New perspective on complex cluster radioactivity of heavy nuclei

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Experimental data of complex cluster radioactivity (^{14}C–^{34}Si) are systematically analyzed and investigated with different models. The half-lives of cluster radioactivity are well reproduced by a new formula between half-lives and decay energies and by a microscopic density-dependent cluster model with the renormalized M3Y nucleon-nucleon interaction. The formula can be considered as a natural extension of both the Geiger-Nuttall law and the Viola-Seaborg formula from simple α decay to complex cluster radioactivity where different kinds of clusters are emitted. It is useful for experimentalists to analyze the data of cluster radioactivity. A new linear relationship between the decay energy of cluster radioactivity and the number of α particles in the cluster is found where the increase of decay energy for an extra α particle is between 15 and 17 MeV. The possible physics behind this new linear relationship is discussed.

I. INTRODUCTION

Nuclear physics originated from the discovery of natural radioactivity. A hundred years ago Becquerel discovered a kind of unknown radiation from uranium. Rutherford identified this radiation as α decay and named other kinds of radiation as β decay and γ transitions. Nearly a century after the discovery of radioactivity by Becquerel, Rose and Jones observed a new kind of radioactivity, ^{14}C from ^{223}Ra [1]. Later experiments by Gales *et al.* and by Price *et al.* [2,3] confirmed the existence of the new radioactivity. Other kinds of heavier cluster radioactivity (such as ^{20}O, ^{24}Ne, ^{28}Mg, ^{34}Si) were also observed [4,5] and primary studies of cluster radioactivity were carried out by some groups [6–13]. Although data on cluster radioactivity from ^{14}C to ^{34}Si have been accumulated in recent years, systematic analysis of the data has not been completed and a general law among the data has not been established for complex cluster radioactivity. In this article experimental data on cluster radioactivity are systematically investigated with an accurate and simple formula and with the microscopic density-dependent cluster model (DDCM) where the realistic M3Y nucleon-nucleon interaction is used. New physics behind cluster radioactivity is explored and discussed.

This article is organized in the following way. In Sec. II we approximately derive a new formula between half-life and decay energy for complex cluster radioactivity. This formula is a natural generalization of the famous Geiger-Nuttall law and Viola-Seaborg formula from simple α decay to complex cluster radioactivity. It can reproduce experimental half-lives within a factor of 4. In Sec. III we use a microscopic density-dependent cluster model to calculate the half-lives where the effective potential between cluster and daughter nucleus is a doubly folded integral between the renormalized M3Y nucleon-nucleon interaction and the density distributions of daughter nucleus and cluster. The common points and differences of different approaches are compared and discussed. In Sec. IV we propose a new linear relationship between the decay energy of cluster radioactivity and the

number of α particles in a cluster and the possible physics related to it is discussed. A summary is given in Sec. V.

II. NEW FORMULA BETWEEN THE HALF-LIVES AND DECAY ENERGIES OF CLUSTER RADIOACTIVITY

The cluster radioactivity of heavy nuclei is described as a quantum tunneling effect through a barrier where the process is determined by the sum of an attractive nuclear potential, a repulsive Coulomb potential, and a centrifugal potential. It is assumed that the potential is a function of only the radial coordinate R where R is the separation between the mass center of the cluster and the mass center of the daughter nucleus. Usually there are three classical turning points for the above potential and they are denoted as R_1, R_2, and R_3 in order of increasing distance from the origin. R_1 is close to the origin (or coincides with the origin). The turning point R_3 lies in a region far from the origin where the attractive nuclear potential is zero. R_1, R_2, and R_3 are obtained by numerical solutions of the equation $V(R)=Q$ where Q is the cluster decay energy. Without loss of generality the decay width (or decay constant) can be written in the following way [14–17]:

$$\Gamma = P_c F_c \exp\left(-2\int_{R_2}^{R_3} dR\, K(R)\right) \qquad (1)$$

where P_c is the preformation probability of the cluster in the parent nucleus, and F_c describes the motion of the cluster between the first and second classical turning points. The exponential factor is the Gamow factor. The wave number $K(R)$ is given by

$$K(R) = \sqrt{(2\mu/\hbar^2)|Q - V(R)|} \qquad (2)$$

where $V(R)$ is the total potential between cluster and daughter nucleus and μ is the reduced mass of cluster and daughter nucleus. The cluster-decay half-life is related to the width by [14–17]

$$T_{1/2} = \hbar \ln 2/\Gamma. \qquad (3)$$

In order to obtain an analytical expression for the half-life let us complete the integral in Eq. (1). We make the same approximation as that for α decay [18–21]. It is assumed that the long-range Coulomb potential $(\sim Z_1 Z_2 / R)$ dominates in the range $R \geqslant R_2$ and the decay energy is significantly lower than the height of the barrier [18–21]. The Gamow factor is approximately $G = \exp(-c_1 Z_1 Z_2 Q^{-1/2} - c_2)$ where c_1 and c_2 are constants [18–21]. Then the half-lives of cluster radioactivity can be given by the following equation:

$$\log_{10}(T_{1/2}) \approx c_3 - \log_{10}(P_c F_c) + c_4(Z_1 Z_2 Q^{-1/2}) \qquad (4)$$

where c_3 and c_4 are constants. Therefore an analytical expression for the half-life is approximately obtained. This is similar to the derivation of the Geiger-Nuttall law for α decay [18,19]. Now we assume that the preformation probability of a cluster is an exponential function of the multiplication of charge numbers $P_c = 10^{(-c_5 Z_1 Z_2 - c_6)}$ where c_5 and c_6 are constants. It is expected that the preformation probability of a heavy cluster such as ^{14}C should be smaller than that of an α cluster. The maximum probability of an α cluster in a heavy nucleus from both experimental data and theoretical calculations is 1 [11,22,23]. We consider that the preformation probability of a heavy cluster such as ^{14}C should be much less than 1 and the probability decreases quickly with increase of the charge number of the cluster. The first leading term in the analytical expression of $\log_{10} T_{1/2}$ is the term $Z_1 Z_2 Q^{-1/2}$ which is directly related to the Gamow factor. Then we write the other terms as the sum of the $Z_1 Z_2$ term and a constant. This is equivalent to an averaging process of a complex problem and this idea is widely used in nuclear physics as in the model of a compound nucleus in reactions. The term related to the preformation probability is also included. Then the equation for the half-life can be written as

$$\log_{10}(T_{1/2}) = a Z_1 Z_2 Q^{-1/2} + c Z_1 Z_2 + d + h \qquad (5)$$

where a, c, and d are the constants to be determined for even-even cluster emitters and h represents a blocking factor of an odd nucleon in odd-A nuclei. This is a new formula to calculate the half-lives of cluster radioactivity. For even-even nuclei there are only three parameters a, c, and d ($h=0$ for even-even parent nuclei). For any calculations on half-lives of cluster radioactivity with a phenomenological potential, the number of input parameters is larger than three because one needs at least three parameters to define a potential (depth, diffuseness, and radius) and its variation for different parent nuclei and for different clusters. One also needs at least one parameter to define the preformation probability for various types of cluster radioactivity. The new formula for cluster radioactivity has the minimum inputs in physics. It has a firm basis in physics as it can be approximately derived. The meaning of each term is also very clear. We also notice that there are simple formulas to calculate the half-lives of cluster radioactivity [6,7]. All of these formulas should be considered as an effective theory for the very complicated process of cluster radioactivity because they are based on different variations of Gamow's theory.

Before presenting numerical results let us compare the formula of complex cluster radioactivity with the famous Geiger-Nuttall law and Viola-Seaborg formula of α decay. The Geiger-Nuttall law is $\log_{10} T_{1/2} = a Q^{-1/2} + b$ where a and b are the given constants for even-even nuclei on an isotopic chain. When we fix the formula of cluster radioactivity for $Z_1 = 2$, the new formula comes naturally back to the Geiger-Nuttall law for an isotopic chain. The Viola-Seaborg formula for α decay is [19]

$$\log_{10}(T_{1/2}) = (aZ + b)Q^{-1/2} + cZ + d + h \qquad (6)$$

where the four parameters a, b, c, d are determined by α decay of even-even nuclei and h is the blocking factor for odd nucleons. In our formula we only use three parameters to describe the complex cluster decay of even-even nuclei from ^{14}C to ^{32}Si. In view of the similarity between the new formula for cluster radioactivity and the Viola-Seaborg formula the new formula can be considered as a natural generalization of the Viola-Seaborg formula from simple α decay to complex cluster radioactivity.

Now we determine the three parameters a, c, and d for even-even emitters with the available data of cluster radioactivity in Table I. The parameters a, c, d are obtained by a linear least-squares fit to experimental half-lives of cluster radioactivity for even-even emitters. The total square deviation is

$$S_{\text{even-even}} = \sum_{i=1}^{i=10} [\log_{10} T_{1/2}(i)(\text{expt}) - \log_{10} T_{1/2}(i)(\text{theor})]^2. \qquad (7)$$

Then the parameter h is obtained by a linear least-squares fit of the experimental half-lives of cluster radioactivity for five odd-A emitters. In this way the parameters a, c, d, and h are determined and their values are as follows:

$$a = 1.517\,99, \quad c = -0.053\,387, \quad d = -92.911\,42, \quad h$$
$$= 1.402. \qquad (8)$$

The total square deviation of both even-even and odd-A nuclei is $S = 1.790$ for 15 cluster emitters. The average square deviation of each nucleus is $S/15 = 0.12$. In the above formula the unit of decay energy is the MeV and that of the half-life is the second. The numerical results are listed in Table I and Fig. 1.

In Table I the first column denotes the mode of cluster radioactivity. The second column shows the experimental decay energies where the data are taken from the nuclear mass table by Audi and Wapstra [5]. The third column is the logarithm of experimental half-lives $(\log_{10} T_{1/2})$ [5]. The numerical results from the formula of cluster radioactivity are listed in column 4. The results from a microscopic DDCM are listed in columns 5 and 6 where the angular momentum and parity of the cluster are taken into account in numerical calculations. We will discuss the model later. It is seen from columns 3 and 4 that the half-lives from the formula agree very well with the data. In many cases the deviation between the data and calculated values is less than 0.5 and this means that calculated half-lives from the formula agree with the

TABLE I. Half-lives of cluster radioactivity calculated from the formula $\log_{10}(T_{1/2})=aZ_1Z_2Q^{-1/2}+(cZ_1Z_2+d)+h$ and from density-dependent cluster model with renormalized M3Y nucleon-nucleon interactions. The total square deviation between the formula and the data is $S=1.790$ for 15 emitters. The total square deviations from two sets of results in the DDCM are $S=2.075$ and $S=1.793$, respectively. The maximum deviation for single cluster decay is $|\log_{10}(T_{1/2})(\text{expt})-\log_{10}(T_{1/2})(\text{theor})|=0.6$ for the formula and this corresponds to a factor of 4 for the absolute half-life of cluster radioactivity. The maximum deviation between the results of the DDCM and data is $|\log_{10}(T_{1/2})(\text{expt})-\log_{10}(T_{1/2})(\text{theor})|=0.7$ and it corresponds to a factor of 5 for the absolute half-life.

Decay	Q (MeV)	$\log_{10}T_{1/2}^{\text{expt}}$ (s)	$\log_{10}T_{1/2}^{formula}$ (s)	$\log_{10}T_{1/2}^{\text{RM3Y}(1)}$ (s)	$\log_{10}T_{1/2}^{\text{RM3Y}(2)}$ (s)
^{221}Fr–^{207}Tl+^{14}C	31.29	14.52	14.43	14.82	14.86
^{221}Ra–^{207}Pb+^{14}C	32.40	13.37	13.43	13.68	13.79
^{222}Ra–^{208}Pb+^{14}C	33.05	11.10	10.73	11.08	11.19
^{223}Ra–^{209}Pb+^{14}C	31.83	15.05	14.60	14.77	14.88
^{224}Ra–^{210}Pb+^{14}C	30.54	15.90	15.97	15.91	16.02
^{226}Ra–^{212}Pb+^{14}C	28.20	21.29	21.46	21.05	21.16
^{228}Th–^{208}Pb+^{20}O	44.72	20.73	20.98	21.46	21.09
^{230}Th–^{206}Hg+^{24}Ne	57.76	24.63	24.17	24.67	24.38
^{231}Pa–^{207}Tl+^{24}Ne	60.41	22.89	23.44	23.09	22.91
^{232}U–^{208}Pb+^{24}Ne	62.31	20.39	21.00	20.41	20.34
^{233}U–^{209}Pb+^{24}Ne	60.49	24.84	24.76	24.30	24.24
^{234}U–^{206}Hg+^{28}Mg	74.11	25.74	25.12	25.42	25.39
^{236}Pu–^{208}Pb+^{28}Mg	79.67	21.65	21.90	20.94	21.20
^{238}Pu–^{206}Hg+^{32}Si	91.19	25.30	25.33	25.80	26.04
^{242}Cm–^{208}Pb+^{34}Si	96.51	23.11	23.19	23.28	23.04

data within a factor of 3. The biggest deviation occurs for ^{234}U and it is 0.62 (see Table I). This corresponds to a factor of 4 between experimental half-life and calculated value. Therefore the experimental half-lives of complex cluster radioactivity can be reproduced by the formula within a factor of 4. The experimental half-lives of complex cluster radioactivity can be reproduced very well by an accurate and simple formula. This formula has a firm basis in physics as it can be approximately derived. The signs and values of the constants in the formula also agree with our expectation. After we make a scale transformation of the parameters between the formula for cluster radioactivity and the Viola-Seaborg formula for α decay, we find that the difference of parameters in the two formulas is small. So the meaning of the terms in the two formulas is similar. The only difference between the two formulas is that Viola and Seaborg introduced a parameter b for α decay. This is because the half-lives of α decay range from a very short time (microseconds) to a very long time (10^{19} yr). So a parameter b is possibly needed. For cluster radioactivity (^{14}C–^{34}Si) the half-lives are very long due to
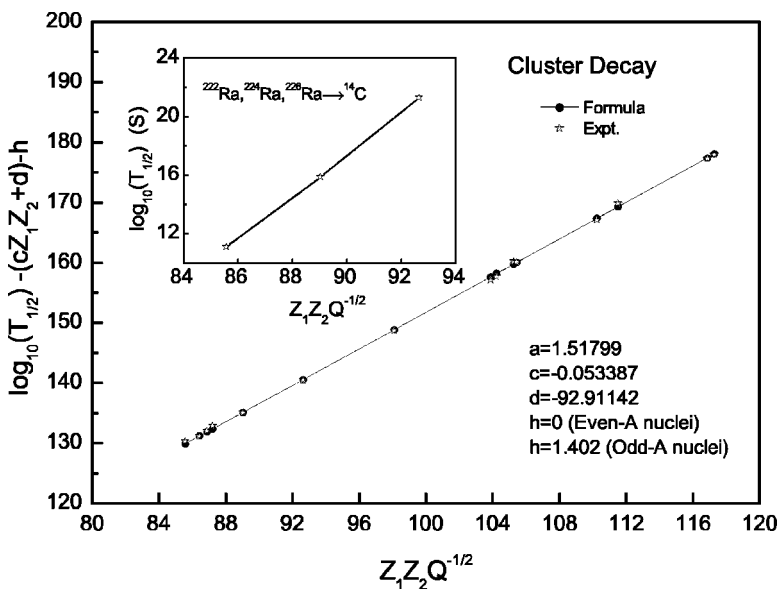

FIG. 1. The small figure in the inset is the Geiger-Nuttall law for the radioactivity of ^{14}C in the even-even Ra isotopic chain. The large figure plots the linear relationship between half-lives ($\log_{10}T_{1/2}-cZ_1Z_2-d-h$) and decay energies ($Z_1Z_2Q^{-1/2}$) for complex cluster radioactivity.

both very low preformation probability and low tunneling probability of a large-mass cluster. Therefore the parameter b is not needed at present. With more and more accumulation of data on cluster radioactivity it will be interesting to see whether this parameter should be introduced.

By the way we would like to point out that we can get the formula for cluster radioactivity in another way. Let us focus on the inset of Fig. 1 where the half-lives of ^{14}C radioactivity for even-even Ra isotopes ($\log_{10} T_{1/2}$) are plotted against decay energies ($Q^{-1/2}$). It is found that there is a linear relationship between the decay half-lives of ^{14}C and decay energies (see the inset in Fig. 1). It can be described by the following expression:

$$\log_{10}(T_{1/2}) = aZ_1Z_2Q^{-1/2} + cZ_1Z_2 + d. \tag{9}$$

This indicates that there is also the Geiger-Nuttall law for ^{14}C radioactivity in the Ra isotopic chain. It is assumed that this equation can be extrapolated to other isotopic chains and to other clusters. Then a general law for cluster radioactivity can be obtained. Although the formula of cluster radioactivity is derived with some approximations, the good agreement between the calculated values and experimental data clearly shows the validity of the accurate and simple formula. This is drawn in Fig. 1 where the x axis is $Z_1Z_2Q^{-1/2}$ and the y axis is the linear combination of a few parts of the formula of complex cluster radioactivity. It is seen clearly that the experimental points lie approximately in a straight line. This is very similar to the results in α decay.

III. DENSITY-DEPENDENT CLUSTER MODEL OF CLUSTER RADIOACTIVITY

After discussing the formula for cluster radioactivity, we present numerical results of half-lives from a microscopic density-dependent cluster model [24]. In the DDCM the cluster-core potential is the sum of the nuclear, Coulomb, and centrifugal potentials,

$$V(R) = V_N(R) + V_C(R) + l(l+1)\hbar^2/(2\mu R^2), \tag{10}$$

where R is the separation between cluster and core and l is the angular momentum of the cluster. μ is the reduced mass of the cluster-core system. The nuclear potential $V_N(R)$ between cluster and daughter nucleus is the double-folded integral of the renormalized M3Y nucleon-nucleon potential [25–27] and the density distributions of cluster and daughter nucleus,

$$V_N(R) = \lambda \int d\mathbf{r}_1 d\mathbf{r}_2 \rho_1(\mathbf{r}_1)\rho_2(\mathbf{r}_2)g(E,|\mathbf{s}|) \tag{11}$$

where $\lambda = 0.55$ is the renormalization factor [27]. ρ_1 and ρ_2 are the density distributions of cluster particle and core (daughter nucleus) [26,27]. The quantity $|\mathbf{s}|$ is the distance between a nucleon in the core and a nucleon in the cluster. $\mathbf{s} = \mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1$ [26,27]. The density distribution of cluster and daughter nucleus has a standard Fermi form $\rho_i(r_i) = \rho_{0i}/\{1 + \exp[(r_i - c_i)/a]\}$. Here $i=1$, 2 corresponds to cluster and daughter nucleus. The value of ρ_{0i} is fixed by integrating the density distribution equivalent to the mass number of the

nucleus. The constants $c_i = 1.07 A_i^{1/3}$ fm and $a = 0.54$ fm are taken from textbooks [28–32]. The matter radius of heavy nuclei with this choice is $R_{\text{rms}} \approx 1.2 \times A^{1/3}$ fm [28–32]. The M3Y nucleon-nucleon interaction [25] is given by two direct terms with different ranges, and by an exchange term with a delta interaction [26,27],

$$g(E,|\mathbf{s}|) = 7999\frac{\exp(-4s)}{4s} - 2134\frac{\exp(-2.5s)}{2.5s} - 276(1 - 0.005\, E_1/A_1)\delta(\mathbf{s}). \tag{12}$$

For the renormalization factor in the nuclear potential its value $\lambda = 0.55$ [27] is taken directly from the reaction model where the scattering data of α particles on nuclei are reasonably reproduced by a double-folded model with the renormalized M3Y interaction (RM3Y).

For the Coulomb potential between daughter nucleus and cluster, a uniform charge distribution of nuclei is assumed [28–32]. The Coulomb potential is [28–30]

$$V_C(R) = \begin{cases} \dfrac{Z_1Z_2e^2}{2R_c}\left[3 - \left(\dfrac{R}{R_c}\right)^2\right] & (R \leqslant R_c), \\ \dfrac{Z_1Z_2e^2}{R} & (R \geqslant R_c), \end{cases} \tag{13}$$

where $R_c = 1.2 A_d^{1/3}$ fm and A_d is the mass number of the daughter nucleus [28–31]. Z_1 and Z_2 are the charge numbers of cluster and daughter nucleus, respectively.

Recently this model has reproduced the experimental half-lives of α decay within a factor of a few times [24]. Here we generalize the model to complex cluster radioactivity without extra adjustment of the potential. Substituting this potential into Eqs. (1) and (2), we obtain the classical turning points and calculate the Gamow factor directly. The factor F_c is given by the expression [14,15,17]

$$F_c \int_{R_1}^{R_2} dR \frac{1}{K(R)}\cos^2\left(\int_{R_1}^{R} dR' K(R') - \frac{\pi}{4}\right) = 1. \tag{14}$$

A detailed discussion of the factor F_c can be found in Refs. [14–17]. For the preformation probability of clusters, two choices are used in the DDCM in order to see the sensitivity of the results. One is $P_c = 10^{-(0.4 A_1 - 2)}$ where A_1 is the mass number of the cluster and the other is $P_c = 10^{-(0.011674 Z_1 Z_2 - 2.035446)}$. The numerical results of the above choices are presented in the last two columns of Table I [denoted as RM3Y (1) and RM3Y (2)]. For calculations of odd-A nuclei the blocking effect of the preformation probability is taken into account by subtracting a constant $h = 1.175$ from the exponential expression of P_c. For even-even nuclei a favored transition is assumed. For odd-A nuclei the variation of angular momentum and parity between parent and daughter nuclei has been included for cluster decay in the DDCM where we assume parity conservation in cluster radioactivity. By the way the preformation probability of ^{14}C in the DDCM is close to that used by Rose and Jones [1]. It is seen from the last two columns that the DDCM can reproduce experimental data well.
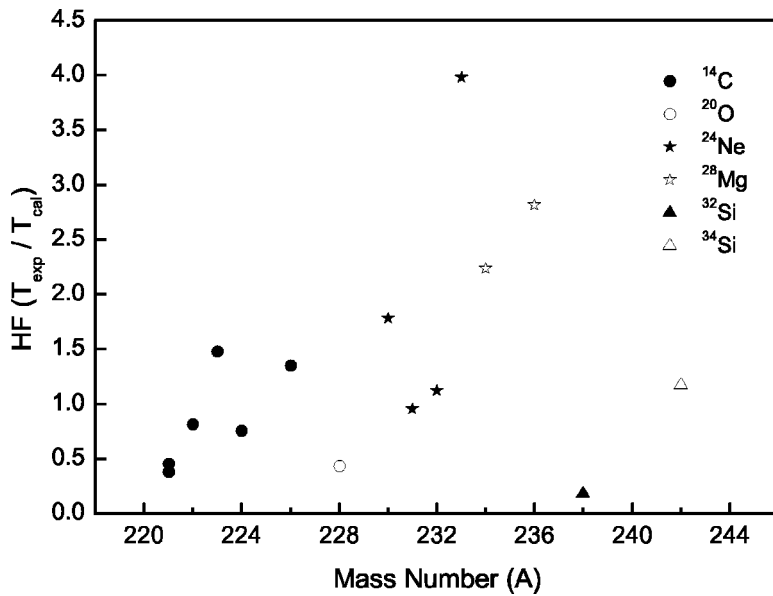
FIG. 2. The variation of the hindrance factor [HF=T(expt)/T(theor)] with mass number for cluster radioactivity where theoretical results are obtained by the DDCM [RM3Y(2)].

The total square deviations in the DDCM are $S=2.075$ and $S=1.793$, respectively. Therefore the results of the DDCM with two adjustable parameters in the preformation probability are as good as the results from the formula of cluster radioactivity. For the two sets of results in the DDCM the second set of results is better than the first one. In order to see the difference between the theoretical results and experimental data, we define the hindrance factor (HF), which is the ratio of experimental half-lives and theoretical ones. The hindrance factors for the second set of results are drawn in Fig. 2 [HF=$T_{1/2}$(expt)/$T_{1/2}$(theor)]. In general the experimental data agree with the theoretical results within a factor of 1–3. For one or two nuclei the agreement is within a factor of 4–5.

In order to see the common points and differences between the DDCM and the formula we fit the theoretical results of the DDCM [RM3Y(2)] with the formula and obtain the parameters $a'=1.498\,15$, $c'=-0.052\,532$, $d'=-91.549\,17$, and $h'=1.402$. These values are very close to the previous values a,c,d, and h in the formula. This shows again that the cluster radioactivity formula of half-lives has a firm basis in physics. Important physics is included in the

formula. The good agreement between theoretical half-lives and experimental data shows that the model and formula are very reliable. In future one can also use the above values a',c',d',h' to estimate the half-lives of cluster radioactivity for unknown emitters as they are based on the DDCM. So we provide two sets of parameters to estimate the half-lives of unknown cluster radioactivity with the formula. It is expected that the ratios between experimental half-lives and estimated values are within a factor of 5–10 for many cluster emitters. An abnormally large deviation (such as 20–1000 times) between the estimated values and the future data will suggest the existence of a new mechanism of cluster radioactivity and it will be interesting to investigate the abnormal behavior in detail.

In Table II, we predict the half-lives of some possible cluster emitters by the formula for half-lives of cluster radioactivity and by the DDCM with two sets of inputs. We consider that they are suitable candidates for new cluster emitters. This is useful for future experiments.

In Figs. 3–5 we plot the variation of the total cluster-core potential for parent nuclei ^{222}Ra, ^{232}U, and ^{242}Cm where the daughter nucleus is ^{208}Pb. It is seen from Figs. 3–5 that the

TABLE II. Predicted half-lives for the candidates for new cluster emitters by the formula and by the DDCM with two sets of inputs.

| Decay	Q (MeV)	$\log_{10}T_{1/2}^{formula}$ (s)	$\log_{10}T_{1/2}^{RM3Y(1)}$ (s)	$\log_{10}T_{1/2}^{RM3Y(2)}$ (s)
^{218}Rn–^{208}Pb+^{10}Be	14.36	20.97	21.51	21.30
^{220}Rn–^{206}Hg+^{14}C	28.54	17.85	18.34	18.31
^{222}Rn–^{208}Hg+^{14}C	26.45	23.14	23.30	23.27
^{236}U–^{212}Pb+^{24}Ne	55.95	29.72	30.38	30.32
^{237}Np–^{207}Tl+^{30}Mg	74.82	27.18	28.16	27.47
^{240}Pu–^{206}Hg+^{34}Si	91.03	25.49	26.76	26.20
^{241}Am–^{207}Tl+^{34}Si	93.93	25.56	26.01	25.61
^{240}Cm–^{208}Pb+^{32}Si	97.34	22.43	21.62	22.18
^{244}Cm–^{210}Pb+^{34}Si	93.14	26.37	27.28	27.05
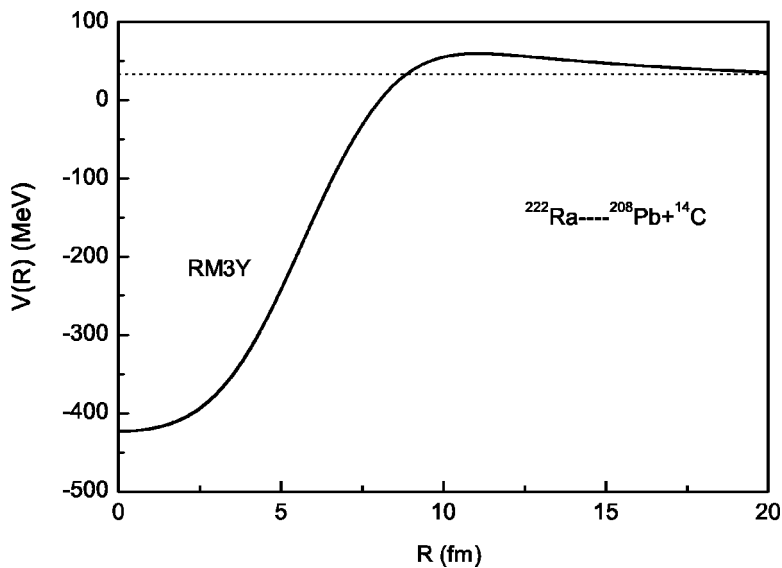
FIG. 3. The variation of the total cluster-core potential in the DDCM (denoted as RM3Y) with radial coordinate R for ^{14}C radioactivity of ^{222}Ra. The dotted horizontal line in the figure is the cluster decay energy.

potential of the cluster is very deep for small R. This is the common behavior of the double-folded potential and it is well known in nuclear reactions. There is a barrier in the range $R=8$–20 fm. The width of the barrier decreases with increase of the mass number of the cluster. The height of the barrier relative to the decay energy is approximately between 25 and 40 MeV for cluster radioactivity.

IV. LINEAR RELATIONSHIP BETWEEN DECAY ENERGIES OF CLUSTER RADIOACTIVITY AND THE NUMBER OF α PARTICLES IN THE CLUSTER

Now let us see the variation of decay energy in Table I. It is seen from Table I that the decay energies for the same kind of cluster such as ^{14}C (or ^{24}Ne) are approximately constant. A linear relation between the decay energy and the number of α particles in the cluster is found and drawn in Fig. 6 where the daughter nuclei are fixed as ^{208}Pb and ^{206}Hg, respectively. The data for large clusters ^{20}O, ^{24}Ne, ^{28}Mg, and ^{32,34}Si are used in the figure. This linear relation of cluster

radioactivity is very different from that of α decay in heavy nuclei. Here it is not completely sure that this linear relationship is a new phenomenon for cluster radioactivity or is from an accident agreement. A definite answer can be given with the further accumulation of experimental data. The approximate relationship between the decay energy and the number of α particles in the cluster is $Q=Q_c(Z1-2)/2$ where Q_c is between 15 and 17 MeV (the fit value of Q_c for all nuclei in Table I is $Q_c=15.378$ MeV). This is very similar to the saturation of the nuclear force where the total binding energy of a nucleus is $B_{\text{tot}}=\bar{B}A$ with $\bar{B}=7.0$–8.8 MeV. In exact meaning this formula should be modified as $B=\bar{B}(A-1)$ for light nuclei because the number of nucleons is at least two in order to define the binding energy of the nuclei. So this is very similar to the expression for the decay energy of cluster radioactivity. The increase of the decay energy for an extra α particle in cluster radioactivity is approximately a constant $Q_c=15$–17 MeV. This increase of the decay energy is much larger than for the α decay energy of the nuclei in this mass range ($Q_\alpha=4$–7 MeV). This energy is less than the binding
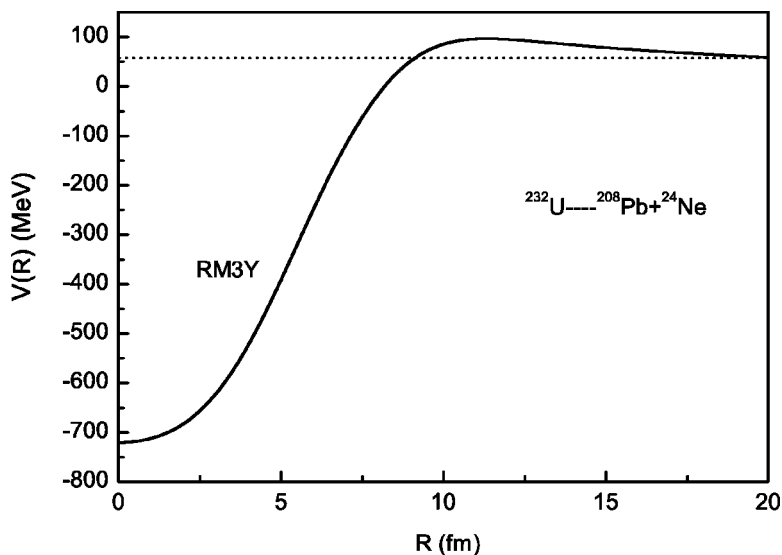

FIG. 4. The variation of the total cluster-core potential in the DDCM (denoted as RM3Y) with radial coordinate R for ^{24}Ne radioactivity of ^{232}U. The dotted horizontal line in the figure is the cluster decay energy.
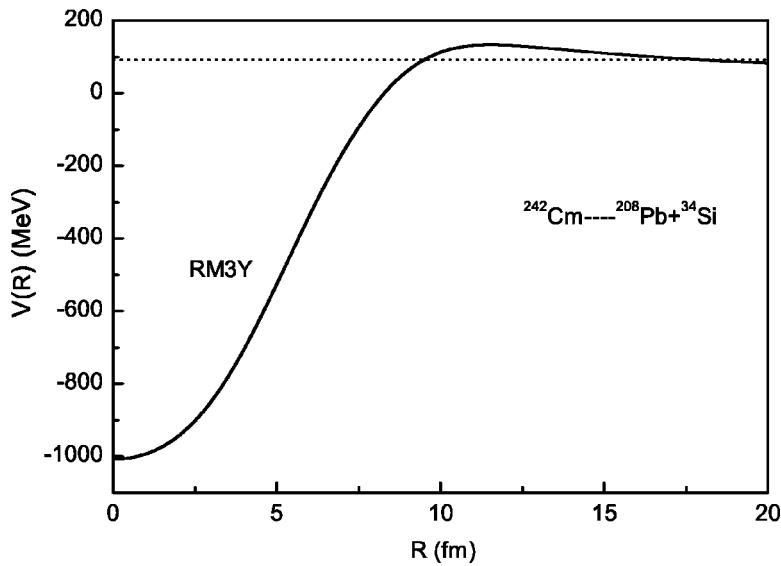
FIG. 5. The variation of the total cluster-core potential in the DDCM (denoted as RM3Y) with radial coordinate R for ^{34}Si radioactivity of ^{242}Cm. The dotted horizontal line in the figure is the cluster decay energy.

energy of an α particle ($B=28.3$ MeV). We may guess that several α particles and a few neutron pairs are correlated into a huge cluster such as ^{14}C, ^{24}Ne, ^{28}Mg, and ^{32,34}Si near the surface of heavy nuclei just before cluster decay. Deep discussions on this are beyond the scope of this article [33–42] because the formation probability of clusters is still an open problem in nuclear physics. It is well known in condensed matter physics that there is the Josephson effect [43,44]. It is stated that the collective Cooper pairs near the Fermi surface of a superconductor naturally move without resistance through a thin insulator to another superconductor with a low Fermi surface [43,44]. The movement of collective Cooper pairs in coherence leads to a macroscopic electric current which is called a supercurrent in condensed matter physics [43,44]. We may expect that the mechanism of cluster radioactivity from nuclei is like the Josephson effect in a finite system where a huge cluster such as ^{24}Ne moves through the barrier without resistance. Of course the ground state of the parent nuclei has definite spin and definite parity [29,45,46]. One can observe some new phenomena such as the variation

of spin and parity between parent and daughter nuclei in cluster radioactivity. Up to date parity conservation has been tested for α decay [29] but it has not been tested for cluster radioactivity. It seems to us that parity should be conserved for cluster radioactivity which starts from the ground state of nuclei and is governed by strong interaction and electromagnetic interaction [46]. The existence of the blocking effect of odd nucleons in cluster radioactivity can be a primary indication of parity conservation in cluster radioactivity. However, any evidence of possible parity violation in atomic spectra and in nuclear spectra is very interesting in physics [47]. Therefore we strongly suggest that experimental physicists check the conservation of parity for cluster radioactivity because it is an exotic phenomenon between simple α decay and very complex spontaneous fission.

V. CONCLUSIONS

In summary we systematically investigated the available data of cluster radioactivity by both phenomenological and
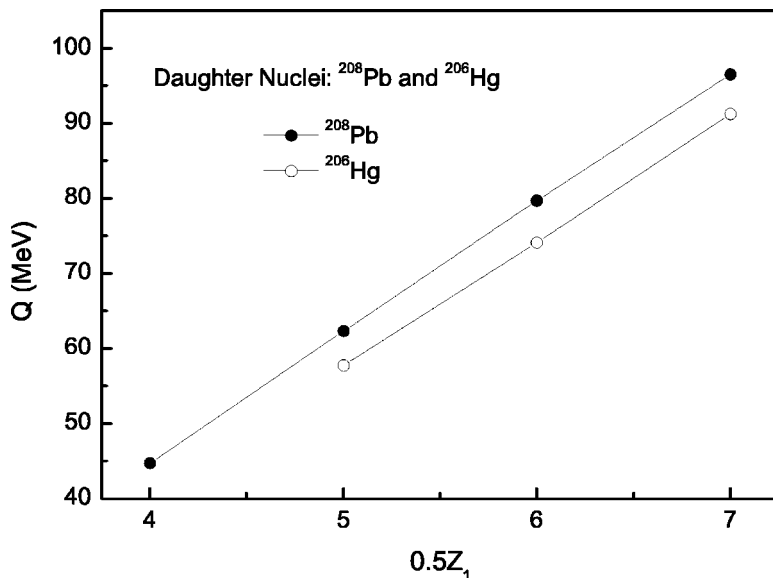

FIG. 6. The variation of the decay energy of cluster radioactivity with the number of α particles in the cluster where the data of large clusters (^{20}O, ^{24}Ne, ^{28}Mg, and ^{32,34}Si) are drawn.

microscopic models. A new formula between the half-lives and decay energies of cluster radioactivity is proposed. Experimental half-lives are well reproduced by this formula which is a natural generalization of the Geiger-Nuttall law and Viola-Seaborg formula from simple α decay to complex cluster radioactivity. Experimental half-lives are also well reproduced by the density-dependent cluster model where a renormalized M3Y nucleon-nucleon interaction is used. The DDCM supports the new formula for half-lives of cluster radioactivity. It is further found that there is a new linear relation between the decay energy of cluster radioactivity and the number of α particles in the cluster. The possible physics of this is discussed.

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