

Pion photoproduction on nucleons in a covariant hadron-exchange modelVladimir Pascalutsa^{1,2,3,*} and John A. Tjon^{1,4,†}¹*Theory Group, Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*²*Department of Physics, The College of William & Mary, Williamsburg, Virginia 23188, USA*³*Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA*⁴*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

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We present a relativistic dynamical model of pion photoproduction on the nucleon in the resonance region. It offers several advances over the existing approaches. The model is obtained by extending our πN -scattering description to the electromagnetic channels. The resulting photopion amplitude is thus unitary in the πN , γN channel space; Watson's theorem is exactly satisfied. At this stage we have included the pion, nucleon, $\Delta(1232)$ resonance degrees of freedom. The ρ and ω meson exchanges are also included, but play a minor role in the considered energy domain (up to $\sqrt{s}=1.5$ GeV). In this energy range the model provides a good description of all the important multipoles. We have allowed for only two free parameters—the photocouplings of the Δ resonance. These couplings are adjusted to reproduce the strength of corresponding resonant-multipoles M_{1+} and E_{1+} at the resonance position.

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I. INTRODUCTION

In recent years there has been significant interest in the pion and kaon *photoproduction* and *electroproduction* off the nucleon. Several excellent experimental programs exploring these reactions in the *resonance region* have been performed at MAMI, MIT Bates, BNL, and Jefferson Lab. To extract the resonance properties from the experimental data a number of sophisticated tools have been developed over the past decade. Most widely exploited are the *partial-wave* photoproduction *solutions* SAID [1] and MAID [2], *K-matrix models* [3], and *dynamical models*, such as DMT [4], the model of Sato and Lee [5], the Gross and Surya model [6], and a number of others [7].

In this paper we present a new dynamical model for *pion photoproduction*. It is an extension of our model of pion-nucleon (πN) interaction [8,9] to include the electromagnetic interaction in a way consistent with the Watson theorem [10] and current conservation. The framework is based on solving a Bethe-Salpeter type of equation for the scattering amplitude in the channel space spanned by πN and γN states. As in the πN case we use the *equal-time* (instantaneous) quasipotential reduction of the Bethe-Salpeter equation. The driving force of the equation to lowest order in interactions is given by single-particle exchange graphs. Here we will restrict our discussion to the force given by the single nucleon, pion, ρ and ω mesons, and $\Delta(1232)$ -resonance exchanges.

In comparing with other approaches based on the hadron-exchange dynamics we note that they differ mainly in the use of relativistic dynamics and the renormalization procedures. Our model bears close analogies to the relativistic model developed by Gross and Surya [6]. In contrast to their work, we do not approximate the hadron exchanges in the *t*- and

u-channels by a separable interaction. As a result, the integral equation for the πN amplitude can be solved only numerically, and hence the task of solving the model is more technically involved. Our models are also different in the choice of a quasipotential reduction of the Bethe-Salpeter equation—equal-time vs pion spectator.

There are important differences of our model with the DMT model. First of all, the dressed resonance exchanges in the *s* channel are represented by a Breit-Wigner form with “unitarity phases” which need to be fitted to the condition of Watson's theorem. In our model these contributions are generated dynamically. This has an advantage of satisfying Watson's theorem automatically. Also, the resonance parameters, apart from the electromagnetic couplings, are thus fully constrained by the πN interaction and need not be fitted separately. The second major difference is again in the choice of relativistic dynamics—the DMT model exploits the Kadyshevsky quasipotential reduction of the Bethe-Salpeter equation.

Sato and Lee [5] apply the Hamiltonian approach and the method of unitary transformations which makes it difficult to compare directly to the Bethe-Salpeter type of approach. The generic feature that distinguishes the two is that in the quantum-mechanical Hamiltonian description the particles are always on the mass shell and intermediate particles are off the energy shell, while in the field-theoretic description it is the other way around. Another difference is that Sato and Lee do not perform any renormalizations of the dressed baryon-pole contributions.

In this paper we shall only briefly present the framework and the results for the photoproduction multipoles. The results for the pion-photoproduction observables as well as the extension to electroproduction of pions will appear in subsequent publications, see, e.g., [11]. The paper is organized as follows. In the following section we briefly summarize the usual arguments for inclusion of the πN final state interaction in the π photoproduction reactions. In Sec. III we construct the pion-photoproduction potential with an emphasis

*Electronic address: vlad@jlab.org

†Electronic address: tjon@jlab.org

on satisfying the gauge-invariance constraints. In Sec. IV the renormalization of the pole terms of the photoproduction amplitude is described. In Sec. V we present some numerical results for the pion-photoproduction multipoles and discussion. Section VI concludes the paper.

II. πN - γN COUPLED CHANNEL EQUATIONS

To include the photon in a way preserving unitarity in the channel space spanned by the πN and γN states we consider the following four processes:

$$\begin{aligned} \pi N &\rightarrow \pi N, \quad \pi N \rightarrow \gamma N, \\ \gamma N &\rightarrow \pi N, \quad \gamma N \rightarrow \gamma N, \end{aligned} \quad (1)$$

and the following coupled-channel scattering equation:

$$\begin{aligned} \begin{pmatrix} T_{\pi\pi} & T_{\pi\gamma} \\ T_{\gamma\pi} & T_{\gamma\gamma} \end{pmatrix} &= \begin{pmatrix} V_{\pi\pi} & V_{\pi\gamma} \\ V_{\gamma\pi} & V_{\gamma\gamma} \end{pmatrix} + \begin{pmatrix} V_{\pi\pi} & V_{\pi\gamma} \\ V_{\gamma\pi} & V_{\gamma\gamma} \end{pmatrix} \begin{pmatrix} G_{\pi} & 0 \\ 0 & G_{\gamma} \end{pmatrix} \\ &\times \begin{pmatrix} T_{\pi\pi} & T_{\pi\gamma} \\ T_{\gamma\pi} & T_{\gamma\gamma} \end{pmatrix}, \end{aligned} \quad (2)$$

where T and V are the suitably normalized amplitudes and driving potentials of the πN scattering ($\pi\pi$), pion photoproduction ($\pi\gamma$), absorption ($\gamma\pi$), and the nucleon Compton effect ($\gamma\gamma$). The propagators G_{π} and G_{γ} are, respectively, the pion-nucleon and photon-nucleon two-particle propagators. With the assumption of Hermiticity of the potential and time-reversal symmetry, which in particular relates the $\gamma\pi$ and $\pi\gamma$ amplitudes, Eq. (2) leads to an exactly unitary S matrix, $S_{fi} = \delta_{fi} + 2iT_{fi}$, in the defined channel space.

Since iterations of the potentials involving the photon give rise to the small electromagnetic corrections, one can simplify the equation by keeping only the leading order in the electric charge e . This leads to

$$T_{\pi\pi} = V_{\pi\pi} + V_{\pi\pi} G_{\pi} T_{\pi\pi}, \quad (3a)$$

$$T_{\pi\gamma} = V_{\pi\gamma} + T_{\pi\pi} G_{\pi} V_{\pi\gamma}, \quad (3b)$$

$$T_{\gamma\pi} = V_{\gamma\pi} + V_{\gamma\pi} G_{\pi} T_{\pi\pi}, \quad (3c)$$

$$T_{\gamma\gamma} = V_{\gamma\gamma} + V_{\gamma\pi} G_{\pi} T_{\pi\gamma}. \quad (3d)$$

In this approximation, the integral equation has to be solved for the πN amplitude only. The rest is determined in a one-loop calculation.

The equation for the πN amplitude, Eq. (3a), has been studied by us previously in the framework of relativistic quasipotential scattering with the πN potential modeled by a number of relevant hadron exchanges [9]. The parameters have been fitted to the πN -scattering partial-wave analysis

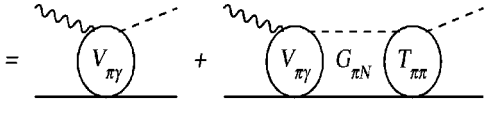
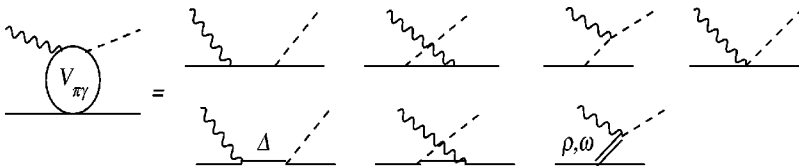


FIG. 1. The unitary model for the photoproduction amplitude.

data. In the present work we obtain the photoproduction amplitude from Eq. (3b) (diagrammatically shown in Fig. 1) using exactly the same quasipotential approach and the πN amplitude as in Ref. [9]. The only free parameters in this calculation will be the electromagnetic couplings of hadrons entering the driving force $V_{\pi\gamma}$, all the rest is fixed through the analysis of πN scattering.

Our resulting photoproduction amplitude obeys the *Watson theorem* [10], which relates the phase of the photoproduction amplitude to the πN elastic phase shift $\delta_{\pi\pi}$:

$$T_{\gamma\pi} = |T_{\gamma\pi}| e^{i\delta_{\pi\pi}}. \quad (4)$$

The phase of the photoproduction amplitude is thus fully determined in terms of the on-shell πN amplitude. The dependence on the off-shell behavior of the πN interaction resides fully in the absolute magnitude of the photoproduction amplitude.

III. THE MODEL POTENTIAL AND GAUGE INVARIANCE

The pion-photoproduction potential of this model is shown diagrammatically in Fig. 2. The first four graphs represent the so-called Born term, where the fourth graph is the Kroll-Ruderman contact term. The latter is obtained by the “minimal substitution” in the pseudovector πNN coupling, and is therefore needed to ensure the current conservation of the Born contribution.

Except for the $\gamma N \Delta$ couplings, the Lagrangian we use is standard. For brevity we only specify here the Feynman rules for corresponding vertices [omitting isospin, the isospin structure will be specified below, cf. Eq. (13)]

$$\Gamma_{\gamma NN}^{\mu}(\kappa; q) = e\gamma^{\mu} - \frac{e\kappa}{2m_N} \gamma^{\mu\nu} q_{\nu}, \quad (5a)$$

$$\Gamma_{\gamma\pi NN}^{\mu} = \frac{e g_{\pi NN}}{2m_N} \gamma^{\mu} \gamma_5, \quad (5b)$$

$$\Gamma_{\gamma\pi\pi}^{\mu}(k', k) = e(k' + k)^{\mu}, \quad (5c)$$

$$\Gamma_{\gamma\pi\nu}^{\mu\alpha}(q, k) = \frac{e g_{\gamma\pi\nu}}{m_{\pi}} \varepsilon^{\mu\alpha\beta\nu} k_{\beta} q_{\nu}, \quad (5d)$$

where e is the proton electric charge ($e^2/4\pi \approx 1/137$), κ is the anomalous magnetic moment of the nucleon, m_N

FIG. 2. The tree-level photoproduction potential.

≈ 0.9383 GeV, $m_\pi \approx 0.139$ GeV are the nucleon and pion masses, $\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$, and q and k denote the momenta of the photon and pion, respectively. The subscript v stands for a vector meson, in this case ρ or ω .

The $\gamma N\Delta$ vertices we obtain from the following Lagrangian:

$$\mathcal{L}_{\gamma N\Delta} = \frac{3e}{2m_N(m_N + m_\Delta)} \bar{N} T_3^\dagger (i g_M \tilde{F}^{\mu\nu} - g_E \gamma_5 F^{\mu\nu}) \partial_\mu \Delta_\nu + \text{H.c.}, \quad (6)$$

where $m_\Delta \approx 1232$ MeV is the Δ -isobar mass, T_3 is the isospin $N\Delta$ transition matrix, with normalization $T_3^\dagger T_3 = \frac{2}{3}$. This $\gamma N\Delta$ coupling is invariant under electromagnetic gauge transformations (to the order to which we work), as well as under the spin-3/2 gauge transformation,

$$\Delta_\mu(x) \rightarrow \Delta_\mu(x) + \partial_\mu \varepsilon(x), \quad (7)$$

with ε a spinor field. Invariance under Eq. (7) ensures the correct spin-degrees-of-freedom counting [12]. In the Δ 's rest frame (where $\Delta_0=0$, $\partial_0 \Delta_i = -im_\Delta \Delta_i$, and $\partial_i \Delta_j = 0$), the coupling (6) becomes

$$\mathcal{L}_{\gamma N\Delta} = - \frac{3em_\Delta}{2m_N(m_N + m_\Delta)} \bar{N} T_3^\dagger (g_M B^i + g_E \gamma_5 E^i) \Delta_i + \text{H.c.}, \quad (8)$$

where B^i is the magnetic and E^i the electric field. Therefore the two terms correspond to $N\Delta$ magnetic and electric transitions, respectively.

However, in the standard convention [13], the electric coupling G_E is defined as the one directly proportional to the $E_{1+}^{(3/2)}$ multipole. On the mass shell of the Δ , our convention and the convention of Jones and Scadron [13] are related as follows:

$$g_M = G_M - G_E, \quad (9a)$$

$$g_E = -2G_E \frac{m_\Delta + m_N}{m_\Delta - m_N}. \quad (9b)$$

The Feynman rule corresponding to the coupling (7) reads

$$\Gamma_{\gamma N\Delta}^{\alpha\mu}(p, q) = \frac{3e}{2m_N(m_N + m_\Delta)} [g_M \varepsilon^{\alpha\mu\beta\nu} p_\beta q_\nu - g_E (p \cdot q g^{\alpha\mu} - q^\alpha p^\mu) i \gamma_5], \quad (10)$$

with p (q) being the 4-momentum of the Δ (photon), and α (μ) the vector index of the Δ (photon) field.

For the ‘‘strong-interaction’’ vertices we use the same forms as in [9], namely,

$$\Gamma_{\pi NN}(k) = \frac{g_{\pi NN}}{2m_N} \gamma_5 \not{k}, \quad (11a)$$

$$\Gamma_{\pi NN^*}(k) = \frac{g_{\pi NN^*}}{2m_N} \gamma_5 \not{k}, \quad (11b)$$

$$\Gamma_{v NN}^\alpha(q) = g_{v NN} \left(\gamma^\alpha - \frac{\kappa_v}{2m_N} \gamma^{\alpha\nu} q_\nu \right), \quad (11c)$$

$$\Gamma_{\pi N\Delta}^\alpha(k, p) = \frac{f_{\pi N\Delta}}{m_\pi m_\Delta} \varepsilon^{\alpha\beta\mu\nu} p_\beta \gamma_\mu \gamma_5 k_\nu. \quad (11d)$$

The Feynman graphs depicted in Fig. 2 correspond to

$$(4\pi) V_{(N),\text{pole}}^{(S,V)\mu} = \Gamma_{\pi NN}(k') S_N(p+q) \Gamma_{\gamma NN}^\mu(\kappa_{S,V}; q), \quad (12a)$$

$$(4\pi) V_{(N),\text{exch}}^{(S,V)\mu} = \Gamma_{\gamma NN}^\mu(\kappa_{S,V}; q) S_N(p-k') \Gamma_{\pi NN}(k'), \quad (12b)$$

$$(4\pi) V_{(\Delta),\text{pole}}^{\mu} = \Gamma_{\pi N\Delta}^\alpha(k', p) S_\Delta^{\alpha\beta}(p+q) \Gamma_{\gamma N\Delta}^{\mu\beta}(p; q), \quad (12c)$$

$$(4\pi) V_{(\Delta),\text{exch}}^{\mu} = \Gamma_{\gamma N\Delta}^{\mu\alpha}(p; q) S_\Delta^{\alpha\beta}(p-k') \Gamma_{\pi N\Delta}^\beta(k', p), \quad (12d)$$

$$(4\pi) V_{(\pi)}^\mu = \Gamma_{\pi NN}(q-k') S_\pi(q-k') \Gamma_{\gamma\pi\pi}^\mu(k', q-k'), \quad (12e)$$

$$(4\pi) V_{(\text{KR})}^\mu = \Gamma_{\gamma\pi NN}^\mu, \quad (12f)$$

$$(4\pi) V_{(v)}^\mu = \Gamma_{v NN}^\alpha(q-k') S_v^{\alpha\beta}(q-k') \Gamma_{\gamma v}^{\mu\beta}(q, k'), \quad v = (\rho, \omega). \quad (12g)$$

These graphs give the following contributions to the isospin $\pi\gamma$ amplitudes:

$$V^{(1/2)\mu} = 3V_{(N),\text{pole}}^{(V)\mu} - V_{(N),\text{exch}}^{(V)\mu} + 2V_{(\pi)}^\mu + 2V_{(\text{KR})}^\mu + \frac{4}{3}V_{(\Delta),\text{exch}}^\mu + V_{(\omega)}^\mu, \quad (13a)$$

$$V^{(3/2)\mu} = 2V_{(N),\text{exch}}^{(V)\mu} - V_{(\pi)}^\mu - V_{(\text{KR})}^\mu + V_{(\Delta),\text{pole}}^\mu + \frac{1}{2}V_{(\Delta),\text{exch}}^\mu + V_{(\omega)}^\mu, \quad (13b)$$

$$V^{(0)\mu} = V_{(N),\text{pole}}^{(S)\mu} + V_{(N),\text{exch}}^{(S)\mu} + V_{(\rho)}^\mu. \quad (13c)$$

The gauge invariance of the electromagnetic interactions imposes the following *current conservation* condition:

$$q_\mu V^{(I)\mu} = 0, \quad (14)$$

for all values of the isospin: $I=0, \frac{1}{2}, \frac{3}{2}$. For the Δ , ρ , and ω exchange graphs this condition is trivially satisfied, since the

corresponding electromagnetic vertices vanish when contracted with the photon momentum. For the nucleon and pion exchange contributions the situation is complicated by the fact that the photon couples minimally and hence the vertices fulfill the Ward-Takahashi (WT) identities:

$$\frac{1}{e}q_\mu \Gamma_{\gamma NN}^\mu(p', p) = S_N^{-1}(p') - S_N^{-1}(p), \quad (15a)$$

$$\frac{1}{e}q_\mu \Gamma_{\gamma \pi \pi}^\mu(k', k) = S_\pi^{-1}(k') - S_\pi^{-1}(k). \quad (15b)$$

Nonetheless, using these identities, it is easy to see that a cancellation among the nucleon, pion, and the KR contact term contributions leads to

$$\begin{aligned} (4\pi)q_\mu V^{(1/2)\mu} = & -3e\Gamma_{\pi NN}(k')S_N(p+q)S_N^{-1}(p) \\ & + eS_N^{-1}(p')S_N(p-k')\Gamma_{\pi NN}(k') \\ & + 2e\Gamma_{\pi NN}(q-k')S_\pi(q-k')S_\pi^{-1}(k'), \end{aligned} \quad (16)$$

and analogously for the other isospin amplitudes. Therefore, the current is conserved up to the terms proportional to the inverse propagators of the external particles, and hence is exactly conserved when the external particles are on the mass shell.

A problem arises when we would like to include the *side-ways* form factors, i.e., cutoff functions dependent on the 4-momentum of the exchanged particle. Obviously, simply introducing them into the pion and nucleon exchange graphs, as we have done for the πN potential in [9], will destroy the current conservation. The cancellation among the graphs does not anymore take place.

The easiest way to implement such cutoff form factors without loss of current conservation is to perform the minimal substitution on the form factors themselves. We follow essentially the method of Gross and Riska [14]. We use the fact that our sideways form factors depend exclusively on the momentum of the exchanged particle and hence it makes no difference whether to include the form factor into the vertex function or the inverse form factor squared into the propagator. In the latter case the minimal substitution is more straightforward.

More specifically, in the nucleon case we start with

$$\mathcal{L} = [f^{-1}(\partial^2)\bar{\Psi}](i\not{\partial} - m_N)f^{-1}(\partial^2)\Psi, \quad (17)$$

where $f(\partial^2)$ is the form factor operator in the coordinate space. Substituting ∂_μ by $D_\mu = \partial_\mu - ieA_\mu$, and linearizing in the electromagnetic field we find the *modified* γNN vertex

$$\begin{aligned} \Gamma_{\gamma NN}^{\mu, \text{mod}}(p', p) = & e\gamma^\mu f^{-1}(p')f^{-1}(p^2) + e(p+p')^\mu [f^{-1}(p')^2 \\ & \times (\not{p}' - m_N) + f^{-1}(p^2)(\not{p} - m_N)]\Xi(p', p^2), \end{aligned} \quad (18)$$

where in general Ξ is the finite difference the inverse form factor

$$\Xi(p', p^2) = \frac{f^{-1}(p') - f^{-1}(p^2)}{p'^2 - p^2}. \quad (19)$$

For instance, for the monopole type [i.e., $f^{-1}(p^2) = (\Lambda^2 - p^2)/(\Lambda^2 - m^2)$] we have simply $\Xi = (\Lambda^2 - m^2)^{-1}$.

Taking the specific nucleon form factor used in our πN model,

$$f(p^2) = \left(\frac{2\Lambda_N^4}{2\Lambda_N^4 + (p^2 - m_N^2)^2} \right)^2 \quad (20)$$

we find $\Xi(p', p^2) = (p'^2 + p^2 - 2m)[f^{-1}(p') + f^{-1}(p^2)]/(2\Lambda_N^4)$.

The anomalous magnetic moment term, $\Gamma_{\text{amm}}^\mu = -(e\kappa/4m_N)[\gamma^\mu, \gamma^\nu]q_\nu$, is explicitly gauge invariant and we choose to leave it unchanged. Adding it to the vertex, and substituting Eq. (19) for Ξ , we obtain

$$\begin{aligned} \Gamma_{\gamma NN}^{\mu, \text{mod}}(p', p) = & e f^{-1}(p') f^{-1}(p^2) \left(g^{\mu\nu} - \frac{(p+p')^\mu q^\nu}{q \cdot (p+p')} \right) \gamma_\nu \\ & + \frac{e(p+p')^\mu}{q \cdot (p+p')} [f^{-2}(p') S_N^{-1}(p') \\ & - f^{-2}(p^2) S_N^{-1}(p)] + \Gamma_{\text{amm}}^\mu. \end{aligned} \quad (21)$$

This equation, together with Eq. (20), defines the γNN vertex of the model.

One needs to keep in mind that since the free Lagrangian is modified by form factors, the propagators take the form

$$S_N^{\text{mod}}(p) = f^2(p^2) S_N(p), \quad (22)$$

where $S_N(p) = (\not{p} - m_N)^{-1}$. Nucleon spinors should be modified accordingly, i.e., multiplied by f . From Eq. (21) it is particularly easy to see that the modified vertex and propagator obey the same WT identity as the unmodified ones. Thus, we have included the cutoff functions in a way consistent with gauge invariance.

Considering the pion case in the same fashion, and using the monopole form of $f(k^2)$, we find

$$\begin{aligned} \Gamma_{\gamma \pi \pi}^{\mu, \text{mod}}(k', k) = & e(k+k')^\mu \left[f^{-1}(k') f^{-1}(k^2) + f^{-1}(k')^2 \frac{k'^2 - m_\pi^2}{\Lambda_\pi^2 - m_\pi^2} \right. \\ & \left. + f^{-1}(k^2) \frac{k^2 - m_\pi^2}{\Lambda_\pi^2 - m_\pi^2} \right]. \end{aligned} \quad (23)$$

Note that the KR term is not modified, since we do not introduce any form factors in the πNN interaction Lagrangian, but rather have them in the propagators. Since the modified propagators and vertices obey the standard WT identities, the proof of current conservation for the model with form factors is exactly the same as before.

IV. RENORMALIZATION OF THE POLE TERMS

One of the effects of the πN final-state interaction is to renormalize the s -channel contributions of the photoproduction potential $V_{\pi\gamma}$. Let us recall that the πN amplitude $T_{\pi\pi}$ amplitude can symbolically be presented as

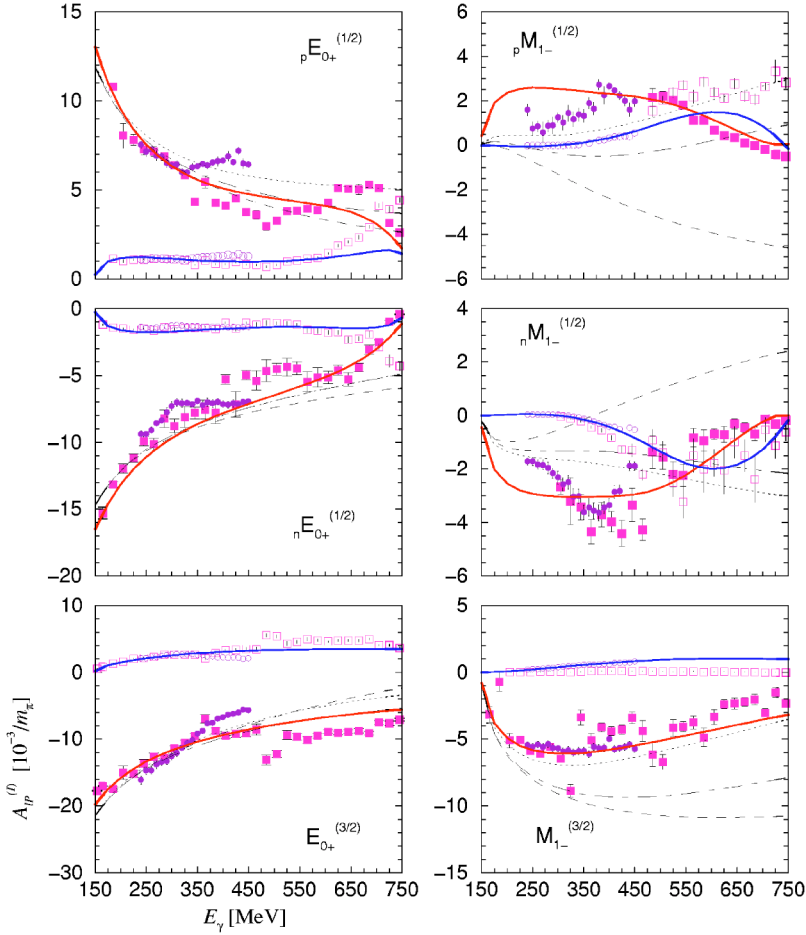


FIG. 3. (Color online) The description of $J = 1/2$ pion-photoproduction multipoles. The dashed curves represent the Born amplitude *without* the sideways form factors. The dashed-dotted curves represent the Born amplitude *with* the sideways form factors. The dotted curves show the tree-level Born $+\rho, \omega$ calculation (with the form factors intact). The solid curves are the full calculation including the final state interaction (Re A , red solid line; Im A , blue solid line). The results are compared to the partial-wave analyses: BD75 [15] (Re A , filled violet circles; Im A , open violet circles), and SAID SM95 solution [1] (Re A , filled purple squares; Im A , open purple squares).

$$T = \Gamma^\dagger \mathbf{S} \Gamma + T_u,$$

$$T_u = V_u + V_u G T_u,$$

$$\Gamma = Z_1 (\Gamma + \Gamma G T_u),$$

$$\begin{aligned} S^{-1} &= S^{-1} - Z_1 \Gamma G \Gamma + Z_2 (m - m_0) + (Z_2 - 1) S^{-1} \\ &= Z_2 S_0^{-1} - Z_1 \Gamma G \Gamma, \end{aligned} \quad (24)$$

where S_0^{-1} is the inverse bare propagator, e.g., for the nucleon it is given by $\not{p} - m_0$.

The photoproduction potential $V_{\pi\gamma}$ and the resulting amplitude $T_{\pi\gamma}$ can also be separated into the ‘‘pole’’ and ‘‘non-pole’’ parts. In order for $T_{\pi\gamma}$ to have the same dressed baryon exchanges as in the $\pi\pi$ amplitude, one ought to use the bare parameters in the pole terms of the $V_{\pi\gamma}$ potential, i.e.,

$$V_{\pi\gamma} = \frac{Z_2}{Z_1} \Gamma S_0 \Gamma_\gamma + V_{\pi\gamma,u}, \quad (25)$$

where Γ_γ is the electromagnetic vertex. Indeed, one then has

$$\begin{aligned} T_{\pi\gamma} &= (1 + \Gamma^\dagger \mathbf{S} \Gamma G + T_u G) \left(\frac{Z_2}{Z_1} \Gamma S_0 \Gamma_\gamma + V_{\pi\gamma,u} \right) \\ &= \Gamma \mathbf{S} \Gamma_\gamma + T_u G V_{\pi\gamma,u}. \end{aligned} \quad (26)$$

We thus construct the nucleon- and Δ -pole contributions by

using the bare parameters known from the πN model, see Table VII of Ref. [9].

V. RESULTS AND DISCUSSION

In Figs. 3 and 4 we present the model predictions for the pion photoproduction multipoles, in units of $10^{-3}/m_{\pi^+}$. The dashed curves represent the Born amplitude without the sideways form factors, while the dashed-dotted curves—with the sideways form factors. The dotted curves show the tree-level Born $+\rho, \omega$ calculation with all the form factors intact. The solid curves (Red—real part, Blue—imaginary part) represent the full calculation defined by Eq. (3a) (see also Fig. 1) with Born $+\rho, \omega, \Delta$ photoproduction potential and the complete πN final state interaction from Ref. [9]. The results are compared with the data from the partial-wave solutions of Berends and Donnachie [15] and SAID [1].

The electromagnetic coupling parameters used in the calculation are given in Table I, with $m_\omega = 0.783$ GeV, $\Lambda_\omega = \Lambda_\rho$. Only the Δ -isobar electromagnetic couplings g_M and g_E were adjusted to for the best description of the resonant multipoles: $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$. In the figures we have plotted the results for the central values of these parameters, given in bold in Table I. The other multipoles are very weakly sensitive to the Δ isobar contribution (recall that the spin-1/2 backgrounds are absent in our model because of the specific form of the $\gamma N \Delta$ vertex). Other parameters have been taken from the literature [3,16].

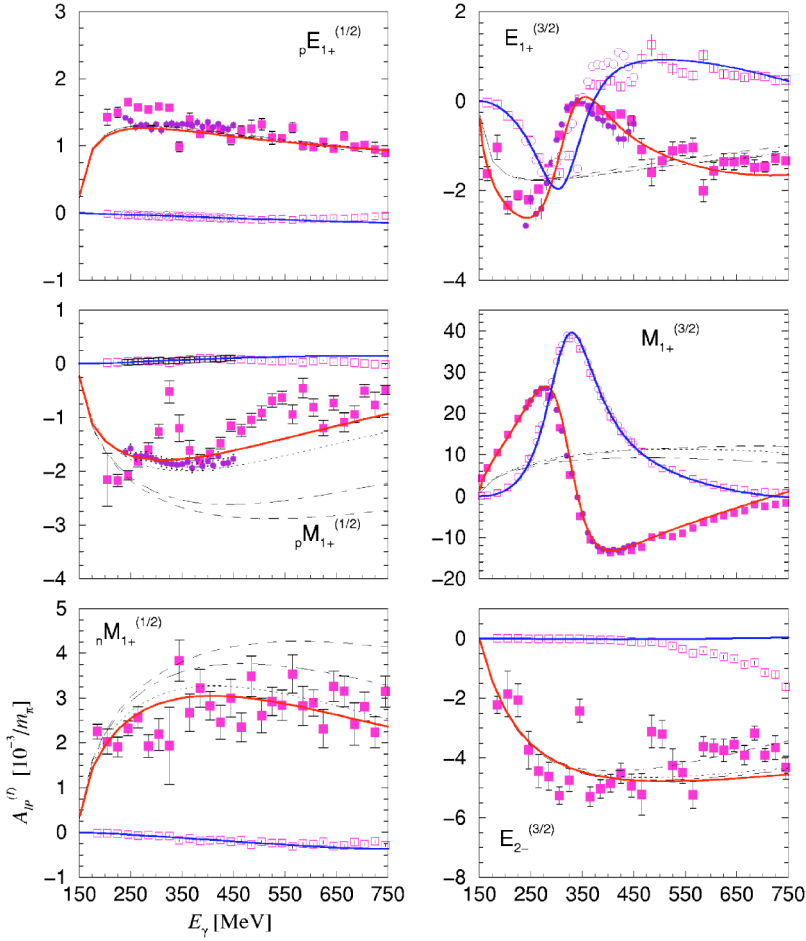


FIG. 4. (Color online) The description of some of the $J=3/2$ pion-photoproduction multipoles. The legend is the same as in Fig. 3.

From the figures we see that the full model calculation for most of the pion-photoproduction multipoles are in a very good agreement with the partial-wave analyses in this energy region. The only problematic ${}_pM_{1-}^{(1/2)}$ and ${}_nM_{1-}^{(1/2)}$ multipoles can possibly be corrected by including an explicit Roper-resonance exchange in the photoproduction potential. It is expected to correct not only the resonance but also lower energy region because of the N -Roper mixing and related renormalization issues, cf. [9] for details.

The difference between the solid and dotted curves, in the nonresonant multipoles, can serve as a good measure of the effect of the final state interaction. One can see that this effect is not dramatically large. However, it does make a significant difference in some channels, as will be demonstrated below.

Let us consider the reaction close to the threshold, $s \simeq (m_N + m_{\pi})^2$. The electric dipole amplitudes, $E_{0+}^{(l)}$ are of primary interest in this regime, all the other multipoles are tiny. There are predictions for E_{0+} from the low-energy theorems

(LET) [17,18] and chiral perturbation theory (ChPT) [19]. The result of the “old” LETs [17] are given simply by the Born-graph contribution expanded in powers of $\mu = m_{\pi}/m_N$:

$$E_{0+}(\pi^+n) = \frac{eg_{\pi NN}}{8\pi m_N} \sqrt{2} \left(1 - \frac{3}{2}\mu\right) + O(\mu^2), \quad (27a)$$

$$E_{0+}(\pi^-p) = \frac{eg_{\pi NN}}{8\pi m_N} \sqrt{2} \left(-1 + \frac{1}{2}\mu\right) + O(\mu^2), \quad (27b)$$

$$E_{0+}(\pi^0p) = -\frac{eg_{\pi NN}}{8\pi m_N} \mu \left[1 - \frac{1}{2}\mu(3 + \kappa_p)\right] + O(\mu^3), \quad (27c)$$

$$E_{0+}(\pi^0n) = -\frac{eg_{\pi NN}}{8\pi m_N} \frac{1}{2} \mu^2 \kappa_n + O(\mu^3). \quad (27d)$$

TABLE I. The electromagnetic coupling constants. The values given in bold were varied for a best fit.

$\kappa_V = 3.71$, $\kappa_S = \kappa_{\omega} = -0.12$
$g_{\rho NN} = 2.66$, $g_{\omega NN} = 9.0$, $g_{\gamma\pi\omega} = 3g_{\gamma\pi\rho} = 0.313$
$g_M = \mathbf{2.8} \pm 0.2$, $g_E = \mathbf{1.5} \pm 0.5$

Bernard *et al.* [18] discovered that at $O(\mu^2)$ there is an important chiral-loop correction to the LET for the *neutral* pion-production channels:

TABLE II. Predictions and experimental data for the threshold electric dipole multipoles for various reaction channels.

Multipole	Born	$V_{\gamma\pi}(1+GT_{\pi\pi})$	Old LET	New LET	Experiment
$E_{0+}(\pi^+n)$	26.1	26.3	25.9	25.9	27.9 ± 0.5 [20], 28.06 ± 0.27 [21]
$E_{0+}(\pi^-p)$	-29.9	-29.6	-30.8	-30.8	-31.4 ± 1.3 [20], -31.5 ± 0.8 [22]
$E_{0+}(\pi^0p)$	-2.4	-1.4	-2.3	1.0	-1.31 ± 0.08 [23], $-1.32 \pm 0.05 \pm 0.06$ [24]
$E_{0+}(\pi^0n)$	0.4	1.0	0.5	3.8	≈ 1.6 [25]

$$E_{0+}(\pi^0p) = E_{0+}^{LET}(\pi^0p) + \frac{eg_{\pi NN}}{8\pi m_N} \left(\frac{m_\pi}{4f_\pi} \right)^2, \quad (28a)$$

$$E_{0+}(\pi^0n) = E_{0+}^{LET}(\pi^0n) + \frac{eg_{\pi NN}}{8\pi m_N} \left(\frac{m_\pi}{4f_\pi} \right)^2, \quad (28b)$$

where $f_\pi \approx 93$ MeV is the pion decay constant. This result is commonly referred to as the ‘‘new LET.’’ Of course at this order there are also loop corrections to the charged multipoles; however, they appear to be less significant numerically than for the neutral channels.

The numerical values of these predictions, together with the predictions of our model and some experimental results are collected in Table II. In all of the theory predictions we have used $g_{\pi NN}^2/4\pi = 13.8$, the value inferred from our pion-nucleon analysis.

In Table II, the second column represents the value of the Born amplitude in our model, while the third column corresponds to the full model calculations. It is reassuring that without need to fit any parameters we obtained a reasonable agreement with experiment in all the channels. It is also good to see that the effect of the FSI is small for the charged pion photoproduction and significant for the π^0 channels in analogy with the chiral loop effect of the new LET. Thus, our results at threshold are in at least qualitative agreement with ChPT. They also are in reasonable quantitative agreement with experiment, and for the π^0 production are even in better agreement than the ‘‘new LET’’ result. Although, it should be noted that in a more complete calculation, including higher order effects and counterterms, ChPT is in better agreement with experiment than our simple model. One can of course try to improve the model by including higher-order contact terms in the photoproduction potential. We, however, have not done that. Our main aim is to apply the model in the resonance region where ChPT is not applicable (yet).

In particular, in the Δ -resonance region we have been able to extract the coupling constants of the $\gamma N \rightarrow \Delta$ transition. A quantity of interest here is the ratio of the electric ($E2$) and magnetic ($M1$) $\gamma N \rightarrow \Delta$ transition strength: $R_{EM} = E2/M1$. The physical significance of these values is attributed to the deformation of the nucleon, see, e.g., [26,27]. For instance, in a naive quark model where the nucleon consists of three constituent quarks in the sphere-shape S state—the $E2/M1$ ratio vanishes.

In terms of the $\gamma N \Delta$ -vertex parameters in our model the $E2/M1$ ratio is defined as (cf. Appendix A of Ref. [28]):

$$R_{EM} = \frac{g_E}{2g_M \frac{m_\Delta + m_N}{m_\Delta - m_N} - g_E} \times 100\%. \quad (29)$$

Using the bare values of g_M and g_E in Table I, we estimate this ratio to be

$$R_{EM}^{\text{bare}} = (3.8 \pm 1.6)\%. \quad (30)$$

We should immediately note that this value only seems to be inconsistent with the PDG value [29]: $R_{EM} = (-2.5 \pm 0.5)\%$, the reason being that the PDG analyses define this ratio as the ratio of corresponding resonant multipoles:

$$R_{EM}^{(\text{multipoles})} = \frac{\text{Im } E_{1+}^{(3/2)}}{\text{Im } M_{1+}^{(3/2)}} \times 100\%. \quad (31)$$

In our model we obtain $\text{Im } E_{1+}^{(3/2)} = -1.0 \pm 0.2$, and $\text{Im } M_{1+}^{(3/2)} = 38.5 \pm 1.5$ (in units of $10^{-3}/m_\pi$) at the Δ resonance position (i.e., where $\text{Re } E_{1+}^{(3/2)} = 0 = \text{Re } M_{1+}^{(3/2)}$). Therefore, we have

$$R_{EM}^{(\text{multipoles})} = (-2.6 \pm 0.6)\% \quad (32)$$

which is consistent with the PDG value.

The definition (31), however, is equivalent to Eq. (29) only assuming that the on-mass-shell renormalized values of g_M and g_E are used in Eq. (29). While this is correct for the M_{1+} , for the E_{1+} the Δ contribution turns out to be relatively small. This multipole is dominated by processes which have nothing to do with the electric quadrupole $\gamma N \rightarrow \Delta$ transition.

Our result that the bare $E2/M1$ ratio is, in fact, small and *positive* is in agreement with other dynamical models [4], which allows us to believe that the model dependence in the extraction of this quantity in a dynamical modeling is rather mild and should be pursued further.

VI. CONCLUSION

We have extended the dynamical modeling of the pion-nucleon system in the first resonance region [8,9] to the process of pion photoproduction on the nucleon. Such an extension is indispensable in testing the πN dynamics beyond the elastic πN scattering.

The presented numerical results are obtained in the model which satisfies unitarity in the $\pi N \otimes \gamma N$ channel space to the leading order in the electromagnetic coupling, and hence Watson’s theorem is exactly fulfilled. We find that the model description of the pion-photoproduction multipoles is in overall agreement with the partial-wave analyses in the region from the threshold up to 650 MeV photon laboratory energy. We have therefore developed a realistic hadron-

exchange model describing the low and intermediate energy pion scattering and photoproduction on the nucleon in a unitary fashion. The model treats the quantum effects due to pion-nucleon loops in a Lorentz-covariant framework. It can be extended to higher energies by including more reaction channels. Furthermore, it is fully compatible and complementary to the relativistic meson-exchange models for the few-nucleon system, and hence can naturally be embedded in these models to describe more complicated processes.

The results for the threshold electric dipoles of the charged pion photoproduction are very close to the low-energy theorem (LET) prediction and in a reasonable agreement with experiment. In contrast, the electric dipole for the neutral pion photoproduction off the proton receives a sizable correction due to the final state interaction and which improves the agreement with experimental as compared to LETs. This correction is found to be in a qualitative agreement with the large chiral-loop correction to LET known from chiral perturbation theory (ChPT).

The two parameters of the $\gamma N \Delta$ vertex, which essentially are the only free parameters of the model, were fitted to E_{1+} and M_{1+} multipoles from the SAID solution. In the future we plan to extract these parameters directly from experimental data. At present, the $E2/M1$ ratio obtained in the model is equal to $3.8 \pm 1.6\%$ for the bare value and -2.6 ± 0.6 for the physical value. This is consistent with other analyses based on dynamical models. The precise value of this ratio is model dependent as it is sensitive on the details of the πN final state interaction. The only possibility of extracting this value in a model-independent way is by using ChPT with explicit Δ degrees of freedom. It would be extremely useful to carry out such analysis.

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APPENDIX: LORENTZ, MULTIPOLE, AND ISOSPIN DECOMPOSITION OF THE PHOTOPRODUCTION AMPLITUDE

The general Lorentz structure of the fully off-shell $\gamma\pi$ amplitude can be written as

$$T_{\lambda's}^{\rho'\rho}(p', k'; p, q) = \bar{u}_{\lambda'}^{\rho'}(\vec{p}') (1, \vec{p}') \left[\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} + \vec{p} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} + \vec{\epsilon} \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} + \vec{p} \vec{\epsilon} \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \right] \times \begin{pmatrix} 1 \\ \vec{p} \end{pmatrix} u_{\lambda}^{\rho}(\vec{p}), \quad (\text{A1})$$

where A, B, C, D are scalar functions, ε_{τ} ($\tau=0, \pm 1$) stands for the photon polarization vector, index $s=\lambda-\tau$ denotes the helicity of the γN state, and P is the total 4-momentum.

The parity-conserving $\gamma\pi$ amplitudes are the transition amplitudes from the γN partial-wave state

$$|J, r, \rho\rangle = \frac{|J, \rho, s\rangle - r\rho|J, \rho, -s\rangle}{\sqrt{2}} \quad (\text{A2})$$

to the πN partial-wave state,

$$|J, r, \rho\rangle = \frac{|J, \rho, \lambda\rangle + r\rho|J, \rho, -\lambda\rangle}{\sqrt{2}}. \quad (\text{A3})$$

In terms of the partial-wave helicity amplitudes $M_{\lambda's}^{\rho'\rho}$, these amplitudes are given by

$$T_r^{\rho'\rho} = T_{\lambda's}^{\rho'\rho} - rT_{\lambda'-s}^{\rho'\rho}. \quad (\text{A4})$$

For real photons s takes the values: $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$. Thus, for each parity r and the ρ -spin values, we find two independent amplitudes, e.g.,

$$\begin{aligned} \mathcal{A}_r^{\rho'\rho} &= T_{1/2,1/2}^{\rho'\rho} - rT_{1/2,-1/2}^{\rho'\rho}, \\ \mathcal{B}_r^{\rho'\rho} &= T_{1/2,3/2}^{\rho'\rho} - rT_{1/2,-3/2}^{\rho'\rho}. \end{aligned} \quad (\text{A5})$$

The *multipole amplitudes* are related to the parity conserving amplitudes in the following way:

$$E_{l+} = \frac{\sqrt{2}}{4(l+1)} [\mathcal{A}_+ + \sqrt{l(l+2)}\mathcal{B}_+], \quad (\text{A6a})$$

$$M_{l-} = \frac{\sqrt{2}}{4l} [-\mathcal{A}_- + \sqrt{(l-1)(l+1)}\mathcal{B}_-], \quad (\text{A6b})$$

$$E_{l-} = \frac{\sqrt{2}}{4l} [\mathcal{A}_- + \sqrt{(l+1)(l-1)}\mathcal{B}_-], \quad J > 1/2, \quad (\text{A6c})$$

$$M_{l+} = \frac{\sqrt{2}}{4(l+1)} [\mathcal{A}_+ - \sqrt{(l+2)l}\mathcal{B}_+], \quad J > 1/2, \quad (\text{A6d})$$

where E or M denotes whether the transition is of electric or magnetic type. Index l_{\pm} stands for the value of the πN state orbital momentum, $l=J-\frac{1}{2}r$, and the value of parity r .

Considering the isospin structure,

$$T = \pi'^a \chi_N' A_a \chi_N, \quad (\text{A7})$$

the following three decompositions are usually made:

$$\begin{aligned} A_a &= \delta_{a3} A^{(+)} + \tau_a A^{(0)} + i\varepsilon_{a3b} \tau_b A^{(-)} \\ &= \frac{1}{3} \tau_a \tau_3 A^{1/2} + \tau_a A^{(0)} + \left(\delta_{a3} - \frac{1}{3} \tau_a \tau_3 \right) A^{3/2} \\ &= \frac{1}{2} \tau_a (1 + \tau_3)_p A^{1/2} + \frac{1}{2} \tau_a (1 - \tau_3)_n A^{1/2} + \left(\delta_{a3} - \frac{1}{3} \tau_a \tau_3 \right) A^{3/2}. \end{aligned} \quad (\text{A8})$$

The relation amongst them is given by

$$A^{3/2} = A^{(+)} - A^{(-)}, \quad A^{1/2} = A^{(+)} + 2A^{(-)}, \quad (\text{A9a})$$

$${}_p A^{1/2} = A^{(0)} + \frac{1}{3}A^{1/2}, \quad {}_n A^{1/2} = A^{(0)} - \frac{1}{3}A^{1/2}. \quad (\text{A9b})$$

It is also possible to relate these to the amplitudes of specific reactions,

$$A(\gamma p \rightarrow \pi^0 p) = A^{(+)} + A^{(0)} = \frac{2}{3}A^{3/2} + {}_p A^{1/2}, \quad (\text{A10a})$$

$$A(\gamma n \rightarrow \pi^0 n) = A^{(+)} - A^{(0)} = \frac{2}{3}A^{3/2} - {}_n A^{1/2}, \quad (\text{A10b})$$

$$A(\gamma p \rightarrow \pi^+ n) = \sqrt{2}(A^{(0)} + A^{(-)}) = \sqrt{2}\left(-\frac{1}{3}A^{3/2} + {}_p A^{1/2}\right), \quad (\text{A10c})$$

$$A(\gamma n \rightarrow \pi^- p) = \sqrt{2}(A^{(0)} - A^{(-)}) = \sqrt{2}\left(\frac{1}{3}A^{3/2} + {}_n A^{1/2}\right). \quad (\text{A10d})$$

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