Limiting fragmentation from the color glass condensate

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We show how the limiting fragmentation phenomenon can arise from the Color Glass Condensate model of high energy QCD. We consider the very forward rapidity region in relativistic heavy ion collisions and argue that in this region, nucleus-nucleus collisions are similar to proton-nucleus collisions (up to shadowing corrections). We then use the known results for proton-nucleus cross sections to show that it leads to the phenomenon of limiting fragmentation in the very forward region of heavy ion collisions as observed at RHIC.

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The Relativistic Heavy Ion Collider (RHIC) has opened up a new frontier in high energy nucleus-nucleus collisions. Many exciting and new phenomena have been observed which have challenged theoretical models and predictions. Suppression of high p_t hadrons in mid rapidity, increase of baryon to pion ratio with p_t and a large, constant anisotropy at high p_t are yet to be explained satisfactorily. In the fragmentation region (very forward rapidity), the PHOBOS experiment [1] has observed the so called limiting fragmentation phenomenon [2], shown in Fig. 1 for two different energies, which clearly shows most central (0–6%) charged particle multiplicities are independent of the center of mass energy. (The error bars shown for $\sqrt{s}=200$ GeV data are the average of positive and negative uncertainties published by PHOBOS [1].) In this note, we show that the Color Glass Condensate model of high energy nuclei can lead to a semi-quantitative understanding of this phenomenon.

It has been suggested that at high energies, due to high gluon density effects, a hadron or nucleus is a Color Glass Condensate and can be described by semiclassical methods [3,4]. This approach has been applied to heavy ion collisions at RHIC with some success [5]. Proton (deuteron) nucleus collisions at RHIC, scheduled to begin shortly, will greatly clarify the role and significance of the gluon saturation at RHIC energies [6]. Here we show that limiting fragmentation observed at RHIC can serve as yet another indication of the importance of high gluon density effects and the Color Glass Condensate at RHIC energies.

Unlike the mid-rapidity region, the fragmentation region (very forward rapidities) in a high energy heavy ion collision, is expected to be quite similar to high energy proton nucleus collisions, up to shadowing corrections. (By shadowing here, we mean any modification of the nuclear parton distributions, be it antishadowing, EMC effect etc.) This is because the Quark Gluon Plasma is expected to be formed only in the mid-rapidity region and will not affect particle production in the very forward rapidity region. Also, in the fragmentation region, one can treat the target nucleus as a dilute system of quarks and gluons while the projectile nucleus must be treated as a Color Glass Condensate due to its large number of gluons. This is formally the same as a proton nucleus system treated in Ref. [6] where one considers scattering of quarks and gluons [7] coming from the proton on the dense nucleus.

In the Color Glass Condensate model, small x gluons in the wave function of a high energy nuclei are described by a classical color field generated by the large x quarks and gluons which constitute a color charge ρ. The classical gluon field of the nucleus $A_a^\mu(x^-,x_t)=\delta^{\mu +}\alpha_a(x^-,x_t)$ is static (light cone time independent) and is given by $\alpha_a=-(1/\partial_t^2)\rho_a$ in the covariant gauge. One can calculate the quark or gluon propagator in the background of this classical field. For definiteness, here we consider the quark propagator in the background field since we will focus on quark nucleus scattering. It is given by (see for example, Refs. [6,8])

$$\tau(q,p)=(2\pi)\delta(p^- - q^-)\gamma^- e^{i(q_t-p_t)z_t}\int d^2z_t[U(z_t)-1], \quad (1)$$

where the matrix U contains multiple scatterings of the quark from the classical background field and is given by

$$U(z_t)\equiv \hat{P}\exp\left[-ig^2\int_{-\infty}^{+\infty}dz^-\frac{1}{\partial_t^2}\rho_a(z^-,z_t)t_a\right]. \quad (2)$$

The quark nucleus scattering cross section is related to the quark propagator via [6]

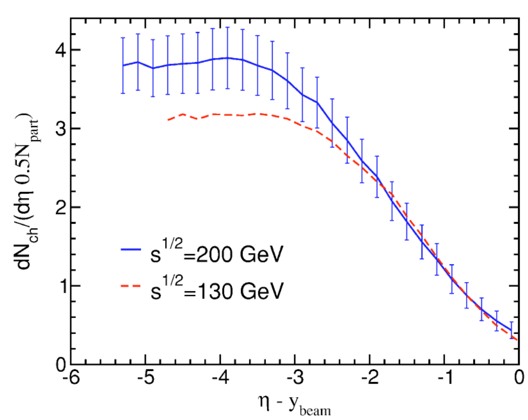

FIG. 1. (Color online) Limiting fragmentation observed at RHIC [1].

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$$d\sigma \sim \langle |\overline{u}(q)\tau(q,p)u(p)|^2\rangle_\rho \tag{3}$$

and is given by

$$\frac{d\sigma^{qA\to qX}}{d^2q_t dq^- d^2b_t} = \frac{1}{(2\pi)^2}\delta(q^- - p^-)\int d^2r_t e^{iq_t\cdot r_t}\frac{1}{N_c}\,\mathrm{Tr}\times\left[\left\langle 1 - U\left(b_t + \frac{r_t}{2}\right) - U^\dagger\left(b_t - \frac{r_t}{2}\right) + U\left(b_t + \frac{r_t}{2}\right)U^\dagger\left(b_t - \frac{r_t}{2}\right)\right\rangle_\rho\right], \tag{4}$$

where $p^-(q^-)$ is the longitudinal momentum of the incoming (outgoing) quark, with a similar equation for gluon scattering. This is the multiple scattering generalization of quark gluon scattering in pQCD and unlike the leading twist (single scattering) result, is finite as $q_t \to 0$ due to higher twist effects. We emphasize the fact that this integral is *finite* and can be done *exactly*. To see this, let us integrate (4) over q_t which gives an overall $\delta(r_t)$ which in turn can be used to do the r_t integration and sets $r_t=0$. Using unitarity of the U matrices and that the color averages of U and $U^\dagger$ go to zero as $\exp[-Q_s^2/\Lambda_{\mathrm{QCD}}^2]$ and can therefore be ignored, trace of the bracket in (4) becomes equal to $2N_c$. To relate this to nucleus nucleus scattering in the very forward rapidity region, we convolute this cross section with the quark and gluon distributions in the target nucleus

$$\frac{d\sigma^{AA\to qX}}{d^2q_t dq^- d^2b_t} = \int dx_q f_{q/A}(x_q)\frac{d\sigma^{qA\to qX}}{d^2q_t dq^- d^2b_t} \tag{5}$$

and since we are interested in total number of produced particles per unit rapidity, we will integrate over the transverse momentum q_t of the scattered quark.

In pQCD, to calculate the hadronic cross section from the partonic one, one needs to convolute the partonic cross section with the appropriate fragmentation function. This requires a hard scale which is typically taken to be the transverse momentum of the produced hadrons. This means that one cannot calculate total hadron multiplicities from pQCD since the cross sections are divergent as $q_t \to 0$ while total multiplicities are dominated by such low q_t scales. This rules out the use of fragmentation functions for calculation of total multiplicities. In our formalism, the partonic cross sections are finite at low q_t. However, we can still not use fragmentation functions since there is no hard scale left in the problem after integrating over the transverse momentum of the produced hadrons. We can avoid this problem if we are willing to treat the absolute normalization of multiplicities as a parameter to be fixed at some energy. In other words, a scattered quark or gluon in our approach can "fragment" into many hadrons. Since we are interested only in the rapidity dependence of multiplicity distributions, we do not need to know how many hadrons a quark or gluon will "fragment" into. So therefore, we do not use a fragmentation function. We then have

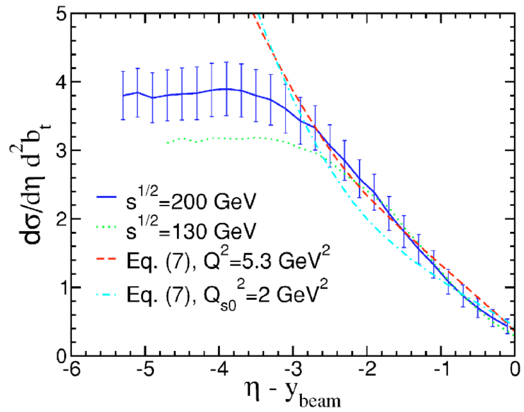

FIG. 2. (Color online) Limiting fragmentation from (7) compared to data from RHIC.

$$\frac{d\sigma^{AA\to hX}}{dq^- d^2b_t} \sim \int dx\,\delta(p^- - q^-)[f_q^A(x) + G^A(x)], \tag{6}$$

where $p^- = x\sqrt{s/2}$ and $q^- \sim e^{\eta_h}$. Using the delta function $\delta(p^- - q^-)$ in (6) to do the x integration leads to

$$\frac{d\sigma^{AA\to \mathrm{hadrons}}}{d\eta_h d^2b_t} \sim [xf_q^A(x) + xG^A(x)], \tag{7}$$

where $x \sim e^{\eta_h - y_{\mathrm{beam}}}$ and f_q^A and G_A are the quark and gluon distributions of the target nucleus.

In Fig. 2 we plot $d\sigma/d\eta_h d^2b_t$ from Eq. (7) shifted for normalization and by the beam rapidity. Since we have assumed a uniform nuclear density and since our formalism is valid for most central collisions where the value of the saturation scale is large, we compare our results with the most central (0–6%) data [1] from RHIC at $\sqrt{s}=200$. We have used GRV98 [9] parton distributions and the EKS98 parameterization of nuclear shadowing [10].

As is seen in Fig. 2, the agreement with the data is quite good for the first three (two) units of rapidity for $\sqrt{s}$ =200 GeV (130 GeV). The physical picture behind the limiting fragmentation phenomenon in the Color Glass Condensate model is quite simple; in the rest frame of the target nucleus, the projectile nucleus is highly Lorenz contracted and due to its large number of gluons, looks black to the partons in the target nucleus which interact with the projectile nucleus with unit probability (the black disk limit). In other words, due to a large boost factor, the projectile nucleus has a saturation momentum scale which is larger than the momenta of most particles produced. In this kinematic region, the partons from the target nucleus interact with the projectile nucleus (Color Glass Condensate) with unit probability.

There are a few caveats to our results. We cannot predict the overall normalizations, only the slope and have to normalize our results to the data at one reference point taken to be the target beam rapidity. Also, the scale dependence of nuclear parton distributions, and in particular the gluon distribution, is very poorly known due to the limited Q^2 coverage of fixed target experiments. The current parameterizations of nuclear gluon distributions are at best an educated guess. Unfortunately, our results are quite sensitive to the change of scale Q^2 [the scale dependence of distribution functions is not written out explicitly in Eq. (7)]. We therefore fix this scale by requiring that Eq. (7) gives a reasonable fit to the RHIC limiting fragmentation data at $\sqrt{s} \sim 20$ GeV for a couple of units of rapidity. It turns out that $Q = 2.3$ GeV works well. We then use the same scale Q in (7) to predict the multiplicities at higher energies of $\sqrt{s} = 130$ GeV and $\sqrt{s} = 200$ GeV. We also show the case when $Q^2 = Q_s^2(y)$ as suggested (during completion of this note, we learned of Ref. [11] which however focuses on a different problem) in Ref. [11]. Here $y = \log 1/x$ and $Q_s^2(y) \equiv Q_{s0}^2 \exp(\lambda y)$ with $Q_{s0}^2 = 2.0$ GeV^2 at mid-rapidity and $\lambda = 0.3$ [5]. The choice of $Q_{s0}^2 = 3.0$ GeV^2 leads to a much better agreement but is disfavored [5] by RHIC data. Alternatively and if one insists on keeping $Q_{s0}^2 = 2.0$ GeV^2 at mid-rapidity, the choice of $\lambda = 0.45$ leads to a good agreement with the data but this value of $\lambda = 0.45$ is too large to fit the HERA data (also, choosing a x dependent scale in parton distributions would seem to violate the sum rules reflecting various conservation laws. We will not pursue this further since we are not doing a detailed quantitative study here.)

There are two principle reasons why our approach should break down as we get closer to the mid-rapidity region. First, as one goes further away from the target nucleus, high gluon density effects in the target nucleus become important. This will show as the growth in the saturation scale of the target nucleus (which is of order Λ_{QCD} right at the target nucleus rapidity). To estimate this, we use $Q_s^2(\Delta\eta) = \Lambda_{\mathrm{QCD}}^2 \exp(\Delta\eta)$. As one goes about three units of rapidity away from the target nucleus, its saturation scale becomes appreciable (~ 1 GeV) and one cannot describe it as a dilute system of partons anymore [12].

Another reason why this approach should break down as one gets closer to mid-rapidity is that the classical fields of both nuclei will become strong and the system will be very different from a proton nucleus collision. Also, in the mid-rapidity region one will have to include the media effects due to the deconfined matter presumably produced in heavy ion collisions at RHIC. The media effects are presently not well understood and are beyond the scope of this work.

To summarize, the underlying physics of limiting fragmentation in the Color Glass Condensate formalism is that since most particles are produced with transverse momenta which are below the saturation scale of the projectile nucleus (in the target nucleus reference frame), their cross sections are transverse momentum independent (the black disk limit). Thus the rise of the particle multiplicities in the very forward rapidity region (near the target nucleus) is due to the blackness of the projectile nucleus and the growth of the target nucleus parton distributions with rapidity.

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