Gauge ambiguities in (e,e') reactions in the quasielastic region

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We investigate the ambiguity in the choice of different gauges for a relativistic single particle model of the inclusive (e,e') reaction in the quasielastic region. Gauge ambiguities lead to different results due to the off-shellness for a bound nucleon. The difference between results using different gauges increases with energy transfer and is larger for heavier nuclei at the same incident electron energy. We compare the theoretical results with the experimental data measured at Bates, Saclay, and SLAC.

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The exact treatment for the off-shellness of a bound nucleon in nuclei remains one of the long standing unsolved problems in theoretical nuclear physics. Many theoretical attempts [1–5] have already estimated off-shell variations for the exclusive $(e,e'p)$ reactions. In particular, de Forest [1] introduced an *ad hoc* recipe for the nonconserved nuclear current in the impulse approximation. This prescription makes it possible to factorize the cross section into a product of an electron kinematic factor and a spectral function.

After de Forest's paper, Picklesimer *et al.* [2] calculated the $(\vec{e},e'p)$ reaction and investigated the initial state off-shell effect, comparing the relativistic calculation with the nonrelativistic result. They also investigated the off-shell effect in the final state interaction within a relativistic framework. The ambiguity due to the current nonconservation was reported to be roughly 20% in the longitudinal response function and 10% at the peak position.

In the middle 1990's, Pollock *et al.* [3] showed that for the $(e,e'p)$ reaction the variations on the prescriptions of the nonconserved nucleon current are explicitly related to the choice of different gauges in a virtual photon propagator and yield different predictions for the cross sections in the plane wave impulse approximation. The ambiguity in the cross section is shown to be about 10% for small angles of the knocked-out proton, i.e., low missing momenta.

Other recent papers by Kelly [4] and by Udias [5] show gauge ambiguities in physical observables for the $(e,e'\vec{p})$ reaction. The ambiguities could be insensitive to the ratio of recoil polarizations for the low missing momentum region, although they are still present in the $(e,e'p)$ reaction. A recent experimental paper [6] discussed nuclear medium effects by measuring the polarization of the outgoing proton. The paper shows the comparison of the experimental data with the theoretical results obtained using the Kelly and Udias codes. At the low momentum transfer [6] the discrepancy between the measured experimental data and the theoretical results is insensitive to the gauge ambiguity, which turned out to give only a few percent difference.

In this work, we apply the off-shell to the inclusive (e,e') reaction and compare the theoretical results with the experimental data measured at Bates [7] for ^{40}Ca, Saclay [8] for ^{208}Pb, and SLAC [9] for ^{12}C, ^{56}Fe, and ^{197}Au, where the quasielastic contribution is kinematically isolated from an inelastic process like pion electroproduction.

We apply three possible prescriptions for conserving the electromagnetic current which are related to the particular choice of gauges in a realistic model. We review briefly the relation between the prescriptions for current conservation and the choice of different gauges. The first method proposed by de Forest [1] is to replace the longitudinal term, which is parallel to the momentum transfer $\mathbf{q}$ along the $\hat{\mathbf{z}}$ direction, by the time part

$$J_z \rightarrow J_z = \frac{\omega J_0}{q}, \tag{1}$$

associated with the Coulomb gauge. The matrix element for the Coulomb gauge is given by

$$M_C = \frac{i}{q^2} j_0 J_0 + \frac{i}{q_\mu^2}\left[\mathbf{j}\cdot\mathbf{J} - \frac{(\mathbf{q}\cdot\mathbf{j})(\mathbf{q}\cdot\mathbf{J})}{q^2}\right], \tag{2}$$

where we use the fact that the electron current is conserved, namely, $q_\mu j^\mu = \omega j_0 - q j_z = 0$. The four momentum square q_μ^2 is $q_\mu^2 = \omega^2 - q^2$.

The second one is to eliminate the charge density part instead of removing the z component [10]

$$J_0 \rightarrow J_0 = \frac{\mathbf{q}\cdot\mathbf{J}}{\omega}, \tag{3}$$

related to the Weyl gauge. The matrix element for the Weyl gauge in which the charge current density does not contribute is given by

$$M_W = \frac{i}{q_\mu^2}\left[\mathbf{j}\cdot\mathbf{J} - \frac{(\mathbf{q}\cdot\mathbf{j})(\mathbf{q}\cdot\mathbf{J})}{\omega^2}\right]. \tag{4}$$

In the third prescription, one subtracts an *ad hoc* term proportional to q_μ [11]:

$$J^\mu \rightarrow J^\mu = J^\mu - \frac{(q_\mu J^\mu)}{q_\mu^2} q_\mu, \tag{5}$$

associated with the Landau gauge. The matrix element for the Landau gauge which adds an *ad hoc* term to the nucleon current is given by

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$$M_L = \frac{i}{q_\mu^2}\left[- j_\mu J^\mu + \frac{(q_\mu j^\mu)(q_\mu J^\mu)}{q_\mu^2}\right]. \tag{6}$$

The earlier discussions are already given in Ref. [3] in detail. Even if different gauges are used, the same results should be obtained for conserved currents. However, inconsistent results are inevitable because of the off-shellness of nucleons in a nucleus. Note that the electron current is conserved in all of the cases.

Now, we apply the earlier nucleon currents to the cross section for the inclusive (e,e') reaction. In the plane wave Born approximation calculation, where the electron wave functions are described by the Dirac solution, the cross section in the Coulomb gauge with Eq. (2) is given by

$$\left(\frac{d^2\sigma}{d\omega d\Omega}\right)_C = \sigma_M\left[\frac{q_\mu^4}{q^4}|J_0|^2 + \left(\tan^2\frac{\theta_e}{2} - \frac{q_\mu^2}{2q^2}\right)(|J_x|^2 + |J_y|^2)\right], \tag{7}$$

where σ_M is the Mott cross section. In the Weyl gauge, the cross section with Eq. (4) is written as the following:

$$\left(\frac{d^2\sigma}{d\omega d\Omega}\right)_W = \sigma_M\left[\frac{q_\mu^4}{q^2\omega^2}|J_z|^2 + \left(\tan^2\frac{\theta_e}{2} - \frac{q_\mu^2}{2q^2}\right)(|J_x|^2 + |J_y|^2)\right]. \tag{8}$$

Similarly, the cross section in the Landau gauge is obtained from Eq. (6) in the following form:

$$\left(\frac{d^2\sigma}{d\omega d\Omega}\right)_L = \sigma_M\left[\left| - J_0 + \frac{\omega}{q}J_z\right|^2 + \left(\tan^2\frac{\theta_e}{2} - \frac{q_\mu^2}{2q^2}\right) \times (|J_x|^2 + |J_y|^2)\right]. \tag{9}$$

The gauge ambiguity contributes only to the longitudinal term, and the structure functions for the transverse part are not changed.

In the analysis of the (e,e') reaction, it is necessary to include the electron Coulomb distortion. Containing the electron Coulomb distortion is difficult, but we use the approximate method developed by Kim and Wright [12–16]. For the nuclear transition current, a relativistic single particle model requires bound state and continuum nucleon wave functions and a transition current operator. The wave functions of the bound state are solutions to the Dirac equation in the presence of the strong scalar and vector potentials of the σ-ω model generated by Horowitz and Serot [17]. Since the early 1990's, for the inclusive (e,e') reaction, the Ohio University group [12,15,16,18] has used the same real potentials for the continuum state nucleons as those used for the bound state nucleons. This ansatz guarantees orthogonality, gauge invariance, and current conservation of initial and final states. Coupled with the free relativistic nucleon current operator, which contains the standard nucleon form factors, this model, with inclusion of the electron Coulomb distortion, described (e,e') Bates [7] and SLAC [16] data very well. Although the gauge ambiguity violates this ansatz, we wish to investigate the off-shell effect explicitly keeping only the orthogonality.

In Figs. 1–3, we compare the theoretical calculations to the experimental data. The solid lines represent the calculations for the Coulomb gauge. The dotted curves represent the calculations for the Landau and the dashed lines are the results for the Weyl gauges. The thin curves in all of these figures correspond to the cross sections and the thick curves represent the longitudinal cross sections.

In Figs. 1 and 2, we show the cross sections corresponding to kinematics for intermediate electron energies for ^{40}Ca from Bates [7] and for ^{208}Pb from Saclay [8], respectively. In Fig. 1, the momentum transfer at the peak positions is about $q=490$ MeV/c for both forward and backward angles. The solid curve and the dotted curve agree with each other. The calculations fit the experimental data relatively well, but for 45.5° the amplitudes are about 10% greater than the experimental data around the peak position. The dashed lines for the Weyl gauge show big differences on the order of 10%, although the shapes have the same form. For the case of the backward angle, the difference between the solid (or dotted) curve and the dashed curve is about 1%. The difference between the solid (or dotted) line and the dashed line for the forward angle is bigger than that for the backward angle because the transverse part is dominant at the backward angle. For both cases the peak positions of the longitudinal cross section for the dashed lines shift about 10 MeV to the right. Furthermore, the difference increases with larger energy transfer. Also, although some theoretical (e,e') calculations claimed the suppression of the longitudinal structure function, there is no evidence of the suppression, as our previous papers [12,15,16] show. Finally, the longitudinal cross sections are enhanced in the Weyl gauge.

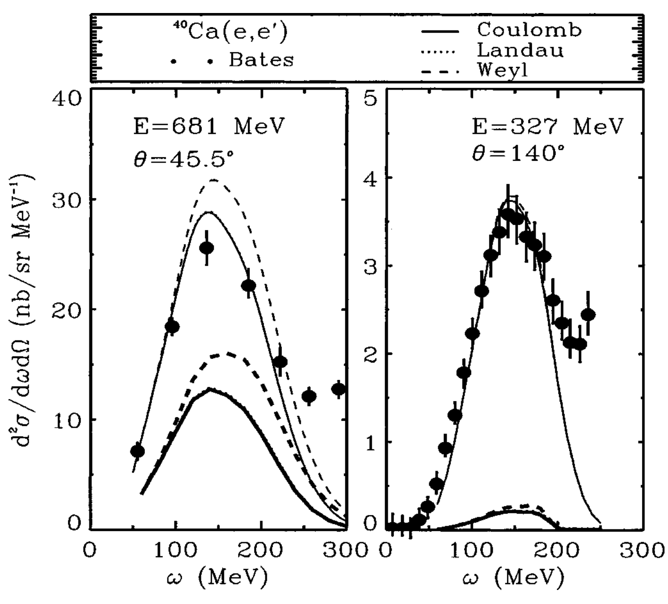

FIG. 1. The cross sections for ^{40}Ca at two different electron energies, $E=327$ MeV and $E=681$ MeV, and scattering angle $\theta=140°$ and $\theta=45.5°$. The solid lines are calculated with the Coulomb gauge. The dotted lines are the results for the Landau gauge and the dashed lines are for the Weyl gauge. The experimental data are from Bates [7].

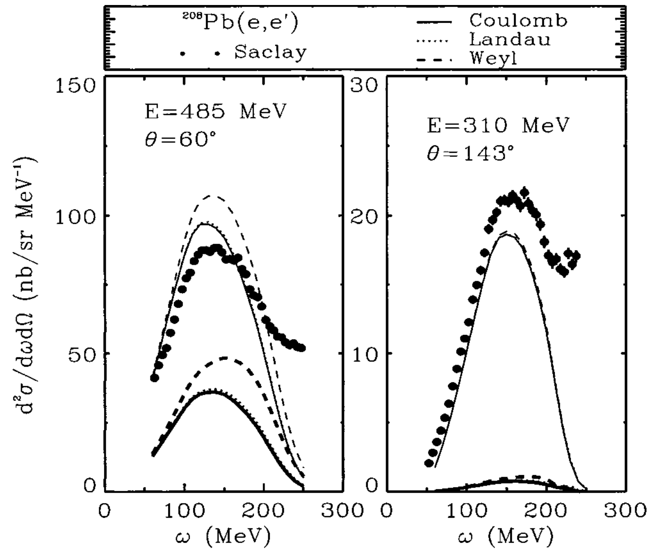

FIG. 2. The cross sections for ^{208}Ca at two different electron energies, $E=310$ MeV and $E=485$ MeV, and scattering angle $\theta=143°$ and $\theta=60°$. The solid lines are calculated with the Coulomb gauge. The dotted lines are the results for the Landau gauge and the dashed lines are for the Weyl gauge. The experimental data are from Saclay [8].

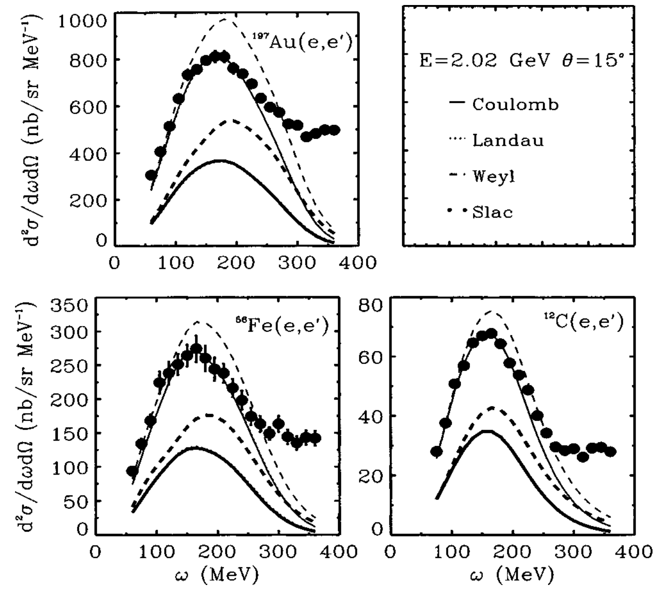

FIG. 3. The cross sections at the same electron kinematics for three different nuclei, ^{12}C, ^{56}Fe, and ^{197}Au. The solid lines are calculated with the Coulomb gauge. The dotted and dashed lines are the results with the Landau gauge and the dashed lines are for the Weyl gauge. The experimental data are from SLAC [9].

In Fig. 2, the momentum transfer is about $q=450$ MeV/c at the peak positions for these cases. Around the peak, the difference between the dashed line and the solid line for the cross section is about 10% at the forward angle and about 1% at the backward angle, like the cases ^{40}Ca in Fig. 1. The results show that the difference also increases with energy transfer and that the peak positions shift about 10 MeV toward large energy transfer. As shown in our previous papers [12,15], our theoretical calculations do not reproduce the Saclay data well.

Figure 3 shows the cross sections for higher electron energy, $E=2.02$ GeV, very forward scattering angle, $\theta=15°$, and three different nuclei, ^{12}C, ^{56}Fe, and ^{197}Au. The experimental data were measured at SLAC [9]. The momentum transfer is about $q=530$ MeV/c at the peak positions in these nuclei. The results using the Coulomb gauge and the Landau gauge show excellent agreement with the experimental data. The differences between the solid lines and the dashed lines increase for heavier nuclei keeping the incident electron energy the same. The peaks also shift about 10 MeV toward larger energy transfer.

In this report, we have examine the gauge ambiguity due to the off-shellness of a bound nucleon for the inclusive (e,e') reaction in the quasielastic region. The ambiguity increases for heavier nuclei with the same incident electron energy. Moreover, the ambiguity also increases with energy transfer. This may be due to the fact that short-range interactions lead to strong nucleon correlations in the mean field so that the Fermi surface is smeared out. This means that large energy transfer contributes to highly excited states. Since these highly excited states are mostly the off-shell, the off-shell sensitivity shows up strongly in the large energy transfer region. While some theoretical (e,e') calculations show the suppression of the longitudinal structure function, here we do not find the suppression with the relativistic single particle model even including the off-shell effect.

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[1] T. de Forest, Jr., Nucl. Phys. **A392**, 232 (1983).
[2] A. Picklesimer, J. W. Van Orden, and S. J. Wallace, Phys. Rev. C **32**, 1312 (1985).
[3] S. Pollock, H. W. L. Naus, and J. H. Koch, Phys. Rev. C **53**, 2304 (1996).
[4] J. J. Kelly, Phys. Rev. C **56**, 2672 (1997); **60**, 044609 (1999).
[5] J. A. Caballero, T. W. Donnelly, E. Moya de Guerra, and J. M. Udias, Nucl. Phys. **A632**, 323 (1998); J. M. Udias and J. R. Vignote, Phys. Rev. C **62**, 034302 (2000).
[6] S. Dieterich *et al.*, Phys. Lett. B **500**, 47 (2001).
[7] C. F. Williamson *et al.*, Phys. Rev. C **56**, 3152 (1997); T. C. Yates *et al.*, Phys. Lett. B **312**, 382 (1993).
[8] A. Zghiche *et al.*, Nucl. Phys. **A572**, 513 (1994).
[9] D. B. Day *et al.*, Phys. Rev. Lett. **59**, 427 (1987); Phys. Rev. C **48**, 1849 (1993).

[10] J. A. Caballero, T. W. Donnelly, and G. I. Poulis, Nucl. Phys. **A555**, 709 (1993).
[11] J. Mougey *et al.*, Nucl. Phys. **A262**, 461 (1976).
[12] K. S. Kim, L. E. Wright, Y. Jin, and D. W. Kosik, Phys. Rev. C **54**, 2515 (1996).
[13] K. S. Kim and L. E. Wright, Phys. Rev. C **56**, 302 (1997).
[14] K. S. Kim and L. E. Wright, Phys. Rev. C **60**, 067603 (1999).
[15] K. S. Kim, L. E. Wright, and D. A. Resler, Phys. Rev. C **64**, 044607 (2001).
[16] K. S. Kim and L. E. Wright, Phys. Rev. C **67**, 054604 (2003).
[17] C. J. Horowitz and B. D. Serot, Nucl. Phys. **A368**, 503 (1981).
[18] Yanhe Jin, D. S. Onley, and L. E. Wright, Phys. Rev. C **45**, 1333 (1992).