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Bose-Einstein condensation of α particles and Airy structure in nuclear rainbow scattering

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It is shown that the dilute density distribution of α particles in nuclei can be observed in the Airy structure in nuclear rainbow scattering. We have analyzed $\alpha+{}^{12}$C rainbow scattering to the 0_2^+ (7.65 MeV) state of ^{12}C in a coupled-channel method with the precise wave functions for ^{12}C. It is found that the enhanced Airy oscillations in the experimental angular distributions for the 0_2^+ state is caused by the dilute density distribution of this state in agreement for the idea of Bose-Einstein condensation of the three alpha particles.

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Bose-Einstein condensation (BEC) has been well known for liquid ^{4}He and recently in a dilute gas [1]. For systems of strong interaction, pion condensation and kaon condensation have been proposed but have not been confirmed experimentally. An α particle, which is a boson composed of two protons and two neutrons, also plays an important role in understanding the structure of nuclei. In fact, the success of the α-cluster model in light-and medium-weight nuclei [2] shows that an α particle behaves as a constituent unit in nuclei. Recently, α-particle condensation in light nuclei, especially in ^{12}C and ^{16}O, has been proposed [3]. Uegaki *et al.* [4] and Kamimura *et al.* [5] have shown that the 0_2^+ (7.65 MeV) state of ^{12}C, which is 0.39 MeV above the α threshold, has a well-developed α-cluster structure with the $^8\mathrm{Be}\otimes\alpha$ configuration. Tohsaki *et al.* [3] conjectured that this may be a BEC state of α particles in that weakly interacting three α particles are sitting in the lowest single-particle 0*s* state like a gas with a dilute density distribution obeying Bose statistics. Its calculated rms radius, 4.29 fm, is far larger than the experimental rms of the ground state, 2.65 fm. However, the experimental rms radius for this excited state is unfortunately not available. This dilute property has not been experimentally confirmed in a direct way. In this paper we show that the dilute density distribution of the 0_2^+ state can be confirmed in the Airy structure observed in $\alpha+{}^{12}$C rainbow scattering.

Nuclear rainbow scattering is well known in α-particle scattering [6] and in some light heavy-ion scattering such as $^{16}\mathrm{O}+{}^{16}\mathrm{O}$, $^{16}\mathrm{O}+{}^{12}\mathrm{C}$, and $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ [7], for which absorption is weak. The rainbow scattering and the associated Airy structure can be well described by a deep folding type potential. The angular distributions of rainbow scattering are sensitive to the potential up to very internal region, which made it possible to determine the interaction potential uniquely and precisely. For example, the equation of state of nuclear matter was concluded to be soft from the study of rainbow scattering using a folding model potential [8]. This shows that the wave function of the nucleus used in deriving the folding potential is very sensitive to rainbow scattering.

Rainbow phenomena are only observed in the limited nuclei where absorption is incomplete. Therefore, the existence of nuclear rainbow in inelastic scattering is rarely expected. In fact, there are only a few measurements of inelastic rainbow scattering involving α particles and light heavy ions. Among them, it is suggested that $^{12}\mathrm{C}+{}^{12}\mathrm{C}(2^+\ 4.44\ \mathrm{MeV})$ scattering shows some similarity to elastic rainbow scattering [9,10]. On the other hand, from the theoretical point of view, inelastic nuclear rainbow scattering has hardly been studied partly because there is no classical counterpart in meteorological rainbow scattering. To reveal the existence, as well as its mechanism, of inelastic nuclear rainbow scattering has been a challenging subject in nuclear physics [9].

Very recently, Michel and Ohkubo [11] have shown the existence of an Airy structure in inelastic scattering and its mechanism by studying the $\alpha+{}^{40}$Ca system systematically. The existence of Airy minima has been clearly shown in the inelastic scattering of $\alpha+{}^{40}\mathrm{Ca}(3^-\ 3.74\ \mathrm{MeV})$ and $\alpha+{}^{40}\mathrm{Ca}(0_2^+\ 3.35\ \mathrm{MeV})$ at the incident energy of $E_\alpha=50$ MeV. The mechanism was explained in the same way as in the elastic scattering, in terms of barrier-wave and internal-wave decomposition. It was shown that the Airy minima in the angular distributions for inelastic scattering are shifted to larger angles compared with those for elastic scattering. Also, the larger the excitation energy of the state, the more the Airy minima are shifted to larger angles. The 0_1^+ (0.0 MeV), 0_2^+ (3.35 MeV), and 3^- (3.74 MeV) states of ^{40}Ca have normal density distributions.

A nucleus behaves like a lens, that is, the trajectories of the incident particles go through the focus of the target nucleus [12]. This is especially clear at the low incident energy region where anomalous large angle scattering (ALAS) [13] is observed. Recently, it has been shown that the nucleus is very similar to a Lunenburg lens and the nuclear rainbow is understood to be caused as an astigmatism of the lens [14]. Also, in the energy region where the effective potential active in the scattering displays a pocket, the origin of the Airy structure is understood in a consistent way in terms of internal-wave and barrier-wave decomposition of the scattering amplitude [14–16]. As illustrated in Fig. 1, this suggests that if the density distribution of the 0_2^+ state of ^{12}C is dilute, its role as a lens should be qualitatively very different from that of the ground state with a normal matter distribution. The size of the lens for the 0_2^+ state would be considerably larger than that for the ground state. If the two lenses, that is, the two potentials for the ground state 0_1^+ and the 0_2^+ are so different, this difference would be observed in the Airy struc-

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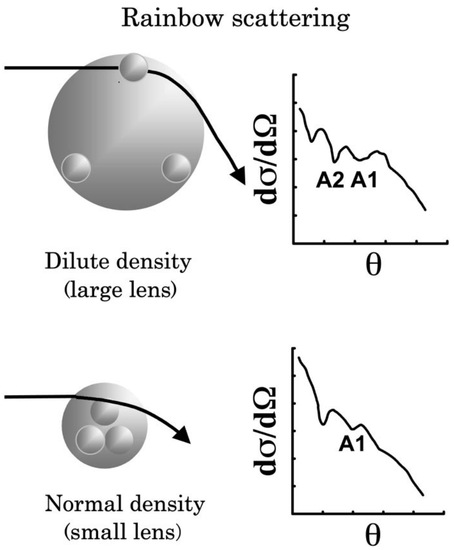

FIG. 1. Illustration of $\alpha+{}^{12}$C rainbow scattering from the 0_1^+ state (with a compact lens) and the 0_2^+ state (with a large lens) and the Airy structure in the angular distributions.

ture of rainbow scattering involving ^{12}C. For the latter, the number of the Airy minima in the angular distributions should certainly increase. According to Ref. [3], the volume of the 0_2^+ state is about five times as large as that of the 0_1^+ state.

In this respect, inelastic rainbow scattering data involving the excitation to the 0_2^+ state of ^{12}C play an important role in extracting the structure information about the unusual behavior of this state. Fortunately there are experimental data of rainbow scattering for the $\alpha+{}^{12}$C system involving the excitation to the relevant 0_2^+ state [17–20]; to the best our knowledge, these data have not received attention from the viewpoint of BEC. We think these data are very useful at this stage in investigating whether the ^{12}C (0_2^+) is in a dilute state or not, considering that there are no direct experimental data concerning the rms radius for this state and that all the papers on the BEC of the 0_2^+ state published up to now [3,21–24] are based on the theoretical conjectures through the elaborate analyses of the wave function of the 0_2^+ state.

Elastic $\alpha+{}^{12}$C scattering was thoroughly measured from a low energy to a high energy region and systematic data are available. However, the well-known difficulty for this system is that there has been no global optical potential which describes the scattering data from a low energy to a high energy region consistently. This situation is in contrast to the $\alpha+{}^{16}$O, $\alpha+{}^{15}$N, and $\alpha+{}^{14}$C systems, for which the global potentials have been uniquely established from systematic analyses of the elastic scattering data. These global potentials are essentially similar and well reproduced by a folding model potential. This contrast seems to be related to the peculiarity of the structure of the ^{12}C nucleus. Although a global $\alpha-{}^{12}$C potential has been an open problem for many years, it has been shown that a folding-type deep potential can reproduce the experimental angular distributions at higher energy region in the coupled channel method [25].

TABLE I. The normalization factor N_R, volume integral per nucleon pair J_V, rms radius $\langle R^2\rangle^{1/2}$, of the folding potential, and the strength of the imaginary potential. The radius R_W=4.7 fm and diffuseness a_W=0.7 fm of the imaginary potentials are used.

E_L (MeV)	N_R	J^π	J_V (MeV fm^3)	$\langle R^2\rangle^{1/2}$ (fm)	W (MeV)
139	1.23	0_1^+	294	3.484	8.0
		2^+	291	3.469	10.0
		3^-	327	3.742	14.0
		0_2^+	366	4.304	28.0
166	1.26	0_1^+	286	3.499	7.0
		2^+	283	3.485	10.0
		3^-	320	3.752	14.0
		0_2^+	358	4.310	27.0
172.5	1.26	0_1^+	292	3.506	7.5
		2^+	289	3.493	9.0
		3^-	327	3.758	12.0
		0_2^+	367	4.315	25.0
240	1.42	0_1^+	264	3.532	7.5
		2^+	261	3.519	12.0
		3^-	295	3.777	17.0
		0_2^+	330	4.328	30.0

We analyze the elastic and inelastic angular distributions of $\alpha+{}^{12}$C rainbow scattering in the coupled-channel method by taking into account simultaneously the 0_1^+ (0.0 MeV), 2^+ (4.43 MeV), 0_2^+ (7.65 MeV), and 3^- (9.63 MeV) states of ^{12}C. The diagonal and coupling potentials for the alpha-^{12}C system are calculated by the double-folding (DF) model,

$$V_{ij}(\mathbf{R}) = \int \rho_{00}^{(\alpha)}(\mathbf{r}_1)\rho_{ij}^{(^{12}\mathrm{C})}(\mathbf{r}_2)v_{\mathrm{NN}}(E,\rho,\mathbf{r}_1+\mathbf{R}-\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2, \tag{1}$$

where $\rho_{00}^{(\alpha)}(\mathbf{r})$ is the ground state density of alpha nucleus, while v_{NN} denotes the density-dependent M3Y effective interaction (DDM3Y) usually used in the DF model. $\rho_{ij}^{(^{12}\mathrm{C})}(\mathbf{r})$ represents the diagonal ($i=j$) or transition ($i\neq j$) nucleon density of ^{12}C which is calculated in the resonating group method by Kamimura *et al.* [5]. This coupled-channel method was successfully used in the analyses of the reactions involving ^{12}C [26]. In the calculation of densities of ^{12}C, the shell-like structure of the ground state 0_1^+, 2^+ (4.43 MeV), and 3^- (9.63 MeV) states, and the well-developed α-cluster structure of the 0_2^+ (7.65 MeV) state are simultaneously well reproduced. These wave functions have been checked for many experimental data, including charge form factors, electric transition probabilities, and reactions involving excitation to the 0_2^+ state [5,26]. (Recently, it was shown that this wave function for the 0_2^+ is almost completely equivalent to the Bose condensate wave function [22].) In the calculation the normalization factor N_R for the real part of the potential is introduced. Imaginary potentials with a Wood-Saxon form factor are introduced phenomenologically for each channel. The properties of the real folding potential and potential parameters used are given in Table I. The energy dependence of

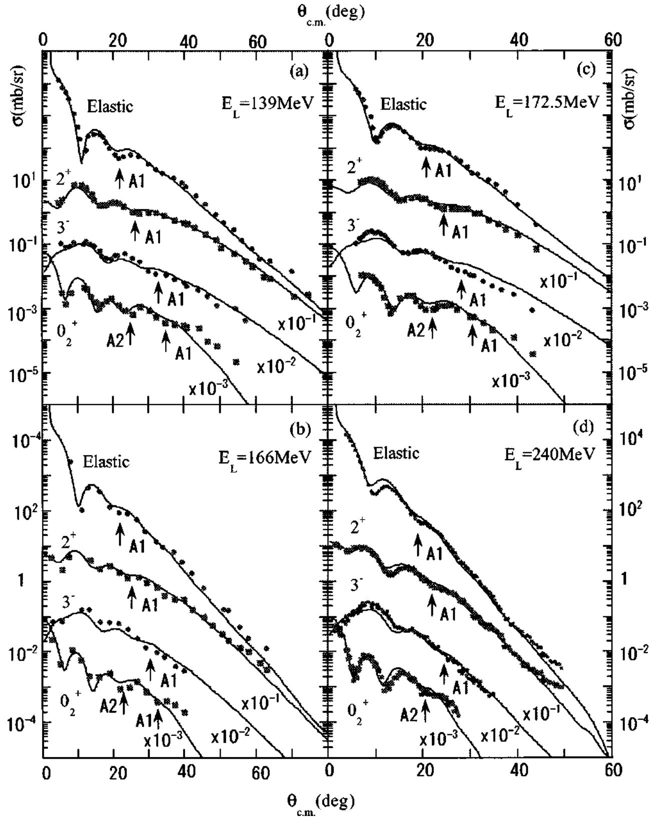

FIG. 2. The elastic and inelastic angular distributions of $\alpha+{}^{12}$C scattering calculated in the coupled channel method (solid lines) are compared with the experimental data (points) [17–20]. Airy minimum is indicaded by an arrow.

the obtained potential parameters is reasonable. Table I shows that the normalization factor increases at higher energies. However, the obtained volume integral shows reasonable energy dependence that is consistent with the phenomenological potentials in α-particle scattering.

Figure 2 shows the calculated results in comparison with the experimental angular distributions of elastic and inelastic α-particle scattering from ^{12}C at the incident energies of $E_L=139$, 166, 172.5, and 240 MeV [17–20]. The data of elastic angular distributions show a typical rainbow falloff followed by the Airy minima in the forward angular region. Similar structures with maxima and minima, as well as the falloff of the angular distributions, are also observed in the inelastic scattering data. We notice the following points in the inelastic scattering data: (1) The angle that the falloff starts is shifted to a larger angle than that of elastic scattering. (2) For the 0_2^+ state, the number of the Airy oscillations is larger than that for the 0_1^+, 2^+, and the 3^- states. The characteristic features of the experimental angular distributions are reproduced by the calculations.

The first Airy minimum $A1(0_1^+)$ for elastic scattering can be determined without ambiguity by calculating the farside cross sections in the optical model that reproduces the experimental data. Also by calculating the deflection function for the real part of the optical potential, the rainbow angle, therefore, the angle of $A1$ can be confirmed. As seen in Fig. 2(c), for example at $E_L=172.5$ MeV, the minimum at about $\theta=20°$ is due to far side scattering, which is consistent with the other calculation from the deflection angle [27] and can be labeled as the first Airy minimum $A1(0_1^+)$. As shown in Ref. [11], the first Airy minimum $A1(2^+)$ for the 2^+ state is slightly shifted to a larger angle than that of the elastic scattering due to its excitation energy 4.44 MeV. Similarly, the $A1(3^-)$ for the 3^- state is a little more shifted to a larger angle than that of the $A1(0_1^+)$ and $A1(2^+)$. Likewise, the $A1(0_2^+)$ for the 0_2^+ state corresponds to the minimum at about 31°. As seen in Table I, this rather large shift to a larger angle is due to its very large volume integral as well as its large excitation energy of this state. Similarly, the Airy minima $A1$ for $E_L=166$ and 139 MeV are determined in the experimental data. Very recent data at 240 MeV [20] are consistent with this assignment.

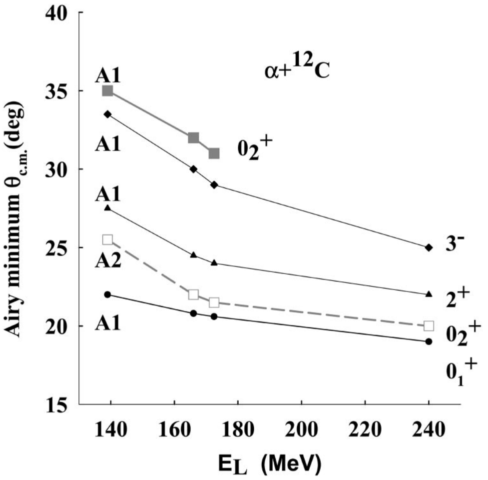

FIG. 3. (Color online) Angular position of the first (filled symbols) and second (open squares) Airy minimum for the 0_1^+, 2^+ (4.44 MeV), 3^- (9.63 MeV), and 0_2^+ (7.65 MeV) states of ^{12}C in $\alpha+{}^{12}$C scattering versus the incident energy. The lines are only to guide the eye.

In Fig. 3 we see that the positions of $A1$ for the 0_1^+, 2^+, and 3^- states are shifted to larger angles as the excitation energy increases. Since the potential for these three states is almost the same (Table I), this shift is understood to be due to its excitation energy. A similar trend is typically seen in inelastic $\alpha+{}^{40}$Ca scattering to the excited states of 0_2^+ (3.35 MeV) and 3^- (3.73 MeV), as recently shown in Ref. [11]. Interestingly, the position of the $A1(0_2^+)$ and $A2(0_2^+)$ is not in this line. If the lens for the 0_2^+ state is not very different from the other three states, the position of the $A1(0_2^+)$ and $A2(0_2^+)$ should be shifted to larger angles than that of the 2^+ state and be smaller than that of the 3^-. However, on the contrary, they are more shifted to larger angles than that of the 3^- state. This shows that the refractive effect is much stronger for the 0_2^+ than the 3^- state. In other words, the potential is much more attractive for the 0_2^+ than the 3^- state as well as the 0_1^+ and 2^+ states. This consideration is supported further by the fact that the second Airy minimum $A2$ is only observed for the 0_2^+ state in the experimental angular distributions. This increase of the number of the Airy minima can be understood reasonably from our theoretical results. As seen in Table I, the volume integral for the 0_2^+ is much larger than those for the other states and the potential belongs to a deeper family. It is well known that as the volume integral becomes larger, the number of the Airy minima increases [28]. The fact that the number of the Airy minima for the 0_2^+ state is increased shows that the lens is extremely strong compared with that for the other states. In fact, as seen in Table I, the calculated rms radius of the potential for the 0_2^+ is very large compared with that for the 0_1^+ state. This means that the rms of the density distribution of the 0_2^+ state of ^{12}C is extremely large. Thus it is shown that the Airy structure in the experimental angular distributions for the 0_2^+ state in $\alpha+^{12}$C rainbow scattering supports that the matter distribution is dilute in agreement for the idea of BEC [3].

To summarize, we have investigated the Airy structure of elastic and inelastic $\alpha+^{12}$C rainbow scattering in the coupled-channel calculations with the folding potential derived by using the precise wave functions for the ^{12}C nucleus. It was shown that the nuclear lens for the 0_2^+ (7.65 MeV) state causes extremely strong refraction, which means the dilute matter distribution of the state and supports the idea of Bose-Einstein condensate of the three α particles. The present approach may be also applied to rainbow scattering to the $n\alpha$-particle states of $4N$ nuclei near the threshold such as the four α-particle state in ^{16}O.

[1] E. A. Cornel and C. E. Wieman, Rev. Mod. Phys. **74**, 875 (2002).
[2] S. Ohkubo, M. Fujiwara, and P. E. Hodgson, Prog. Theor. Phys. Suppl. **132**, 1 (1998).
[3] A. Tohsaki *et al.*, Phys. Rev. Lett. **87**, 192501 (2001).
[4] E. Uegaki *et al.*, Prog. Theor. Phys. **57**, 1262 (1977); **59**, 1031 (1978); **62**, 1621 (1979).
[5] M. Kamimura, Nucl. Phys. **A351**, 456 (1981); Y. Fukushima and M. Kamimura, Proceedings of the International Conference on Nuclear Structure, Tokyo, 1977, edited by T. Marumori [J. Phys. Soc. Jpn. **44**, 225 (1978)].
[6] D. A. Goldberg and S. M. Smith, Phys. Rev. Lett. **33**, 715 (1974).
[7] S. Szilner *et al.*, Phys. Rev. C **64**, 064614 (2001) and references therein.
[8] D. T. Khoa *et al.*, Phys. Rev. Lett. **74**, 34 (1995).
[9] M. P. Nicoli *et al.*, *Proceedings of the 7th International Conference on Clustering Aspects of Nuclear Structure and Dynamics*, edited by Korolija *et al.* (World Scientific, Singapore, 2000), p. 151.
[10] A. Morsad, F. Haas, C. Beck, and R. M. Freeman, Z. Phys. A **338**, 61 (1991).
[11] F. Michel and S. Ohkubo, Nucl. Phys. **A738**, 231 (2004).
[12] F. Brau, F. Michel, and G. Reidemeister, Phys. Rev. C **57**, 1386 (1998).
[13] F. Michel, S. Ohkubo, and G. Reidemeister, Prog. Theor. Phys. Suppl. **132**, 7 (1998).
[14] F. Michel, G. Reidemeister, and S. Ohkubo, Phys. Rev. Lett. **89**, 152701 (2002).
[15] F. Michel, F. Brau, G. Reidemeister, and S. Ohkubo, Phys. Rev. Lett. **85**, 1823 (2000).
[16] F. Michel, G. Reidemeister, and S. Ohkubo, Phys. Rev. C **63**, 034620 (2001).
[17] A. Kiss *et al.*, J. Phys. G **13**, 1067 (1987).
[18] B. Tatischeff and I. Brissaud, Nucl. Phys. **A155**, 89 (1970).
[19] S. M. Smith *et al.*, Nucl. Phys. **A207**, 273 (1973).
[20] B. John *et al.*, Phys. Rev. C **68**, 014305 (2003).
[21] Y. Suzuki and M. Takahashi, Phys. Rev. C **65**, 064318 (2002).
[22] Y. Funaki *et al.*, Phys. Rev. C **67**, 051306 (2003).
[23] T. Yamada and P. Schuck, Phys. Rev. C **69**, 024309 (2004).
[24] H. Matsumura and Y. Suzuki, Nucl. Phys. **A739**, 238 (2004).
[25] Y. Hirabayashi and S. Ohkubo, Phys. At. Nucl. **65**, 683 (2002).
[26] Y. Hirabayashi, Y. Sakuragi, and Y. Abe, Phys. Rev. Lett. **74**, 4141 (1995).
[27] A. Ingemarsson, A. Auce, and R. Johansson, Phys. Rev. C **49**, 1609 (1994).
[28] Y. Kondo *et al.*, Phys. Lett. B **365**, 17 (1996).