
\widetilde{\mathcal{L}}_{ij}^L = \widetilde{\mathcal{K}}_{ij}^L - \widetilde{\mathcal{K}}_{in}^L - \widetilde{\mathcal{K}}_{nj}^L + \widetilde{\mathcal{K}}_{nn}^L, \tag{59}
$$

$$
\widetilde{\mathcal{M}}_i^L = \widetilde{\mathcal{K}}_{nn}^L - \widetilde{\mathcal{K}}_{in}^L, \tag{60}
$$

and

$$
\widetilde{\mathcal{K}}_{ij}^L = \int \widetilde{u}^L(R', S_i) \mathcal{L}^L(R', R) \widetilde{u}^L(R, S_j) R' R dR' dR, \quad (61)
$$

where the RGM kernel  $\mathcal{L}^{L}(R',R)$  is defined in Eq. (55). Before solving Eq. (58), we have to calculate the kernel  $\widetilde{\mathcal{K}}_{ij}^L$ . Considering the asymptotic form of spherical Hankel functions,  $\widetilde{\mathcal{K}}_{ij}^L$  can be written as

$$
\widetilde{\mathcal{K}}_{ij}^L = p_i p_j (\mathcal{L}_{ij}^L - K_{ij}^{L(ex)}), \qquad (62)
$$

$$
K_{ij}^{L(ex)} = \int_{R_C}^{\infty} u^L(R, S_i) \left( -\frac{\hbar^2}{2\mu_{KN}} \frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu_{KN}} \frac{L(L+1)}{R^2} - E_{rel} \right) \times u^L(R, S_j) dR.
$$
 (63)

Having solved Eq. (58), the *S*-matrix element  $S^L$  and the phase shifts  $\delta_L$  are given by

$$
S^{L} \equiv e^{2i\delta_{L}} = \sum_{i=1}^{n} c_{i} s_{i}.
$$
 (64)



FIG. 1. *KN S*-wave phase shifts as a function of the laboratory momentum of kaon meson. The solid lines represent the results obtained by considering  $\theta^S$ =35.264° while the dotted lines  $\theta^S$ =−18°. The hole circles and the triangles correspond respectively to the phase shifts analysis of Hyslop *et al.* [29] and Hashimoto [30].

# **III. RESULTS OF** *KN* **PHASE SHIFTS AND DISCUSSIONS**

A RGM dynamical calculation is made to study the partial wave phase shifts of *KN* scattering by using the Hamiltonian, Eq. (18), and the calculated phase shifts of *S* and *P* waves with isospin  $I=0$  and  $I=1$  are shown in Figs. 1 and 2 with solid lines.

For the  $S_{01}$ ,  $P_{01}$ ,  $P_{11}$ , and  $P_{03}$  waves (here the first subscript refers to the isospin quantum number *I* and the second one to twice of the total angular momentum of the system 2*J*), our results are in agreement with the experimental data. While for the  $P_{13}$  channel our numerical phase shifts are too repulsive when the laboratory momentum of the kaon meson in greater than 500 MeV and  $S_{11}$  channel a little repulsive. Comparing with the results of the recent resonating group method calculation of Lemaire *et al.* [16] based on a constituent quark model (CQM), in which the calculated phase shifts of  $S_{01}$ ,  $P_{03}$ ,  $P_{11}$  waves have opposite sign and the  $P_{01}$ channel is too repulsive for the experimental data, we obtained the correct sign and reproduced the experimental data quite well. This means that a reasonable interaction between *K* and *N* can be obtained from the chiral SU(3) quark model when the mixing of  $\sigma_0$  and  $\sigma_8$  mesons is considered as ideal mixing and the mass of the  $\sigma$  meson is taken to be 675 MeV, which is closely consistent with the relation  $m_{\sigma}$  $i = \sqrt{m_{\pi}^2 + (2m_u)^2}$  from the dynamical vacuum spontaneous breaking mechanism [31]. We also compare our results with those of the previous work of Black [18]. Although our calculation achieves a considerable improvement on the theoretical phase shifts in the magnitude for  $S_{01}$ ,  $S_{11}$ ,  $P_{01}$ ,  $P_{11}$ ,  $P_{03}$  waves, the results of the  $P_{13}$  channel are too repulsive in both Black's work and our present work. Maybe the



FIG. 2. *KN P*-wave phase shifts. Same notation as in Fig. 1.

Since there is something uncertain in the annihilation interaction part, its influence on the phase shifts should be investigated. We omitted the annihilation part entirely to see the effect, and found that the numerical phase shifts only have very small changes. This is because the annihilation part acts in the very short range, so that it plays a nonsignificant role in the *KN* scattering process.

One thing should be mentioned: in our present one channel calculation for *KN* scattering process the confinement potential contributes pimping interactions between the two color singlet clusters *K* and *N*. Thus our numerical results will almost remain unchanged; even the color quadratic confinement is replaced by the color liner confinement or an improved one which is presently unknown.

Recently we became aware of Ref. [32] written by Dai and Wu, in which an investigation based on a dynamically spontaneous symmetry breaking mechanism predicted that the mass of  $\sigma$  meson is  $m_{\sigma}$ =677 MeV and the mixing angle between  $\sigma_0$  and  $\sigma_8$  is  $\theta$ <sup>S</sup>=−18°. Using this  $m_\sigma$  and  $\theta$ <sup>S</sup>, we calculated the *KN* phase shifts and the results are shown as dotted lines in Figs. 1 and 2. One can see that the *KN* phase shifts can be also explained quite well by taking this group of parameters. It is comprehensible because in both of these two cases the attraction of  $\sigma$  is reduced, just in different approaches. When  $\theta$ <sup>S</sup>=35.264° (ideal mixing), the reduction comes from the interaction between  $u(d)$  and *s* quarks vanished, while  $θ^s = -18°$ , the interaction of  $σ$  between two *u*, *d* quarks, is strongly reduced.

From the phase shifts of *KN* (Figs. 1 and 2) one can see that there is no signal for an existing *KN* resonance state both in *S* and *P* waves until the laboratory momentum of the kaon meson stretches to 1 GeV. For studying the existence of bound states of the *KN* system, we solved the RGM equation for the bound state problem [Eq. (53)]. The results showed that the energies of the *KN* system for both *S* and *P* waves are located above the *KN* threshold, which means that there is no bound state. As a consequence, it can be said that the newly observed exotic baryon  $\Theta^+$  cannot be explained as a *KN* resonance state or a *KN* bound state in our present calculation.

#### **IV. CONCLUSIONS**

The chiral SU(3) quark model is extended to the system with an antiquark, and the *KN* scattering process is studied by using this model in the framework of the resonating group method. We take the same initial input parameters as in our previous work, which successfully explained the existing *NN* and *YN* experimental data. The difference is that in the present work the mass of the scalar meson  $\sigma$  is chosen to be 675 MeV (in our precious work  $m_{\sigma}$ =595 or 625 MeV) and the mixing of  $\sigma_0$  and  $\sigma_8$  is considered. Except for the case of *P*<sub>13</sub>, the numerical results of different partial waves are in agreement with the experimental data. In comparison with the previous results, our calculation achieves a considerable improvement on the theoretical phase shifts. It seems that our model can work well for the *KN* system, in which an antiquark  $\bar{s}$  is there besides four  $u(d)$  quarks, and the interactions between two quarks obtained from this model might be reasonable, which would be useful to study the structure of the  $\Theta^+(1540)$  pentaquark state from the constituent quark model point of view.

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- [1] Z. Y. Zhang, Y. W. Yu, P. N. Shen, L. R. Dai, A. Faessler, and U. Straub, Nucl. Phys. **A625**, 59 (1997).
- [2] L. R. Dai, Z. Y. Zhang, Y. W. Yu, and P. Wang, Nucl. Phys. **A727**, 321 (2003).
- [3] P. N. Shen, Z. Y. Zhang, Y. W. Yu, S. Q. Yuan, and S. Yang, J. Phys. G **25**, 1807 (1999).
- [4] K. Imai, in *Hadrons and Nuclei with Strangeness*, Proceedings of First Sino-Japan Symposium on Strangeness Physics, 1999, and references therein.
- [5] LEPS Collaboration, T. Nakano *et al.*, Phys. Rev. Lett. **91**, 012002 (2003).
- [6] DIANA Collaboration, V. V. Barmin *et al.*, Phys. At. Nucl. **66**, 1715 (2003).
- [7] CLAS Collaboration, S. Stepanyan *et al.*, Phys. Rev. Lett. **91**, 252001 (2003).
- [8] SAPHIR Collaboration, J. Barth *et al.*, Phys. Lett. B **572**, 127 (2003).
- [9] A. E. Asratyan, A. G. Dolgolenko, and M. A. Kubantsev, Phys. At. Nucl. **67**, 682 (2004).
- [10] CLAS Collaboration, V. Kubarovsky et al., Phys. Rev. Lett. **92**, 032001 (2004).
- [11] HERMES Collaboration, A. Airapetian *et al.*, Phys. Lett. B **585**, 213 (2004).
- [12] SDV Collaboration, A. Aleev *et al.*, hep-ex/0401024.
- [13] W. R. Coker, J. D. Lumpe, and L. Ray, Phys. Rev. C **31**, 1412 (1985).
- [14] Y. Abgrall, R. Belaidi, and J. Labarsouque, Nucl. Phys. **A462**, 781 (1987).
- [15] R. Buettgen, K. Holinde, A. Mueller-Groeling, J. Speth, and P. Wyborny, Nucl. Phys. **A506**, 586 (1990).
- [16] S. Lemaire, J. Labarsouque, and B. Silvestre-Brac, Nucl. Phys. **A714**, 265 (2003).
- [17] H. J. Wang, H. Yang, and J. C. Su, Phys. Rev. C **68**, 055204 (2003).
- [18] N. Black, J. Phys. G **28**, 1953 (2002).
- [19] H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, California, 1984).
- [20] I. T. Obukhovsky and A. M. Kusainov, Phys. Lett. B **238**, 142

(1990).

- [21] A. M. Kusainov, V. G. Neudatchin, and I. T. Obukhovsky, Phys. Rev. C **44**, 2343 (1991).
- [22] A. Buchmann, E. Fernandez, and K. Yazaki, Phys. Lett. B **269**, 35 (1991).
- [23] E. M. Henley and G. A. Miller, Phys. Lett. B **251**, 453 (1991).
- [24] Z. Y. Zhang, A. Faessler, U. Straub, and L. Ya. Glozman, Nucl. Phys. **A578**, 573 (1994).
- [25] F. Huang, Z. Y. Zhang, Y. W. Yu, and B. S. Zou, Phys. Lett. B **586**, 69 (2004).
- [26] M. Kamimura, Suppl. Prog. Theor. Phys. **62**, 236 (1977).
- [27] M. Oka and K. Yazaki, Prog. Theor. Phys. **62**, 556 (1981).
- [28] U. Straub, Z. Y. Zhang, K. Brauer, A. Faessler, S. B. Kardkikar, and G. Lubeck, Nucl. Phys. **A483**, 686 (1988).
- [29] J. S. Hyslop, R. A. Arndt, L. D. Roper, and R. L. Workman, Phys. Rev. D **46**, 961 (1992).
- [30] K. Hashimoto, Phys. Rev. C **29**, 1377 (1984).
- [31] M. D. Scadron, Phys. Rev. D **26**, 239 (1982).
- [32] Y. B. Dai and Y. L. Wu, hep-ph/0304075.