Renormalization of the σ-ω model within the framework of U(1) gauge symmetry

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It is shown that the σ-ω model which is widely used in the study of the nuclear relativistic many-body problem can be exactly treated as an Abelian massive gauge field theory. The quantization of this theory can perfectly be performed by means of the general methods described in the quantum gauge field theory. Especially, the local U(1) gauge symmetry of the theory leads to a series of Ward-Takahashi identities satisfied by Green's functions and proper vertices. These identities form a uniquely correct basis for the renormalization of the theory. The renormalization is carried out in the mass-dependent momentum space subtraction scheme and by the renormalization group approach. With the aid of the renormalization boundary conditions, the solutions to the renormalization group equations are given in definite expressions without any ambiguity and renormalized *S*-matrix elements are exactly formulated in forms as given in a series of tree diagrams provided that the physical parameters are replaced by the running ones. As an illustration of the renormalization procedure, the one-loop renormalization is concretely carried out and the results are given in rigorous forms which are suitable in the whole energy region. The effect of the one-loop renormalization is examined by the two-nucleon elastic scattering.

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I. INTRODUCTION

The quantum hadrodynamics (QHD), as a relativistic quantum field theory for baryons and mesons, has been widely applied to studying various nuclear phenomena including the hadron-hadron interaction, the hadron-nucleus scattering, the bulk and single-particle properties of nuclei, etc. [1–5]. It is commonly recognized that although the quantum chromodynamics is a fundamental theory for strong interaction, the QHD, as an effective field theory formulated in terms of hadronic degrees of freedom, provides a simple and reliable approach to produce the nuclear observables that are insensitive to the short-range dynamics. There are various QHD models, renormalizable and nonrenormalizable, which were tested in the past to reproduce the empirical nuclear properties and the experimental data. Among these models, the σ-ω model proposed by Walecka [1] has been raising particular interest. This model contains proton, neutron, and isoscalar, Lorentz scalar and vector mesons σ and ω, and in the tree diagram and nonrelativistic approximations leads to a nucleon-nucleon interaction potential which behaves as short-range repulsion and medium-range attraction. The early development of this model is based on the relativistic mean-field and Hartree approximation and shows that the model is quite successful in the applications to the infinite nuclear matter and atomic nuclei. Since the model is renormalizable, it is necessary to consider higher order perturbative corrections to the results given in the mean field approximation by a certain renormalization procedure. Along this line, a number of efforts were made previously [6–19]. Especially, the efforts were mostly concentrated on the renormalization of the model in the study of the nuclear matter at finite temperature and density. In this renormalization, the loop expansion and spectral function methods were applied to evaluate the loop corrections. However, there are various difficulties to occur in the renormalization [8–16]. For example, in Ref. [12], the authors calculated the nuclear matter energy density up to the two loop level and found enormous contributions arising from the loop terms that alter the description of the nuclear bound state qualitatively. Therefore it was concluded that "the loop expansion does not provide a reliable approximation scheme in renormalizable QHD" [19]. To this end, one may ask what is the correct procedure of performing the renormalization for a model of QHD, and how to assess the applicability of a renormalizable model of QHD for which the renormalization is carried out? To answer these questions, it is meaningful to examine the renormalization of a QHD model from different angles and, as suggested in Ref. [19], "to develop and apply systematic and consistent *power counting* schemes that lead to more general conserving approximations and to study renormalization group methods that could determine the analytic structure of the ground-state energy functional."

In this paper, we confine ourself to discussing the renormalization of the σ-ω model by the renormalization group method in the case of zero temperature. The procedure is very similar to that described in our previous work on the QED and QCD renormalizations [20]. The main features of the renormalization given in this paper contain two aspects: (i) the renormalization is based on the U(1) gauge symmetry because the σ-ω model, as argued in the next section, is exactly of the U(1) local gauge symmetry; (ii) The renormalization is carried out by a mass-dependent momentum space subtraction [21–24] which will lead to rigorous renormalized results by the renormalization group method [25–28]. Ordinarily, the massive vector fields such as the ω meson field, the ρ meson field, and so on, are not viewed as gauge fields because the mass term in the Lagrangian is not gauge invari-

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ant [29–31]. On the contrary, it was pointed out in Refs. [32–34] that a massive vector field must be viewed as a constrained system in the whole space of the vector potential $A_\mu(x)$. This is because a massive vector meson has only three polarization states which need only three spatial components of the vector potential $A_\mu(x)$ to describe them, while the remaining component of the $A_\mu(x)$ appears to be a redundant degree of freedom which must be eliminated by introducing the Lorentz condition. According to the general principle for constrained systems, the gauge invariance of a massive Abelian or non-Abelian gauge field should be seen from its action given in the physical space defined by the Lorentz condition. This viewpoint will be explained in more detail in the next section. From this viewpoint, it is easy to see that the σ-ω model is surely of U(1) local gauge symmetry. Therefore the model may be quantized by the method as used in the gauge field theory. In this paper, we will describe the Lorentz-covariant quantization performed in both the Hamiltonian and Lagrangian path-integral formalisms by following the procedure proposed in Refs. [32–34]. From this quantization, we obtain an effective action which contains a gauge-fixing term and a ghost term in it and manifests itself to be invariant under a set of Becchi-Rouet-Stora-Tyutin (BRST) transformations [35]. It should be mentioned that the quantum theory of the σ-ω model was set up previously by the method of canonical quantization and in the path-integral formalism [29–31,36,37]. Especially, with the time paths being generalized to a manifestly covariant form, a covariant path-integral formulation for the model at finite temperature was achieved in Ref. [37] and led to manifestly covariant Feynman rules for both real and imaginary times. Nevertheless, owing to the lack of the gauge-fixing term and ghost term in the effective action, the generating functional given in these quantizations would not exhibit the BRST symmetry.

As emphasized in Ref. [20], a correct renormalization procedure for a gauge field theory must respect the gauge-symmetry (the Ward-Takahashi identities [38,39]), the Lorentz invariance (the energy-momentum conservation), and the mathematical convergence principles. Otherwise, the renormalization would be incorrect. From the gauge invariance (or say, the BRST symmetry) of the σ-ω model, we derive a set of Ward-Takahashi (WT) identities satisfied by the generating functionals, Green's functions, and vertices which provide a firm basis for the renormalization of the model. As mentioned before, in this paper, the renormalization of the σ-ω model will be performed in the mass-dependent momentum space subtraction. The prominent advantage of such a subtraction is that it naturally provides boundary conditions satisfied by the renormalized wave functions, propagators, and proper vertices for the quantum σ-ω model. These boundary conditions enable us to uniquely determine the solutions to the renormalization group equations for those renormalized quantities. With the solutions of the renormalization group equations, a S-matrix element can be expressed in the form as given in the tree diagrams provided that the physical parameters in the S-matrix element are replaced by the effective (running) ones. To specify the procedure of the renormalization group method, the one-loop effective physical parameters are concretely calculated and given exact and analytical expressions.

The remainder of this paper is arranged as follows. In Sec. II, we present arguments for the gauge invariance of the σ-ω model. In Sec. III, the σ-ω model will be respectively quantized in the Hamiltonian and Lagrangian path-integral formalisms. In Sec. IV, we will derive a set of WT identities obeyed by the generating functionals. In Sec. V, a WT identity satisfied by the ω meson propagator will be derived and the renormalization of the propagator will be discussed. In Sec. VI, we will derive a WT identity satisfied by the vectorial vertex (nucleon-nucleon-ω meson vertex) and discuss the renormalizations of the vertex and the nucleon propagator. In Sec. VII, the renormalizations of the σ meson propagator and the scalar coupling vertex (nucleon-nucleon-σ meson vertex) will be derived and discussed. Section VIII is used to sketch the renormalization group method and the renormalized S-matrix elements. Section IX serves to derive the one-loop effective coupling constants and masses. In the last section, summary and discussions will be made. In Appendix A, the gauge independence of the S-matrix elements given in the one-loop level will be proved. In Appendix B, we will show the differential cross section of the two-nucleon elastic scattering in the approximation of order g^2 and examine the effect of the one-loop renormalization on it.

II. ARGUMENT OF GAUGE INVARIANCE FOR THE σ-ω MODEL

The σ-ω model is described by the following Lagrangian density [1]:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - M)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2 A^\mu A_\mu + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m_\sigma^2\varphi^2, \tag{2.1}$$

where

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \tag{2.2}$$

is the nucleon isospin doublet in which ψ_p and ψ_n are the proton and neutron field functions, respectively,

$$D_\mu = \partial_\mu - ig_v A_\mu - \frac{i}{4}g_s\gamma_\mu\varphi \tag{2.3}$$

is the covariant derivative in which A_μ and φ stand for the ω and σ meson fields, g_v and g_s designate the vectorial and scalar coupling constants,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.4}$$

is the vector field strength, and M, m_ω, and m_σ are the masses of nucleon, ω meson, and σ meson, respectively. In the above Lagrangian, the scalar self-couplings are ignored as was done originally in the Walecka model [1].

In the previous, the σ-ω model was considered to be gauge-noninvariant with respect to the following local U(1) gauge transformations [29–31]:

$$\psi'(x) = e^{ig_v\theta(x)}\psi(x),$$

$$\overline{\psi'}(x) = e^{-ig_v\theta(x)}\overline{\psi}(x),$$

$$A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x),$$

$$\varphi'(x) = \varphi(x), \tag{2.5}$$

where $\theta(x)$ is the scalar parametric function of U(1) group since the mass term of the ω meson in the Lagrangian is not gauge invariant. But, this does not mean that the dynamics of the ω meson system is not gauge invariant. As mentioned in the Introduction, the ω meson field must be viewed as a constrained system in the space spanned by the four-dimensional vector potential $A_\mu(x)$. As we know, a massive gauge field has three polarization states which need only three spatial components of the four-dimensional vector potential A_μ to describe them. In the Lorentz-covariant formulation, a full vector potential $A^\mu(x)$ can be split into two Lorentz-covariant parts: the transverse vector potential $A_T^\mu(x)$ and the longitudinal vector potential $A_L^\mu(x)$,

$$A^\mu(x) = A_T^\mu(x) + A_L^\mu(x), \tag{2.6}$$

where

$$A_T^\mu(x) = \left(g^{\mu\nu} - \frac{1}{\Box}\partial^\mu\partial^\nu\right)A_\nu(x), \tag{2.7}$$

$$A_L^\mu(x) = \frac{1}{\Box}\partial^\mu\partial^\nu A_\nu(x) \tag{2.8}$$

with $\Box = \partial^\mu\partial_\mu$ being the D'Alembertian operator. The vector potentials $A_T^\mu(x)$ and $A_L^\mu(x)$ satisfy the following transverse and longitudinal field conditions (identities):

$$\partial_\mu A_T^\mu(x) = 0, \tag{2.9}$$

$$\left(g_{\mu\nu} - \frac{1}{\Box}\partial_\mu\partial_\nu\right)A_L^\nu(x) = 0 \tag{2.10}$$

and the orthogonality relation

$$\int d^4x A_T^\mu(x)A_{L\mu}(x) = 0 \tag{2.11}$$

which characterizes the linear independence of the two field variables. Since the Lorentz-covariant transverse vector potential $A_T^\mu(x)$ contains three-independent spatial components, it is sufficient to represent the polarization states of a massive vector boson, whereas the Lorentz-covariant longitudinal vector potential A_L^μ appears to be a redundant unphysical variable which must be constrained by introducing the Lorentz condition

$$\chi \equiv \partial^\mu A_\mu = 0 \tag{2.12}$$

whose solution is

$$A_L^\mu = 0. \tag{2.13}$$

With this solution, the σ-ω model Lagrangian may be expressed in terms of the independent dynamical variables $A_T^\mu(x)$,

$$\mathcal{L} = \bar{\psi}\left[\gamma^\mu\left(i\partial_\mu + g_v A_{T\mu} + \frac{1}{4}g_s\gamma_\mu\varphi\right) - M\right]\psi - \frac{1}{4}F_T^{\mu\nu}F_{T\mu\nu}$$
$$+ \frac{1}{2}m_\omega^2 A_T^\mu A_{T\mu} + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m_\sigma^2\varphi^2, \tag{2.14}$$

where $F_T^{\mu\nu}$ is defined as in Eq. (2.4) with replacing the $A^\mu(x)$ by $A_T^\mu(x)$. The Lagrangian represented above gives a complete description of the dynamics of the σ-ω model. If we want to represent the dynamics in the whole space of the full vector potential as described by the Lagrangian in Eq. (2.1), the ω field must be treated as a constrained system. In this case, according to the general procedure for constrained systems as formulated in mechanics, the Lorentz condition in Eq. (2.12), as a constraint, must be introduced from the onset and imposed on the Lagrangian in Eq. (2.1) so as to guarantee the redundant degree of freedom to be eliminated from the Lagrangian. Otherwise, the Lagrangian in Eq. (2.1) itself cannot give a complete description for the ω field system. From the Lagrangian in Eq. (2.14), one may derive an equation of motion satisfied by the ω meson field as follows:

$$\partial_\mu F_T^{\mu\nu} + m_\omega^2 A_T^\nu = -j^\nu, \tag{2.15}$$

where

$$j^\nu = g_v\bar{\psi}\gamma^\nu\psi \tag{2.16}$$

is the current generated from the nucleon field. The above equation describes the evolution of the independent variable A_T^μ with time. In particular, when we take divergence of the both sides of Eq. (2.15), considering the identities in Eq. (2.9) and $\partial_\nu\partial_\mu F_T^{\mu\nu} \equiv 0$, we immediately obtain the current conservation

$$\partial^\mu j_\mu = 0 \tag{2.17}$$

which shows that the current is transverse.

Ordinarily, the Lorentz condition is viewed as a consequence of the following ω field equation of motion which is derived from the Lagrangian in Eq. (2.1) [29,30,36],

$$\partial_\mu F^{\mu\nu} + m_\omega^2 A^\nu = -j^\nu. \tag{2.18}$$

The argument of this viewpoint is as follows. When we take divergence of Eq. (2.18) and notice the current conservation, it is found that

$$m_\omega^2\partial^\mu A_\mu = 0. \tag{2.19}$$

Since $m_\omega \neq 0$, the above equation leads to the Lorentz condition which implies that one component of the vector potential is not independent. It is pointed out here that the above viewpoint actually is an ill concept and the procedure leading to the Lorentz condition logically is not consistent with the principle established well in the mechanics for constrained systems. In fact, the aforementioned derivation seems to imply that the Lorentz condition has already been included in the Lagrangian denoted in Eq. (2.1). If so, when the Lagrangian is written in the first order form, we should see a term in the Lagrangian which is given by incorporating the Lorentz condition with the aid of the Lagrange multiplier method. Nevertheless, as will be shown in the next section, there is no such term to appear in the Lagrangian. Moreover, as we know, equations of motion should describe the evolution of the independent variables with time as the equation given in Eq. (2.15) does and should not lead to a constraint condition which implies that some variable in the equation is not independent. Therefore the viewpoint stated above is not reasonable. In accordance with the general principle for constrained systems, the correct procedure is to treat the Lorentz condition as a primary constraint and to impose this condition on the Lagrangian in Eq. (2.1) from the beginning. The necessity of introducing the Lorentz condition can also be seen from the derivation mentioned in Eqs. (2.18) and (2.19). Equation (2.19) can be understood in such a way that if the Lorentz condition is not introduced, there would appear a contradiction that the right hand side of the equation is zero, but the left hand side is not. Only when the Lorentz condition is introduced, the contradiction disappears. In this case, due to the Lorentz condition, Eq. (2.19), as a trivial identity, naturally holds and the equation of motion (2.18) can naturally go over to Eq. (2.15), exhibiting the self-consistency of the theory. Particularly in the latter case, when the divergence of Eq. (2.18) is taken and the Lorentz condition is employed, one immediately obtains the current conservation in Eq. (2.17). In addition, we would like to note that for the quantum theory, in the zero-mass limit: $m_\omega \to 0$, the vector field part of the Lagrangian in Eq. (2.1) naturally goes over to the one for the massless vector meson, but, as shown in Sec. V, the vector meson propagator does not and a worse singularity occurs, revealing a severe inconsistence of the theory. Only when the Lorentz condition is introduced initially and incorporated into the Lagrangian by the Lagrange multiplier method can a consistent quantum theory be constructed.

Now, let us turn to address the gauge invariance of the σ-ω model. Usually, the gauge invariance is required to the Lagrangian. From the dynamical viewpoint, as pointed out in Refs. [32–34], the action is of more essential significance than the Lagrangian. This is why in mechanics and field theory, to investigate the dynamical and symmetric properties of a system, one always starts from the action of the system. Similarly, when we examine the gauge-symmetric property of a field system, in general, we should also see whether the action for the system is gauge invariant or not. In particular, for a constrained system such as the massive vector field, we should see whether or not the action represented in terms of the independent dynamical variables is gauge invariant. This point of view is easy to understand from the mechanics for constrained systems. Suppose a mechanical system is described by a Hamiltonian

$$H(p_i,q_i)(i=1,2,\cdots,n) \tag{2.20}$$

which is given in the $2n$-dimensional phase space and constraint conditions

$$\varphi_a(p_i,q_i)=0(\alpha=1,2,\cdots,2m<2n) \tag{2.21}$$

which define a physical phase space of dimension $2(n-m)$ where the system exists and moves only. If the constrained variables can be solved out from the constraint conditions, we may write a Hamiltonian

$$H^*(p_j^*,q_j^*)(j=1,2,\cdots,n-m) \tag{2.22}$$

which is expressed via the independent variables and gives a complete formulation of the constrained system. Obviously, to examine some symmetry of the constrained system, it is only necessary to see if the Hamiltonian $H^*(p_j^*,q_j^*)$ other than the Hamiltonian $H(p_i,q_i)$ has the desired symmetry because in contrast to the $H^*(p_j^*,q_j^*)$, the $H(p_i,q_i)$ is not complete for describing the system.

Certainly, in some special cases, the Lagrangian given in the physical space itself is locally gauge invariant so that the gauge invariance of the corresponding action is ensured. This situation happens for the massless gauge fields and the massive Abelian gauge field. The gauge transformation of an Abelian gauge field was shown in the third equality in Eq. (2.5). Since $\partial_\mu\theta(x)$ acts as a longitudinal field, according to the decomposition denoted in Eq. (2.6) and considering the independence of the fields $A_T^\mu(x)$ and $A_L^\mu(x)$, the gauge transformation of the ω field can be equivalently divided into two transformations:

$$A_T'^\mu(x)=A_T^\mu(x), \tag{2.23}$$

$$A_T'^\mu(x)=A_L^\mu(x)+\partial^\mu\theta(x). \tag{2.24}$$

Equations (2.23) and (2.24) clearly express the fact that the gauge transformation only changes the unphysical longitudinal part of the vector potential, while the physical transverse vector potential is a gauge-invariant quantity. Furthermore, it is easy to verify that the longitudinal vector potential $A_L^\mu(x)$, which may be expressed as $A_L^\mu(x)=\partial^\mu\varphi(x)$ where $\varphi(x)$ is a scalar function, is canceled in the field strength tensor so that

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu=\partial^\mu A_T^\nu-\partial^\nu A_T^\mu=F_T^{\mu\nu}. \tag{2.25}$$

This indicates that the longitudinal part of the vector potential has no kinetic energy term in the Lagrangian and hence has no dynamical meaning. Such a vector potential can only be viewed as a constrained variable. Since the transverse field variable A_T^μ is gauge invariant, the Lagrangian (2.14) which is written in the physical space is manifestly gauge invariant. Therefore the action given by this Lagrangian is gauge invariant. Alternatively, the gauge invariance may also be seen from the action given by the Lagrangian in Eq. (2.1) which is now constrained by the Lorentz condition. Under the gauge transformation written in Eq. (2.5) and the Lorentz condition denoted in Eq. (2.12), it is easy to find that

$$\delta S=-m_\omega^2\int d^4x\,\theta\partial^\mu A_\mu=0. \tag{2.26}$$

This indicates that the σ-ω model can surely be set up on the basis of gauge-invariance principle.

III. PATH-INTEGRAL QUANTIZATION OF THE σ-ω MODEL

A. Quantization in the Hamiltonian path-integral formalism

According to the general procedure of dealing with constrained systems, the Lorentz condition (2.12) may be incorporated into the Lagrangian (2.1) by the Lagrange undetermined multiplier method to give a generalized Lagrangian [32,40]. In the first order formalism [32,40,41], this Lagrangian can be written as

$$\mathcal{L}_\lambda = \bar{\psi}(i\gamma^\mu D_\mu - M)\psi + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m_\sigma^2\varphi^2 + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{2}m_\omega^2 A^\mu A_\mu + \lambda\partial^\mu A_\mu, \quad (3.1)$$

where A_μ and $F_{\mu\nu}$ are now treated as the mutually independent variables and λ is chosen to represent the Lagrange multiplier. Using the canonically conjugate variables defined by

$$\Pi_\psi = \frac{\partial\mathcal{L}}{\partial\dot{\psi}} = i\bar{\psi}\gamma^0, \quad (3.2)$$

$$\Pi_{\bar{\psi}} = \frac{\partial\mathcal{L}}{\partial\dot{\bar{\psi}}} = 0, \quad (3.3)$$

$$\Pi_\varphi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = \dot{\varphi}, \quad (3.4)$$

and

$$\Pi_\mu(x) = \frac{\partial\mathcal{L}}{\partial\dot{A}^\mu} = F_{\mu 0} + \lambda\delta_{\mu 0} = \begin{cases} F_{k0} = E_k, & \text{if } \mu = k = 1,2,3; \\ \lambda = -E_0, & \text{if } \mu = 0, \end{cases} \quad (3.5)$$

the Lagrangian in Eq. (2.1) may be rewritten in the canonical form

$$\mathcal{L} = E^\mu\dot{A}_\mu + \Pi_\psi\dot{\psi} + \Pi_\varphi\dot{\varphi} + A_0 C - E_0\chi - \mathcal{H}, \quad (3.6)$$

where $E_\mu = (E_0, E_k)$ is a Lorentz vector,

$$C \equiv \partial^\mu E_\mu + m^2 A_0 + g_\omega\bar{\psi}\gamma^0\psi, \quad (3.7)$$

χ was defined in Eq. (2.12), and $\mathcal{H}$ is the Hamiltonian density expressed by

$$\mathcal{H} = \frac{1}{2}(E_k)^2 + \frac{1}{4}(F_{ij})^2 + \frac{1}{2}m_\omega^2[(A_0)^2 + (A_k)^2] + \frac{1}{2}[\Pi_\varphi^2 + (\nabla\varphi)^2 + m_\sigma^2\varphi^2] - i\bar{\psi}\vec{\gamma}\cdot\nabla\psi + M\bar{\psi}\psi - g_v\bar{\psi}\gamma^k\psi A_k - g_s\bar{\psi}\psi\varphi \quad (3.8)$$

in which F_{ij} was defined in Eq. (2.4). In the above, the four-dimensional and the spatial indices are respectively denoted by the Greek and Latin letters. Equation (3.6) clearly shows that the terms $A_0 C$ and $E_0\chi$ are respectively given by incorporating the constraint condition

$$C = 0 \quad (3.9)$$

and the Lorentz condition into the Lagrangian by the Lagrange multiplier method and the Lagrange multipliers A_0 and E_0 are just the constrained variables themselves in this case. Since the A_0 and E_0 are a pair of the canonically conjugate unphysical variables, their constraint conditions in Eqs. (2.12) and (3.9) should simultaneously occur in the Lagrangian (3.6). Otherwise, if the Lorentz condition is not introduced, the term $E_0\chi$ does not appear in the Lagrangian shown in Eq. (3.6) or in Eq. (2.1). In this case, the Lagrangian could not be complete for describing the constrained system under consideration.

From the stationary condition of the action constructed by the Lagrangian (3.6), one may derive the following first-order canonical equations of motion:

$$\dot{A}_k = \partial_k A_0 - E_k, \quad (3.10)$$

$$\dot{E}_k = \partial^i F_{ik} + m_\omega^2 A_k + \partial_k E_0 + g_v\bar{\psi}\gamma_k\psi, \quad (3.11)$$

$$\dot{\varphi} = \Pi_\varphi, \quad (3.12)$$

$$\dot{\Pi}_\varphi = \nabla^2\varphi - m_\sigma^2\varphi + g_s\bar{\psi}\psi, \quad (3.13)$$

$$(i\gamma^\mu\partial_\mu - M + g_v\gamma^\mu A_\mu + g_s\varphi)\psi = 0, \quad (3.14)$$

$$\bar{\psi}(i\gamma^\mu\overleftarrow{\partial}_\mu + M - g_v\gamma^\mu A_\mu - g_s\varphi) = 0, \quad (3.15)$$

as well as the constraint equations written in Eqs. (2.12) and (3.9). Equations (3.10) and (3.11) act as the equations of motion satisfied by the independent canonical variables A_k and $E_k (k=1,2,3)$ which precisely describe the three degrees of freedom of polarization for the massive ω field, while Eqs. (2.12) and (3.9) can only be regarded as the constraint equations obeyed by the constrained variables A_0 and E_0 because in these equations, there are no time derivatives of the dynamical variables A_k and E_k. It is clear to see that in Eqs. (3.10)–(3.15), (2.12), and (3.9), there are altogether 12 equations. They are sufficient to determine the 12 variables including the dynamical canonical variables ψ, $\bar{\psi}$, Π_φ, φ, A_k, and $E_k (k=1,2.3)$ and one pair of constrained variables A_0 and E_0, showing the completeness of the equations.

Now, we turn to formulate the quantization performed in the Hamiltonian path-integral formalism for the σ-ω model. In accordance with the general procedure of the quantization, we should first write a generating functional of Green's functions in terms of the independent canonical variables which are ψ, $\bar{\psi}$, Π_φ, φ, and the transverse parts of the vectors A_μ and E_μ for the ω meson field [32,40,41],

$$Z[J^\mu, J, \bar{\eta}, \eta] = \frac{1}{N}\int D(A_T^\mu, E_T^\mu, \psi, \bar{\psi}, \Pi_\varphi, \varphi) \times \exp\left\{ i\int d^4x [E_T^\mu \dot{A}_{T\mu} + \Pi_\psi\dot{\psi} + \Pi_\varphi\dot{\varphi} - \mathcal{H}^*(A_T^\mu, E_T^\mu, \psi, \bar{\psi}, \Pi_\varphi, \varphi) + J_T^\mu A_{T\mu} + J\varphi + \bar{\eta}\psi + \bar{\psi}\eta] \right\}, \quad (3.16)$$

where $\mathcal{H}^*(A_T^\mu, E_T^\mu, \psi, \bar{\psi}, \Pi_\varphi, \varphi)$ is the Hamiltonian which is obtained from the Hamiltonian (3.8) by replacing the constrained variables A_L^μ and E_L^μ with the solutions of Eqs. (2.12) and (3.9),

$$\mathcal{H}^*(A_T^\mu, E_T^\mu, \ldots) = \mathcal{H}(A^\mu, E^\mu, \ldots)|_{\chi=0, C=0}, \tag{3.17}$$

and J_μ, J, η, and $\bar{\eta}$ are the external sources coupled to the ω meson, σ meson, and nucleon fields, respectively. As mentioned before, Eq. (2.12) leads to $A_L^\mu = 0$. Noticing this solution and the decomposition

$$E^\mu(x) = E_T^\mu(x) + E_L^\mu(x), \tag{3.18}$$

when setting

$$E_L^\mu(x) = \partial_x^\mu Q(x), \tag{3.19}$$

where $Q(x)$ is a scalar function, one may get from Eq. (3.9) an equation obeyed by the scalar function $Q(x)$,

$$\Box_x Q(x) = W(x), \tag{3.20}$$

where

$$W(x) = -g_v \bar{\psi}(x)\gamma^0\psi(x) - m_\omega^2 A_T^0(x). \tag{3.21}$$

With the aid of the Green's function $G(x-y)$ (the ghost particle propagator) which satisfies the equation

$$\Box_x G(x-y) = \delta^4(x-y), \tag{3.22}$$

one may find the solution to Eq. (3.20) as follows:

$$Q(x) = \int d^4y G(x-y) W(y). \tag{3.23}$$

From the expressions given in Eqs. (3.19), (3.21), and (3.23), we see that the $E_L^\mu(x)$ is a complicated functional of the variables A_T^μ and E_T^μ so that the Hamiltonian $\mathcal{H}^*(A_T^\mu, E_T^\mu, \ldots)$ is of a much more complicated functional structure which is not convenient for constructing the diagram technique in perturbation theory. Therefore it is better to express the generating functional in Eq. (3.16) in terms of the variables A_μ and E_μ. For this purpose, it is necessary to insert the following delta functional into Eq. (3.16) [32,40,41]:

$$\delta[A_L^\mu]\delta[E_L^\mu - E_L^\mu(A_T^0, \psi, \bar{\psi})] = \det M \delta[C]\delta[\chi], \tag{3.24}$$

where M is the matrix whose elements are

$$\begin{aligned} M(x,y) &= \{C(x), \chi(y)\} \\ &\equiv \int d^4z \left\{ \frac{\delta C(x)}{\delta A_\mu(z)} \frac{\delta \chi(y)}{\delta E^\mu(z)} - \frac{\delta \chi(y)}{\delta E_\mu(z)} \frac{\delta C(x)}{\delta A^\mu(z)} \right\} \\ &= \Box_x \delta^4(x-y), \end{aligned} \tag{3.25}$$

where $\{C(x), \chi(y)\}$ is the Poisson bracket as defined in the second equality in Eq. (3.25). The relation in Eq. (3.24) is easily derived from Eqs. (2.12) and (3.9) by applying the property of the delta functional. Upon inserting Eq. (3.24) into Eq. (3.16) and utilizing the Fourier representation of the delta functional,

$$\delta[C] = \int D(\rho/2\pi) e^{i\int d^4x \rho(x) C(x)}, \tag{3.26}$$

we have

$$\begin{aligned} &Z[J^\mu, J, \bar{\eta}, \eta] \\ &= \frac{1}{N} \int D(A_\mu, E_\mu, \psi, \bar{\psi}, \Pi_\varphi, \varphi, \rho/2\pi) \det M \delta[\chi] \\ &\quad \times \exp\left\{ i \int d^4x [E^\mu \dot{A}_\mu + \Pi_\psi \dot{\psi} + \Pi_\varphi \dot{\varphi} + \rho C \right. \\ &\quad \left. - \mathcal{H}(A_\mu, E_\mu, \psi, \bar{\psi}, \Pi_\varphi, \varphi) + J^\mu A_\mu + J\varphi + \bar{\eta}\psi + \bar{\psi}\eta] \right\}. \end{aligned} \tag{3.27}$$

In the above exponent, there is a E_0-related term $E_0(\partial_0 A_0 - \partial_0 \rho)$ which permits us to perform the integration over E_0, giving a delta functional

$$\delta[\partial_0 A_0 - \partial_0 \rho] = \det|\partial_0|^{-1} \delta[A_0 - \rho]. \tag{3.28}$$

The determinant $\det|\partial_0|^{-1}$, as a constant, may be put in the normalization constant N and the delta functional $\delta[A_0 - \rho]$ will disappear when the integration over ρ is carried out. The integrals over E_k Π_φ are of Gaussian type and hence easily calculated. After these computations and noticing the expression in Eq. (3.2), we arrive at

$$\begin{aligned} Z[J^\mu, J, \bar{\eta}, \eta] = \frac{1}{N} \int D(A_\mu, \psi, \bar{\psi}, \varphi,) \det M \delta[\partial^\mu A_\mu] \\ \times \exp\left\{ i \int d^4x [\mathcal{L} + J^\mu A_\mu + J\varphi + \bar{\eta}\psi + \bar{\psi}\eta] \right\}, \end{aligned} \tag{3.29}$$

where $\mathcal{L}$ was written in Eq. (2.1). When employing the familiar expression [41,42]

$$\begin{aligned} \det M &= \int D(\bar{C}, C) e^{i\int d^4x d^4y \bar{C}(x) M(x,y) C(y)} \\ &= \int D(\bar{C}, C) e^{i\int d^4x \bar{C}(x) \Box C(x)}, \end{aligned} \tag{3.30}$$

where $\bar{C}(x)$ and $C(x)$ are the mutually conjugate ghost field variables and the following limit for the Fresnel functional:

$$\delta[\partial^\mu A_\mu] = \lim_{\alpha \to 0} C[\alpha] e^{-(i/2\alpha)\int d^4x (\partial^\mu A_\mu)^2}, \tag{3.31}$$

where $C[\alpha] \sim \Pi_x (i/2\pi\alpha)^{1/2}$ and supplementing the external source terms for the ghost fields, the generating functional in Eq. (3.29) is finally given in the form

$$\begin{aligned} Z[J^\mu, J, \bar{\eta}, \eta, \bar{\xi}, \xi] = \frac{1}{N} \int D(A_\mu, \psi, \bar{\psi}, \varphi, \bar{C}, C) \\ \times \exp\left\{ i \int d^4x [\mathcal{L}_{eff} + J^\mu A_\mu + J\varphi + \bar{\eta}\psi \right. \\ \left. + \bar{\psi}\eta + \bar{\xi}C + \bar{C}\xi] \right\}, \end{aligned} \tag{3.32}$$

where

$$\mathcal{L}_{eff} = \mathcal{L} - \frac{1}{2\alpha}(\partial^\mu A_\mu)^2 + \bar{C}\Box C \tag{3.33}$$

which is the effective Lagrangian for the quantized σ-ω model in which the last two terms are the so-called gauge-fixing term and the ghost term, respectively. In Eq. (3.32), the limit $\alpha \to 0$ is implied. Certainly, the theory may be given in arbitrary gauges $(\alpha \neq 0)$. In this case, as will be seen shortly, the ghost particle will acquire a spurious mass $\nu = \sqrt{\alpha} m_\omega$.

B. Quantization in the Lagrangian path-integral formalism

Now let us quantize the σ-ω model in the (second order) Lagrangian path-integral formalism following the procedure proposed in Refs. [32–34,40]. For later convenience, the Lagrangian in Eq. (2.1) and the Lorentz constraint condition in Eq. (2.12) are respectively generalized to the following forms:

$$\mathcal{L}_\lambda = \mathcal{L} - \frac{1}{2}\lambda^2 \tag{3.34}$$

and

$$\partial^\mu A_\mu + \alpha\lambda = 0, \tag{3.35}$$

where $\lambda(x)$ is an extra function which will be identified with the Lagrange multiplier and α is an arbitrary constant playing the role of gauge parameter. According to the general procedure for constrained systems, Eq. (3.35) may be incorporated into Eq. (3.34) by the Lagrange multiplier method to give a generalized Lagrangian,

$$\mathcal{L}_\lambda = \mathcal{L} + \lambda\partial^\mu A_\mu + \frac{1}{2}\alpha\lambda^2. \tag{3.36}$$

This Lagrangian is obviously not gauge invariant. However, for building up a correct gauge field theory, it is necessary to require the dynamics of the gauge field to be gauge invariant. In other words, the action given by the Lagrangian (3.36) is required to be invariant under the gauge transformations shown in Eq. (2.5). By this requirement and applying the constraint condition (3.35), we have

$$\delta S_\lambda = -\frac{1}{\alpha}\int d^4x \partial^\nu A_\nu(x)(\Box_x + \nu^2)\theta(x) = 0, \tag{3.37}$$

where $\nu^2 = \alpha m_\omega^2$. From Eq. (3.35) we see $(1/\alpha)\partial^\nu A_\nu = -\lambda \neq 0$. Therefore, to ensure the action to be gauge invariant, the following constraint condition on the gauge group is necessary to be required:

$$(\Box_x + \nu^2)\theta(x) = 0. \tag{3.38}$$

The constraint condition in Eq. (3.38) may also be incorporated into the Lagrangian in Eq. (3.36) by the Lagrange undetermined multiplier method. In doing this, it is convenient, as is usually done, to introduce the ghost field variable $C(x)$ in such a fashion,

$$\theta(x) = \varsigma C(x), \tag{3.39}$$

where ς is an infinitesimal Grassmann's number. Based on the above definition, the constraint condition (3.38) can be rewritten as

$$(\Box_x + \nu^2)C = 0, \tag{3.40}$$

where the number ς has been dropped. This constraint condition usually is called ghost equation. When the condition (3.40) is incorporated into the Lagrangian (3.36) by the Lagrange multiplier method, we obtain a more generalized Lagrangian as follows:

$$\mathcal{L}_\lambda = \mathcal{L} + \lambda\partial^\mu A_\mu + \frac{1}{2}\alpha\lambda^2 + \bar{C}(\Box_x + \nu^2)C, \tag{3.41}$$

where $\bar{C}(x)$, acting as a Lagrange undetermined multiplier, is the new scalar variable conjugate to the ghost variable $C(x)$. At present, we are ready to formulate the quantization of the σ-ω model. As we learn from the Lagrange undetermined multiplier method, the dynamical and constrained variables as well as the Lagrange multiplier in the Lagrangian (3.41) can all be treated as free ones, varying arbitrarily. Therefore we are allowed to use this kind of Lagrangian to construct the generating functional of Green's functions,

$$\begin{aligned} Z[J^\mu, J, \bar{\eta}, \eta, \bar{\xi}, \xi] = \frac{1}{N}\int & D(A_\mu, \psi, \bar{\psi}, \varphi, \bar{C}, C, \lambda) \\ & \times \exp\Big\{ i \int d^4x[\mathcal{L}_\lambda(x) J^\mu A_\mu + J\varphi + \bar{\eta}\psi \\ & + \bar{\psi}\eta + \bar{\xi}C + \bar{C}\xi]\Big\}. \end{aligned} \tag{3.42}$$

Looking at the expression of the Lagrangian in Eq. (3.41), we see, the integral over $\lambda(x)$ is of Gaussian type. Upon completing the calculation of this integral, we finally obtain

$$\begin{aligned} Z[J^\mu, J, \bar{\eta}, \eta, \bar{\xi}, \xi] = \frac{1}{N}\int & D(A_\mu, \psi, \bar{\psi}, \varphi, \bar{C}, C) \\ & \times \exp\Big\{ i \int d^4x[\mathcal{L}_{eff}(x) J^\mu A_\mu + J\varphi + \bar{\eta}\psi \\ & + \bar{\psi}\eta + \bar{\xi}C + \bar{C}\xi]\Big\}, \end{aligned} \tag{3.43}$$

where

$$\mathcal{L}_{eff} = \mathcal{L} - \frac{1}{2\alpha}(\partial^\mu A_\mu)^2 + \bar{C}(\Box_x + \nu^2)C \tag{3.44}$$

is the effective Lagrangian given in the general gauges. In the Landau gauge $(\alpha \to 0)$, the Lagrangian (3.44) just goes over to the one given in Eq. (3.33). As proved in Ref. [32], the quantization described in Eqs. (3.34) and (3.44) is equivalent to the quantization performed by the Faddeev-Popov approach [42]. At last in this section, we would like to emphasize that the ghost term in the $\mathcal{L}_{eff}$ does not couple to the other fields. But, we do not integrate it out in the generating functional. Keeping this term in the effective action and in the generating functional is helpful to later derivations of WT identities.

IV. WARD-TAKAHASHI IDENTITIES FOR GENERATING FUNCTIONALS

A. BRST transformation

In this section, we show that the action and the generating functional in Eq. (3.43) are invariant with respect to a set of BRST transformations which include the infinitesimal gauge transformations of the nucleon, σ meson, and ω meson fields as well as the transformations for the ghost fields [29–31,35]. The BRST transformations can be written in the form

$$\begin{aligned}
\delta\psi &= \varsigma\,\Delta\,\psi,\\
\delta\bar{\psi} &= \varsigma\,\Delta\,\bar{\psi},\\
\delta A_\mu &= \varsigma\,\Delta\,A_\mu,\\
\delta\bar{C} &= \varsigma\,\Delta\,\bar{C},\\
\delta C &= 0,\\
\delta\varphi &= 0,
\end{aligned} \tag{4.1}$$

where

$$\begin{aligned}
\Delta\,\psi &= ig_v C\psi,\\
\Delta\,\bar{\psi} &= -\,ig_v C\bar{\psi},\\
\Delta A_\mu &= \partial_\mu C,\\
\Delta\,\bar{C} &= \frac{1}{\alpha}\partial^\mu A_\mu.
\end{aligned} \tag{4.2}$$

The above transformations for the nucleon, σ meson, and ω meson fields can directly be written out from Eqs. (2.5) and (3.39). The transformations for the ghost fields may be found from the stationary condition of the effective action under the BRST transformations for the nucleon, σ meson, and ω meson fields,

$$\begin{aligned}
\delta S_{eff} &= \int d^4x\,\delta\mathcal{L}_{eff}\\
&= \int d^4x\left\{\left(\delta\bar{C} - \frac{\varsigma}{\alpha}\partial^\nu A_\nu\right)(\Box_x + \nu^2)C + \bar{C}(\Box_x + \nu^2)\,\delta C\right\}\\
&= 0.
\end{aligned} \tag{4.3}$$

This expression suggests that when the ghost fields undergo the transformations shown in Eqs. (4.1) and (4.2), the effective action is invariant. It is easy to prove that the integration measure in Eq. (3.43) is also invariant under the BRST transformations owing to the Jacobian of the transformations being unity.

B. WT identity satisfied by the generating functionals for Green's functions

When we make the BRST transformations shown in Eqs. (4.1) to the generating functional in Eq. (3.43) and consider the invariance of the generating functional, the action, and the integration measure under the transformations, we obtain an identity such that [29–31]

$$\frac{1}{N}\int \mathcal{D}(A_\mu,\psi,\bar{\psi},\varphi,\bar{C},C)\int d^4x\{J^\mu(x)\,\delta A_\mu(x) + \bar{\eta}(x)\,\delta\psi(x) + \delta\bar{\psi}(x)\,\eta(x) + \delta\bar{C}(x)\xi(x)\}e^{iS_{eff}+iE\cdot\Phi} = 0, \tag{4.4}$$

where $E\cdot\Phi$ with $E=(J_\mu,J,\bar{\eta},\eta,\bar{\xi},\xi)$ and $\Phi=(A_\mu,\varphi,\psi,\bar{\psi},C,\bar{C})$ stands for the external source terms appearing in Eq. (3.43). The Grassmann number ς contained in the BRST transformations in Eq. (4.1) may be eliminated by performing a partial differentiation of Eq. (4.4) with respect to ς. As a result, we get a WT identity as follows:

$$\frac{1}{N}\int \mathcal{D}(A_\mu,\psi,\bar{\psi},\varphi,\bar{C},C)\int d^4x\{J^\mu(x)\,\Delta\,A_\mu(x) - \bar{\eta}(x)\,\Delta\,\psi(x) + \Delta\,\bar{\psi}(x)\,\eta(x) + \Delta\,\bar{C}(x)\xi(x)\}e^{iS_{eff}+iE\cdot\Phi} = 0. \tag{4.5}$$

In order to represent the composite field functions ΔA_μ, $\Delta\bar{\psi}$, and $\Delta\psi$ in Eq. (4.5) in terms of derivatives of the functional Z with respect to external sources, we may, as usual, construct a generalized generating functional by introducing new external sources (called BRST sources later on) into the generating functional written in Eq. (3.43),

$$\begin{aligned}
Z[J^\mu,J,\bar{\eta},\eta,\bar{\xi},\xi;u^\mu,\bar{v},v]
&= \frac{1}{N}\int D(A_\mu,\psi,\bar{\psi},\varphi,\bar{C},C)\\
&\quad\times\exp\left\{iS_{eff} + iE\cdot\Phi + i\int d^4x[u^\mu\,\Delta\,A_\mu + \bar{v}\,\Delta\,\psi + \Delta\,\bar{\psi}v]\right\},
\end{aligned} \tag{4.6}$$

where u^μ, $\bar{v}$, and v are the sources coupled to the functions ΔA_μ, $\Delta\Psi$, and $\Delta\bar{\Psi}$, respectively. Obviously, u^μ and ΔA_μ are anticommuting quantities, while $\bar{v}$, v, $\Delta\bar{\psi}$, and $\Delta\psi$ are commuting ones. It is easy to verify that the BRST-source terms are invariant under the BRST transformation because the functions ΔA_μ, $\Delta\bar{\psi}$, and $\Delta\psi$ are nilpotent with respect to the BRST transformations. Thus we may start from the above generating functional to re-derive the WT identity. The result is the same as given in Eq. (4.5) except that the external source terms are now extended to include the BRST sources,

$$\frac{1}{N}\int D(A_\mu,\psi,\bar{\psi},\varphi,\bar{C},C)\int d^4x\{J^\mu(x)\,\Delta A_\mu(x)-\bar{\eta}(x)\,\Delta\,\psi(x)+\Delta\,\bar{\psi}(x)\,\eta(x)+\Delta\,\bar{C}(x)\xi(x)\}\exp\left\{iS_{eff}+iE\cdot\Phi+i\int d^4x[u^\mu\,\Delta A_\mu+\bar{v}\Delta\,\psi+\Delta\,\bar{\psi}v]\right\}=0. \quad (4.7)$$

Clearly, Eq. (4.7) may be represented as [29–31]

$$\int d^4x\left[J^\mu(x)\frac{\delta}{\delta u^\mu(x)}-\bar{\eta}(x)\frac{\delta}{\delta\bar{v}(x)}+\eta(x)\frac{\delta}{\delta v(x)}+\frac{1}{\alpha}\xi(x)\partial_x^\mu\frac{\delta}{\delta J^\mu(x)}\right]Z[J^\mu,\ldots,v]=0. \quad (4.8)$$

This is the WT identity satisfied by the generating functional of full Green's functions.

Apart from the identity in Eq. (4.8), there is another identity called ghost equation. The ghost equation may easily be derived by first making the translation transformation: $\bar{C}\to\bar{C}+\bar{\lambda}$ in Eq. (4.6) where $\bar{\lambda}$ is an arbitrary Grassmann variable, then differentiating Eq. (4.6) with respect to $\bar{\lambda}$ and finally setting $\bar{\lambda}=0$. The result is

$$\frac{1}{N}\int D(A_\mu,\psi,\bar{\psi},\varphi,\bar{C},C)\{\xi(x)+(\Box_x+\nu^2)C(x)\}\times\exp\left\{iS_{eff}+iE\cdot\Phi+i\int d^4x[u^\mu\,\Delta A_\mu+\bar{v}\Delta\,\psi+\Delta\,\bar{\psi}v]\right\}=0 \quad (4.9)$$

which may be represented in the form [29–31]

$$\left[\xi(x)+(\Box_x+\nu^2)\frac{\delta}{i\delta\bar{\xi}(x)}\right]Z[J_\mu,\ldots,v]=0. \quad (4.10)$$

On substituting into Eqs. (4.8) and (4.10) the relation $Z=e^{iW}$, where W denotes the generating functional of connected Green's functions, one may obtain a WT identity and a ghost equation satisfied by the functional W such that

$$\int d^4x\left[J^\mu(x)\frac{\delta}{\delta u^\mu(x)}-\bar{\eta}(x)\frac{\delta}{\delta\bar{v}(x)}+\eta(x)\frac{\delta}{\delta v(x)}+\frac{1}{\alpha}\xi(x)\partial_x^\mu\frac{\delta}{\delta J^\mu(x)}\right]W[J^\mu,\ldots,v]=0 \quad (4.11)$$

and

$$\xi(x)+(\Box_x+\nu^2)\frac{\delta}{\delta\bar{\xi}(x)}W[J_\mu,\ldots,v]=0. \quad (4.12)$$

C. WT identity obeyed by the generating functional for proper vertex functions

The WT identity in Eq. (4.11) and the ghost equation in Eq. (4.12) may be represented in terms of the generating functional Γ for proper (one-particle-irreducible) vertex functions. The functional Γ is usually defined by the following Legendre transformation [29–31]:

$$\Gamma[A^\mu,\bar{C},C,\varphi,\bar{\psi},\psi;u_\mu,\bar{v},v]=W[J_\mu,\bar{\xi},\xi,J,\bar{\eta},\eta;u_\mu,\bar{v},v]-\int d^4x[J_\mu A^\mu+\bar{\xi}C+\bar{C}\xi+J\varphi+\bar{\eta}\psi+\bar{\psi}\eta], \quad (4.13)$$

where A_μ, $\bar{C}$, C, φ, $\bar{\psi}$, and ψ are field variables defined by the following functional derivatives:

$$A_\mu(x)=\frac{\delta W}{\delta J^\mu(x)},\ \bar{C}(x)=-\frac{\delta W}{\delta\xi(x)},\ C(x)=\frac{\delta W}{\delta\bar{\xi}(x)},$$

$$\bar{\psi}(x)=-\frac{\delta W}{\delta\eta(x)},\ \psi(x)=\frac{\delta W}{\delta\bar{\eta}(x)},\ \varphi(x)=\frac{\delta W}{\delta J(x)}. \quad (4.14)$$

From Eq. (4.13), it is not difficult to get the inverse transformations,

$$J^\mu(x)=-\frac{\delta\Gamma}{\delta A_\mu(x)},\ \bar{\xi}(x)=\frac{\delta\Gamma}{\delta C(x)},\ \xi(x)=-\frac{\delta\Gamma}{\delta\bar{C}(x)},$$

$$\bar{\eta}(x)=\frac{\delta\Gamma}{\delta\psi(x)},\ \eta(x)=-\frac{\delta\Gamma}{\delta\bar{\psi}(x)},\ J(x)=-\frac{\delta\Gamma}{\delta\varphi(x)}. \quad (4.15)$$

It is obvious that

$$\frac{\delta W}{\delta u_\mu}=\frac{\delta\Gamma}{\delta u_\mu},\ \frac{\delta W}{\delta v}=\frac{\delta\Gamma}{\delta v},\ \frac{\delta W}{\delta\bar{v}}=\frac{\delta\Gamma}{\delta\bar{v}}. \quad (4.16)$$

Employing Eqs. (4.15) and (4.16), Eqs. (4.11) and (4.12) will be represented as

$$\int d^4x\left\{\frac{\delta\Gamma}{\delta A_\mu(x)}\frac{\delta\Gamma}{\delta u^\mu(x)}+\frac{\delta\Gamma}{\delta\psi(x)}\frac{\delta\Gamma}{\delta\bar{v}(x)}+\frac{\delta\Gamma}{\delta\bar{\psi}(x)}\frac{\delta\Gamma}{\delta v(x)}+\frac{1}{\alpha}\partial_x^\mu A_\mu(x)\frac{\delta\Gamma}{\delta\bar{C}(x)}\right\}=0 \quad (4.17)$$

and

$$\frac{\delta\Gamma}{\delta\bar{C}(x)}-(\Box_x+\nu^2)C(x)=0. \quad (4.18)$$

When we define a new functional $\hat{\Gamma}$ in such a manner,

$$\hat{\Gamma}=\Gamma+\frac{1}{2\alpha}\int d^4x(\partial^\mu A_\mu)^2, \quad (4.19)$$

it follows that

$$\frac{\delta\Gamma}{\delta A_\mu}=\frac{\delta\hat{\Gamma}}{\delta A_\mu}+\frac{1}{\alpha}\partial^\mu\partial^\nu A_\nu \quad (4.20)$$

and

$$\frac{\delta\Gamma}{\delta\Phi}=\frac{\delta\hat{\Gamma}}{\delta\Phi}, \tag{4.21}$$

where $\Phi=\psi$, $\bar{\psi}$, u^{μ}, v, and $\bar{v}$. Upon inserting Eqs. (4.18)–(4.21) into Eq. (4.17) and noticing $\delta\Gamma/\delta u_{\mu}=\partial^{\mu}C$, we arrive at

$$\int d^4x\left\{\frac{\delta\hat{\Gamma}}{\delta A_{\mu}}\frac{\delta\hat{\Gamma}}{\delta u^{\mu}}+\frac{\delta\hat{\Gamma}}{\delta\psi}\frac{\delta\hat{\Gamma}}{\delta\bar{v}}+\frac{\delta\hat{\Gamma}}{\delta\bar{\psi}}\frac{\delta\hat{\Gamma}}{\delta v}+m^2\partial^{\nu}A_{\nu}C\right\}=0. \tag{4.22}$$

The ghost equation represented through the functional $\hat{\Gamma}$ may be written as

$$\frac{\delta\hat{\Gamma}}{\delta\bar{C}(x)}-\partial_x^{\mu}\frac{\delta\hat{\Gamma}}{\delta u^{\mu}(x)}-\nu^2C(x)=0. \tag{4.23}$$

In the Landau gauge, since $\nu=0$ and $\partial^{\nu}A_{\nu}=0$, Eqs. (4.22) and (4.23) are respectively reduced to

$$\int d^4x\left\{\frac{\delta\hat{\Gamma}}{\delta A_{\mu}}\frac{\delta\hat{\Gamma}}{\delta u^{\mu}}+\frac{\delta\hat{\Gamma}}{\delta\psi}\frac{\delta\hat{\Gamma}}{\delta\bar{v}}+\frac{\delta\hat{\Gamma}}{\delta\bar{\psi}}\frac{\delta\hat{\Gamma}}{\delta v}\right\}=0 \tag{4.24}$$

and

$$\frac{\delta\hat{\Gamma}}{\delta\bar{C}}-\partial^{\mu}\frac{\delta\hat{\Gamma}}{\delta u^{\mu}}=0. \tag{4.25}$$

These equations formally are the same as those for the massless Abelian gauge field theory [29–31].

From the WT identities formulated in this section, we may derive various WT identities obeyed by Green's functions and vertices, as will be illustrated later.

V. WT IDENTITY FOR ω MESON PROPAGATOR AND RENORMALIZATION OF THE PROPAGATOR

The WT identity satisfied by the ω meson propagator can be derived from the identities shown in Eqs. (4.8) and (4.10). By successive differentiations of the identity in Eq. (4.8) with respect to the sources $J^{\nu}(y)$ and $\xi(x)$ and then setting all the sources to be zero, one may obtain

$$\partial_x^{\mu}\frac{\delta^2 Z}{\delta J^{\mu}(x)\,\delta J^{\nu}(y)}\bigg|_{J^{\nu}=\xi=\cdots=v=0}=-\alpha\frac{\delta^2 Z}{\delta\xi(x)\,\delta u^{\nu}(y)}\bigg|_{J^{\nu}=\xi=\cdots=v=0}. \tag{5.1}$$

Noticing the definitions of the ω meson and ghost particle propagators,

$$\begin{aligned}iD_{\mu\nu}(x-y)&=\frac{\delta^2 Z}{i^2\,\delta J^{\mu}(x)\,\delta J^{\nu}(y)}\bigg|_{J^{\nu}=\xi=\cdots=v=0}\\&=\langle 0^+|T\{\mathbf{A}_{\mu}(x)\mathbf{A}_{\nu}(y)\}|0^-\rangle,\end{aligned} \tag{5.2}$$

$$i\,\Delta\,(x-y)=\frac{\delta^2 Z}{\delta\bar{\xi}(x)\,\delta\xi(y)}\bigg|_{J^{\nu}=\xi=\cdots=v=0}=\langle 0^+|T\{\mathbf{C}(x)\bar{\mathbf{C}}(y)\}|0^-\rangle \tag{5.3}$$

(here and afterward the bold letters represent the operators), and interchanging the coordinate variables and Lorentz indices, Eq. (5.1) may be written as

$$\begin{aligned}\partial_y^{\nu}D_{\mu\nu}(x-y)&=i\alpha\langle 0^+|T*\{\Delta\mathbf{A}_{\mu}(x)\bar{\mathbf{C}}(y)\}|0^-\rangle\\&=-\alpha\partial_{\mu}^{x}\,\Delta\,(x-y),\end{aligned} \tag{5.4}$$

where $T*$ symbolizes the covariant time-ordering product and the definition of ΔA_{μ} given in Eq. (4.2) has been considered. Similarly, when taking the derivative of Eq. (4.10) with respect to the source $\xi(y)$ and then letting all the sources vanish, we get

$$(\Box_x+\nu^2)\,\Delta\,(x-y)=\delta^4(x-y). \tag{5.5}$$

This is the equation obeyed by the ghost particle propagator. Differentiating Eq. (5.4) with respect to x and utilizing Eq. (5.5), we find

$$\partial_x^{\mu}\partial_y^{\nu}D_{\mu\nu}(x-y)=-\alpha\Box_x(\Box_x+\nu^2)^{-1}\delta^4(x-y). \tag{5.6}$$

This just is the WT identity satisfied by the full ω meson propagator.

By the Fourier transformation

$$D_{\mu\nu}(x-y)=\int d^4xD_{\mu\nu}(k)e^{-ik(x-y)}, \tag{5.7}$$

Eq. (5.6) becomes

$$k^{\mu}k^{\nu}D_{\mu\nu}(k)=-\frac{\alpha k^2}{k^2-\nu^2}. \tag{5.8}$$

The propagator $D_{\mu\nu}(k)$ may be decomposed into a transverse part and a longitudinal part:

$$D_{\mu\nu}(k)=D_T(k^2)\left(g_{\mu\nu}-\frac{k_{\mu}k_{\nu}}{k^2}\right)+D_L(k^2)\frac{k_{\mu}k_{\nu}}{k^2}. \tag{5.9}$$

Substitution of Eq. (5.9) into Eq. (5.8) gives rise to

$$D_L(k^2)=-\frac{\alpha}{k^2-\nu^2}. \tag{5.10}$$

In comparison of the above expressions with the free propagator which was given in the indefinite-metric approach previously [29] and may easily be derived from the generating functional in Eq. (3.43) by the perturbation method [32],

$$D_{\mu\nu}^{(0)}(k)=-\left\{\frac{g_{\mu\nu}-k_{\mu}k_{\nu}/k^2}{k^2-m_{\omega}^2+i\epsilon}+\frac{\alpha k_{\mu}k_{\nu}/k^2}{k^2-\nu^2+i\epsilon}\right\}, \tag{5.11}$$

one can see that the longitudinal parts in Eqs. (5.9) and (5.11) are the same, implying that the longitudinal part of the ω meson propagator does not undergo renormalization.

To derive the expression of the function $D_T(k^2)$, it is convenient to start from the Dyson equation satisfied by the full ω meson propagator [29–31,36],

$$D_{\mu\nu}(k) = D^{(0)}_{\mu\nu}(k) + D^{(0)}_{\mu\lambda}(k)\Pi^{\lambda\tau}(k)D_{\tau\nu}(k), \qquad (5.12)$$

where $\Pi^{\lambda\tau}(k)$ stands for the vacuum polarization operator, or say, the self-energy operator of the ω meson. Contraction of Eq. (5.12) with k^μ and use of the expressions in Eqs. (5.9)–(5.11) yield [1]

$$k_\lambda \Pi^{\lambda\tau}(k) = 0. \qquad (5.13)$$

This is the WT identity obeyed by the vacuum polarization operator which is a consequence of the gauge symmetry of the theory. The above identity indicates that the operator $\Pi^{\mu\nu}(k)$ is transverse and therefore can be written in the form

$$\Pi^{\mu\nu}(k) = (k^2 g_{\mu\nu} - k_\mu k_\nu)\Pi(k^2), \qquad (5.14)$$

where $\Pi(k^2)$ is a scalar function characterizing the vacuum polarization. With the above representation, it is easy to find from Eq. (5.12) that

$$D_T(k^2) = -\frac{1}{k^2[1+\Pi(k^2)] - m_\omega^2 + i\epsilon}. \qquad (5.15)$$

Thus the full propagator in Eq. (5.9) can be written as

$$D_{\mu\nu}(k) = -\left\{\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2[1+\Pi(k^2)] - m_\omega^2 + i\epsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2 - \nu^2 + i\epsilon}\right\}. \qquad (5.16)$$

When the gauge parameter α is taken to be 0 and 1, we obtain the propagators given in the Landau gauge and in the Feynman gauge, respectively. When the α tends to infinity, we have the propagator given in the so-called unitary gauge. In the lowest order perturbative approximation, the latter propagator is of the form [1–5,29–31]

$$D^{(0)}_{\mu\nu}(k) = -\frac{g_{\mu\nu} - k_\mu k_\nu/m_\omega^2}{k^2 - m_\omega^2 + i\epsilon}. \qquad (5.17)$$

This propagator was originally derived in the canonical quantization from the vacuum expectation value of the time-ordered product of the transverse field operators, $iD^{(0)}_{\mu\nu}(x-y) = \langle 0|T\{\mathbf{A}_{T\mu}(x)\mathbf{A}_{T\nu}(y)\}|0\rangle$, and by making use of the Fourier representation of the transverse field operator $\mathbf{A}_{T\mu}(x)$ in which the ω meson momentum k is put on the mass shell, $k^2 = m_\omega^2$, so that the propagator in Eq. (5.17) is transverse only for this momentum [29,36]. However, due to the on-shell property of the momentum in Eq. (5.17), when evaluating the contraction $k^\mu D^{(0)}_{\mu\nu}(k)$, as we see, there appears an indefinite result since the numerator and the denominator in Eq. (5.17) all come to zero. Especially in the zero-mass limit, there is a serious contradiction that the vector field part of the Lagrangian in Eq. (2.1) is converted to the massless one, but the propagator in Eq. (5.17) does not and is of an awful singularity. In contrast, for the propagator in Eq. (5.11), the momentum is off shell, $k^2 \neq m_\omega^2$. Therefore, for the transverse part of the propagator (or say, the propagator given in the Laudau gauge), we have $k^\mu D^T_{\mu\nu}(k) = 0$, showing a definite result. Moreover, in the calculation of a loop diagram involving internal ω meson lines in which the momentum of the ω meson line is off shell, it is necessary to use the propagator in Eq. (5.11). In particular, the good ultraviolet property of the propagator allows us to perform the renormalization safely (in spite of whether the current conservation holds or not). In the zero-mass limit, the propagator in Eq. (5.11) and the vector field part of the Lagrangian in Eq. (3.44) simultaneously go over to the massless ones, exhibiting the logical consistency of the theory.

Now let us discuss renormalization of the ω meson propagator. According to the conventional procedure of renormalization, the divergence included in the functions $\Pi(k^2)$ may be subtracted at a renormalization point, say, $k^2 = \mu^2$ where μ may be real or imaginary, corresponding to the subtraction point being timelike or spacelike,

$$\Pi(k^2) = \Pi(\mu^2) + \Pi_c(k^2), \qquad (5.18)$$

where $\Pi(\mu^2)$ and $\Pi_c(k^2)$ are, respectively, the divergent part and the finite part of the functions $\Pi(k^2)$. The divergent part can be absorbed in the renormalization constant Z_3 which is defined as

$$Z_3^{-1} = 1 + \Pi(\mu^2). \qquad (5.19)$$

With this definition, on inserting Eq. (5.18) into Eq. (5.16), the ω meson propagator will be renormalized as

$$D_{\mu\nu}(k) = Z_3 D_{R\mu\nu}(k), \qquad (5.20)$$

where

$$D_{R\mu\nu}(k) = -\left\{\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - (m_\omega^R)^2 + \Pi_R(k^2) + i\varepsilon} + \frac{\alpha_R k_\mu k_\nu/k^2}{(k^2 - \nu^2 + i\varepsilon)}\right\} \qquad (5.21)$$

is the renormalized propagator in which m_ω^R is the renormalized mass, α_R the renormalized gauge parameter and $\Pi_R(k^2)$ denotes the finite correction coming from the loop diagrams. They are defined as

$$m_\omega^R = \sqrt{Z_3} m_\omega, \alpha_R = Z_3^{-1}\alpha, \Pi_R(k^2) = Z_3 k^2 \Pi_c(k^2). \qquad (5.22)$$

It is noted that the spurious mass ν is a renomalization-invariant quantity, $\nu^2 = \alpha m_\omega^2 = \alpha_R m_{\omega R}^2 = \nu_R^2$. Especially, at the renormalization point, $\Pi_R(\mu^2) = 0$, as seen from Eqs. (5.18) and (5.22). In this case, we have a renornalization boundary condition such that

$$D_{R\mu\nu}(k)|_{k^2=\mu^2} = -\left\{\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - (m_\omega^R)^2 + i\epsilon} + \frac{\alpha_R k_\mu k_\nu/k^2}{k^2 - \nu^2 + i\epsilon}\right\} \qquad (5.23)$$

which is of the form of free propagator except that the parameters are replaced by the renormalized ones.

VI. WT IDENTITY FOR THE VECTORIAL VERTEX FUNCTION AND RENORMALIZATION OF THE VERTEX AND THE NUCLEON PROPAGATOR

A. WT identity for the vectorial vertex function

The WT identity for the vectorial vertex (nucleon–nucleon-ω meson vertex) can be derived by differentiating the identity in Eq. (4.8) with respect to the sources $\xi(x)$, $\overline{\eta}(y)$, and $\eta(z)$ and then turning off all the sources. The result derived, written in the operator form, is

$$\frac{1}{\alpha g_\omega}\partial^\mu\langle 0^+|T\{\mathbf{A}_\mu(x)\psi(y)\overline{\psi}(z)\}|0^-\rangle$$
$$= i\langle 0^+|T\{\overline{\mathbf{C}}(x)\psi(y)\mathbf{C}(z)\overline{\psi}(z)\}|0^-\rangle$$
$$+ i\langle 0^+|T\{\overline{\mathbf{C}}(x)\mathbf{C}(y)\psi(y)\overline{\psi}(z)\}|0^-\rangle, \tag{6.1}$$

where the definitions written in Eq. (4.2) have been used. Similarly, by differentiating the ghost equation in Eq. (4.10) with respect to the sources $\xi(y)$, $\overline{\eta}(y)$, and $\eta(z)$ and then letting the sources vanishing, one may derive the following equation:

$$\delta^4(x-y)\langle 0^+|T\{\psi(y)\overline{\psi}(z)\}|0^-\rangle$$
$$= -i(\Box_x+\nu^2)\langle 0^+|T\{\mathbf{C}(x)\overline{\mathbf{C}}(y)\psi(\mathbf{y})\overline{\psi}(z)\}|0^-\rangle. \tag{6.2}$$

Here it is noted that since there is no coupling between the ghost field and the fermion field, the two fields cannot construct a connected Green's function. Therefore we can write

$$\langle 0^+|T\{\mathbf{C}(x)\overline{\mathbf{C}}(y)\psi(y)\overline{\psi}(z)\}|0^-\rangle$$
$$= \langle 0^+|T\{\mathbf{C}(x)\overline{\mathbf{C}}(y)\}|0^-\rangle\langle 0^+|T\{\psi(y)\overline{\psi}(z)\}|0^-\rangle$$
$$= -\Delta(x-y)S_F(y-z), \tag{6.3}$$

where

$$\langle 0^+|T\{\psi(x)\overline{\psi}(y)\}|0^-\rangle = iS_F(x-y) \tag{6.4}$$

is the nucleon propagator. It is easy to verify that once Eq. (6.3) is substituted into the right hand side of Eq. (6.2) and applying Eq. (5.5), we just obtain the expression on the left hand side of Eq. (6.2). Acting on both sides of Eq. (6.1) with the operator $\Box_x+\nu^2$ and employing the decomposition in Eq. (6.3) and the ghost equation in Eq. (5.5), we obtain the following WT identity:

$$\frac{1}{\alpha g_v}(\Box_x+\nu^2)\partial_x^\mu G_\mu(x,y,z) = i[\delta^4(x-y)-\delta^4(x-z)]S_F(y-z), \tag{6.5}$$

where

$$G_\mu(x,y,z) = \langle 0^+|T\{\mathbf{A}_\mu(x)\psi(y)\overline{\psi}(z)\}|0^-\rangle \tag{6.6}$$

is the three-point Green's function which is connected. This Green's function has the following one-particle irreducible decomposition [29–31]:

$$G_\mu(x,y,z) = \int d^4x'd^4y'd^4z' iD_{\mu\nu}(x-x')iS_F(y-y')$$
$$\times i\Gamma^\nu(x',y',z')iS_F(z'-z) \tag{6.7}$$

in which $\Gamma^\nu(x',y',z')$ is the vectorial proper vertex. On inserting Eq. (6.7) into Eq. (6.5), through the Fourier transformation, we get in the momentum space that

$$\frac{1}{\alpha g_v}(k^2-\nu^2)k^\mu D_{\mu\nu}(k)S_F(p)\Gamma^\nu(p,q,k)S_F(q)$$
$$= (2\pi)^4\delta^4(k+p-q)[S_F(q)-S_F(p)]. \tag{6.8}$$

Considering that the energy-momentum conservation holds at the vertex, we can write

$$\Gamma_\mu(p,q,k) = (2\pi)^4\delta^4(k+p-q)ig_v[\gamma_\mu+\Lambda_\mu(p,q)]. \tag{6.9}$$

With this representation and noticing

$$(k^2-\nu^2)k^\mu D_{\mu\nu}(k) = -\alpha k_\nu = \alpha(p-q)_\nu, \tag{6.10}$$

one may obtain from Eq. (6.8) that

$$(p-q)^\mu[\gamma_\mu+\Lambda_\mu(p,q)] = S_F^{-1}(p) - S_F^{-1}(q). \tag{6.11}$$

It is well known that the general expression of the nucleon propagator $S_F(p)$ can be found from the Dyson equation satisfied by the propagator, as was similarly done in Eqs. (5.12)–(5.16) for the ω meson propagator. The inverse of the propagator can be written as

$$S_F^{-1}(p) = \not{p} - M - \Sigma(p), \tag{6.12}$$

where $\not{p}=\gamma^\mu p_\mu$ and $\Sigma(p)$ is the nucleon self-energy. Noticing this expression, when we differentiate both sides of Eq. (6.11) with respect to p^μ and then set $q=p$, it is found that [1]

$$\Lambda_\mu(p,p) = -\frac{\partial\Sigma(p)}{\partial p^\mu}. \tag{6.13}$$

This is the WT identity which establishes the relation between the vectorial proper vertex and the nucleon self-energy.

It is interesting to note that the above identity determines the subtraction fashion of the nucleon self-energy. As one knows, the divergence in the vertex $\Lambda_\mu(p,q)$ may be subtracted at the renormalization point μ in such a way,

$$\Lambda_\mu(p,p) = L\gamma_\mu + \Lambda_\mu^c(p), \tag{6.14}$$

where

$$L = \Lambda_\mu(p,p)|_{\not{p}=\mu} \tag{6.15}$$

is a divergent constant. Substituting Eq. (6.14) into Eq. (6.13) and then integrating Eq. (6.13) over p_μ from p_μ^0 to p_μ, we get

$$\Sigma(p) = \Sigma(\mu) - L(\not{p}-\mu) + (\not{p}-\mu)C(p^2), \tag{6.16}$$

where we have chosen the p_μ^0 to meet $\gamma^\mu p_\mu^0=\mu$ and set the integral $\int_{p^0}^p dp^\mu\Lambda_\mu^c(p^2)=(\not{p}-\mu)C(p^2)$ with the consideration that when $p=p^0$, the integral vanishes and $\Lambda_\mu^c(p)$ is finite, satisfying the boundary condition $\Lambda_\mu^c(p)|_{\not{p}=\mu}=0$ so that the $C(p^2)$ is also finite, having the boundary condition $C(\mu^2)=0$. When the divergent constants $\Sigma(\mu)$ and L are set to be

$$\Sigma(\mu) = A \tag{6.17}$$

and

$$L = -B, \tag{6.18}$$

Eq. (6.16) will be written in the form

$$\Sigma(p) = A + (\not{p} - \mu)[B - C(p^2)]. \tag{6.19}$$

This is the formula that gives the uniquely correct way for the subtraction of the nucleon self-energy.

B. Renormalization of the nucleon propagator and the vectorial vertex

Based on the representation of the self-energy in Eq. (6.19), the full nucleon propagator may be written as

$$S_F(p) = \frac{1}{\not{p}(1-B) - (M + A - \mu B) + (\not{p} - \mu)C(p^2)}. \tag{6.20}$$

With the renormalization constant of the nucleon propagator defined by

$$Z_2^{-1} = 1 - B, \tag{6.21}$$

the nucleon propagator will be renormalized as

$$S_F(p) = Z_2 S_F^R(p), \tag{6.22}$$

where

$$S_F^R(p) = \frac{1}{\not{p} - M_R - \Sigma_R(p) + i\epsilon} \tag{6.23}$$

is the renormalized propagator in which M_R and $\Sigma_R(p)$ are the renormalized mass and the finite renormalization correction, respectively. They are separately represented in the following:

$$M_R = Z_M^{-1} M, \tag{6.24}$$

where Z_M is the nucleon mass renormalization constant defined by

$$Z_M^{-1} = 1 + Z_2\left[\frac{A}{M} + \left(1 - \frac{\mu}{M}\right)B\right] \tag{6.25}$$

and

$$\Sigma_R(p) = -Z_2(\not{p} - \mu)C(p^2) \tag{6.26}$$

with the boundary condition $\Sigma_R(p)|_{\not{p}=\mu}=0$ which leads to the boundary condition of nucleon propagator like this:

$$S_F^R(p)|_{\not{p}=\mu} = \frac{1}{\not{p} - M_R + i\epsilon}. \tag{6.27}$$

Clearly, this propagator is formally the same as the free propagator.

We would like to mention here the renormalization of the vertex function defined by $\hat{\Gamma}_\mu(p,q) = \gamma_\mu + \Lambda_\mu(p,q)$. In view of the subtraction in Eq. (6.14) and the following definition of vertex renormalization constant Z_1:

$$Z_1^{-1} = 1 + L, \tag{6.28}$$

the vertex function will be renormalized as

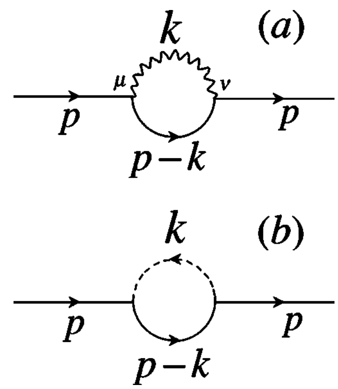

FIG. 1. The one-loop nucleon self-energy in the σ-ω model. The solid, wavy, and dashed lines represent the free nucleon, ω meson, and σ meson propagators, respectively.

$$\hat{\Gamma}_\mu(p,q) = Z_1^{-1}\hat{\Gamma}_\mu^R(p,q) = Z_1^{-1}[\gamma_\mu + \Lambda_\mu^R(p,q)], \tag{6.29}$$

where

$$\Lambda_\mu^R(p,q) = Z_1 \Lambda_\mu^c(p,q) \tag{6.30}$$

is the finite renormalization correction to the vertex. From the boundary condition $\Lambda_\mu^c(p)|_{\not{p}=\mu}=0$ mentioned before, it follows that $\Lambda_\mu^R(p,q)|_{\not{p}=\not{q}=\mu}=0$ by which we have

$$\hat{\Gamma}_\mu^R(p,q)|_{\not{p}=\not{q}=\mu} = \gamma_\mu. \tag{6.31}$$

This just is the boundary condition for the renormalized vertex function $\hat{\Gamma}_\mu^R(p,q)$ under which the vertex is of the form of the bare vertex. In particular, from Eqs. (6.18), (6.21), and (6.28), it is clear to see that

$$Z_2 = Z_1. \tag{6.32}$$

This is the Ward identity satisfied by the nucleon propagator renormalization constant and the vertex one.

At last, it is pointed out that the identities shown in Eqs. (6.13) and (6.32) and the subtraction represented in Eq. (6.19) formally are the same as those in QED because they are all the consequence of U(1) gauge symmetry. Originally, the identities mentioned above follow from the current conservation. This result is natural because the current conservation, as generally proved in the gauge field theory [29–31], can be derived from the global gauge symmetry or the local gauge symmetry. It would be emphasized that the aforementioned identities hold not only for the case where the ω meson is considered only, but also for the general case that the ω meson and the σ meson are taken into account together. We take one-loop diagrams to illustrate this point. The one-loop nucleon self-energy in the σ-ω model is represented in Figs. 1(a) and (b). The one-loop vectorial vertex is shown in Figs. 2(a) and (b). In the figures, the solid line designates the free nucleon propagator $iS_F^{(0)}(p)$ represented in Eq. (6.27), the wavy line stands for the free ω meson propagator $iD_{\mu\nu}^{(0)}(k)$ written in Eq. (5.11), the dashed line denotes the free σ meson propagator which is of the form

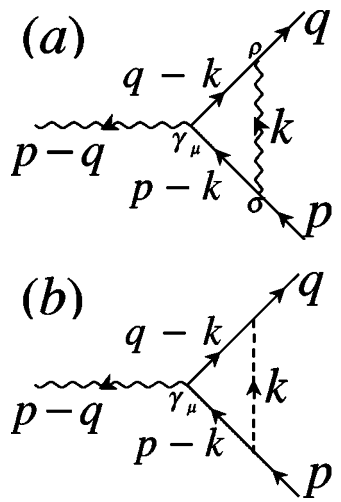

FIG. 2. The one-loop vectorial vertices in the σ-ω model. The lines represent the same as in Fig. 1.

$$i\Delta(q) = \frac{i}{q^2 - m_\sigma^2 + i\varepsilon}, \tag{6.33}$$

the bare vectorial vertex $\Gamma_\mu^{(0)}$ and the bare scalar vertex $\Gamma^{(0)}$ are

$$\Gamma_\mu^{(0)} = ig_v\gamma_\mu,$$

$$\Gamma^{(0)} = ig_s. \tag{6.34}$$

The above Feynman rules are easily derived from the generating functional in Eq. (3.43) by the perturbation method. Applying the Feynman rules, for the one-loop nucleon self-energy defined by $-i\Sigma(p)$ we have the following expression:

$$\Sigma(p) = i\int d^4k[g_v^2\gamma^\mu S_F^{(0)}(p-k)\gamma^\nu D_{\mu\nu}^{(0)}(k) + g_s^2 S_F^{(0)}(p-k)\Delta^{(0)}(k)], \tag{6.35}$$

where the first term and the second one are respectively given by Figs. 1(a) and (b) while for the one-loop vectorial vertex, in accordance with the definition in Eq. (6.9), we have

$$\Lambda_\mu(p,q) = i\int d^4k[g_v^2\gamma^\nu S_F^{(0)}(q-k)\gamma_\mu S_F^{(0)}(p-k)\gamma^\lambda D_{\nu\lambda}^{(0)}(k) + g_s^2 S_F^{(0)}(q-k)\gamma_\mu S_F^{(0)}(p-k)\Delta^{(0)}(k)], \tag{6.36}$$

where the first and second terms are given by Figs. 2(a) and (b), respectively. By making use of the derivative

$$\frac{\partial}{\partial p^\mu}S_F(p-k) = -S_F(p-k)\gamma_\mu S_F(p-k), \tag{6.37}$$

it is easy to find that the identity in Eq. (6.13) holds. Thus the correctness of the WT identity in Eq. (6.13) which follows from the U(1) gauge symmetry of the model is verified by the perturbative calculation. The identities in Eqs. (6.13) and (6.32) will be helpful to facilitate calculations of the renomalization of the σ-ω model.

VII. RENORMALIZATION OF THE σ MESON PROPAGATOR AND THE SCALAR VERTEX

For later convenience, it is necessary to give a general description for the renormalization of the σ meson propagator and the scalar vertex. We start from the Dyson equation satisfied by the σ meson full propagator $i\Delta(q)$,

$$\Delta(q) = \Delta^{(0)}(q) + \Delta^{(0)}(q)\Omega(q)\Delta(q), \tag{7.1}$$

where $\Delta^{(0)}(q)$ is the σ meson free propagator shown in Eq. (6.33) and $-i\Omega(q)$ represents the σ meson self-energy. From Eq. (7.1) it may be solved that

$$\Delta(q) = \frac{1}{q^2 - m_\sigma^2 - \Omega(q) + i\varepsilon}. \tag{7.2}$$

The self-energy can be Lorentz-covariantly decomposed into

$$\Omega(q) = \Omega_1(q^2)q^2 + \Omega_2(q^2)m_\sigma^2. \tag{7.3}$$

The divergence in the $\Omega(q)$ can be subtracted at the renormalization point μ in such a way:

$$\Omega_1(q^2) = \Omega_1(\mu^2) + \Omega_1^c(q^2),$$

$$\Omega_2(q^2) = \Omega_2(\mu^2) + \Omega_2^c(q^2). \tag{7.4}$$

On substituting Eq. (7.4) in Eq. (7.2), the propagator $\Delta(q)$ will be renormalized as

$$\Delta(q) = Z_3'\Delta_R(q), \tag{7.5}$$

where

$$Z_3'^{-1} = 1 - \Omega_1(\mu^2) \tag{7.6}$$

is the renormalization constant of the σ meson propagator,

$$\Delta_R(q) = \frac{1}{q^2 - (m_\sigma^R)^2 - \Omega_R(q) + i\varepsilon} \tag{7.7}$$

is the renormalized propagator in which

$$m_\sigma^R = Z_m^{\sigma-1}m_\sigma \tag{7.8}$$

is the renormalized σ meson mass with

$$Z_m^\sigma = \{Z_3'[1 + \Omega_2(\mu^2)]\}^{-1/2} \tag{7.9}$$

being the renormalization constant of the σ meson mass and

$$\Omega_R(q) = Z_3'[q^2\Omega_1^c(\mu^2) + m_\sigma^2\Omega_2^c(q^2)] \tag{7.10}$$

is the finite correction to the renormalized propagator. Obviously, the $\Omega_R(q)$ has the boundary condition $\Omega_R(q)|_{q^2=\mu^2} = 0$ which leads to the boundary condition of the propagator as follows:

$$\Delta_R(q)|_{q^2=\mu^2} = \frac{1}{q^2 - (m_\sigma^R)^2 + i\varepsilon}. \tag{7.11}$$

This propagator formally is the same as the free propagator in Eq. (6.33).

Analogous to Eq. (6.9) for the vectorial vertex, the scalar vertex can be written as

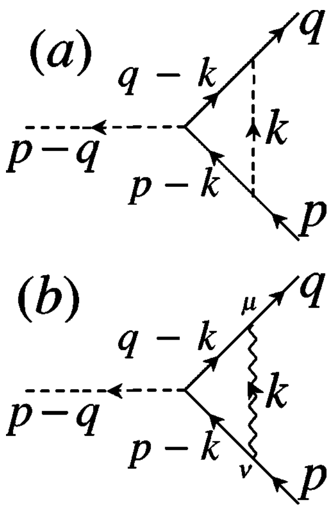

FIG. 3. The one-loop scalar vertices in the σ-ω model. The lines mark the same as in Fig. 1.

$$\Gamma(p,q,k) = (2\pi)^4\delta^4(k+p-q)ig_s\hat{\Gamma}(p,q), \qquad (7.12)$$

where

$$\hat{\Gamma}(p,q) = 1 + \Lambda(p,q) \qquad (7.13)$$

in which $\Lambda(p,q)$ denotes the contribution of all higher order diagrams. When the divergence in the $\Lambda(p,q)$ is subtracted at the renormalization point μ, we have

$$\Lambda(p,q) = L' + \Lambda_c(p,q), \qquad (7.14)$$

where

$$L' = \Lambda(p,p)|_{\not p=\mu} \qquad (7.15)$$

is the divergent constant and $\Lambda_c(p,q)$ is the finite correction of the $\Lambda(p,q)$. With the above subtraction, the vertex in Eq. (7.13) will be renormalized as

$$\hat{\Gamma}(p,q) = Z_1'^{-1}\hat{\Gamma}_R(p,q) = Z_1'^{-1}[1+\Lambda_R(p,q)], \qquad (7.16)$$

where Z_1' is the renormalization constant of the scalar vertex defined by

$$Z_1'^{-1} = 1 + L' \qquad (7.17)$$

and $\Lambda_R(p,q)=Z'\Lambda_c(p,q)$ is the finite correction of the $\hat{\Gamma}_R(p,q)$ with the boundary condition $\Lambda_R(p,q)|_{\not p=\not q=\mu}=0$ which yields the boundary condition for the renormalized vertex $\hat{\Gamma}_R(p,q)$ as follows:

$$\hat{\Gamma}_R(p,q)|_{\not p=\not q=\mu} = 1. \qquad (7.18)$$

This shows that at the renormalization point, the renormalized vertex is reduced to the form of bare vertex.

It is interesting to note that there is an identity which holds between the nucleon self-energy and the scalar vertex. For example, from the expression of the one-loop scalar vertex $\Lambda(p,q)$ shown in Figs. 3(a) and (b),

$$\Lambda(p,q) = i\int d^4k[g_s^2 S_F^{(0)}(q-k)\Delta^{(0)}(k)S_F(p-k) + g_v^2\gamma^\mu S_F^{(0)}(q-k)S_F^{(0)}(p-k)\gamma^\nu D_{\mu\nu}^{(0)}(k)] \qquad (7.19)$$

and the following derivative:

$$\frac{\partial}{\partial M}S_F(p-k) = S_F(p-k)S_F(p-k), \qquad (7.20)$$

we find

$$\Lambda(p,p) = \frac{\partial\Sigma(p)}{\partial M}. \qquad (7.21)$$

Based on this identity and the expression in Eq. (6.19), the constant defined in Eq. (7.15) can be computed by

$$L' = \left.\frac{\partial\Sigma(p)}{\partial M}\right|_{\not p=\mu} = \frac{\partial A}{\partial M}. \qquad (7.22)$$

This relation will be used to simplify the calculation of renormalization of the σ-ω model.

VIII. RENORMALIZATION GROUP EQUATION AND RENORMALIZED *S*-MATRIX ELEMENTS

Suppose F_R is a renormalized quantity. In the multiplicative renormalization, it is related to the unrenormalized one F in such a way,

$$F = Z_F F_R, \qquad (8.1)$$

where Z_F is the renormalization constant of F. The Z_F and F_R are all functions of the renormalization point $\mu=\mu_0 e^t$ where μ_0 is a fixed renormalization point corresponding the zero value of the group parameter t. Differentiating Eq. (8.1) with respect to μ and noticing that the F is independent of μ, we immediately obtain a renormalization group equation (RGE) satisfied by the function F_R [27–30]

$$\mu\frac{dF_R}{d\mu} + \gamma_F F_R = 0, \qquad (8.2)$$

where γ_F is the anomalous dimension defined by

$$\gamma_F = \mu\frac{d}{d\mu}\ln Z_F. \qquad (8.3)$$

We first note here that because the renormalization constant is dimensionless, the anomalous dimension can only depend on the ratio $\sigma=m_R/\mu$, $\gamma_F=\gamma_F(g_R,\sigma)$, where m_R and g_R are the renormalized mass and coupling constant, respectively. Next, we note that Eq. (8.2) is suitable for a physical parameter (mass or coupling constant), a propagator, a vertex, a wave function or some other Green function. If the function F_R stands for a renormalized Green function, vertex or wave function, in general, it depends explicitly not only on the scale μ, but also on the renormalized coupling constant g_R, mass m_R and gauge parameter α_R which are all functions of μ, $F_R=F_R(p,g_R(\mu),m_R(\mu),\alpha_R(\mu);\mu)$ where p symbolizes all the momenta. Considering that the function F_R is homogeneous in the momentum and mass, it may be written, under the scaling transformation of momentum $p=\lambda p_0$, as follows:

$$F_R(p;g_R,m_R,\alpha_R;\mu)=\lambda^{D_F}F_R\left(p_0;g_R,\frac{m_R}{\lambda},\alpha_R;\frac{\mu}{\lambda}\right), \quad (8.4)$$

where D_F is the canonical dimension of F. Since the renormalization point is a momentum taken to subtract the divergence, we may set $\mu=\mu_0\lambda$ where $\lambda=e^t$ which is taken to be the same as in $p=p_0\lambda$. Noticing the above transformation, the solution to the RGE in Eq. (8.2) can be expressed as

$$F_R(p;g_R,m_R,\alpha_R,\mu_0) = \lambda^{D_F}e^{\int_1^{\lambda}(d\lambda/\lambda)\gamma_F(\lambda)}F_R(p_0;g_R(\lambda),m_R(\lambda)\lambda^{-1},\alpha_R(\lambda);\mu_0), \quad (8.5)$$

where $g_R(\lambda)$, $m_R(\lambda)$, and $\alpha_R(\lambda)$ are the effective (running) coupling constant, mass, and gauge parameter, respectively. The solution written above describes the behavior of the function F_R under the scaling of momenta.

How do we determine the function $F_R(p_0;\ldots,\mu_0)$ on the right-hand side of Eq. (8.5) when the $F_R(p_0,\ldots)$ stands for a wave function, a propagator, or a vertex? This question can be unambiguously answered in the momentum space subtraction scheme. Noticing that the momentum p_0 and the renormalization point μ_0 are fixed, but may be chosen arbitrarily, we can, certainly, set $\mu_0^2=p_0^2$. With this choice, by making use of the boundary condition satisfied by the propagator, the vertex, or the wave function as denoted in Eqs. (5.23), (6.27), (6.31), (7.11), and (7.18), we may write

$$F_R(p_0;g_R,m_R,\alpha_R,\mu)|_{p_0^2=\mu^2}=F_R^{(0)}(p_0;g_R,m_R,\alpha_R), \quad (8.6)$$

where the function $F_R^{(0)}(p;g_R,m_R,\alpha_R)$ is of the form of a free propagator, a bare vertex (if the vertex is fundamental, i.e., follows directly from the interaction Lagrangian) or a free wave function and independent of the renormalization point (see the examples given in the preceding sections). In light of the boundary condition in Eq. (8.6) and considering the homogeneity of the function F_R as mentioned in Eq. (8.4), one can write

$$\lambda^{D_F}F_R(p_0;g_R(\lambda),m_R(\lambda)\lambda^{-1},\alpha_R(\lambda),\mu_0)|_{p_0^2=\mu_0^2} = F_R^{(0)}(p;g_R(\lambda),m_R(\lambda),\alpha_R(\lambda)), \quad (8.7)$$

where the renormalized coupling constant, mass, and gauge parameter in the function $F_R^{(0)}(p,\ldots)$ become the running ones. With the expression given in Eq. (8.7), Eq. (8.5) will finally be written in the form [20]

$$F_R(p;g_R,m_R,\alpha_R)=e^{\int_1^{\lambda}(d\lambda/\lambda)\gamma_F(\lambda)}F_R^{(0)}(p;g_R(\lambda),m_R(\lambda),\alpha_R(\lambda)). \quad (8.8)$$

For a gauge field theory, the anomalous dimensions shown in Eq. (8.8) are all canceled out in S-matrix elements. To show this point more specifically, let us take the two-nucleon scattering taking place via ω meson exchanges as an example. The exact matrix element for the two-nucleon scattering can be written out from the well-known reduction formula which establishes the relation between an on-mass-shell S-matrix element and the corresponding off-mass-shell connected Green's function [29,36]. A connected Green's function may conveniently be derived from the generating functional W for connected Green's functions as mentioned in Sec. IV. For the nucleon-nucleon scattering, the S-matrix element is related to the following four-point connected Green's function,

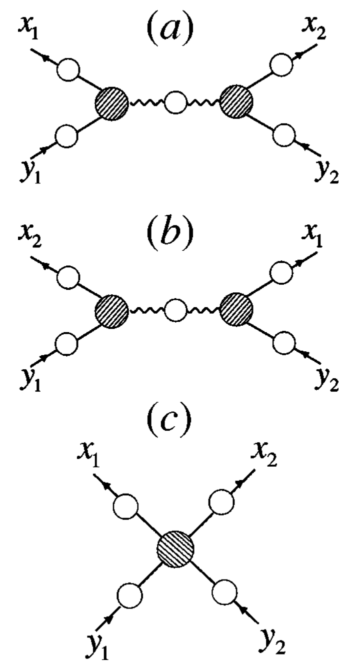

FIG. 4. The diagrams represent the nucleon four-point one-particle irreducible Green's function. The solid line with a white blob represents the full nucleon propagator. The wavy line with a white blob denotes the full ω meson propagator; the shaded blobs represent the proper vertices.

$$G_c(x_1,x_2;y_1,y_2)=\langle 0^+|T[\psi(x_1)\psi(x_2)\bar{\psi}(y_1)\bar{\psi}(y_2)]|0^-\rangle_C, \quad (8.9)$$

where the subscript C marks the connectivity of the Green's function. According to the familiar procedure of irreducible decomposition [29–31], the connected Green's function in Eq. (8.9) can be decomposed into three one-particle irreducible ones as represented graphically in Figs. 4(a)–(c). In each of the diagrams, there are four external legs which represent the full off-mass-shell nucleon propagators. These propagators will be converted to the full on-mass-shell nucleon wave functions by the reduction formula. The shaded blobs in the diagrams stand for the exact proper (one-particle irreducible) vertices. Let us first concentrate our attention on the diagrams in Figs. 4(a) and (b). These two diagrams represent the two-nucleon scattering taking place in the t channel via a ω meson exchange. The wavy line with a white blob in the figures denotes the full ω meson propagator. Considering the well-known fact that a S-matrix element expressed in terms of unrenormalized quantities is equal to that represented by the corresponding renormalized quantities, the scattering amplitude given by Figs. 4(a) and (b) may be written as [20,29,36]

$$T_{fi}^{(1)} = \overline{u}_R^{\sigma}(q_1)\Gamma_R^{\mu}(q_1,p_1)u_R^{\alpha}(p_1)iD_{\mu\nu}^R(k)\overline{u}_R^{\rho}(q_2)\Gamma_R^{\nu}(q_2,p_2)u_R^{\beta}(p_2) - \overline{u}_R^{\rho}(q_2)\Gamma_R^{\mu}(q_2,p_1)u_R^{\alpha}(p_1)iD_{\mu\nu}^R(k)\overline{u}_R^{\sigma}(q_1)\Gamma_R^{\nu}(q_1,p_2)u_R^{\beta}(p_2), \tag{8.10}$$

where $k=q_1-p_1=p_2-q_2$, $u_R^{\alpha}(p)$, $\Gamma_R^{\mu}(q_i,p_i)$, and $iD_{\mu\nu}^R(k)$ are the nucleon wave function, the proper vectorial vertex, and the ω meson propagator, respectively, which are all renormalized. The renormalization constants of the wave function, the propagator, and the vertex are denoted by $\sqrt{Z_2}$, Z_3, and Z_Γ, respectively. The constant Z_Γ is defined by

$$Z_\Gamma = Z_2^{-1}Z_3^{-1/2} \tag{8.11}$$

because the vertex in Eq. (8.10) is now defined by containing a vectorial coupling constant g_v^R multiplied with an imaginary number i in it. The renormalized coupling constant is defined as

$$g_v^R = \frac{Z_2\sqrt{Z_3}}{Z_1}g_v. \tag{8.12}$$

On the basis of the formula given in Eq. (8.8), the renormalized nucleon wave function, meson propagator, and vertex can be represented in the forms as shown separately in the following. For the nucleon wave function, we have

$$u_R^{\alpha}(p) = e^{\int_1^{\lambda}(d\lambda/\lambda)\gamma_w(\lambda)}u_{R\alpha}^{(0)}(p), \tag{8.13}$$

where

$$u_{R\alpha}^{(0)}(p) = \left(\frac{E+M_R(\lambda)}{2M_R(\lambda)}\right)^{1/2}\left(\frac{\vec{\sigma}\cdot\vec{p}}{E+M_R(\lambda)}\right)\varphi_\alpha(\vec{p}) \tag{8.14}$$

is the renormalized wave function which formally is the same as the free wave function, but the $M_R(\lambda)$ in it is a running mass and

$$\gamma_w = \frac{1}{2}\mu\frac{d}{d\mu}\ln Z_2 \tag{8.15}$$

is the anomalous dimension of the nucleon wave function.

For the renormalized ω meson propagator, we can write

$$iD_{\mu\nu}^R(k) = e^{\int_1^{\lambda}(d\lambda/\lambda)\gamma_3(\lambda)}iD_{\mu\nu}^{R(0)}(k), \tag{8.16}$$

where

$$iD_{\mu\nu}^{R(0)}(k) = -\frac{i}{k^2-m_\omega^R(\lambda)+i\varepsilon}\left[g_{\mu\nu}-[1-\alpha_R(\lambda)]\frac{k_\mu k_\nu}{k^2-\nu^2+i\varepsilon}\right] \tag{8.17}$$

is the free propagator with $m_\omega^R(\lambda)$ and $\alpha_R(\lambda)$ in it being the running ω meson mass and gauge parameter and

$$\gamma_3(\lambda) = \mu\frac{d}{d\mu}\ln Z_3 \tag{8.18}$$

is the anomalous dimension of the propagator.

For the renormalized vertex, it reads

$$\Gamma_R^{\mu}(q_i,p_j) = e^{\int_1^{\lambda}(d\lambda/\lambda)\gamma_v(\lambda)}\Gamma_R^{(0)\mu}(q_i,p_j), \tag{8.19}$$

where i, $j=1$, 2,

$$\Gamma_R^{(0)\mu}(q_i,p_j) = ig_v^R(\lambda)\gamma^\mu \tag{8.20}$$

is the bare vertex in which $g_v^R(\lambda)$ is the running coupling constant, and

$$\gamma_v(\lambda) = \mu\frac{d}{d\mu}\ln Z_v = -\mu\frac{d}{d\mu}\ln Z_2 - \frac{1}{2}\mu\frac{d}{d\mu}\ln Z_3 \tag{8.21}$$

is the anomalous dimension of the vertex here the relation in Eq. (8.11) has been used.

Upon substituting Eqs. (8.13), (8.16), and (8.19) into Eq. (8.10) and noticing Eqs. (8.15), (8.18), and (8.21), we find that the anomalous dimensions in the S-matrix element are all canceled out with each other. As a result of the cancellation, we arrive at

$$T_{fi}^{(1)} = \overline{u}_{R\sigma}^{(0)}(q_1)\Gamma_R^{(0)\mu}(q_1,p_1)u_{R\alpha}^{(0)}(p_1)iD_{\mu\nu}^{R(0)}(k)\overline{u}_{R\rho}^{(0)} \times(q_2)\Gamma_R^{(0)\nu}(q_2,p_2)u_{R\beta}^{(0)}(p_2) - \overline{u}_{R\rho}^{(0)}(q_2)\Gamma_R^{(0)\mu}(q_2,p_1)u_{R\alpha}^{(0)} \times(p_1)iD_{\mu\nu}^{R(0)}(k)\overline{u}_{R\sigma}^{(0)}(q_1)\Gamma_R^{(0)\nu}(q_1,p_2)u_{R\beta}^{(0)}(p_2). \tag{8.22}$$

This expression clearly shows that the exact t-channel S-matrix element of the two-nucleon scattering can be represented in the form as given by the tree diagrams shown in Figs. 5(a) and (b) provided that all the physical parameters in the matrix elements are replaced by their effective (running) ones.

Next, let us turn to the diagram in Fig. 4(c). In the diagram, the shaded blob with four amputated external legs represents the nucleon four-line proper vertex. The direct term of the scattering amplitude given by Fig. 4(c) can be represented in terms of the renormalized quantities as follows:

$$T_{fi}^{(2)} = \overline{u}_R^{\sigma}(q_1)\overline{u}_R^{\rho}(q_2)\Gamma_R(p_1,p_2;q_1,q_2)u_R^{\alpha}(p_1)u_R^{\beta}(p_2). \tag{8.23}$$

In accordance with Eq. (8.8), the renormalized vertex $\Gamma_R(p_1,p_2;q_1,q_2)$ is of the form

$$\Gamma_R(p_1,p_2;q_1,q_2) = e^{\int_1^{\lambda}(d\lambda/\lambda)\gamma_\Gamma(\lambda)}\Gamma_R^{(0)}(p_1,p_2;q_1,q_2), \tag{8.24}$$

where $\gamma_\Gamma(\lambda)$ is the anomalous dimension of the vertex which is determined by the renomalization constant $Z_\Gamma=Z_2^{-2}$ (which is the inverse of the renormalization constant of the nucleon four-point Green's function). According to Eq. (8.3),

$$\gamma_\Gamma(\lambda) = \mu\frac{d}{d\mu}\ln Z_\Gamma = -2\mu\frac{d}{d\mu}\ln Z_2. \tag{8.25}$$

Substituting Eqs. (8.13) and (8.24) into Eq. (8.23) and noticing Eqs. (8.15) and (8.25), we also find that the anomalous dimensions are all canceled out. Thus we have

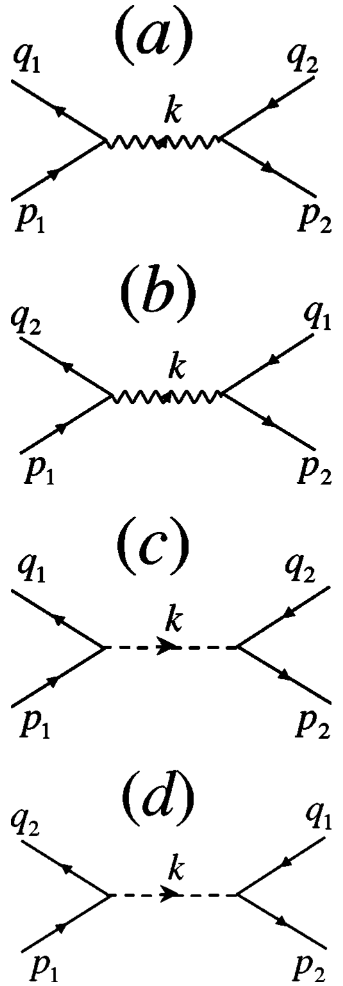

FIG. 5. The tree diagrams of nucleon-nucleon scattering. The first two diagrams represent the interaction generated by the ω meson exchange. The remaining two diagrams represent the interaction mediated by the σ meson exchange.

$$T_{fi}^{(2)} = \bar{u}_{R\sigma}^{(0)}(q_1)\bar{u}_{R\rho}^{(0)}(q_2)\Gamma_R^{(0)}(p_1,p_2;q_1,q_2)u_{R\alpha}^{(0)}(p_1)u_{R\beta}^{(0)}(p_2). \tag{8.26}$$

As mentioned in Eq. (8.8), the vertex $\Gamma_R^{(0)}(p_1,p_2;q_1,q_2)$ is given at $p_{i0}^2=q_{i0}^2=\mu^2$ $(i=1,2)$ and the physical parameters in it are all running ones. Since the unrenormalized vertex $\Gamma(p_1,p_2;q_1,q_2)$ is not fundamental, it has a complicated structure, containing a series of tree and loop diagrams [36]. The expression of the vertex $\Gamma_R^{(0)}(p_1,p_2;q_1,q_2)$ can be determined by the perturbation method. Unlike the loop expansion, the perturbation series of the S-matrix usually is expanded in powers of the coupling constant g_v. The lowest order approximation of the vertex $\Gamma(p_1,p_2;q_1,q_2)$ is of the order of g^4 and contains two terms which are given by the truncated subdiagrams (the box and crossed box diagrams) obtained from Figs. 6(e) and (f) by amputating the external lines. The scattering amplitude given by the tree diagrams in Figs. 6(e) and (f) and their corresponding exchanged counterparts are convergent. We may dress these diagrams by replacing the free wave functions, the free propagators, and the bare vertices with the exact wave functions, the full propagators, and the rigorous proper vertices. In this way, we obtain a series of loop diagrams. As mentioned before, the dressed wave functions, propagators, and vertices in the S-matrix element can all be replaced by the renormalized ones. Therefore they can be expressed in the forms as given in Eqs. (8.13), (8.16), and (8.19). Due to the cancellation of the anomalous dimensions, we will obtain an expression of the renormalized scattering amplitude which is formally the same as that written from the tree diagrams in Figs. 6(e) and (f) and their exchanged ones. For this reason, the tree diagrams are called skeletons of the dressed diagrams. There are a series of skeleton diagrams (or called tree diagrams) of the vertex $\Gamma(p_1,p_2;q_1,q_2)$ such as the ladder diagrams and some others. But, in practical calculations, it is only feasible to consider the skeleton diagrams given in lower order perturbative approximations. We would like to stress that the skeleton diagrams can all be dressed. The S-matrix elements given by the dressed diagrams can be written out from the corresponding skeleton (tree) diagrams provided that the physical parameters are replaced by the solutions of their RGE's. For other S-matrix elements representing other processes, the conclusion is completely the same. It is noted that a S-matrix element evaluated by the σ-ω model is indepen-

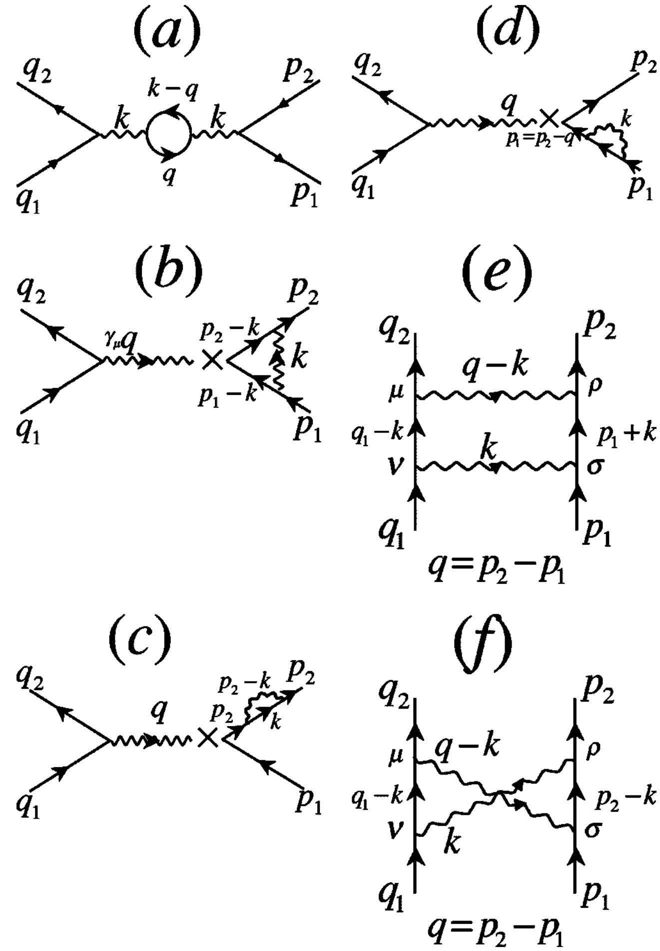

FIG. 6. Some two-nucleon one-loop Feynman diagrams which are chosen to demonstrate the gauge-independence of the nucleon scattering matrix elements.

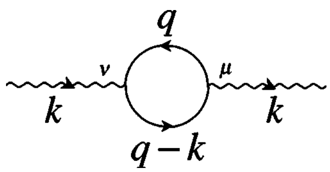

FIG. 7. The one-loop diagram of the effective ω meson self-energy. The solid line marks the free nucleon propagator and the wavy line denotes the free ω meson propagator.

dent of the gauge parameter as illustrated in Appendix A in the one-loop approximation (this is the so-called gauge independence of the S matrix which is implied by the unitarity of S-matrix elements). This fact indicates that the task of renormalization for the σ-ω model is reduced to find the effective coupling constants and the effective masses by solving their RGE's. These effective quantities completely describe the effect of higher order loop corrections. As an illustration, the effective coupling constants and the effective masses given in the one-loop approximation will be derived and discussed in detail in the next section.

IX. ONE-LOOP EFFECTIVE COUPLING CONSTANTS AND MASSES

For the renornalization of the σ-ω model, we need to derive the effective vectorial coupling constant, the effective scalar coupling constant, the effective nucleon mass, and the effective ω meson and σ meson masses. The one-loop expressions of these effective quantities will be derived and discussed in the following subsections.

A. Effective vectorial coupling constant

The RGE for the renormalized vectorial coupling constant g_v^R which appears in the vectorial vertex may be immediately written out from Eq. (8.2) by setting $F=g_v$,

$$\mu\frac{d}{d\mu}g_v^R(\mu)+\gamma_g^v(\mu)g_v^R(\mu)=0. \tag{9.1}$$

In view of the definition shown in Eq. (8.1) and the relation in Eq. (6.32), the renormalization constant defined in Eq. (8.12) will be represented as

$$Z_g^v=\frac{Z_1}{Z_2Z_3^{1/2}}=Z_3^{-1/2}. \tag{9.2}$$

According to the definition in Eq. (8.3), the anomalous dimension $\gamma_g^v(\mu)$ in Eq. (9.1) can be calculated by

$$\gamma_g^v=\lim_{\varepsilon\to 0}\mu\frac{d}{d\mu}\ln Z_g^v=-\frac{1}{2}\lim_{\varepsilon\to 0}\mu\frac{d}{d\mu}\ln Z_3, \tag{9.3}$$

where $\varepsilon=2-n/2$ with n being the dimension of the space in which the regularization is performed. Based on the definition denoted in Eq. (5.19), the renormalization constant Z_3 will be given by the subtraction of the ω meson vacuum polarization (self-energy) operator $-i\Pi_{\mu\nu}(k)$. From the one-loop diagram represented in Fig. 7, one may write [1,29]

$$\Pi_{\mu\nu}(k)=-2ig_v^2\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}\left[\gamma_\mu\frac{1}{\not l-\not k-M+i\varepsilon}\gamma_\nu\frac{1}{\not l-M+i\varepsilon}\right], \tag{9.4}$$

where the factor 2 comes from the fact that the fermion loop can be formed by both of the proton and neutron loops. By the dimensional regularization [28–30,43], the divergent integral shown above is easily calculated. Then, from the definitions in Eqs. (5.14) and (5.19), it is not difficult to find that in the n-dimensional space, the renormalization constant Z_3 is expressed as

$$\begin{aligned}Z_3&=1-\Pi(\mu^2)\\&=1+\frac{g_v^2}{2\pi^2}(4\pi m_g^2)^\varepsilon(2-\varepsilon)\frac{\Gamma(1+\varepsilon)}{\varepsilon}\int_0^1\frac{dx x(x-1)}{[\mu^2x(x-1)+M^2]^\varepsilon},\end{aligned} \tag{9.5}$$

where m_g is a mass introduced to make the coupling constant to be dimensionless in the n-dimensional space. It is noted here that the factors $(4\pi m_g^2)^\varepsilon$ and $\Gamma(1+\varepsilon)$ may all be set to unity because they do not give an effect on the anomalous dimension when we set $\varepsilon\to 0$ in the final step of the calculation for the anomalous dimension. Inserting Eq. (9.5) into Eq. (9.3), it can be found that

$$\gamma_g^v=-\frac{g_v^2}{6\pi^2}\left\{1+6\sigma^2+\frac{12\sigma^4}{\sqrt{1-4\sigma^2}}\ln\frac{1+\sqrt{1-4\sigma^2}}{1-\sqrt{1-4\sigma^2}}\right\}, \tag{9.6}$$

where $\sigma=M/\mu$. In this expression, the coupling constant g_v and the nucleon mass M are unrenormalized. In the approximation of order g_v^2, they can be replaced by the renormalized ones g_v^R and M_R because in this approximation, as pointed out in the previous literature [28], the lowest order approximation of the relation between the $g_v(M)$ and the $g_v^R(M_R)$ is only necessary to be taken into account. Furthermore, when we introduce the scaling variable λ defined by $\mu=\mu_0\lambda$ for the renormalization point and set $\mu_0=M_R$ (this can always be done since the μ_0 is fixed, but may be chosen at will; the above choice amounts to taking the renormalization scale parameter to be the nucleon mass), we have $\sigma=M_R/\mu_0\lambda=1/\lambda$. Thus, with the γ_g^v expressed in Eq. (9.6) and noticing $\mu(d/d\mu)=\lambda(d/d\lambda)$, Eq. (9.1) may be rewritten in the form

$$\lambda\frac{dg_v^R(\lambda)}{d\lambda}=\frac{[g_v^R(\lambda)]^3}{6\pi^2}F_g^v(\lambda), \tag{9.7}$$

where

$$F_g^v(\lambda)=1+\frac{6}{\lambda^2}+\frac{12}{\lambda^4}f(\lambda) \tag{9.8}$$

in which

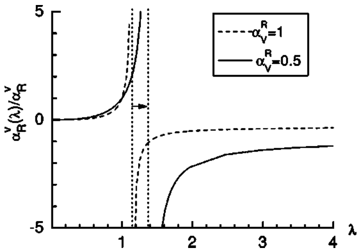

FIG. 8. The effective one-loop vectorial coupling constants $\alpha_R^v(\lambda)$ given by the timelike momentum space subtraction. The solid line represents the coupling constant given by taking $\alpha_R^v=0.5$. The dashed line denotes the coupling constant given by $\alpha_R^v=1$.

$$f(\lambda) = \frac{\lambda}{\sqrt{\lambda^2-4}} \ln \frac{\lambda+\sqrt{\lambda^2-4}}{\lambda-\sqrt{\lambda^2-4}} = \begin{cases} \dfrac{2\lambda}{\sqrt{4-\lambda^2}} \cot^{-1} \dfrac{\lambda}{\sqrt{4-\lambda^2}}, \text{ if } \lambda \leqslant 2, \\ \dfrac{2\lambda}{\sqrt{\lambda^2-4}} \operatorname{Coth}^{-1} \dfrac{\lambda}{\sqrt{\lambda^2-4}}, \text{ if } \lambda \geqslant 2. \end{cases} \tag{9.9}$$

Upon substituting Eqs. (9.8) and (9.9) into Eq. (9.7) and then integrating Eq. (9.7) by applying the familiar integration formulas, the effective (running) coupling constant will be found to be

$$\alpha_R^v(\lambda) = \frac{\alpha_R^v}{1 - \dfrac{4\alpha_R^v}{3\pi} G_v(\lambda)}, \tag{9.10}$$

where $\alpha_R^v(\lambda)=[g_R^v(\lambda)]^2/4\pi$, $\alpha_R^v=\alpha_R^v(1)$ which is the coupling constant that should be determined by experiment and

$$G_v(\lambda) = \int_1^\lambda \frac{d\lambda}{\lambda} F_g^v(\lambda) = 2 + \sqrt{3}\pi - \frac{2}{\lambda^2} + \left(1 + \frac{2}{\lambda^2}\right) \frac{1}{\lambda} \varphi(\lambda) \tag{9.11}$$

in which

$$\varphi(\lambda) = \sqrt{\lambda^2-4} \ln \frac{1}{2}(\lambda + \sqrt{\lambda^2-4}) = \begin{cases} -\sqrt{4-\lambda^2} \cos^{-1} \dfrac{\lambda}{2}, \text{ if } \lambda \leqslant 2, \\ \sqrt{\lambda^2-4} \cosh^{-1} \dfrac{\lambda}{2}, \text{ if } \lambda \geqslant 2. \end{cases} \tag{9.12}$$

As mentioned before, the variable λ is also the scaling parameter of momenta, $p=\lambda p_0$, and it is convenient to put $p_0^2=\mu_0^2$ so as to apply the boundary condition. Thus, owing to the choice $\mu_0=M_R$, we have $p_0^2=M_R^2$ and $\lambda=(p^2/M_R^2)^{1/2}$. In this case, it is apparent that when $\lambda=1$, Eq. (9.10) will be reduced to the result given on the nucleon mass shell with the value $\alpha_R^v(1)=\alpha_R^v$. The behaviors of the effective coupling constants obtained in the timelike and spacelike momentum space subtractions are separately represented in Figs. 8 and 9. The figures show that the effective coupling constant $\alpha_R^v(\lambda)$ has a singularity λ_0 (the Landau pole). The position of the pole strongly depends on the parameter α_R^v. By our numerical test, we find, when the α_R^v is getting smaller and smaller, the λ_0 is getting larger and larger. If the α_R^v goes to zero, the λ_0 approaches a value near infinity, similar to the case of QED [20]. While, when the α_R^v is getting larger and larger, the λ_0 moves toward unity; but it cannot arrive at unity because $\alpha_R^v(1)=\alpha_R^v$. In the region [0,1] of λ, the $\alpha_R^v(\lambda)$ has no singularity to appear. In Fig. 8, there are two lines representing the $\alpha_R^v(\lambda)$ given in the timelike momentum subtraction: one is given by taking $\alpha_R^v=1$ and has a singularity at $\lambda \simeq 1.1385$; another is obtained by taking $\alpha_R^v=0.5$ and has a singularity at $\lambda \simeq 1.3885$. When λ goes from λ_0 to zero, the $\alpha_R^v(\lambda)$ decreases and tends to zero, exhibiting an asymptotically free behavior as we met in QED. In Fig. 9, the two lines represent the $\alpha_R^v(\lambda)$ given in the spacelike momentum subtraction: one line is obtained by taking $\alpha_R^v=1$ and has a singularity at $\lambda_0 \simeq 26.4689$; another is given by $\alpha_R^v=0.5$ and has a singularity at $\lambda_0 \simeq 280.431$. When λ goes to zero, the $\alpha_R^v(\lambda)$ approaches the constant α_R^v. As one knows, the Landau poles mentioned above give a limitation of applicability of the one loop renormalization. That is to say, beyond the region $[0,\lambda_0]$, the $\alpha_R^v(\lambda)$ is meaningless even though in the limit: $\lambda \to \infty$, the $\alpha_R^v(\lambda)$ tends to zero from an opposite direction. In comparison of Fig. 8 with Fig. 9, it is clear to see that the range of applicability for the $\alpha_R^v(\lambda)$ given in the spacelike momentum subtraction is much larger than the range for the $\alpha_R^v(\lambda)$ given in the timelike momentum subtraction.

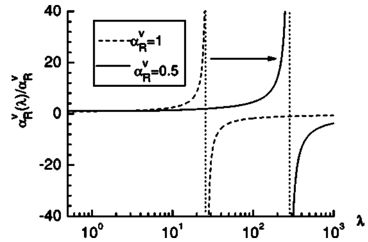

FIG. 9. The effective one-loop vectorial coupling constants $\alpha_R^v(\lambda)$ given by the spacelike momentum space subtraction. The solid line represents the coupling constant given by taking $\alpha_R^v=0.5$. The dashed line denotes the coupling constant given by $\alpha_R^v=1$.

B. Effective ω meson mass

In Eq. (8.2), when we set $F=m_\omega$, we have a RGE for the renormalized mass of ω meson such that

$$\mu \frac{d}{d\mu} m_\omega^R(\mu) + \gamma_m^\omega(\mu) m_\omega^R(\mu) = 0. \tag{9.13}$$

From the definitions given in Eq. (8.1) and the first equality in Eq. (5.22), we find

$$Z_m^\omega = Z_3^{-1/2} = Z_g^v \tag{9.14}$$

so that

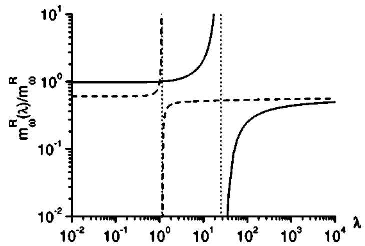

FIG. 10. The effective one-loop ω meson masses $m_\omega^R(\lambda)$ given by taking $\alpha_R^v=1$. The solid line and the dashed line represent the effective masses obtained in the spacelike momentum subtraction and the timelike momentum, respectively.

$$\gamma_m^\omega(\mu)=\gamma_g^v(\mu)=\frac{[g_v^R(\lambda)]^2}{6\pi^2}F_g^v(\lambda), \tag{9.15}$$

where $F_g^v(\lambda)$ was given in Eq. (9.8). With the above expression, Eq. (9.13) can be written as

$$\frac{dm_\omega^R}{m_\omega^R}=\frac{2\alpha_R^v(\lambda)}{3\pi}F_g^v(\lambda)\frac{d\lambda}{\lambda}. \tag{9.16}$$

Integrating the above equation, one gets

$$m_\omega^R(\lambda)=m_\omega^R e^{(2/3\pi)\int_1^\lambda (d\lambda/\lambda)\alpha_R^v(\lambda)F_g^v(\lambda)}, \tag{9.17}$$

where $m_\omega^R=m_\omega^R(1)$ is the observed ω meson mass. This is just the one-loop result of the effective ω meson mass. If we take the approximation $\alpha_R^v(\lambda)\simeq\alpha_R^v$, the above expression becomes

$$m_\omega^R(\lambda)\simeq m_\omega^R e^{(2/3\pi)G_v(\lambda)}, \tag{9.18}$$

where $G_v(\lambda)$ was given in Eq. (9.11).

To have an insight into the behavior of the effective masses in Eq. (9.17), we take the $m_\omega^R(\lambda)$ given by taking $\alpha_R^v=1$ as an example. This $m_\omega^R(\lambda)$ is shown in Fig. 10. In the figure, the solid line represents the effective mass obtained in the spacelike momentum subtraction and the dashed line describes the one given in the timelike momentum subtraction. Comparing Fig. 10 with Figs. 8 and 9, we see that the both effective masses have the same singularities and the same scopes of applicability as the corresponding effective coupling constants $\alpha_R^v(\lambda)$. Particularly, the position of the singularity strongly depends on the choice of α_R^v as the $\alpha_R^v(\lambda)$ does. When λ tends to zero, the $m_\omega^R(\lambda)$ for the spacelike momentum approaches a nonvanishing value near the m_ω^R, while the $m_\omega^R(\lambda)$ for the timelike momentum goes to zero.

C. Effective nucleon mass

The RGE for the renormalized nucleon mass, according to Eq. (8.2), can be written as

$$\mu\frac{d}{d\mu}M_R(\mu)+\gamma_M(\mu)M_R(\mu)=0, \tag{9.19}$$

where $\gamma_M(\mu)$ is the anomalous dimension of nucleon mass. In view of Eqs. (6.25) and (8.3), we have

$$\gamma_M(\mu)=\mu\frac{d}{d\mu}\ln Z_M$$
$$=-\mu\frac{d}{d\mu}\ln\left\{1+Z_2\left[\frac{A}{M}+\left(1-\frac{\mu}{M}\right)B\right]\right\}. \tag{9.20}$$

To determine the constants A, B, and Z_2 in the one-loop approximation, it is necessary to compute the nucleon self-energy written in Eq. (6.35). By the dimensional regularization, it is not difficult to derive from Eq. (6.35) the following expression:

$$\Sigma(p)=(\not p-\mu)\Sigma_1(p)+\Sigma_2(p), \tag{9.21}$$

where

$$\Sigma_1(p)=\frac{g_v^2}{(4\pi)^2}\int_0^1 dx\frac{2(x-1)}{\varepsilon\Theta_v(x)^\varepsilon}+(1-\alpha)\frac{g_v^2}{(4\pi)^2}\times\left[\int_0^1 dx\int_0^1 dyy\left(\frac{(1+3xy)}{\varepsilon\Theta_v(x,y)^\varepsilon}+\frac{1}{\Theta_v(x,y)}x^2y^2\{(1-xy)\times[p^2+\mu(\not p+\mu)]+M(\not p+\mu)\}\right)-\frac{1}{2}\right]+\frac{g_s^2}{(4\pi)^2}\int_0^1 dx\frac{x-1}{\varepsilon\Theta_s(x)^\varepsilon} \tag{9.22}$$

and

$$\Sigma_2(p)=\frac{g_v^2}{(4\pi)^2}\int_0^1 dx\frac{2[(x-1)\mu+2M]}{\varepsilon\Theta_v(x)^\varepsilon}+(1-\alpha)\frac{g_v^2}{(4\pi)^2}\times\left(\int_0^1 dx\int_0^1 dyy\times\left\{\frac{(1+3xy)\mu-2M}{\varepsilon\Theta_v(x,y)^\varepsilon}+\frac{1}{\Theta_v(x,y)}\mu^2x^2y^2[(1-xy)\mu+M]\right\}+\frac{1}{2}(M-\mu)\right)+\frac{g_s^2}{(4\pi)^2}\int_0^1 dx\frac{(x-1)\mu-M}{\varepsilon\Theta_s(x)^\varepsilon}, \tag{9.23}$$

where

$$\Theta_v(x)=p^2x(x-1)+M^2x+m_\omega^2(1-x),$$
$$\Theta_v(x,y)=p^2xy(xy-1)+M^2xy+m_\omega^2[(1-x)y+\alpha(1-y)], \tag{9.24}$$
$$\Theta_s(x)=p^2x(x-1)+M^2x+m_\sigma^2(1-x).$$

From the definition $A=\Sigma(\mu)$ written in Eq. (6.17) and the expressions in Eqs. (9.21)–(9.23), we find

$$A=A_1+A_2+A_3, \tag{9.25}$$

where

$$A_1=\frac{g_v^2}{(4\pi)^2}\int_0^1 dx\frac{2[(x-1)\mu+2M]}{\varepsilon\Omega_v(x)^\varepsilon}, \tag{9.26}$$

$$A_2 = (1-\alpha)\frac{g_v^2}{(4\pi)^2}\left(\int_0^1 dx\int_0^1 dyy\left\{\frac{(1+3xy)\mu - 2M}{\varepsilon\Omega_v(x,y)^\varepsilon} + \frac{1}{\Omega_v(x,y)}x^2y^2[(1-xy)\mu^3 + M\mu^2]\right\} + \frac{1}{2}(M-\mu)\right) \tag{9.27}$$

and

$$A_3 = \frac{g_s^2}{(4\pi)^2}\int_0^1 dx\frac{(x-1)\mu - M}{\varepsilon\Omega_s(x)^\varepsilon} \tag{9.28}$$

in which

$$\Omega_v(x) = \mu^2 x(x-1) + M^2 x + m_\omega^2(1-x),$$

$$\Omega_v(x,y) = \mu^2 xy(xy-1) + M^2 xy + m_\omega^2[(1-x)y + \alpha(1-y)], \tag{9.29}$$

$$\Omega_s(x) = \mu^2 x(x-1) + M^2 x + m_\sigma^2(1-x).$$

The constant B appearing in Eqs. (6.21) and (6.25), according to Eq. (6.19), ought to be computed by

$$B = (\not p - \mu)^{-1}[\Sigma(p) - A]|_{\not p=\mu}. \tag{9.30}$$

On inserting Eqs. (9.21)–(9.29) into Eq. (9.30) and employing the formula

$$\frac{1}{a^\varepsilon} - \frac{1}{b^\varepsilon} = \int_0^1 dx\frac{\varepsilon(b-a)}{[ax + b(1-x)]^{1+\varepsilon}}, \tag{9.31}$$

one may derive

$$B = B_1 + B_2 + B_3, \tag{9.32}$$

where

$$B_1 = \frac{g_v^2}{(4\pi)^2}\int_0^1 dx\left\{\frac{2(x-1)}{\varepsilon\Omega_v(x)^\varepsilon} - \frac{4}{\Omega_v(x)}x(x-1)\times[(x-1)\mu^2 + 2M\mu]\right\}, \tag{9.33}$$

$$B_2 = (1-\alpha)\frac{g_v^2}{(4\pi)^2}\left(\int_0^1 dx\int_0^1 dyy\left\{\frac{1+3xy}{\varepsilon\Omega_v(x)^\varepsilon} + \frac{1}{\Omega_v(x)}xy[\mu^2(2 + 7xy - 9x^2y^2) - 2M\mu(2-3xy)] + \frac{2}{\Omega_v(x)^2}x^3y^3(1-xy)\times[(1-xy)\mu^4 + M\mu^3]\right\} - \frac{1}{2}\right), \tag{9.34}$$

and

$$B_3 = \frac{g_s^2}{(4\pi)^2}\int_0^1 dx\left\{\frac{(x-1)}{\varepsilon\Omega_s(x)^\varepsilon} - \frac{2}{\Omega_s(x)}x(x-1)\times[(x-1)\mu^2 - M\mu]\right\}. \tag{9.35}$$

The terms expressed in Eqs. (9.27) and (9.34) are dependent on the gauge parameter α and look more complicated. However, as demonstrated in the Appendix A, the S-matrix elements evaluated in the σ-ω model are gauge independent. Therefore for simplicity, these terms will not be taken into account later on. This means that we limit ourselves to working in the Feynman gauge.

When Eqs. (9.25) and (9.32) with the expressions given in Eqs. (9.26), (9.28), (9.33), and (9.35) are substituted into Eq. (9.20), noticing that $Z_2 \simeq 1$ should be taken in Eq. (9.20) in the approximation of order g_i^2, one may find an explicit expression of the anomalous dimension $\gamma_M(\mu)$ through a tedious calculation,

$$\gamma_M(\lambda) = \gamma_M^{(1)}(\lambda) + \gamma_M^{(2)}(\lambda), \tag{9.36}$$

where $\gamma_M^{(1)}(\lambda)$ is derived from the constants in Eqs. (9.26) and (9.33), while $\gamma_M^{(2)}(\lambda)$ is given by the constants in Eqs. (9.28) and (9.35). The expressions of the $\gamma_M^{(1)}(\lambda)$ and $\gamma_M^{(2)}(\lambda)$ are separately described in the following. For the anomalous dimension $\gamma_M^{(1)}(\lambda)$, we have

$$\gamma_M^{(1)}(\lambda) = \frac{\alpha_R^v}{\pi}\Sigma_v(\lambda) \tag{9.37}$$

in which

$$\Sigma_v(\lambda) = \xi_0^v(\lambda) + \sum_{i=1}^4 \xi_i^v(\lambda)J_i(\lambda), \tag{9.38}$$

where the functions $\xi_i^v(\lambda)$ are

$$\xi_0^v(\lambda) = \frac{1}{2}(3+\lambda) - \frac{2}{\lambda} + \frac{1-\beta^2}{\lambda^2}(1-\lambda), \tag{9.39}$$

$$\xi_1^v(\lambda) = \frac{2}{\lambda^4}[2(1-\beta^2)^2 - 3\beta^2\lambda^2]\left(1-\frac{1}{\lambda}\right) + \frac{1}{\lambda^6}[2(1-\beta^2)^3 - 5\beta^2(1-\beta^2)\lambda^2 - \beta^2\lambda^4](1-\lambda), \tag{9.40}$$

$$\xi_2^v(\lambda) = \frac{1}{\lambda^4}[\beta^2\lambda^2 - 3(1-\beta^2)^2](1-\lambda) - \frac{6(1-\beta^2)}{\lambda^2}\left(1-\frac{1}{\lambda}\right), \tag{9.41}$$

$$\xi_3^v(\lambda) = \frac{4\beta^2}{\lambda^6}[\beta^2\lambda^2 - (1-\beta^2)^2]\left(1-\frac{1}{\lambda}\right) + \frac{2\beta^2}{\lambda^8}[\beta^2\lambda^4 + 2(1-\beta^2)\beta^2\lambda^2 - (1-\beta^2)^3](1-\lambda), \tag{9.42}$$

and

$$\xi_4^v(\lambda) = \frac{4}{\lambda^6}[(1-\beta^4)\lambda^2 - (1-\beta^2)^3]\left(1-\frac{1}{\lambda}\right) - \frac{2}{\lambda^8}[(1-\beta^2)^4 - (1-\beta^2)^2(1+2\beta^2)\lambda^2 + \beta^4\lambda^4](1-\lambda) \tag{9.43}$$

with $\beta = m_\omega/M$ and the functions $J_i(\lambda)$ are given by the integrals shown below. With defining $a = (1-\beta^2)\lambda^{-2}$ and $b = \beta^2\lambda^{-2}$, we can write

$$J_1(\lambda) = \int_0^1 dx \frac{1}{x(x-1)+ax+b} = \frac{\lambda^2}{\sqrt{q(\lambda)}} \ln \frac{\lambda^2 - 1 - \beta^2 - \sqrt{q(\lambda)}}{\lambda^2 - 1 - \beta^2 + \sqrt{q(\lambda)}}, \tag{9.44}$$

where

$$q(\lambda) = \lambda^4 - 2(1+\beta^2)\lambda^2 + (1-\beta^2)^2, \tag{9.45}$$

$$J_2(\lambda) = \int_0^1 dx \frac{x}{x(x-1)+ax+b} = -\ln \beta + \frac{\lambda^2 - 1 + \beta^2}{2\sqrt{q(\lambda)}} \ln \frac{\lambda^2 - 1 - \beta^2 - \sqrt{q(\lambda)}}{\lambda^2 - 1 - \beta^2 + \sqrt{q(\lambda)}}, \tag{9.46}$$

$$J_3(\lambda) = \int_0^1 dx \frac{1}{[x(x-1)+ax+b]^2} = [(1-\beta^2)^2 - (1+\beta^2)\lambda^2] \frac{\lambda^4}{\beta^2 q(\lambda)} - \frac{2\lambda^6}{q(\lambda)^{3/2}} \ln \frac{\lambda^2 - 1 - \beta^2 - \sqrt{q(\lambda)}}{\lambda^2 - 1 - \beta^2 + \sqrt{q(\lambda)}}, \tag{9.47}$$

and

$$J_4(\lambda) = \int_0^1 dx \frac{x}{[x(x-1)+ax+b]^2} = -(\lambda^2 + 1 - \beta^2)\frac{\lambda^4}{q(\lambda)} - (\lambda^2 - 1 + \beta^2) \times \frac{\lambda^4}{q(\lambda)^{3/2}} \ln \frac{\lambda^2 - 1 - \beta^2 - \sqrt{q(\lambda)}}{\lambda^2 - 1 - \beta^2 + \sqrt{q(\lambda)}}. \tag{9.48}$$

For the anomalous dimension $\gamma_M^{(2)}(\lambda)$, we can write

$$\gamma_M^{(2)}(\lambda) = \frac{\alpha_R^s}{2\pi} \Sigma_s(\lambda) \tag{9.49}$$

in which $\alpha_R^s = (g_s^R)^2/4\pi$ and

$$\Sigma_s(\lambda) = \xi_0^s(\lambda) + \sum_{i=1}^{4} \xi_i^s(\lambda) J_i^0(\lambda), \tag{9.50}$$

where the functions $\xi_i^s(\lambda)$ are

$$\xi_0^s(\lambda) = -\frac{3}{2} + \frac{\lambda}{2} + \frac{1}{\lambda^2} - \frac{\beta_0^2}{\lambda^2}(1-\lambda) \tag{9.51}$$

with $\beta_0 = m_\sigma / M$,

$$\xi_1^s(\lambda) = \frac{1}{\lambda^6}[2(1-\beta_0^2)^3 - 5\beta_0^2(1-\beta_0^2)\lambda^2 - \beta_0^2\lambda^4](1-\lambda) - \frac{1}{\lambda^4}[2(1-\beta_0^2)^2 - 3\beta_0^2\lambda^2]\left(1 - \frac{1}{\lambda}\right), \tag{9.52}$$

$$\xi_2^s(\lambda) = \frac{1}{\lambda^4}[\beta_0^2\lambda^2 - 3(1-\beta_0^2)^2](1-\lambda) + \frac{3(1-\beta_0^2)}{\lambda^2}\left(1 - \frac{1}{\lambda}\right), \tag{9.53}$$

$$\xi_3^s(\lambda) = \frac{2\beta_0^2}{\lambda^6}[(1-\beta_0^2)^2 - \beta_0^2\lambda^2]\left(1 - \frac{1}{\lambda}\right) + \frac{2\beta_0^2}{\lambda^8}[\beta_0^2\lambda^4 + 2(1-\beta_0^2)\beta_0^2\lambda^2 - (1-\beta_0^2)^3](1-\lambda), \tag{9.54}$$

and

$$\xi_4^s(\lambda) = \frac{2}{\lambda^6}[(1-\beta_0^2)^3 - (1-\beta_0^4)\lambda^2]\left(1 - \frac{1}{\lambda}\right) - \frac{2}{\lambda^8}[(1-\beta_0^2)^4 - (1-\beta_0^2)^2(1+2\beta_0^2)\lambda^2 + \beta_0^4\lambda^4](1-\lambda), \tag{9.55}$$

and the functions $J_i^0(\lambda)$ formally are the same as the functions $J_i(\lambda)$ except that the parameter β in the $J_i(\lambda)$ is now replaced by β_0,

$$J_i^0(\lambda) = J_i(\lambda)|_{\beta \to \beta_0}. \tag{9.56}$$

Substituting the $\gamma_M(\lambda)$ as expressed in Eqs. (9.36)–(9.56) into Eq. (9.19) and solving the equation with noticing $\mu d/\mu = \lambda d/\lambda$, we obtain

$$M_R(\lambda) = M_R e^{-\int_1^\lambda d\lambda/\lambda \gamma_M(\lambda)} = M_R e^{-\int_1^\lambda d\lambda/\lambda [\alpha_R^v(\lambda)/\pi \Sigma_v(\lambda) + \alpha_R^s(\lambda)/2\pi \Sigma_s(\lambda)]}, \tag{9.57}$$

where $M_R = M_R(1)$ is the observed nucleon mass. The coupling constants in the above have been taken to be running ones. The $\alpha_R^v(\lambda)$ was given in Eq. (9.10), while the $\alpha_R^s(\lambda)$ will be derived in the next subsection.

It would be emphasized that in the timelike momentum space subtraction, the scaling parameter λ is real so that the effective nucleon mass is real, while in the spacelike momentum space subtraction, the λ is imaginary so that the effective nucleon mass becomes complex one. In the latter case, the λ in the $\gamma_M(\lambda)$ should be set to be $i\lambda$. Observing the expressions in Eqs. (9.39)–(9.48) and (9.51)–(9.55), we see that in the both subtractions, the functions $J_i(\lambda)$ are always real. The real and imaginary parts of the $\gamma_M(\lambda)$ are distinguished by the real and imaginary parts of the functions $\xi_i^v(\lambda)$ and $\xi_i^s(\lambda)$. In Figs. 11 and 12, we show the behaviors of the effective nucleon masses given by the expression in Eq. (9.57) for which the coupling constants be taken as the constants α_R^v and α_R^s for simplicity of computation. The effective mass $M_R(\lambda)$ obtained in the timelike momentum space subtraction is exhibited in Fig. 11. In the figure, the dashed, dotted and solid lines represent the $M_R(\lambda)$ given by taking $(\alpha_R^v, \alpha_R^s) = (0, 0.5)$, (0.5,0), and (0.5,0.5), respectively. From the figure, we see that in the region [0,1] of λ, the $M_R(\lambda)$ almost keeps a constant M_R. Beyond this region, if only the scalar coupling is considered, the $M_R(\lambda)$ increases up to a maximum at $\lambda_0 = 4.21$ and then decreases to zero rather rapidly when $\lambda \to \infty$. While, if the vectorial coupling enters, the $M_R(\lambda)$ increases a little when λ goes to a smaller λ_0 and then

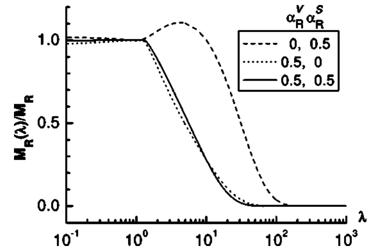

FIG. 11. The effective one-loop nucleon masses $M_R(\lambda)$ obtained in the timelike momentum subtraction. The solid, dashed, and dotted lines represent the effective masses given by $(\alpha_R^v, \alpha_R^s) = (0.5, 0)$, (0,0.5), and (0.5,0.5), respectively.

decreases much rapidly down to zero when λ varies from λ_0 to ∞. The effective mass $M_R(\lambda)$ obtained in the spacelike momentum space subtraction is shown in Fig. 12. In the figure, the real and imaginary parts of the $M_R(\lambda)$ are displayed separately. The dashed, solid, and dotted lines in the figure represent the real and imaginary parts of the $M_R(\lambda)$ which are obtained by taking $(\alpha_R^v, \alpha_R^s) = (0.5, 0)$, (0.5,0.5), and (1,1), respectively. The figure indicates that in the region [0,1] of λ, the real part of the $M_R(\lambda)$ keeps almost a constant equal to M_R, while the imaginary part of the $M_R(\lambda)$ is almost zero. When λ varies from unity to infinity, the real part of the $M_R(\lambda)$ at first increases smoothly and the imaginary part of the $M_R(\lambda)$ decreases, then, both of them drastically oscillate and damp to zero. The figure also shows that the stronger the couplings (especially, the vectorial coupling), the larger is the frequency of the oscillation. The appearance of the oscillation implies that the $M_R(\lambda)$ is invalid to use in the region that the oscillation appears.

D. Effective scalar coupling constant

When setting $F = g_s$ in Eq. (8.2), one obtains the RGE for the renormalized scalar coupling constant g_s^R which is included in the scalar (nucleon–nucleon-σ meson) vertex,

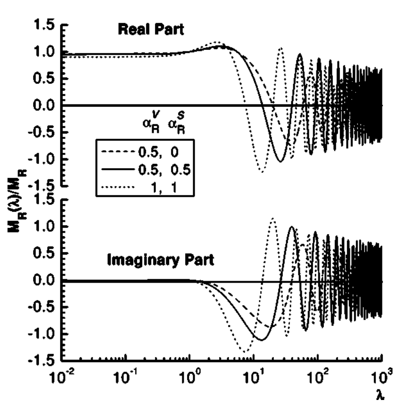

FIG. 12. The effective one-loop nucleon masses $M_R(\lambda)$ obtained in the spacelike momentum subtraction. The dashed, solid, and dotted lines represent the effective masses given by taking $(\alpha_R^v, \alpha_R^s) = (0.5, 0)$, (0.5,0.5), and (1,1), respectively. The upper figure describes the real part of the $M_R(\lambda)$. Another figure shows the imaginary part of the $M_R(\lambda)$.

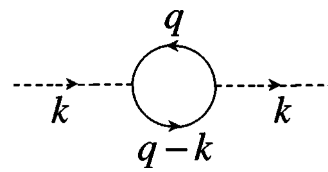

FIG. 13. The σ meson one-loop self-energy. The solid line represents the free nucleon propagator and the dashed line denotes the free σ meson propagator.

$$\mu \frac{d}{d\mu} g_s^R(\mu) + \gamma_g^s(\mu) g_s^R(\mu) = 0. \tag{9.58}$$

Analogous to the case of vectorial coupling, the anomalous dimension $\gamma_g^v(\mu)$ determined by

$$\gamma_g^s = \lim_{\varepsilon \to 0} \mu \frac{d}{d\mu} \ln Z_g^s \tag{9.59}$$

should be calculated from the renormalization constant Z_g^s which is represented as

$$Z_g^s = \frac{Z_1'}{Z_2 Z_3'^{1/2}} \tag{9.60}$$

where Z_2, Z_3', and Z_1' were defined, respectively, in Eqs. (6.21), (7.6), and (7.17). According to these definitions, in the approximation of order g^2 and in the Feynman gauge, the Z_g^s will be written as

$$Z_g^s = 1 + B_1 + B_3 - L_1' - L_3' + \Omega_1, \tag{9.61}$$

where B_1 and B_3 were represented in Eqs. (9.33) and (9.35), respectively, L_1' and L_3' are the parts of the constant L' which can conveniently be determined by the identity in Eq. (7.22). From the identity and the representations written in Eqs. (9.26) and (9.28), it is easy to get

$$L_1' = \frac{\partial A_1}{\partial M} = \frac{g_v^2}{4\pi^2} \int_0^1 dx \left\{ \frac{1}{\varepsilon \Omega_v(x)^\varepsilon} - [x(x-1)\mu + 2Mx] \frac{M}{\Omega_v(x)} \right\},$$

$$L_3' = \frac{\partial A_3}{\partial M} = -\frac{g_s^2}{16\pi^2} \int_0^1 dx \left\{ \frac{1}{\varepsilon \Omega_s(x)^\varepsilon} + [x(x-1)\mu - Mx] \frac{2M}{\Omega_s(x)} \right\}. \tag{9.62}$$

The one-loop expression of the divergent constant $\Omega_1(\mu^2)$ in Eq. (9.61) can be derived from the σ meson one-loop self-energy depicted in Fig. 13. From Fig. 13, it reads

$$\Omega(q) = -2ig_s^2 \int \frac{d^4 l}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{(l - \not{q} - M + i\varepsilon)} \frac{1}{(\not{q} - M + i\varepsilon)} \right], \tag{9.63}$$

where the factor 2 also arises from nucleon doublet. By the dimensional regularization, the above integral is easily calculated and expressed as

$$\Omega(q) = \frac{g_s^2}{(4\pi)^2}\int_0^1 dx \frac{24[q^2x(x-1)+M^2]}{\varepsilon[q^2x(x-1)+M^2]^\varepsilon}$$
$$= \Omega_1(q^2)q^2 + \Omega_2(q^2)m_\sigma^2 \tag{9.64}$$

which gives rise to

$$\Omega_1(\mu^2) = \frac{g_s^2}{(4\pi)^2}\int_0^1 dx \frac{24x(x-1)}{\varepsilon[\mu^2x(x-1)+M^2]^\varepsilon} \tag{9.65}$$

and

$$\Omega_2(\mu^2) = \frac{g_s^2}{(4\pi)^2}\int_0^1 dx \frac{24M^2/m_\sigma^2}{\varepsilon[\mu^2x(x-1)+M^2]^\varepsilon}. \tag{9.66}$$

On substituting Eq. (9.61) into Eq. (9.59), we get

$$\gamma_g^s(\lambda) = \gamma_{g1}^s(\lambda) + \gamma_{g2}^s(\lambda) + \gamma_{g3}^s(\lambda), \tag{9.67}$$

where $\gamma_{g1}^s(\lambda)$, $\gamma_{g2}^s(\lambda)$, and $\gamma_{g3}^s(\lambda)$ are separately defined and described in the following.

For the $\gamma_{g1}^s(\lambda)$, we have

$$\gamma_{g1}^s(\lambda) = \mu\frac{d}{d\mu}(B_1 - L_1') = \frac{\alpha_v^R}{\pi}\Gamma^v(\lambda), \tag{9.68}$$

where

$$\Gamma^v(\lambda) = \eta_0^v(\lambda) + \sum_{i=1}^4 \eta_i^v(\lambda)J_i(\lambda) \tag{9.69}$$

in which

$$\eta_0^v(\lambda) = \frac{3}{2} - \frac{3}{\lambda} + \frac{1-\beta^2}{\lambda^2},$$

$$\eta_1^v(\lambda) = \frac{1}{\lambda^6}[2(1-\beta^2)^3 - 6(1-\beta^2)^2\lambda + (1-\beta^2)(4-5\beta^2)\lambda^2 + 9\beta^2\lambda^3 - 3\beta^2\lambda^4],$$

$$\eta_2^v(\lambda) = -\frac{3}{\lambda^4}[(1-\beta^2)^2 - 3(1-\beta^2)\lambda + (2-\beta^2)\lambda^2],$$

(9.70)

$$\eta_3^v(\lambda) = \frac{2\beta^2}{\lambda^8}[\beta^2\lambda^4 - 3\beta^2\lambda^3 - 2(1-\beta^2)^2\lambda^2 + 3(1-\beta^2)^2\lambda - (1-\beta^2)^3],$$

$$\eta_4^v(\lambda) = \frac{2}{\lambda^8}[(2-\beta^2)\lambda^4 - 3(1-\beta^4)\lambda^3 - (1-\beta^2)(1-2\beta^2)\lambda^2 + 3(1-\beta^2)^3\lambda - (1-\beta^2)^4],$$

and the functions $J_i(\lambda)$ were given in Eqs. (9.44)–(9.48).

For the $\gamma_{g2}^s(\lambda)$ we can write

$$\gamma_{g2}^s(\lambda) = \mu\frac{d}{d\mu}(B_3 - L_3') = \frac{(g_s^R)^2}{8\pi^2}\Gamma_1^s(\lambda), \tag{9.71}$$

where

$$\Gamma_1^s(\lambda) = \eta_0^s(\lambda) + \sum_{i=1}^4 \eta_i^s(\lambda)J_i^0(\lambda) \tag{9.72}$$

in which

$$\eta_0^s(\lambda) = -\frac{3}{2} + \frac{1-\beta_0^2}{\lambda^2},$$

$$\eta_1^s(\lambda) = \frac{1}{\lambda^6}[2(1-\beta_0^2)^2 - (1-\beta_0^2)(2+5\beta_0^2)\lambda^2],$$

$$\eta_2^s(\lambda) = \frac{1}{\lambda^4}[3\lambda^2 - 2(1-\beta_0^2)^2], \tag{9.73}$$

$$\eta_3^s(\lambda) = \frac{2\beta_0^2}{\lambda^8}[\beta_0^2\lambda^4 + (1-\beta_0^2)(1+2\beta_0^2)\lambda^2 - (1-\beta_0^2)^3],$$

$$\eta_4^s(\lambda) = -\frac{2}{\lambda^8}[(1+\beta_0^4)\lambda^4 - 2(1-\beta_0^2)^2(1+\beta_0^2)\lambda^2 + (1-\beta_0^2)^4],$$

and the functions $J_i^0(\lambda)$ were defined in Eq. (9.56).

For the $\gamma_{g3}^s(\lambda)$, by virtue of the expression given in Eq. (9.65), one can get

$$\gamma_{g3}^s(\lambda) = \mu\frac{d}{d\mu}\Omega_1 = -\frac{(g_s^R)^2}{8\pi^2}\Gamma_2^s(\lambda), \tag{9.74}$$

where

$$\Gamma_2^s(\lambda) = 2\left[1 + \frac{6}{\lambda^2} - \frac{12}{\lambda^3\sqrt{\lambda^2-4}}\ln\frac{\lambda-\sqrt{\lambda^2-4}}{\lambda+\sqrt{\lambda^2-4}}\right]. \tag{9.75}$$

Based on the anomalous dimension $\gamma_g^s(\lambda)$ given in Eqs. (9.67), (9.68), (9.71), and (9.74), the RGE in Eq. (9.58) may be represented in the form

$$\frac{dg_s^R(\lambda)}{d\lambda} + P(\lambda)g_s^R(\lambda) + Q(\lambda)[g_s^R(\lambda)]^3 = 0, \tag{9.76}$$

where

$$P(\lambda) = \frac{\alpha_R^v(\lambda)}{\pi\lambda}\Gamma^v(\lambda),\ Q(\lambda) = \frac{1}{8\pi^2\lambda}[\Gamma_1^s(\lambda) - \Gamma_2^s(\lambda)]. \tag{9.77}$$

In the above equation, the $\alpha_R^v(\lambda)$ is a known quantity as given in Eq. (9.10). So, Eq. (9.76) is the equation used to determine the unknown quantity $g_s^R(\lambda)$ only. To solve the nonlinear equation, we may set

$$g_s^R(\lambda) = u(\lambda)^{-1/2} \tag{9.78}$$

which leads Eq. (9.76) to a linear equation obeyed by the function $u(\lambda)$,

$$\frac{du(\lambda)}{d\lambda} - 2P(\lambda)u(\lambda) - 2Q(\lambda) = 0. \tag{9.79}$$

When setting $Q(\lambda)=0$, we obtain a homogeneous equation whose solution is

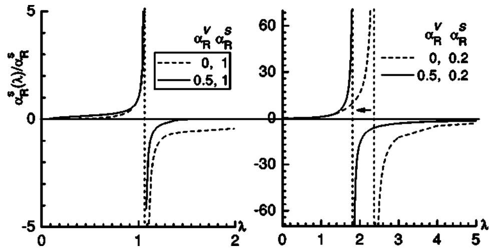

FIG. 14. The effective one-loop scalar coupling constants $\alpha_R^s(\lambda)$ obtained in the timelike momentum subtraction. The dashed and solid lines on the left represents the effective coupling constants given by $(\alpha_R^v,\alpha_R^s)=(0,1)$ and (0.5,1). The dashed and solid lines on the right denote the effective coupling constants given by $(\alpha_R^v,\alpha_R^s)=(0,0.2)$ and (0.5,0.2).

$$u(\lambda)=u(1)e^{2\int_1^\lambda d\lambda P(\lambda)}. \tag{9.80}$$

In order to seek the solution of Eq. (9.79), we assume

$$u(\lambda)=v(\lambda)e^{2\int_1^\lambda d\lambda P(\lambda)}, \tag{9.81}$$

where $v(\lambda)$ is an unknown function needs to be determined from Eq. (9.79). Inserting Eq. (9.81) into Eq. (9.79), we get

$$\frac{dv(\lambda)}{d\lambda}=2Q(\lambda)e^{-2\int_1^\lambda d\lambda P(\lambda)}. \tag{9.82}$$

Integrating the above equation, one obtains

$$v(\lambda)=u(1)+2\int_1^\lambda d\lambda Q(\lambda)e^{-2\int_1^\lambda d\lambda P(\lambda)}. \tag{9.83}$$

Combining the expressions in Eqs. (9.78), (9.81), and (9.83), the solution of Eq. (9.76) is finally given in the form

$$\alpha_R^s(\lambda)=\frac{\alpha_R^s K(\lambda)}{1+\alpha_R^s/\pi G_s(\lambda)}, \tag{9.84}$$

where $\alpha_R^s(\lambda)=[g_s^R(\lambda)]^2/4\pi$, $\alpha_R^s=\alpha_R^s(1)$ which is a parameter needed to be determined by fitting the experimental data,

$$K(\lambda)=e^{-(2/\pi)\int_1^\lambda d\lambda/\lambda\alpha_R^v(\lambda)\Gamma^v(\lambda)}, \tag{9.85}$$

and

$$G_s(\lambda)=\int_1^\lambda d\lambda/\lambda\Gamma^s(\lambda)K(\lambda) \tag{9.86}$$

in which

$$\Gamma^s(\lambda)=\Gamma_1^s(\lambda)-\Gamma_2^s(\lambda). \tag{9.87}$$

The behaviors of the effective coupling constant $\alpha_R^s(\lambda)$ obtained in the timelike and spacelike momentum subtractions are separately displayed in Figs. 14 and 15. For timelike momenta, the $\alpha_R^s(\lambda)$ is real. In Fig. 14, there are four lines representing this $\alpha_R^s(\lambda)$ which are given by four groups of the parameters $(\alpha_R^v,\alpha_R^s)=(0,1)$, (0.5,1), (0,0.2), and (0.5,0.2) respectively. The figure indicates that the $\alpha_R^s(\lambda)$ has a Landau pole λ_0. The poles for the four lines are respectively located about at $\lambda_0\approx 1.075\,23$ (for the first two lines), 1.8237, and 2.3967. Clearly, the λ_0 is larger if the both parameters (α_R^v,α_R^s) are smaller. In particular, the pole moves to the point near infinity when the both parameters (α_R^v,α_R^s) tend to zero. When λ goes from λ_0 to zero and infinity, each $\alpha_R^s(\lambda)$ in Fig. 14 abruptly falls to zero from the opposite directions. For spacelike momenta, the behavior of the effective coupling constant is described by the lines in Fig. 15. The two lines in Figs. 15(a) and (b) are given by considering the scalar coupling only with taking $\alpha_R^s=1$ and 0.2, respectively. In this case, the $\alpha_R^s(\lambda)$ is real and has a pole λ_0. The poles for the aforementioned lines are located respectively at $\lambda_0=5.725$ and 234.6. However, when the vectorial coupling is included, the $\alpha_R^s(\lambda)$ becomes complex. In this case, the pole disappears; instead, there is a maximum to appear as shown in Figs. 15(c) and (d). The lines in Figs. 15(c) and (d) are given by taking $(\alpha_R^v,\alpha_R^s)=(0.5,1)$ and (0.5,0.2), respectively. In the figures, the real part of the $\alpha_R^s(\lambda)$ is represented by the solid line and the imaginary part by the dashed line. From Fig. 15(c), we see that the real and imaginary parts of the $\alpha_R^s(\lambda)$ have sharp peaks corresponding to the pole of the upper line. The peaks are manifested more clearly by the right amplified lines. Figure 15(d) exhibits that either the real part or the imaginary part varies rather smoothly due to the decrement of the parameters (α_R^v,α_R^s) and the larger effect of the vectorial coupling. It is noted that when $\lambda\to 0$, the $\alpha_R^s(\lambda)$ reaches a constant, while in the limit of $\lambda\to\infty$, the $\alpha_R^s(\lambda)$ goes to zero. In particular, the behaviors of the $\alpha_R^s(\lambda)$ tell us that the smaller the parameters (α_R^v,α_R^s), the larger will be the range of applicability and the $\alpha_R^s(\lambda)$ for the spacelike momenta have a larger range of applicability than that for the timelike momenta.

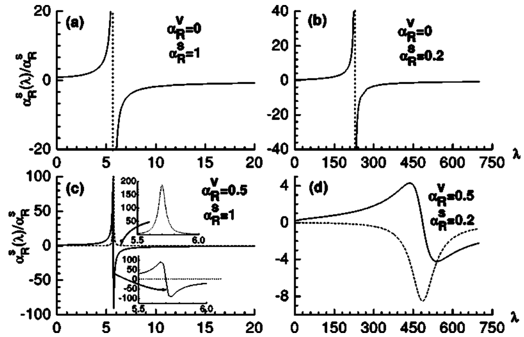

FIG. 15. The effective one-loop scalar coupling constants $\alpha_R^s(\lambda)$ obtained in the spacelike momentum subtraction. The four lines represent the effective coupling constants given by taking $(\alpha_R^v,\alpha_R^s)=(0,1)$, (0,0.2), (0.5,1), and (0.5,0.2), respectively. The solid and dashed lines denote the real parts and the imaginary parts of the coupling constants, respectively.

E. Effective σ meson mass

In accordance with Eq. (8.2), the RGE for the renormalized σ meson mass is

$$\lambda\frac{d}{d\lambda}m_\sigma^R(\lambda)+\gamma_m^\sigma(\lambda)m_\sigma^R(\lambda)=0, \tag{9.88}$$

where

$$\gamma_m^\sigma(\lambda) = \mu \frac{d}{d\mu} \ln Z_m^\sigma. \tag{9.89}$$

From the definitions given in Eqs. (7.6) and (7.9), it is found that in the approximation of order g_s^2, the renormalization constant Z_m^σ can be written as

$$Z_m^\sigma = 1 - \frac{1}{2}[\Omega_1(\mu^2) + \Omega_2(\mu^2)]. \tag{9.90}$$

The one-loop expressions of the divergent constants $\Omega_1(\mu^2)$ and $\Omega_2(\mu^2)$ were given in Eqs. (9.65) and (9.66). With these expressions, the renormalization constants Z_m^σ in Eq. (9.90) can explicitly be written out. Use of this renormalization constant in Eq. (9.89) yields the σ meson mass anomalous dimension as follows:

$$\gamma_m^\sigma = -\frac{g_s^2}{4\pi} G_s(\lambda), \tag{9.91}$$

where

$$G_s(\lambda) = \frac{6}{\pi}\left[\frac{1}{6} - \frac{1}{\beta_0^2} + \frac{1}{\lambda^2} - \frac{2}{\lambda}\left(\frac{1}{\lambda^2} - \frac{1}{\beta_0^2}\right)\eta(\lambda)\right] \tag{9.92}$$

in which $\beta_0 = m_\sigma / M$ and

$$\eta(\lambda) = \frac{1}{\sqrt{\lambda^2 - 4}} \ln \frac{\lambda - \sqrt{\lambda^2 - 4}}{\lambda + \sqrt{\lambda^2 - 4}}. \tag{9.93}$$

Substituting the above anomalous dimension into Eq. (9.88) and solving the equation, we obtain an expression of the effective σ meson mass such that

$$m_\sigma^R(\lambda) = m_\sigma^R e^{\int_1^\lambda (d\lambda/\lambda)\alpha_R^s(\lambda) G_s(\lambda)}, \tag{9.94}$$

where $m_\sigma^R = m_\sigma^R(1)$ is a mass parameter which needs to be determined by experiment. If the coupling constant $\alpha_R^s(\lambda)$ is approximately taken to be a constant α_R^s, the integral over λ can easily be calculated. In this case, we have

$$m_\sigma^R(\lambda) = m_\sigma^R e^{S_\sigma(\lambda)}, \tag{9.95}$$

where

$$S_\sigma(\lambda) = \frac{2\alpha_R^s}{\pi}\left[1 - \frac{1}{\lambda^2} + \frac{\sqrt{3}}{2}\left(1 - \frac{2}{\beta_0^2}\right)\pi + \left(\frac{1}{\lambda^2} + \frac{1}{2} - \frac{3}{\beta_0^2}\right) \times \frac{\sqrt{\lambda^2 - 4}}{\lambda} \ln \frac{1}{2}(\lambda + \sqrt{\lambda^2 - 4})\right]. \tag{9.96}$$

It is seen from Eq. (9.94) that the behavior of the effective mass $m_\sigma^R(\lambda)$ is intimately related to property of the effective coupling constant $\alpha_R^s(\lambda)$. To give a view of the behavior of the $m_\sigma^R(\lambda)$, we take the $m_\sigma^R(\lambda)$ evaluated from Eq. (9.94) by taking $(\alpha_R^v, \alpha_R^s) = (1,1)$ as an example. This $m_\sigma^R(\lambda)$ is shown in Fig. 16. In the figure, the dashed line represents the $m_\sigma^R(\lambda)$ given in the timelike momentum subtraction. This $m_\sigma^R(\lambda)$ is real and has a singularity at $\lambda_0 \approx 1.097\,58$ which implies that the range of applicability of the $m_\sigma^R(\lambda)$ is less than $\lambda = 1$, the nucleon mass scale. When λ tends to zero, the $m_\sigma^R(\lambda)$ approaches a value which does not deviate from the constant m_σ^R so much. The $m_\sigma^R(\lambda)$ given in the spacelike momentum subtraction is complex. The solid line in Fig. 16 represents the real part of the $m_\sigma^R(\lambda)$ which has a maximum near $\lambda_0 \approx 5$ which indicates that the $m_\sigma^R(\lambda)$ is applicable in a wide region of [0,5]. Similar to the coupling constant $\alpha_R^s(\lambda)$, when the parameters (α_R^v, α_R^s) are taken to be smaller, either the pole or the maximum will be shifted to the point of a large λ_0.

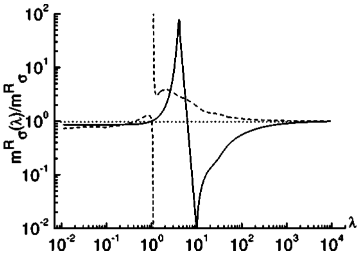

FIG. 16. The effective one-loop σ meson masses $m_\sigma^R(\lambda)$ obtained by taking $(\alpha_R^v, \alpha_R^s) = (1,1)$. The solid line represents the effective mass given in the spacelike momentum subtraction, the dashed line shows the real part of the $m_\sigma^R(\lambda)$ given in the timelike momentum subtraction.

X. SUMMARY AND DISCUSSIONS

In this paper, it has been argued that the σ-ω model, as a constrained system, is really of U(1) local gauge symmetry. This enables us to quantize the σ-ω model by means of the method used for quantizing the gauge field theory. In particular, the gauge symmetry allows us, in a consistent way, to derive various WT identities which provide a faithful basis for performing the renormalization of the model. As shown in Sec. V, the WT identity in Eq. (5.6) satisfied by the ω meson propagator and the WT identity in Eq. (5.13) for the vacuum polarization operator determine not only the structures of the propagator and the vacuum polarization operator, but also the renormalization fashion of the propagator and the ω meson mass as shown in Eqs. (5.14), (5.16), (5.21), and (5.22). Especially, the WT identity in Eq. (6.13) obeyed by the vertex gives rise to the correct manner of subtraction of the nucleon self-energy as denoted in Eq. (6.19). As shown in Sec. VI, the subtraction in Eq. (6.19) leads to the correct representations for the renormalization constants of nucleon propagator and nucleon mass as shown in Eqs. (6.21) and (6.25). Moreover, the identity in Eq. (6.13) directly yields the important relation between the renormalization constants Z_1 and Z_2 as written in Eq. (6.32). This relation together with the relation in Eq. (7.22) which follows from the identity in Eq. (7.21) greatly simplify the calculation of the renormalization. It would be mentioned here that in some previous works [7,15,17], the subtraction based on the expression $\Sigma(p) = A\not{p} + BM$ was ever used. This subtraction gives the nucleon propagator renormalization constant as $Z_2 = [1 - A(\mu^2)]^{-1}$ and nucleon mass renormalization constant as $Z_M = \{Z_2[1 + B(\mu^2)]\}^{-1}$ which are different from the renormalization constants written in Eqs. (6.21) and (6.25) and therefore the relation in Eq. (6.32) could not be fulfilled in this case. The renormalization of the model under consideration is performed in the mass-dependent momentum space subtraction scheme by the renormalization group approach. The prominent advantage of the subtraction scheme is that it naturally leads to the boundary conditions for the renormalized propagators, the vertices, and the wave functions. The boundary conditions allow us to give a unique determination of the solutions to the renormalization group equations for the renormalized propagators, vertices, and wave functions without any ambiguity. As claimed in the Introduction, we limit ourselves in this paper to examine the renormalization of the model at zero temperature by means of the renormalization group method. Since the perturbative series expanded in the powers of coupling constants is chosen to be the starting point of this renormalization, the results of the renormalization would be different from those obtained in the study of nuclear matter by using the loop expansion and the spectral function methods. Hopefully, the renormalization procedure described in this paper will be helpful for applying the renormalization group approach to study the nuclear matter at finite temperature and finite density.

The procedure of renormalization group method was demonstrated by the one-loop renormalization in this paper. Since the renormalization exactly respects the WT identities, the results obtained are faithful. Especially, the one-loop effective coupling constants and masses are given in the rigorous forms as they are derived from the mass-dependent momentum space subtraction. The subtraction scheme used is, in principle, suitable not only for high energy, but also for low energy, unlike the minimal subtraction scheme [28–31] which is only appropriate in the large momentum limit. In addition, the expressions of the one-loop effective physical quantities derived in this paper are applicable for the both of timelike momentum subtraction and spacelike momentum subtraction. As seen from Figs. 8–12 and 14–16, the behaviors of the effective quantities given in the timelike subtraction and the spacelike subtraction are much different from one another. In which case we should use the results given in the timelike momentum subtraction or in the spacelike momentum subtraction? The answer to this question depends on what process is discussed. For example, when we study the nucleon-nucleon scattering taking place in the t channel, as mentioned in Appendix B, the transfer momentum in the boson propagator is spacelike. In this case, it is suitable to take the effective coupling constants and boson masses given in the spacelike momentum subtraction. If we investigate the nucleon-antinucleon annihilation process which takes place in the s channel, since the transfer momentum is timelike, the effective coupling constants and boson masses given in the timelike momentum subtraction should be used. The effect of the one-loop renormalization is examined by the nucleon-nucleon elastic scattering whose differential cross section given in the order of g^2 is described in Appendix B and plotted in Fig. 17. In the figure, we only take the differential cross sections given at the laboratory kinetic energies T_{lab} $=491.9$ and 575.5 MeV as an example. The figure shows that consideration of the one-loop renormalization requires the coupling constants to be smaller in order to fit the experimental data. This actually is a general feature of considering the renormalization effect.

As exhibited in Sec. IX, the one-loop effective physical parameters given in the timelike momentum subtraction and

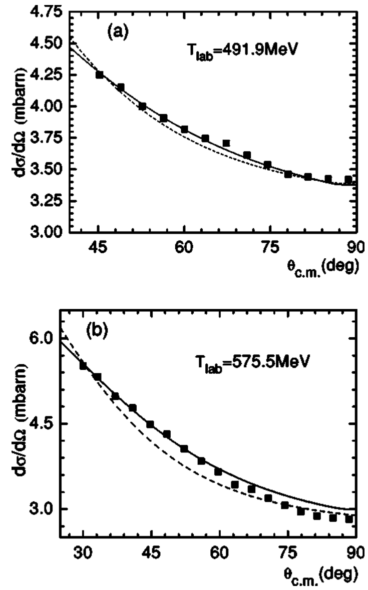

FIG. 17. The two-proton elastic differential cross sections given at the laboratory kinetic energies T_{lab}=491.9 and 575.5 MeV. The black squares show the experimental data [48]. The solid lines represent the theoretical values calculated by considering the one-loop renormalization effect. The dashed lines represent the theoretical values without considering the one-loop renormalization effect.

the ones given in the spacelike momentum subtraction not only behave differently, but also have different ranges of applicability because the singularities of the effective quantities given by the two subtractions appear at the different momenta. Especially, the positions of the singularities are strongly dependent on the coupling constants α_R^v and α_R^s. The smaller the coupling constants, the larger are the ranges of applicability. It would be mentioned that since the propagators written in Eqs. (5.16), (6.12), and (7.2) are solved from the Dyson equations, the one-loop renormalization actually contains the contribution given by partially summing up a set of chain loop diagrams. Just due to the partial summation, as mentioned before, the coupling constants must be set to be smaller for fitting the experimental data of the nucleon scattering. To this end, it is natural to ask if and how the behaviors of the one-loop effective physical parameters can be modified by considering higher order loop renormalizations, in other words, if the coupling constants would be smaller and the ranges of applicability of the renormalization would be enlarged when the contributions arising from more higher order loop diagrams are summed up. Obviously, this is an interesting problem worthy of pursuing further. In addition, we would like to address that the σ-ω model should be viewed as a restrictive model in which the σ field is introduced as a phenomenological field. Aside from the σ-ω model, there are some other models in QHD which are of a certain gauge symmetry. Especially, the model proposed by Sakurai in the early time [44], in our opinion, is most promising to describe the nuclear force because in this model, exchanges of the light mesons, such as pion and ρho meson dominate the strong interaction between nucleons. In view of the argument given in Refs. [32–34,45], Sakurai's model is a SU(2) gauge field theory which is not only gauge invariant, but also renormalizable. Certainly, the renormalization of this model may be investigated along the same line as described in this paper. We will discuss this subject in the future. But, it cannot be expected that a perturbative investigation could give an ultimate solution to the strong interaction. Just as said in Refs. [46,47], to resolve the strong interaction, it is adequate to perform a nonperturbative study of the interaction kernel appearing in the relativistic equation whose closed expression can be derived by the procedure as described in Refs. [46,47].

ACKNOWLEDGMENT

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APPENDIX A: GAUGE INDEPENDENCE OF S-MATRIX ELEMENTS

The gauge independence of S-matrix elements computed by a gauge field theory is a well-known fact. For the σ-ω model, as argued in this paper, it actually is a U(1) gauge field theory. So, the same conclusion should hold for the σ-ω model. To convince oneself of this fact, we take the nucleon-nucleon scattering amplitudes up to the one-loop approximation as examples to show that the matrix elements given by the σ-ω model are surely independent of the gauge parameter α. The typical Feynman diagrams representing the scattering amplitudes are depicted in Figs. 5 and 6. In Fig. 6, only the diagrams with the internal ω meson line are necessary to be considered.

For the tree diagram in Fig. 5(a), the gauge-independence of its S-matrix element is well known. In fact, the S-matrix element

$$S_1 = \overline{u_{s'}}(q_2) i g_v \gamma_\mu u_{r'}(q_1) i D^{\mu\nu}(k) \overline{u_s}(p_2) i g_v \gamma_\nu u_r(p_1), \tag{A1}$$

where $u_s(p)$, the free nucleon wave function, can be divided into two parts according to the decomposition of free ω meson propagator,

$$D^{\mu\nu}(k) = D_F^{\mu\nu}(k) + D_\alpha^{\mu\nu}(k), \tag{A2}$$

where

$$D_F^{\mu\nu}(k) = -\frac{g^{\mu\nu}}{k^2 - m_\omega^2 + i\varepsilon} \tag{A3}$$

which is the propagator given in the Feynman gauge and

$$D_\alpha^{\mu\nu}(k) = (1-\alpha) D_\alpha(k^2) k^\mu k^\nu \tag{A4}$$

which is the α-dependent part of the propagator in which

$$D(k^2) = \frac{1}{(k^2 - m_\omega^2 + i\varepsilon)(k^2 - \nu^2 + i\varepsilon)}. \tag{A5}$$

For the α-dependent part of S_1, applying the energy-momentum conservation $k = q_1 - q_2 = p_2 - p_1$ and Dirac equation $(\not{p} - M) u_s(p) = 0$, it is found that

$$\begin{aligned} S_1^\alpha &= -(1-\alpha) i g_v^2 \overline{u_{s'}}(q_2) \not{k} u_{r'}(q_1) \overline{u_s}(p_2) \not{k} u_r(p_1) D_\alpha(k^2) \\ &= -(1-\alpha) i g_v^2 \overline{u_{s'}}(q_2) (\not{q}_1 - \not{q}_2) u_{r'}(q_1) \overline{u_s}(p_2) (\not{p}_2 - \not{p}_1) \\ &\quad \times u_r(p_1) D_\alpha(k^2) = 0. \end{aligned} \tag{A6}$$

This shows that the ω meson propagator given in the Feynman gauge is sufficient to use for evaluating the tree diagram matrix element. In the same way, one may prove that the S-matrix element given by the tree diagram in Fig. 5(b) is independent of the gauge parameter as well.

Let us focus on the one-loop diagrams in Fig. 6 where only the direct diagrams are plotted and necessary to be examined for our purpose. The gauge independence of the matrix element of Fig. 6(a) which contains a ω meson self-energy in it can easily be proved by using Eq. (A6). So, we only need to examine the gauge independence of Figs. 6(b)–(f). The matrix element of Fig. 6(b) with a vertex correction in it can be written as

$$S_2^1 = M_\mu(q_1, q_2) A^\mu(p_1, p_2), \tag{A7}$$

where

$$M_\mu(q_1, q_2) = \overline{u_{s'}}(q_2) i g_v \gamma_\nu u_{r'}(q_1) i D^{\nu\mu}(k) \tag{A8}$$

which is gauge independent as shown in Eq. (A6) and

$$\begin{aligned} A^\mu(p_1, p_2) = \int \frac{d^4k}{(2\pi)^4} \overline{u_s}(p_2) & \\ \times i g_v \gamma_\rho i S_F(p_2 - k) & i g_v \gamma^\mu i S_F(p_1 - k) i \\ \times g_v \gamma_\sigma u_r(p_1) & i D^{\rho\sigma}(k). \end{aligned} \tag{A9}$$

Replacing $D^{\rho\sigma}(k)$ by the $D_\alpha^{\rho\sigma}(k)$ shown in Eq. (A4), we have the following gauge-dependent part of $A^\mu(p_1, p_2)$:

$$\begin{aligned} A_\alpha^\mu(p_1, p_2) = -(1-\alpha) g_v^3 \int \frac{d^4k}{(2\pi)^4} & \overline{u_s}(p_2) \not{k} S_F(p_2 - k) \\ & \times \gamma^\mu S_F(p_1 - k) \not{k} u_r(p_1) D_\alpha(k^2), \end{aligned} \tag{A10}$$

where $\not{k} = \gamma^\mu k_\mu$ can be written in the form

$$\not{k} = S_F^{-1}(p_i) - S_F^{-1}(p_i - k), \tag{A11}$$

where $i = 1, 2$. Using this relation and the Dirac equation, Eq. (A10) becomes

$$A_\alpha^\mu(p_1, p_2) = -(1-\alpha) g_v^3 \overline{u_s}(p_2) \gamma^\mu u_r(p_1) J, \tag{A12}$$

where

$$J = \int \frac{d^4k}{(2\pi)^4} D_\alpha(k^2) = \lim_{\varepsilon \to 0} \frac{i}{(4\pi)^2} \int_0^1 \frac{dx}{\varepsilon \{[\alpha + (1-\alpha)x] m_\omega^2\}^\varepsilon}, \tag{A13}$$

where the last equality is given by the dimensional regularization. This integral gives a divergent constant without containing any finite number in it. Therefore it may completely be canceled out by a counterterm in a renormalization program and gives no contribution to the renormalized S-matrix element. On the other hand, since the integral is independent of momentum, it would not contribute to the anomalous dimension and hence to any physical quantity.

For the matrix element of Fig. 6(c) which contains a nucleon self-energy, it may be written as

$$S_2^2 = M_\mu(q_1,q_2)B^\mu(p_1,p_2), \quad \text{(A14)}$$

where

$$B^\mu(p_1,p_2) = \int \frac{d^4k}{(2\pi)^4}\overline{u_s}(p_2) \times ig_v\gamma_\rho iS_F(p_2-k)ig_v\gamma_\sigma iS_F(p_1-k) \times ig_v\gamma^\mu u_r(p_1)iD^{\rho\sigma}(k) \quad \text{(A15)}$$

in which the α-dependent part is of the form

$$B_\alpha^\mu(p_1,p_2) = -(1-\alpha)g_v^3\int \frac{d^4k}{(2\pi)^4}\overline{u_s}(p_2)\not{k}S_F(p_2-k) \times \not{k}S_F(p_1+q)\gamma^\mu u_r(p_1)D_\alpha(k^2). \quad \text{(A16)}$$

By employing the relation in Eq. (A11) and Dirac equation, one may find

$$B_\alpha^\mu(p_1,p_2) = (1-\alpha)g_v^3\overline{u_s}(p_2)\gamma_\nu S_F(p_2)\gamma^\mu u_r(p_1)J_2^\nu = 0. \quad \text{(A17)}$$

This is because the integral in it vanishes,

$$J_2^\nu = \int \frac{d^4k}{(2\pi)^4}k^\nu D_\alpha(k^2) = 0, \quad \text{(A18)}$$

due to that the integrand is an odd function. Similarly, the α-dependent part of Fig. 6(d) can also be proved to give no contribution to the S-matrix element.

Let us turn to Figs. 6(e) and (f). The matrix elements of both figures can be respectively represented as

$$S_2^3 = g_v^4\int \frac{d^4k}{(2\pi)^4}\overline{u_{s'}}(q_2)\gamma^\mu S_F(q_1-k)\gamma^\nu u_{r'}(q_1) \times \overline{u_s}(p_2)\gamma^\rho S_F(p_1+k)\gamma^\sigma u_r(p_1)D_{\mu\rho}(q-k)D_{\nu\sigma}(k) \quad \text{(A19)}$$

and

$$S_2^4 = g_v^4\int \frac{d^4k}{(2\pi)^4}\overline{u_{s'}}(q_2)\gamma^\mu S_F(q_1-k)\gamma^\nu u_{r'}(q_1) \times \overline{u_s}(p_2)\gamma^\rho S_F(p_2-k)\gamma^\sigma u_r(p_1)D_{\mu\sigma}(q-k)D_{\nu\rho}(k), \quad \text{(A20)}$$

where $q=p_2-p_1=q_1-q_2$. Their α-dependent parts are denoted by $S_{2\alpha}^3$ and $S_{2\alpha}^4$. By making use of the relation in Eq. (A11) and the relation $q-k=(q_1-k)-q_2=p_2-(k+p_1)$ as well as the Dirac equation, it is easy to find

$$S_{2\alpha}^3 = -2(1-\alpha)g_v^4\overline{u_{s'}}(q_2)\gamma^\mu u_{r'}(q_1)\overline{u_s}(p_2)\gamma_\mu u_r(p_1) \times \int \frac{d^4k}{(2\pi)^4}\frac{D_\alpha(k^2)}{(q-k)^2-m_\omega^2+i\varepsilon} - (1-\alpha)^2g_v^4\overline{u_{s'}}(q_2) \times \gamma^\mu u_{r'}(q_1)\overline{u_s}(p_2)\gamma^\nu u_r(p_1) \times \int \frac{d^4k}{(2\pi)^4}k_\mu k_\nu D_\alpha[(q-k)^2]D_\alpha(k^2) = -S_{2\alpha}^4 \quad \text{(A21)}$$

which gives $S_{2\alpha}^3+S_{2\alpha}^4=0$ so that the sum of S_2^3 and S_2^4 is independent of the gauge parameter.

APPENDIX B: CROSS SECTION OF NUCLEON-NUCLEON SCATTERING

To illustrate the effect of the renormalization described in this paper, we evaluate the cross section of the nucleon-nucleon elastic scattering. Here we limit ourself to first consider the cross section given in the approximation of order g^2. In this approximation, only the tree diagrams denoted in Fig. 5 are concerned. From these diagrams, in the center of mass frame, the differential cross section is easily calculated and represented as follows:

$$\frac{d\sigma}{d\Omega(\theta,\varphi)} = \frac{1}{S}(\alpha_v^2T_v + \alpha_s^2T_s - \alpha_v\alpha_sT_{vs}), \quad \text{(B1)}$$

where $S=4(p^2+M^2)$ with p being the nucleon momentum is the total energy of the system, T_v is contributed from the ω meson exchange interaction, T_s is given by the σ meson exchange interaction and T_{vs} is the crossed term related to both of the ω meson and σ meson exchanges. They are separately represented as follows:

$$T_v = \frac{R_1^v}{(\Delta_1^v)^2} + \frac{R_2^v}{(\Delta_2^v)^2} + (-1)^{1+I}\frac{R_3^v}{\Delta_1^v\Delta_2^v}, \quad \text{(B2)}$$

where

$$R_1^v = M^4 + 2p^2M^2\cos\theta + 2M^2p^2 + 2p^4 + 2p^4\cos^4\theta/2, \quad \text{(B3)}$$

$$R_2^v = M^4 - 2p^2M^2\cos\theta + 2M^2p^2 + 2p^4 + 2p^4\sin^4\theta/2, \quad \text{(B4)}$$

$$R_3^v = 16(p^4 - M^4), \quad \text{(B5)}$$

$$\Delta_1^v = 4p^2\sin^2\theta/2 + m_\omega^2, \quad \text{(B6)}$$

and

$$\Delta_2^v = 4p^2\cos^2\theta/2 + m_\omega^2. \quad \text{(B7)}$$

$$T_s = \frac{R_1^s}{(\Delta_1^s)^2} + \frac{R_2^s}{(\Delta_2^s)^2} + (-1)^I\frac{R_3^s}{\Delta_1^s\Delta_2^s}, \quad \text{(B8)}$$

where

$$R_1^s = 4(4p^2\sin^2\theta/2 + M^2)^2, \quad \text{(B9)}$$

$$R_2^s = 4(4p^2\cos^2\theta/2 + M^2)^2, \quad \text{(B10)}$$

$$R_3^s = 2[2M^2(p^2+M^2) + p^4(\sin^4\theta/2 + \cos^4\theta/2)], \quad \text{(B11)}$$

$$\Delta_1^s = 4p^2\sin^2\theta/2 + m_\sigma^2, \quad \text{(B12)}$$

and

$$\Delta_2^s = 4p^2 \cos^2 \theta/2 + m_\sigma^2. \tag{B13}$$

$$T_{vs} = \frac{R_1^{vs}}{\Delta_1^v \Delta_1^s} + \frac{R_2^{vs}}{\Delta_2^v \Delta_2^s} + (-1)^I \frac{R_3^{vs}}{\Delta_1^v \Delta_2^s} + (-1)^I \frac{R_4^{vs}}{\Delta_2^v \Delta_1^s}, \tag{B14}$$

where

$$R_1^{vs} = 4M^2[(M^2 + 2p^2)^2 + (M^2 + 2p^2 \cos^2 \theta/2)^2], \tag{B15}$$

$$R_2^{vs} = 4M^2[(M^2 + 2p^2)^2 + (M^2 + 2p^2 \sin^2 \theta/2)^2], \tag{B16}$$

$$R_3^{vs} = 4[2(M^2 + p^2 \cos^2 \theta/2)^2 - M^2(M^2 + p^2 + p^2 \sin^2 \theta/2)], \tag{B17}$$

and

$$R_4^{vs} = 4[2(M^2 + p^2 \sin^2 \theta/2)^2 - M^2(M^2 + p^2 + p^2 \cos^2 \theta/2)]. \tag{B18}$$

In the above, (θ, φ) are the scattering angles, I is the isospin of the two-nucleon system, Δ_k^v and Δ_k^s $(k=1,2)$ are respectively given by the ω meson and σ meson propagators, and R_i^α $(\alpha=v,s,i=1,2,3)$ are the functions coming from the nucleon spinor matrix elements. In Eqs. (B2), (B8), and (B14), the isospin-related terms arise from the exchanged diagram, while the remaining terms represent the contribution of the direct diagram.

To consider the renormalization effect on the two-nucleon scattering, as mentioned in Sec. VIII, we may directly replace the coupling constant α_v and α_s in Eq. (B1) by their effective ones $\alpha_R^v(\lambda)$ and $\alpha_R^s(\lambda)$ and the particle masses M, m_ω, and m_σ appearing in the propagators and the functions R_i^α by their effective counterparts $M_R(\lambda)$, $m_\omega^R(\lambda)$, and $m_\sigma^R(\lambda)$. To this end, it should be noted that the nucleon spinors in the S-matrix element under consideration are on the mass shell, satisfying the free nucleon Dirac equation. The momenta p_i in the spinors are timelike because they meet the relation $p^2=M^2$ where M is real. For the renormalized spinor wave function shown in Eq. (8.14), as easily verified, it also satisfies the Dirac equation and the momentum in the spinor fulfills the relation $p(\lambda)^2=M_R(\lambda)^2$ where $p(\lambda)=(E(\lambda),\vec{p})$ with $E(\lambda)=[\vec{p}^2+M_R(\lambda)^2]^{1/2}$. Therefore, for the nucleon-nucleon scattering, it is adequate to take the effective nucleon mass given in the timelike momentum space subtraction, while the momenta in the ω meson and σ meson propagators, as one knows, are off shell and spacelike in the t-channel scattering. Therefore it is appropriate to take the effective coupling constants $\alpha_R^v(\lambda)$ and $\alpha_R^s(\lambda)$ and the effective meson masses $m_\omega^R(\lambda)$ and $m_\sigma^R(\lambda)$ given in the spacelike momentum space subtraction. In this paper, we only examine the effect of the one-loop renormalization on the two-proton scattering by using the effective coupling constants and masses presented in Sec. IX. The differential cross sections given at the laboratory kinetic energies $T_{lab}=491.9$ and 575.5 MeV are shown in Figs. 17(a) and (b). In the figures, the solid lines and the dashed lines represent, respectively, the calculated results with and without considering the renormalization effect and the experimental data are taken from Ref. [48]. As shown in Figs. 17(a) and (b), in the case without considering the renormalization effect, the theoretical parameters are taken to be $M_R=938$ MeV, $\alpha_\omega^R=1.1$, $\alpha_\sigma^R=1.4$, $m_\omega^R=782$ MeV, and $m_\sigma^R=580$ MeV; while in the case of considering the renormalization effect, the parameters must be taken to be $M_R=938$ MeV, $\alpha_\omega^R=0.55$, $\alpha_\sigma^R=0.62$, $m_\omega^R=782$ MeV, and $m_\sigma^R=670$ MeV. It is clearly seen from the figures that consideration of the renormalization has an effect that to fit the experimental data, the coupling constants must be set to be smaller.

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