

Coulomb distortion in high- Q^2 elastic e - p scattering

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Recently, there has been a significant amount of activity to try and understand the discrepancy between Rosenbluth and polarization transfer measurements of the proton form factors. It has been suggested that the standard use of plane-wave Born approximation in extracting the form factors is insufficient, and that higher-order terms must also be included. Of the corrections missing in standard prescriptions, Coulomb distortion is the most well understood. In this paper, we examine the effect of Coulomb distortion on the extraction of the proton form factors.

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I. INTRODUCTION

The determination of the proton charge form factor G_E recently has received extensive attention. The determination via the traditional “Rosenbluth” separation of the longitudinal (L , charge) and transverse (T , magnetic) contributions to the electron-proton elastic cross section disagrees at large momentum transfer q with the results obtained via measurements of the proton recoil polarization [1]. It was immediately suspected that this could come from the fact that at large q the magnetic cross section σ_M becomes much larger than the electric cross section σ_E . Therefore, small corrections to the cross section yield significant corrections to the small contribution proportional to G_E^2 .

A reanalysis of the available data on e - p scattering with careful consideration of the systematic errors has confirmed the presence of the discrepancy [2]. A recent experiment [3] exploiting a technique less sensitive to systematic errors — recoil detection in the L/T separation — has also confirmed the discrepancy between the values of G_E from L/T separation and recoil-polarization measurement

Traditionally, G_E and G_M have been determined using the Rosenbluth technique and the plane-wave Born approximation. Several papers have recently pointed out that second-order effects could play an important role [4–6]. Among the second-order effects two particular ones can be singled out.

(1) The effect of the proton charge to Coulomb distortion of the ingoing and outgoing electron waves. This distortion has traditionally been included in the analysis of electron scattering experiments for nuclei with $Z > 1$, but is almost always neglected for $Z=1$ where it has been effectively absorbed into the form factors. For $Z=1$, Coulomb distortion has been shown to have significant effects on the proton rms radius [7,8] and to remove a long-standing discrepancy for the deuteron [9]. Diagrammatically, Coulomb distortion corresponds to the exchange of one hard and one (or several) soft photon.

(2) The exchange of two hard photons. This contribution plays a role mainly at the larger q 's. Because $\sigma_M \gg \sigma_E$, one could expect that the dominant term comes from two successive magnetic (i.e., spin flip) interactions which therefore

contribute to the small electric (i.e., non-spin-flip) term. Indeed, some exploratory calculations [4,10] support this scenario. Tjon has calculated the second-order contribution with the proton magnetic moment, μ_p , reduced from 2.79 to 1. This modification greatly reduces the second-order effect, the leftover contribution being of the order of what is expected from Coulomb distortion alone. This qualitatively can be understood in terms of the above model; if two successive magnetic scatterings were the only process, the contribution would be expected to scale with μ_p^2 .

Coulomb distortion is an established effect and fairly straightforward to calculate [8,9]. More difficulties arise for the case of the exchange of two hard photons. Here not only the proton ground state (treated in [4]), but also the proton excited states come in. These dispersive effects are much more difficult to calculate. In this paper we study the effect of Coulomb distortion *alone*. Among the various second-order contributions, this is the one which is best established.

II. COULOMB DISTORTION

We have calculated the Coulomb distortion correction for electron-proton scattering using the second Born approximation, following the approach presented in Ref. [9] for electron-deuteron scattering. This series in $Z\alpha$ is expected to be very accurate for $Z\alpha \sim 0.01$ of interest for hydrogen.

The corrections have been calculated using an exponential charge distribution for the proton, in accordance with the fact that the proton charge form factor is close to a dipole. The deviations from the dipole shape found at very high q are not expected to have consequences on the Coulomb distortion, which is a long-range effect. One finds that the effect of the Coulomb distortion is of order 1% of the cross section. It is mainly dependent on the angle, and thus does have effects in Rosenbluth separations which compare data at the same q but different angles.

III. RESULTS

Figure 1 shows the effect of the Coulomb distortion on the cross section as a function of ε for several different Q^2

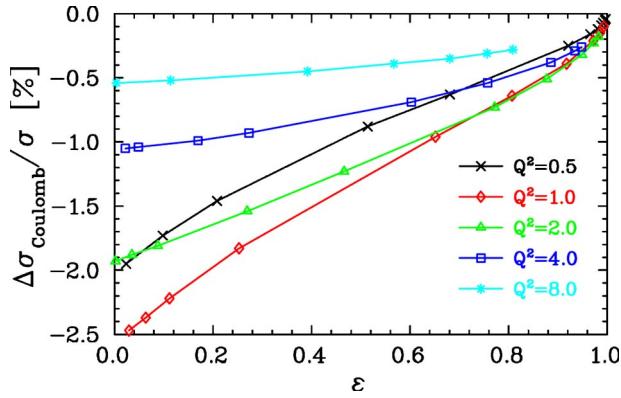


FIG. 1. (Color online) Coulomb distortion to the elastic electron-proton cross section.

values. The effect is maximum near $Q^2=1 \text{ GeV}^2$, and has a significant ε dependence. Note that the effect is very nearly linear for these Q^2 values, except for the very largest ε values.

Figure 2 shows the results obtained in an effective momentum approximation (EMA) calculation. The EMA yields similar results for large ε and large Q^2 , but is significantly different from the second-order Born calculation elsewhere. The correction is calculated by increasing the energy of the incoming and scattered electrons at the interaction vertex by the Coulomb potential, and evaluating the cross section using form factors at the modified Q^2 value, but leaving the Mott cross section unchanged. For nuclei, the Coulomb potential is usually determined by assuming a uniform sphere with an rms radius that matches electron scattering measurements, and then using the potential at the surface (or center) of the sphere. For the proton, the uniform sphere will tend to underestimate the effect since the charge is more highly concentrated in the center, so we use 1.9 MeV, the potential at the center.

Alternative prescriptions for the EMA use the modified kinematics for the full cross section, rather than just the form factors, or else include a focusing factor. However, in both cases one obtains a significant reduction in the size of the correction at low ε , improving the agreement somewhat for very large Q^2 values, but making it significantly worse for

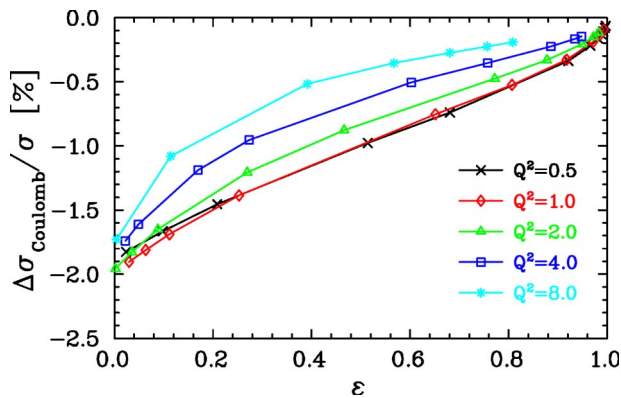


FIG. 2. (Color online) Coulomb distortion to the elastic electron-proton cross section in the EMA.

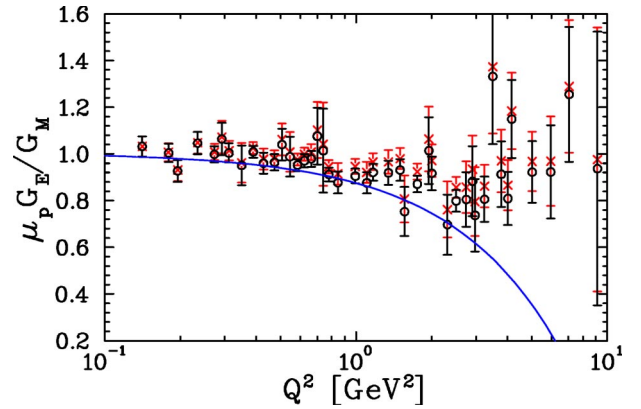


FIG. 3. (Color online) Rosenbluth extraction of $\mu_p G_E/G_M$ before (\times) and after (circle) correcting for Coulomb distortion. The solid line shows the parametrization of the polarization transfer measurements.

lower Q^2 values. One can also use a different value for the value of the Coulomb potential to increase or decrease in the correction, but the overall agreement would not be any better. While the EMA prescription can be “tuned” to give good agreement for a range in ε or Q^2 , none of these approaches yield an adequate approximation to the exact calculation.

The effect of the Coulomb distortion on the form factors at very low Q^2 values has been studied in detail [7]. For larger Q^2 values, the distortion grows with Q^2 until approximately 1 GeV^2 , and then begin to decrease. However, as Q^2 increases, the electric form factor yields a decreasing ε dependence in the reduced cross section, and so the effect of the Coulomb distortion on G_E is magnified as Q^2 increases. Figure 3 shows the Rosenbluth extraction of $\mu_p G_E/G_M$ from the global analysis of Ref. [11] before and after correcting the cross-section data for Coulomb distortion. Applying this correction reduces the extracted form factor ratio, improving the agreement with polarization transfer results, but the ratio is still well above the polarization transfer result. Figure 4 shows the change in the ratio $\mu_p G_E/G_M$ as a function of Q^2 . Because the correction is not precisely linear in ε , the effect on G_E depends somewhat on the ε range covered at each Q^2 value, and thus the correction shows a significant amount of scatter. For intermediate Q^2 values, the change in the ex-

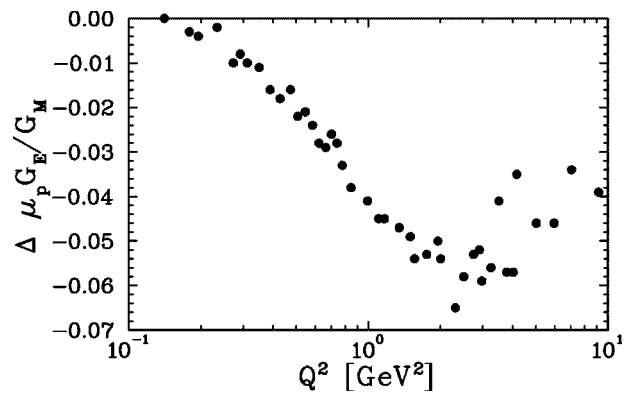


FIG. 4. The change in the extracted value of $\mu_p G_E/G_M$ resulting from correcting for the effects of Coulomb distortion.

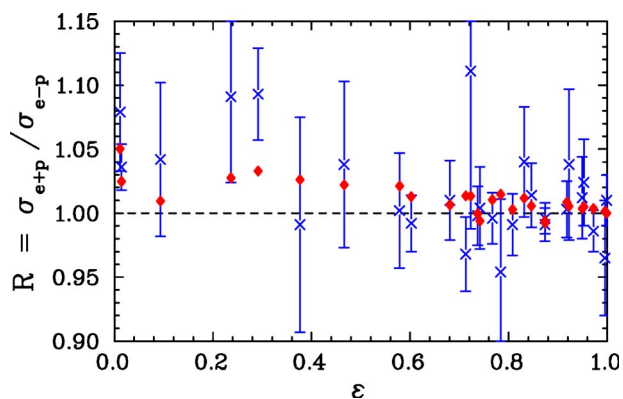


FIG. 5. (Color online) Measurements of the ratio of positron-proton to electron-proton scattering cross section (blue \times) along with the ratio predicted by the effects of Coulomb distortion (red diamonds).

tracted value of $\mu_p G_E / G_M$ can be as large or larger than the experimental uncertainties. Coulomb distortion has a smaller effect on the extraction of G_M , approximately 1% for intermediate Q^2 values, but this is comparable to the experimental uncertainties on G_M .

Finally, because the sign of the Coulomb distortion depends on the product of the beam and target charge, a comparison of electron-proton and positron-proton scattering is sensitive to these corrections. Figure 5 shows the ratio of the positron-proton cross section to the electron-proton cross

section for a series of comparisons made in the 1960s (see Ref. [12] and references therein). While the uncertainties are large and the results consistent with $R=1$ ($\chi^2=23.9$ for 28 data points), the data are in better agreement with the values expected when including Coulomb distortion ($\chi^2=14.7$). In particular, Coulomb distortion reproduces the systematic enhancement of the ratio at low Q^2 and ϵ .

IV. CONCLUSIONS

In this paper we have shown that Coulomb distortion has a non-negligible effect on the proton elastic cross section. The main effect is a change in the ϵ dependence of the cross section. The ϵ -dependent correction behaves approximately as $1/Q^2$ for $Q^2 > 2 \text{ GeV}^2$, as does the contribution from G_E . Thus, the effect on the extraction of G_E decreases very slowly for large Q^2 values. While Coulomb distortion explains only a portion of the discrepancy and appears to be small compared to the effect of a two hard photon exchange [4,6], its inclusion is rather straightforward and should be done.

ACKNOWLEDGMENTS

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