Simple statistical model for analysis of quark-gluon plasma droplet (fireball) formation

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We construct the density of states for quarks and gluons using the “Thomas-Fermi model” for atoms and the “Bethe model” for nucleons as templates. With parameters to take care of the plasma (hydrodynamical) features of the quark-gluon plasma (QGP) with a thermal potential for the interaction, we find a window in the parametric space of the model where observable QGP droplets of ∼5 fm radius can occur with transition temperature in the range 140–250 MeV. By matching with the expectations of lattice gauge estimates of the QGP-hadron transitions, we can further narrow the window, thereby restricting the allowed values of the flow-parameters of the model.

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There are increasing expectations of hadron transition to a quark-gluon plasma phase at about 150–170 MeV [1] from lattice calculations. There is also the natural possibility of QGP droplet (Fireball) formation in ultra relativistic heavy ion collisions (URHIC) [2].

The physics of such a QGP droplet is too complicated to be understood with a rigorous application of QCD to the problem of QGP droplet formation within a hadronic medium. This has forced several attempts at modeling the phenomenon to gain insight into the physical process of droplet formation using equation of state [2–4], microscopic transport equation [5,6], hydrodynamical approaches [7,8], etc. In this paper we briefly report on a simple statistical model of QGP [11,12] which captures a good chunk of the the physics of the QGP-hadron phase transition which can be used in the phenomenological analysis of “fireball” data as and when they are available from the URHIC experiments going on at various laboratories at present. Further, we use the thermal model potential in the construction of the density of states for the quarks and gluons in the QGP as the thermal model [9,10] has proved to be very successful in explaining the particle multiplicities measured in URHIC at the SPS.

In this Brief Report we will only give an outline of our approach, reserving a detailed version for a later date. Using the Thomas-Fermi model for the atom [13] and the Bethe model for the nucleons [14] as templates, we construct the density of states of relativistic quarks and gluons as

$$\rho_{q,g}(k) = (v/\pi^2)\left[[-V_{conf}(k)]^2\left(\frac{dV_{conf}(k)}{dk}\right)\right]_{q,g}, \qquad (1)$$

where v is the volume occupied by the QGP and k is the relativistic four-momentum in natural units. $V_{conf}(k)$ could be any confining potential for quarks and gluons, but for the present we choose to work with a modified thermal potential.

The thermal potential [10] is

$$[V_{\rm conf}(k)]_{q,g} = (1/2k)\gamma_{q,g}g^2(k)T^2 - m_0^2/2k, \qquad (2)$$

where $g^2(k)$ is the QCD coupling constant, which for quarks with three flavors is

$$g^2(k) = (4/3)(12\pi/27)[1/\ln(1+k^2/\Lambda^2)] \qquad (3)$$

with the QCD parameter $\Lambda = 150$ MeV. $\gamma_{q,g}$ are the phenomenological [10] flow parameters introduced to take care of the hydrodynamical aspects of the hot QGP droplet (fireball).

The model has a low energy cutoff

$$k_{min} = (\gamma_{q,g}N^{1/3}T^2\Lambda^2/2)^{1/4} \qquad (4)$$

with

$$N = (4/3)(12\pi/27).$$

With the further simplifying assumption of a pure pionic medium surrounding the QGP droplet [3], we compute the free energy of the system of noninteracting fermions (upper sign) or bosons (lower sign) at temperature T as

$$F_i = \mp Tg_i \int dk\rho_i(k)\ln(1 \pm e^{-(\sqrt{m_i^2+k^2})/T}), \qquad (5)$$

where $\rho_i(k)$ is the density of states of a particle i (quarks, gluons, interface, pion, etc.) being the number of states with

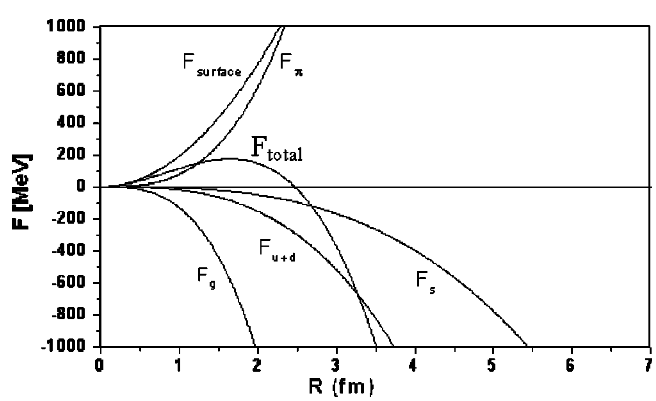

FIG. 1. Individual contribution to free energy F from the quarks, gluons, pions, and the interface leading to the total free energy F_{total} at $T = 152$ MeV for $\gamma_g = 10\gamma_q$, $\gamma_q = 1/6$.

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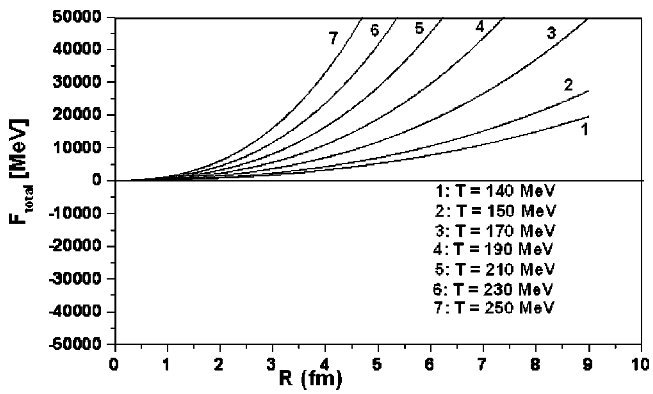

FIG. 2. F_{total} at $\gamma_g = 2\gamma_q$, $\gamma_q = 1/6$ for various temperatures.

momentum between k and $k+dk$ in a spherically symmetric situation, and g_i is the degeneracy factor (color and spin degeneracy) which is six for quarks, eight for gluons, and one for the pions and the interface.

The interface is assumed to be a modified Weyl surface [15],

$$F_{surface} = \frac{1}{4} R^2 T^3 \gamma, \tag{6}$$

where R is the radius of the droplet and γ is a modification sought to be introduced to take care of the plasma (hydrodynamical) nature of the droplet and is consciously chosen as

$$\gamma = \sqrt{2} \times \sqrt{(1/\gamma_g)^2 + (1/\gamma_q)^2}, \tag{7}$$

which is the inverse rms value of the flow parameter of the quarks and gluons, respectively.

The pion free energy is [3]

$$F_\pi = (3T/2\pi^2) v \int_0^\infty k^2 dk \ln(1 - e^{-\sqrt{m_\pi^2 + k^2}/T}). \tag{8}$$

For the quark masses we use the current (dynamic) quark masses $m_u = m_d = 0$ MeV and $m_s = 150$ MeV.

RESULTS AND CONCLUSION

With all the above numerical and theoretical inputs we have computed the free energy contributions of the $u+d$

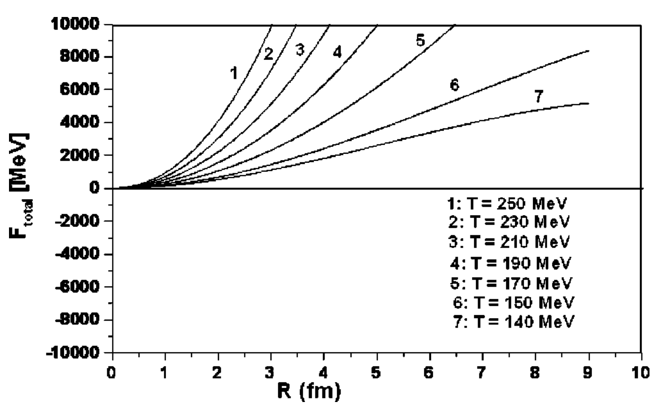

FIG. 3. F_{total} at $\gamma_g = 4\gamma_q$, $\gamma_q = 1/6$ for various temperatures.

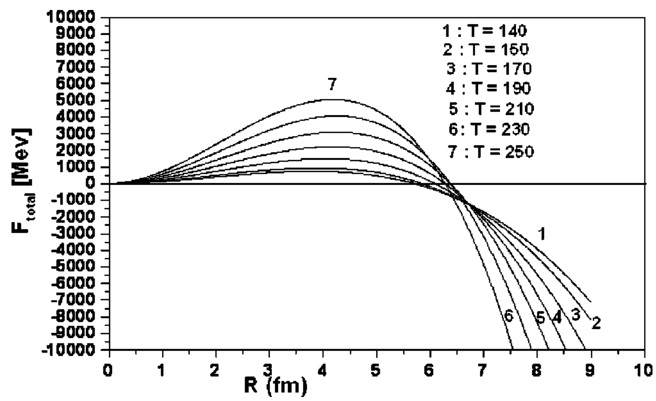

FIG. 4. F_{total} at $\gamma_g = 6\gamma_q$, $\gamma_q = 1/6$ for various temperatures.

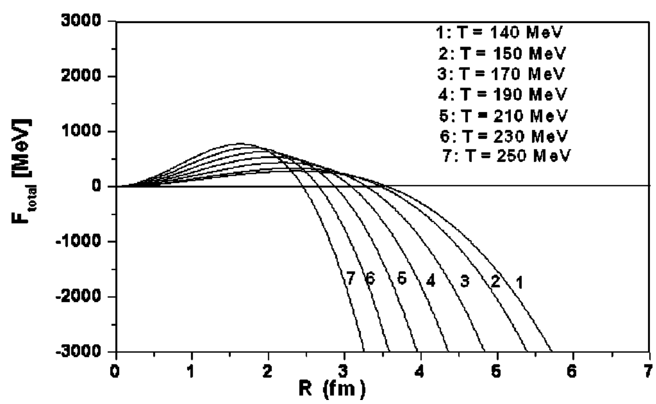

FIG. 5. F_{total} at $\gamma_g = 8\gamma_q$, $\gamma_q = 1/6$ for various temperatures.

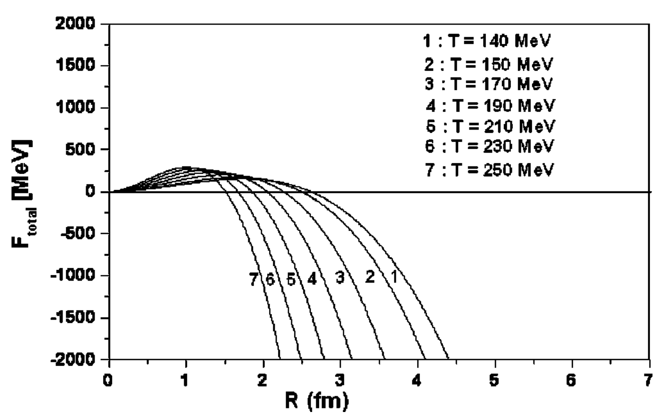

FIG. 6. F_{total} at $\gamma_g = 10\gamma_q$, $\gamma_q = 1/6$ for various temperatures.

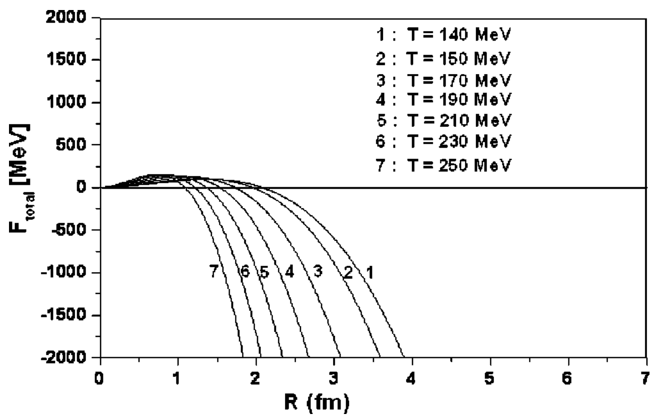

FIG. 7. F_{total} at $\gamma_g=12\gamma_q$, $\gamma_q=1/6$ for various temperatures.

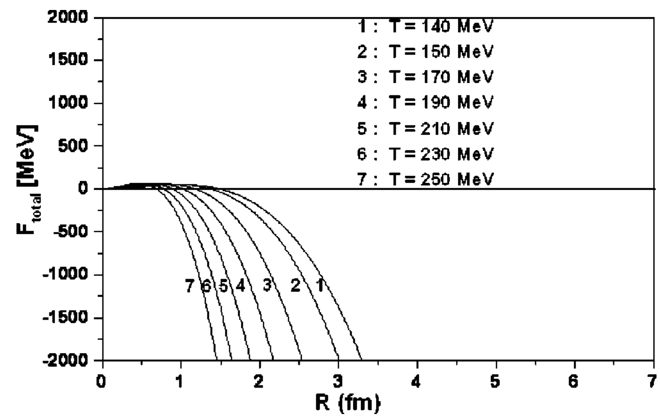

FIG. 8. F_{total} at $\gamma_g=16\gamma_q$ and $\gamma_q=1/6$ for various temperatures.

quarks, s-quarks, and for the gluons while retaining the same behavior for the pions as in [3,4]. All the energy integrations involved for the quark sector have a low energy cutoff at approximately 100 MeV (for example, it is 94.76 MeV at $T=152$ MeV) by virtue of Eq. (4), and the integral saturates at an upper cutoff at nearly four times the low energy cutoff energy.

In the present approach the bag energy is replaced by the interface energy (6) and the individual free-energy contributions are shown in Fig. 1 for a particular temperature, viz., $T=152$ MeV for $\gamma_g=10\gamma_q$, $\gamma_q=1/6$. The behavior of the total free energy of the droplets with increasing droplet size for various temperatures in the range 120 MeV$<T<$250 MeV for the various sets of flow parameters $\gamma_q \leqslant \gamma_g \leqslant 12\gamma_q$ with $\gamma_q=1/6$ (Peshier value) are illustrated by Figs. 2–8.

It can be seen that the QGP-droplet-hadron free energy goes on increasing without any stable droplet, forming for a choice of the flow parameters $\gamma_q \leqslant \gamma_g \leqslant 5\gamma_q$, with γ_q fixed at the value 1/6 as is evident from the graphs in Figs. 2 and 3. Large stable QGP droplets of $R>6$ fm start appearing for the value of $\gamma_g=6\gamma_q$ at $T>140$ MeV (Fig. 4) Stable QGP droplets with smaller radii less than 6 fm start appearing for a choice of $\gamma_g>6\gamma_q$, albeit with much lower barrier heights, indicating that the droplets are highly unstable and the QGP-hadron phase transition occurs at lower temperatures of $T\sim 170$ MeV (Figs. 5–7). At $\gamma_g>12\gamma_q$ (Fig. 8) the droplets become highly unstable with the barrier height almost vanishing, so that the system spontaneously passes into a QGP phase without the intermediate state of QGP droplet formation at much lower temperatures of $T<100$ MeV. The crucial role played by the hydrodynamical flow parameters indicates both their need and primacy in adapting a statistical model meant for a cold system of electrons or nucleons to an essentially hot plasma system of QGP. Also the smooth cut at the phase boundary is indicative of a first-order phase transition as suggested by earlier authors using other models [3,4]. In short, the model gives a simple and robust mechanism for the transition from the hadronic phase to the QGP phase with a minimal phenomenological input in terms of the hydrodynamical flow parameters and the current quark masses. But as to which of the scenarios occurs in actuality, only experiments can tell. The occurrence of droplets with relative stability with a radius of $\sim$6 fm at $\gamma_q \sim 1/6$ and $\gamma_g \sim 1$ with transition temperature $>$150 MeV makes this choice of the flow parameter values most appropriate and in agreement with lattice gauge expectations.

Note added. Since our submission of this paper, it has been brought to our attention that several authors have used other approximation schemes to estimate the droplet size and growth rate [16–19]. Obviously, there is a correlation between their parameters and ours. We are figuring out as to how they map onto each other and we shall endeavor to report it in a future publication.

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