Hidden evidence of nonexponential nuclear decay

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The framework to describe natural phenomena at their basics being quantum mechanics, there exist a large number of common global phenomena occurring in different branches of natural sciences. One such global phenomenon is spontaneous quantum decay. However, its long time behavior is experimentally poorly known. Here we show, that by combining two genuine quantum mechanical results, it is possible to infer on this large time behavior, directly from data. Specifically, we find evidence for nonexponential behavior of alpha decay of ^{8}Be at large times from experiments.

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I. INTRODUCTION

The decay law in quantum mechanics is necessarily nonexponential [1] and deviates from a simple exponential form at small and large times (for reviews see [2–4]). Though there exists some experimental evidence of the deviation from exponential behavior at short times [5], this is not the case for the large time behavior which is often not directly observable [6] (indeed, we are not aware of any such successful experiment). The information about resonance time evolution, in general, and the long time behavior, in particular, should as a matter of principle be encoded in the resonant scattering data $A+a \to \text{resonance} \to B+b$. The survival amplitude, $A_\Psi(t)$, (related to the survival probability, $P_\Psi(t) = |\mathcal{A}_\Psi(t)|^2/|\mathcal{A}_\Psi(0)|^2$) can be written as a Fourier transform [7], namely

$$\mathcal{A}_\Psi(t) = \int_{E_{\text{th.}}}^{\infty} dE \rho_\Psi(E) e^{-iEt}, \tag{1}$$

(where $E_{\text{th.}}$ is the minimum sum of the masses of the decay products) of an energy dependent quantity [the spectral function, $\rho_\Psi(E)$] which can be constructed on the premises of the scattering matrix, S. Since the S-matrix itself for a resonant reaction can be extracted from experiment, the information encoded in the scattering allows us to infer on the time evolution of the resonance produced as an intermediate state in the process. The spectral function $\rho_\Psi(E)$, is the probability density to find the eigenstates $|E\rangle$ of the decay products in $|\Psi\rangle$, or, in other words, it is the continuum probability density of states in a resonance which can hence be written as: $\rho_\Psi(E) = d\text{Prob}_\Psi(E)/dE = |\langle E|\Psi\rangle|^2$.

In the present work, we attempt to extract $\rho_\Psi(E)$ from experimental data and Fourier transform it to conclude, now from experiment, on the long tail of the survival amplitude of the resonance under consideration. This is probably the closest that one can ever come to pinning down the large times in a decay directly from experiment albeit by an indirect method.

II. DENSITY OF STATES

We start by noting that one important result originating from statistical mechanics seems to have been overlooked, at least in connection with unstable states. In calculating the second virial coefficients B and C for the equation of states in a gas, $pV = RT[1 + B/V + C/V^2 + \dots]$, Beth and Uhlenbeck [8] (see also [9–11]) found that the difference between the density of states with interaction, n_l, and without, $n_l^{(0)}$, is given by the derivative of the scattering phase shift δ_l as

$$n_l(k_{\text{cm}}) - n_l^{(0)}(k_{\text{cm}}) = \frac{2l+1}{\pi} \frac{d\delta_l(E_{\text{cm}})}{dE_{\text{cm}}}, \tag{2}$$

where l is the angular momentum of the l^{th} partial wave and k_{cm} and E_{cm} are the momentum and energy in the center-of-mass (cm) system of the scattering particles, respectively. If a resonance is formed during the scattering process, E_{cm} becomes the energy of the resonance in its rest frame. Certainly, the density of states and the probability density mentioned above are connected. Switching off the interaction (by, say, letting the coupling constants go to zero), n_l will not become zero, but tend to $n_l^{(0)}$ from above. Therefore, as long as $n_l - n_l^{(0)} \geqslant 0$ (this is always the case for an *isolated* resonance), we can write for the continuum probability density of states of the decay products in a resonance

$$\frac{d\text{Prob}_{\Psi_l}(E_{\text{cm}})}{dE_{\text{cm}}} = \text{const} \frac{d\delta_l(E_{\text{cm}})}{dE_{\text{cm}}}, \tag{3}$$

which is the sought after connection between data, here in the form of δ_l, and the survival amplitude. This method is a general (i.e., without any further restrictions with the exception of our belief in quantum mechanics) feasible tool for studying the time evolution of isolated resonances from data. In reality, overlapping resonances can cause $d\delta_l/dE_{\text{cm}}$ to have several maxima and minima (see Fig. 1), even negative. As noted by Wigner, the negative regions are bound to appear between resonances [12]. The realistic situation of several overlapping resonances implies that the identification (3) is operative starting from threshold and extending over one resonance region, but often not beyond. However, one very useful feature remains when we restrict ourselves to large

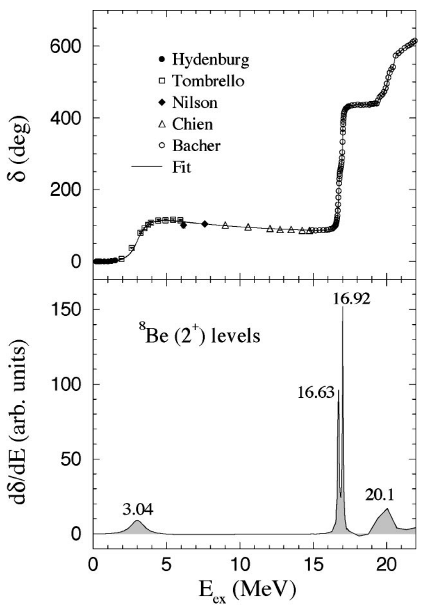

FIG. 1. D-wave phase shifts (upper half) in α-α elastic scattering from Ref. [15], polynomial fit to these data (solid line) and the derivative of phase shift (lower half) calculated from the fit showing all established ^{8}Be(2^+) levels, as a function of the excitation energy $E_{\rm ex}=E_{\rm cm}-E_{^8{\rm Be(ground\ state)}}$ and plotted here in arbitrary units (arb. units). The negative region in derivative of phase shift (lower half) between 5 and 15 MeV, due to the slowly falling phase shift is not obvious in the plot due to the scale of the vertical axis.

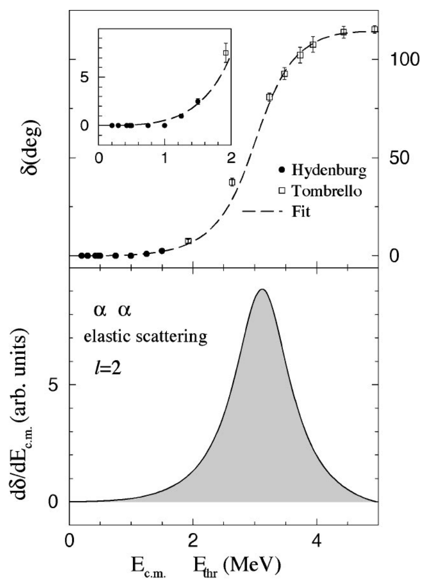

FIG. 2. D-wave phase shift (upper half) and its derivative (lower half) in α-α elastic scattering as a function of $E_{\rm cm}-E_{\rm threshold}$, in the region of the first 2^+ level of ^{8}Be. The dashed line shows the fit mentioned in the text. The inset displays the accuracy of the fit near the threshold energy region which is crucial for the large time behavior of the decay law.

times. Since large times correspond to small energies, in order to experimentally extract information on this region, all we need to know is δ_l at threshold and in the vicinity of the resonance. The exact form of how $\rho_\Psi(E)$ falls off at large E, well beyond the resonance region, is not important to conclude on the behavior of $\mathcal{A}_\Psi(t)$ as $t\to\infty$.

To see the connection between $\rho_\Psi(E)$ and $d\delta_l/dE$ as in (3), we note that starting from the Breit-Wigner form of the amplitude, one gets [13], $d\delta/dE=[\Gamma/2]/\left[(E_R-E)^2+\frac{1}{4}\Gamma^2\right]$. The right-hand side, up to a constant, is commonly taken as $\rho_\Psi(E)$ to display the fact that a one-pole approximation for the amplitude leads to the exponential decay law [14].

III. THE LONG TAIL OF QUANTUM DECAY FROM EXPERIMENT

With the aim of making a fit to the data on phase shifts, δ_l, with a reasonable scan of the threshold/resonance region which would determine the large time behaviour of the decay law, we opted for an experiment with many data points at threshold and relatively small error bars. We chose the $\alpha-\alpha$ D-wave resonant scattering in nuclear physics [15], namely, $\alpha+\alpha\to{}^8{\rm Be}(2^+)\to\alpha+\alpha$. In Fig. 1 we display the phase shift and its derivative over a wide region, using a simple polynomial fit to the phase shift. We find the established ^{8}Be levels, shown in the figure. Motivated by a Lorentzian form [16] with an energy dependent width $\Gamma(E_{\rm cm})$, we parametrized the data in the region of the first 2^+ resonance, in the following form:

$$\delta_l(E_{\rm cm}) = \tan^{-1}\left[\frac{\Gamma(E_{\rm cm})}{E_0-E_{\rm cm}}\right]e^{-\beta E_{\rm cm}}, \tag{4}$$

with

$$\Gamma(E_{\rm cm}) = \Gamma_0\left(\frac{E_{\rm cm}^2-E_{\rm th.}^2}{E_0^2-E_{\rm th.}^2}\right)^{\kappa/2} \tag{5}$$

which is valid for the elastic case. In particular, $\kappa=2\gamma+2$ and the resonance pole is highlighted by the expected [17] peak structure (see Fig. 2). We fitted κ, β, Γ_0, and E_0 simultaneously taking the error bars into consideration. Our best fit gives $\kappa=6.36$, $\beta=0.003\,59$ GeV^{-1}, $\Gamma_0=0.0009$ GeV, and $E_0=7.458\,38$ GeV and is shown also in Fig. 2. In the fitting procedure, special attention was paid to the mandatory threshold. Taking the derivative of the parametrization and numerically performing the integration to obtain the survival amplitude, we get our main result depicted in Fig. 3. We conclude that at large times, the survival probability of the unstable ^{8}Be(2^+) state at 3.04 MeV excitation energy, behaves as,

$$P_{^8{\rm Be}}(t) \sim \frac{1}{t^{6.36}}. \tag{6}$$

One could in principle use the fit along with some known mathematical formulas for survival amplitude (see the Appendix) to arrive at the above conclusion.

Theoretically we would expect that near threshold [16], $\tan\delta_l(E_{\rm cm})\sim k_{\rm cm}^{2l+1}$, which implies, $d\delta_l(E_{\rm cm})/dE_{\rm cm}\sim(E_{\rm cm}-E_{\rm th.})^{l-1/2}$ also near threshold.

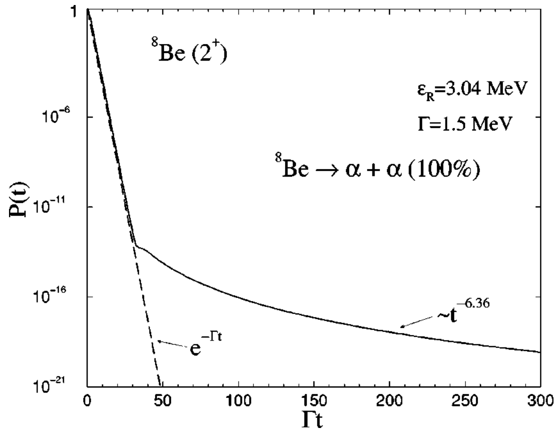

FIG. 3. Survival probability $P(t)$ of the decay $^8\mathrm{Be}(2^+)\to\alpha+\alpha$, as a function of the number of lifetimes after decay. $P(t)$ is evaluated numerically using $d\delta_l/dE_{\mathrm{cm}}$ of Fig. 2 as the spectral density. The dashed line ($e^{-\Gamma t}$) shows that the decay law for the $^8\mathrm{Be}(2^+)$ state (solid line) is exponential up to about 30 lifetimes after which it proceeds as $t^{-6.36}$. ϵ_R and Γ are the resonance mass and width, respectively.

This amounts to saying that κ is expected to be $2l+1$. For the $^8\mathrm{Be}(2^+)$ resonance, one then gets an inverse power law t^{-5} for the survival probability. The data on the phase shift do not seem to follow the standard threshold behaviour and hence we get (6). The discrepancy, however, does not look serious. Indeed, re-calculated the "l"-value of the fitted κ is 2.68. Interestingly, the exponent 6.36 is close to the theoretical prediction of 7 for the case of $l=2$ made in [2]. The deviation from the exponential decay law starts around 30 lifetimes after the onset of the decay. By this time, one could say that the sample with which one started has depleted by about 13 orders of magnitude ($\sim e^{-30}$), making a direct measurement of such a phenomenon not feasible. The above is the case of a strong decay with short lifetime. In the case of weak decays, the onset of the nonexponential law at large times is expected to be much later [2,3] making the direct measurement even less feasible.

In summary, we combined the Fock-Krylov method (to calculate the survival amplitude of an unstable state in terms of the Fourier transform of a spectral function), with a result in statistical mechanics of Beth and Uhlenbeck. This result identifies the energy derivative of the two body scattering phase shift (2) to be proportional to the continuum probability density of states. Using experimental phase shifts, the method allowed us to compute the nonexponential long time behavior (inverse power law to be specific) of an unstable quantum system, namely ^{8}Be, which decays to two α particles. Asked as to why the above method has been overlooked so far, we can only speculate by noting that even the old results by Eisenbud [18], Wigner [12] and Smith [19] concerning the phase shift derivative have been neglected for a long time and came back into vogue only recently [20–24].

We close by quoting from [25]; "Thus it seems unlikely that nuclear decays will show deviations from the exponential decay law which they made famous." We have shown that it is possible, as the information on the time evolutions is encoded in the scattering data.

APPENDIX: THEORETICAL SURVIVAL AMPLITUDE

Parametrizing the spectral function to account for the threshold behavior, we have, $\rho_\Psi(E)=\mathcal{G}(E)(E-E_{\mathrm{th.}})^{\gamma(l)}$, where $\gamma(l)$ is an integer. Imposing standard conditions on $G(E)$ [17,26] the survival probability can be computed by going to the complex plane. Though this method of finding the survival probability is standard and known [2,4], we describe it here as it is not exactly equivalent to that in [2,4,27]. Choosing the closed path $C_{\mathrm{R}}=C_{\mathfrak{I}}+C_{\mathfrak{R}}+C_{\mathrm{R}}^{1/4}$, along the real axis ($C_{\mathfrak{R}}$) attaching to it a quarter of a circle with radius R ($C_{\mathrm{R}}^{1/4}$) in the clockwise direction and completing the path by going upward the imaginary axis up to zero ($C_{\mathfrak{I}}$), using Cauchy's theorem and considering the conditions imposed on $\mathcal{G}(E)$, one can see that $\mathcal{A}_{\Psi_l}(t)=\mathcal{A}^{\mathrm{E}}_{\Psi_l}(t)+\mathcal{A}^{\mathrm{P}}_{\Psi_l}(t)$, with $\mathcal{A}^{\mathrm{E}}_{\Psi_l}(t)$ and $\mathcal{A}^{\mathrm{P}}_{\Psi_l}(t)$ given by,

$$\mathcal{A}^{\mathrm{E}}_{\Psi_l}(t) = e^{-iE_{th.}t}\lim_{\mathrm{R}\to\infty}\oint_{C_{\mathrm{R}}} dz e^{-izt}z^{\gamma}\mathcal{G}(z+E_{\mathrm{th.}}) = C_1 e^{-iE_{\mathrm{R}}t}e^{-\Gamma/2t}$$

$$\mathcal{A}^{\mathrm{P}}_{\Psi_l}(t) = C_2 e^{-iE_{th.}t}\int_0^\infty dx e^{-xt}x^{\gamma}\mathcal{G}(-ix+E_{\mathrm{th.}})$$

$$\simeq \frac{C_2}{t^{\gamma+1}}\Gamma(\gamma+1)e^{-iE_{th.}t}(E_{\mathrm{th.}}),$$

where $\Gamma(x)$ is the Euler's gamma function and C_1 and C_2 are constants. The approximation is valid for large times t. The general case [4] of noninteger γ leads to the same result.

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