Microscopic determination of the nuclear incompressibility within the nonrelativistic framework

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The nuclear incompressibility K_∞ is deduced from measurements of the isoscalar giant monopole resonance (ISGMR) in medium-heavy nuclei, and the resulting value turns out to be model dependent. Since the considered nuclei have neutron excess, it has been suggested that the model dependence is due to the different behavior of the symmetry energy in different models. To clarify this issue, we make a systematic and careful analysis based on new Skyrme forces, which span a wide range of values for K_∞, for the value of the symmetry energy at saturation and for its density dependence. By calculating, in a fully self-consistent fashion, the ISGMR centroid energy in ^{208}Pb, we reach three important conclusions: (i) the monopole energy, and consequently the deduced value of K_∞, depend on a well-defined parameter related to the shape of the symmetry energy curve and called K_{sym}; (ii) Skyrme forces of the type of SLy4 predict K_∞ around 230 MeV, in agreement with the Gogny force (previous estimates using Skyrme interactions having been plagued by a lack of full self-consistency); (iii) it is possible to build forces which predict K_∞ around 250 MeV, although part of this increase is due to our poor knowledge of the density dependence and effective mass.

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I. INTRODUCTION

The question about the proper value of the nuclear incompressibility K_∞ is still open. The model dependence of this quantity amounts to a difference of the order of ~10–20% among the values obtained within different theoretical models. There is a renewed interest in this issue, motivated both by the improved quality of the recent experimental measurements of the isoscalar giant monopole resonance (ISGMR), and by the progress of relativistic mean-field models (RMF), which are to be confronted with more traditional nonrelativistic models based on Skyrme or Gogny effective forces.

Skyrme energy functionals have been widely used in nuclear structure calculations since the 1970's. The first Skyrme effective forces were built in the pioneering work of Vautherin and Brink [1], by fitting their parameters to nuclear matter properties (the saturation point) and to selected observables (binding energies and charge radii) of closed-shell nuclei calculated in the Hartree-Fock (HF) approximation. Later, many improvements of the Skyrme energy functional were devised. These have been possible also because the mean-field approach was extended to the time-dependent case [time-dependent HF (TDHF)] and to its small amplitude limit [random phase approximation (RPA)]. Within this scheme, it is possible to calculate the collective nuclear excitations and to explore the correlations between their properties and the force parameters, or some physically meaningful combinations of them. The relation between the ISGMR and the nuclear incompressibility is one of such relations.

The introduction of reliable RMF effective Lagrangians is more recent. However, the progress in this field has been quite fast [2], and we can nowadays discuss the properties of the RMF parametrizations on the same footing as the Skyrme [3] and Gogny [4] functionals.

The nuclear incompressibility K_∞ is related to the curvature of the energy per particle E/A in symmetric nuclear matter around the minimum ϱ_0, i.e., at the saturation point

$$K_\infty \equiv 9\varrho_0^2 \frac{d^2}{d\varrho^2}\frac{E}{A}\bigg|_{\varrho_0}. \qquad (1)$$

The interest of determining the value of K_∞ stems also from its impact on the physics of supernovas and neutron stars.

Until a year ago, the status of the nuclear incompressibility problem could be summarized as follows. From calculations based on Skyrme functionals, different groups have ex-

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tracted values of K_∞ of the order of 210–220 MeV. Using the Gogny functionals, a value of K_∞ around 230 MeV was obtained. Finally, the relativistic calculations predicted values in the range 250–270 MeV. All these results made use of the measured value of the ISGMR, e.g., in ^{208}Pb, as explained below.

This situation for the nuclear incompressibility "puzzle" has been reviewed in Ref. [5]. There, it was shown that the accuracy of the ISGMR data obtained at Texas A&M [6] allows for extracting K_∞ with an experimental error of no more than ±12 MeV. Moreover, a rather important conclusion was reached. The previous works based on the Skyrme forces consisted of not fully self-consistent HF plus RPA calculations in which the two-body residual Coulomb and spin-orbit forces were neglected. These terms are rather small since they affect the monopole energy in ^{208}Pb by only 4–5%, but this produces a change of 8–10% in the extracted value of the nuclear incompressibility. By considering this effect, the value of K_∞ from the Skyrme functionals turns out to be 235 MeV, in very good agreement with that extracted from the Gogny calculations. Consequently, there is no discrepancy between the results of the different nonrelativistic calculations. On the other hand, the gap with the relativistic results remains significant.

The most recent attempts in the literature [7–11] to attack this problem are focused on the possible relation(s) between the monopole energy in systems with a neutron excess, like ^{208}Pb, and the density dependence of the symmetry energy $S(\varrho)$. In fact, one of the clear differences between the Skyrme and RMF functionals concerns the behavior of the symmetry energy around the saturation point ϱ_0. The Skyrme energy functionals are characterized by smaller values of the symmetry energy at saturation, and of the corresponding slope, as compared with the RMF functionals. In this sense it may be said that the RMF functionals are "stiff" compared with the "soft" nonrelativistic ones.

In Ref. [7] some effective Lagrangians whose symmetry energy has different density dependences are built. This is easy to achieve, since, by adjusting the ρ-meson coupling constant, one can at the same time soften the symmetry energy $S(\varrho)$ and lower its value at the saturation point, $J \equiv S(\varrho_0)$. It is thus found that the extracted values of K_∞ indeed differ and can even become close to the Skyrme force values if J is around 28 MeV. However, in Ref. [7] no systematic treatment of finite nuclei is attempted. In Ref. [8] it is pointed out that RMF parametrizations with J lower than 32 MeV cannot describe satisfactorily the $N \neq Z$ nuclei. The authors conclude from their calculations that the lower limit for the RMF value of K_∞ is around 250 MeV. In Ref. [10], using a markedly improved version of the model of Ref. [7], this lower limit is confirmed since K_∞ results to be 248 MeV.

While in the relativistic framework it seems impossible to push the value of K_∞ below this lower limit, the recent results of Ref. [11] suggest that one can build at least one Skyrme-type interaction having $K_\infty = 255$ MeV and reproducing the correct ISGMR energy in ^{208}Pb. This is at variance with all other nonrelativistic calculations quoted in Ref. [5]. Moreover, the origin of the result of Ref. [11] is unclear. Does it correspond to an effective force with a symmetry energy, which is as stiff as that associated with the RMF Lagrangians? Or is some other parameter playing a role?

It seems necessary to make a systematic analysis of what is the upper limit for K_∞ within the nonrelativistic framework. The value of 230 MeV is extracted from a subset of the existing Skyrme parametrizations. Our main goal is to answer the question of whether it is possible to build interactions in such a way that K_∞ becomes closer to the RMF value, as well as to study what the crucial quantities are that control this variation of K_∞. In particular, we would like to understand the (so far, singular) result of Ref. [11].

The structure of the paper is the following. In Sec. II we review how the nuclear incompressibility is extracted from the microscopic calculations using the ISGMR experimental data, and what are the plausible quantitative arguments in favor of the idea that the density dependence of the symmetry energy plays a role. In Sec. III we describe our fitting of new Skyrme interactions suitable for the microscopic ISGMR calculations, and in Sec. IV we describe the results obtained and the implications for the value of K_∞. In Sec. V we present our conclusions.

II. DEDUCING THE NUCLEAR INCOMPRESSIBILITY FROM MONOPOLE DATA AND THE ROLE OF THE SYMMETRY ENERGY

In all the discussions of the relationship between the nuclear matter incompressibility and the ISGMR in finite nuclei, the starting point is the definition given by Blaizot [12] of the finite nucleus incompressibility K_A as

$$K_A = \frac{m\langle r^2\rangle_0 E_{\rm ISGMR}^2}{\hbar^2}, \tag{2}$$

where m is the nucleon mass and $\langle r^2\rangle_0$ is the ground-state mean-square radius. This expression has a well-defined meaning in medium-heavy nuclei, where the ISGMR is associated to a single peak at the energy $E_{\rm ISGMR} \approx 80\, A^{-1/3}$. In light nuclei the monopole strength is very much fragmented, and many states show up whose microscopic structure does not correspond to the simple picture of the radial "breathing mode" according to theoretical calculations (see, for example, Ref. [13]). In the case of the nuclei studied in [6], the existence of a single, collective monopole state is quite evident from the measured cross sections. In particular, in the case of ^{208}Pb, which is the object of our present study, the experimental peak energy and the centroid energies E_0 and E_{-1} (defined, respectively, as m_1/m_0 and $\sqrt{m_1/m_{-1}}$, where m_k is the kth moment of the strength function) essentially coincide, leaving out any ambiguity about the correct value of $E_{\rm ISGMR}$ to be used for determining the experimental value of K_A.

However, finding a theoretical relation between K_A and K_∞ is less simple. In Ref. [12], the generic expression of the energy functional associated with Skyrme HF has been written in the case of a finite spherical system. At variance with that of infinite matter, the density is not uniform and cannot be reduced to a simple number. Therefore, to minimize the energy functional and find its second derivative around the minimum, one has to resort to various simplifying hypotheses. The main one is the use of the so-called scaling model, in which a simple shape of the ground-state density ϱ_0 is assumed and its changes are associated to a single parameter λ, i.e., they are of the type $\varrho_0(\vec{r}) \rightarrow \varrho_\lambda(\vec{r}) = (1/\lambda^3)\varrho_0(\vec{r}/\lambda)$. In this way, the expression for the finite system incompressibility can be found. By isolating the terms corresponding to the volume, surface, symmetry, and Coulomb contributions, the result can be written as

$$K_A = K_\infty + K_{surf}A^{-1/3} + K_{sym}\delta^2 + K_{Coul}\frac{Z^2}{A^{4/3}}, \qquad (3)$$

where $\delta \equiv (N-Z)/A$ (cf. Sec. 6.2 of Ref. [12]).

We recall here that, in the past, many authors have used the formula (3) as an ansatz and have tried to obtain the parameters of the right-hand side (r.h.s.) from a numerical fit, using as input the experimental values of K_A in different nuclei. This procedure is not stable and leads to ill-defined values of the parameters [14], so that it is nowadays abandoned.

Instead, the microscopic method to deduce K_∞ relies on the fact that RPA calculations of the ISGMR can be performed by using functionals characterized by different values of K_∞. If the calculations done with a given functional reproduce the experimental ISGMR energy, the associated value of K_∞ should be chosen as the best one. Let us examine this in more detail.

Mainly one nucleus has been used so far, that is, ^{208}Pb. In the first work in which the microscopic procedure has been applied [15], the RPA values for K_A obtained from the RPA centroid energies E_{-1} have been plotted versus the K_∞ of the force used. Then, an empirical linear fit of the results was performed, namely

$$K_A = aK_\infty + b. \qquad (4)$$

This relation allows us to extract the best value for K_∞ by inserting the experimental K_A. In [15] the explicit form of Eq. (4) in the case of ^{208}Pb is $K_A = 0.64K_\infty - 3.5$ [MeV]. The second term of the r.h.s. is much smaller than the first term. Consequently, even if in principle the last formula together with Eq. (2) would lead to $E_{\text{ISGMR}} = 1.16\sqrt{0.64K_\infty - 3.5}$, this equation can be approximated by $0.928\sqrt{K_\infty}$ (neglecting the second term under the square root). This explains why, in many of the works quoted in [5], a successful interpolation of the type

$$E_{\text{ISGMR}} = a'\sqrt{K_\infty} + b' \qquad (5)$$

was done: in practice, Eqs. (4) and (5) are equivalent. It is from either of these relations, using the experimental ISGMR energy in ^{208}Pb which is 14.17 ± 0.28 MeV, that the values for K_∞ mentioned in Sec. I were obtained.

The uncertainity of the value of K_∞, which is deduced, is

$$\frac{\delta K_\infty}{K_\infty} = 2\frac{\delta E_{\text{ISGMR}}}{E_{\text{ISGMR}}}.$$

The experimental error on the monopole energy, plus a theoretical error of the same order (see [5]), produce a global error bar of ±12 MeV on K_∞.

One may argue why the linear relations just introduced are valid. So far, Eq. (3) has not played, in fact, any explicit role in the deduction of K_∞. However, this expression can be taken as a rather useful guideline. Given a microscopic functional, the different terms K_{surf}, K_{sym} and K_{Coul} (in addition to K_∞) entering this formula can be calculated as described shortly below. The resulting value of K_A differs from the microscopic outcome of RPA, as a rule, by about 5%. Therefore, we make in the rest of this section a detailed analysis of the role of the different terms in Eq. (3). If, for a family of functionals, K_A depends linearly on K_∞ as written in Eq. (4), it means that the other terms do not vary significantly. This is what happens for a large subset of the Skyrme and Gogny parametrizations, as it is evident from Fig. 6 of Ref. [15]. However, the role of the surface, symmetry, and Coulomb terms should be critically reexamined if new functionals, including relativistic ones, enter into the discussion.

The expression for these terms have been given in Ref. [12]. K_{surf} cannot be calculated analytically, but numerical estimates are possible within both the quantal and semiclassical scheme. We refer the reader to Ref. [16] for an example of a quantal derivation (which is a scaled Hartree-Fock calculation of semi-infinite nuclear matter). The most recent semiclassical, i.e., extended Thomas-Fermi (ETF) calculations, have been performed both in the nonrelativistic and in the relativistic scheme and have shown that the quantity K_{surf} is well approximated by cK_∞ with $c \approx -1$ (however, it should be noted that c tends to grow with K_∞) [17]. We have checked that this approximation is valid in the case of all the forces used in this work: we have seen that, e.g., $c = -1.03$ if $K_\infty = 230$ MeV and $c = -1.07$ if $K_\infty = 250$ MeV.

In order to study K_{sym}, we first give some necessary definitions of the symmetry energy and of the parameters related to its density dependence. We define the symmetry energy by writing the total energy density $\mathcal{E}$ as the sum of an isoscalar part $\mathcal{E}_0(\varrho)$ which depends only on the total density $\varrho \equiv \varrho_n + \varrho_p$, and an isovector part,

$$\mathcal{E}(\varrho,\varrho_-) = \mathcal{E}_0(\varrho) + \varrho S(\varrho)\left(\frac{\varrho_-}{\varrho}\right)^2, \qquad (6)$$

where $\varrho_- \equiv \varrho_n - \varrho_p$. We remind in this context that in a homogeneous system, $E/A = \mathcal{E}/\varrho$. The symmetry energy $S(\varrho)$ can be expanded up to second order around ϱ_0,

$$S(\varrho) = S(\varrho_0) + S'(\varrho_0)(\varrho - \varrho_0) + \frac{1}{2}S''(\varrho_0)(\varrho - \varrho_0)^2. \qquad (7)$$

The value of the symmetry energy at saturation $S(\varrho_0)$ is often denoted as J and we are following the use the same notation in this paper. Other notations, like a_τ or a_4, are also employed in the literature. The first and second derivatives of $S(\varrho)$ at the saturation point have been written many times in terms of the so-called parameters L and K_{sym} (see, e.g., Ref. [18]), as $S'(\varrho_0) = L/3\varrho_0$ and $S''(\varrho_0) = K_{sym}/9\varrho_0^2$. It is quite unfortunate that the symbol K_{sym} has been used in the literature with such different meanings, either in connection with $S''(\varrho_0)$ or in Eq. (3). Here, we will always use K_{sym} to mean the symmetry term of K_A in Eq. (3).

The expression of K_{sym} is

TABLE I. Nuclear matter properties calculated with the different Skyrme parameter sets characterized by $\alpha=1/6$ and by different values of K_∞ and J (these two quantities identify the parameter set and are shown in the first column). All quantities are defined in the text. In the last column, the χ^2 per point is displayed.

	ϱ_0 (fm^{-3})	E/A (MeV)	m^*/m	L (MeV)	K_{sym} (MeV)	K_{Coul} (MeV)	χ^2
230/26	0.161	−15.89	0.69	−39.06	−178.95	−4.92	4.38
230/28	0.162	−15.96	0.70	−11.23	−228.62	−4.91	4.50
230/30	0.161	−15.98	0.70	22.88	−281.05	−4.90	5.67
230/32	0.161	−16.03	0.70	36.22	−314.95	−4.90	5.24
230/34	0.161	−16.06	0.70	56.15	−354.24	−4.89	5.86
230/36	0.161	−16.10	0.71	71.55	−389.53	−4.88	6.46
230/38	0.161	−16.14	0.71	87.60	−424.86	−4.88	7.15
230/40	0.161	−16.16	0.71	106.07	−462.20	−4.87	8.28
240/26	0.160	−15.91	0.63	−15.95	−176.41	−5.05	7.36
240/28	0.161	−15.94	0.63	3.97	−202.07	−5.04	7.04
240/30	0.159	−15.95	0.63	34.05	−273.93	−5.04	7.75
240/32	0.166	−16.15	0.65	34.43	−300.39	−5.00	12.41
240/34	0.164	−16.12	0.65	62.60	−350.02	−5.01	10.22
240/36	0.164	−16.15	0.65	75.67	−384.99	−5.00	11.17
240/38	0.163	−16.19	0.65	98.62	−429.77	−5.00	11.28
240/40	0.165	−16.24	0.65	108.15	−460.88	−4.99	13.76
250/28	0.165	−16.10	0.59	32.99	−238.76	−5.13	14.33
250/30	0.164	−16.09	0.59	30.02	−255.78	−5.13	11.65
250/32	0.164	−16.14	0.59	43.59	−293.94	−5.12	13.02
250/34	0.163	−16.14	0.59	60.33	−334.94	−5.12	12.30
250/36	0.162	−16.17	0.59	80.19	−379.80	−5.12	12.78
250/38	0.162	−16.20	0.59	97.50	−421.63	−5.12	13.36
250/40	0.162	−16.25	0.59	112.18	−460.37	−5.11	15.30

$$K_{sym} = 9\varrho_0^2 S''(\varrho_0) + 9\varrho_0 S'(\varrho_0) - \frac{81\varrho_0^3 S'(\varrho_0)}{K_\infty}\frac{d^3\mathcal{E}}{d\varrho^3}\bigg|_{\varrho 0}, \tag{8}$$

and from this expression it is evident that this parameter contains some relevant information about the density dependence of the symmetry energy.

The values of J are, as a rule, larger in the case of the RMF functionals than for the Skyrme ones. A larger value of J is correlated with a larger value of $S'(\varrho_0)$, which is usually a positive quantity, although it may sometimes become negative (cf. [19] and Fig. 4 of [8], as well as Tables I and II in this paper). In the next section we show that a larger J is also correlated with a more negative value of K_{sym}, at least for the forces we have studied. The explanation which is given for the correlation between J and $S'(\varrho_0)$ is that the fits to finite nuclei observables constrain the symmetry energy at some average density $\langle\varrho\rangle$ lower than ϱ_0 (see, e.g., Ref. [20], and references therein). In the case of one set of forces introduced in this paper (see Sec. III), this typical behavior of the symmetry energy is shown in Fig. 1. In a narrow region around $\varrho=0.10$ fm^{-3} (± 0.001 fm^{-3}) all curves cross one another at a value $S(\varrho)=25\pm 1$ MeV. When the symmetry energy at saturation is larger, the slope is also larger. The other sets of forces show qualitatively the same trend.

TABLE II. The same as Table I for the different Skyrme parameter sets characterized by $\alpha=0.3563$.

	ϱ_0 (fm^{-3})	E/A (MeV)	m^*/m	L (MeV)	K_{sym} (MeV)	K_{Coul} (MeV)	χ^2
250/30	0.153	−15.87	0.77	12.66	−339.16	−5.09	14.49
250/32	0.152	−15.89	0.77	36.57	−377.81	−5.09	11.58
250/34	0.151	−15.91	0.77	58.81	−415.36	−5.08	14.03
250/36	0.150	−15.92	0.77	72.00	−447.51	−5.09	16.69
250/38	0.149	−15.95	0.77	95.20	−485.39	−5.08	20.75
250/40	0.148	−15.96	0.77	110.19	−518.58	−5.08	22.35
250/42	0.148	−15.97	0.77	126.53	−552.32	−5.08	25.80
260/28	0.157	−15.96	0.67	−7.30	−318.87	−5.20	5.57
260/30	0.157	−16.00	0.68	16.92	−326.92	−5.19	6.42
260/32	0.159	−16.08	0.69	29.53	−362.73	−5.17	5.63
260/34	0.159	−16.13	0.70	46.05	−399.70	−5.16	6.50
260/36	0.160	−16.19	0.71	64.11	−437.67	−5.15	7.60
260/38	0.160	−16.25	0.72	78.40	−471.97	−5.14	8.96
260/40	0.161	−16.30	0.73	90.97	−505.06	−5.13	11.13
270/28	0.157	−15.97	0.58	−4.60	−262.94	−5.32	9.43
270/30	0.157	−16.01	0.58	21.47	−311.34	−5.31	9.87
270/32	0.157	−16.05	0.58	43.36	−355.80	−5.30	10.40
270/34	0.157	−16.10	0.59	63.76	−398.68	−5.29	11.10
270/36	0.157	−16.14	0.59	81.41	−438.67	−5.29	11.75
270/38	0.158	−16.19	0.60	98.06	−477.62	−5.28	12.47
270/40	0.157	−16.21	0.60	115.06	−516.21	−5.27	13.48
270/42	0.157	−16.23	0.60	133.90	−556.22	−5.27	14.53

The last term of Eq. (3) is the Coulomb contribution, which is unlikely to be very much model dependent. It is written as

$$K_{Coul} = \frac{3}{5}\frac{e^2}{r_0}\left(1 - \frac{27\varrho_0^2}{K_\infty}\frac{d^3\mathcal{E}}{d\varrho^3}\bigg|_{\varrho 0}\right), \tag{9}$$

where r_0 is the average interparticle spacing.

In summary, if we want to compare two models, say I and II (they could be, for instance, a nonrelativistic and a RMF functional, respectively), we will write, by using $K_{surf}=cK_\infty$,

$$K_A \sim K_\infty^{(I)}(1 + cA^{-1/3}) + K_{sym}^{(I)}\delta^2 + K_{Coul}^{(I)}\frac{Z^2}{A^{4/3}},$$

$$K_A \sim K_\infty^{(II)}(1 + cA^{-1/3}) + K_{sym}^{(II)}\delta^2 + K_{Coul}^{(II)}\frac{Z^2}{A^{4/3}}. \tag{10}$$

We have already mentioned that K_{sym} is negative, and the same is true for K_{surf} and K_{Coul}. All can be viewed as corrections to the leading term K_∞. It is clear that a more negative value of K_{surf} or K_{sym} leads to extracting from the experimental K_A a larger value of K_∞. We will develop this argument in Sec. IV.

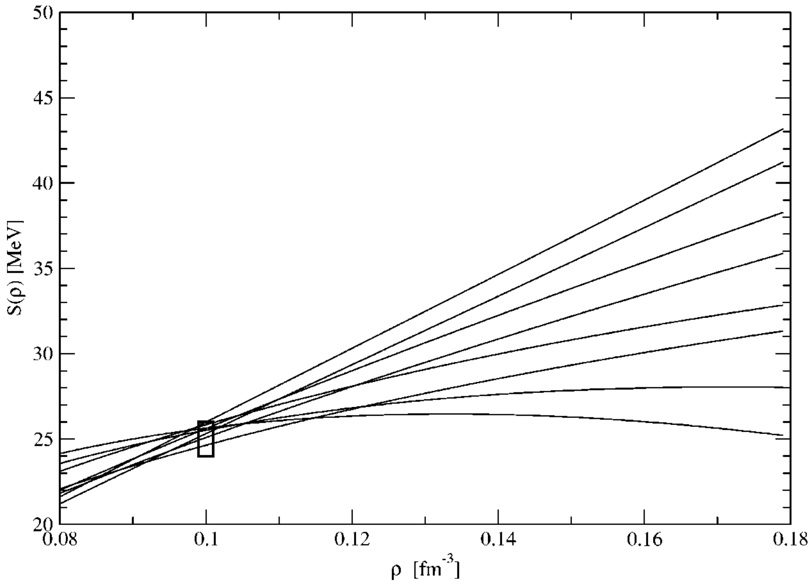

FIG. 1. Density dependence of the symmetry energy for one of the set of forces ($\alpha=1/6$ and $K_\infty=240$ MeV). The symmetry energy J is varied from 26 to 40 MeV.

III. CONSTRUCTION OF NEW SKYRME PARAMETER SETS

The different forces used in this study have been built using a procedure that is quite similar to the one discussed in Ref. [21]. The starting point is the standard form of a Skyrme interaction as given in Eq. (2.1) of [21].

In the case of the first set of forces that we have constructed, the density-dependent term has $\varrho^\alpha=\varrho^{1/6}$. The spin-gradient terms occuring in the Skyrme functional are neglected and the Coulomb exchange term is included within the Slater approximation. The center-of-mass motion is taken into account with the usual $A/(A-1)$ correction in the kinetic term, which means that only the one-body part of the center-of-mass (c.m.) energy is subtracted before variation.

The parameters of the forces have been determined by minimizing a χ^2 built on:

(1) the infinite nuclear matter properties ϱ_0, $E/A(\varrho_0)$ (while K_∞, J, and the enhancement factor κ of the Thomas-Reiche-Kuhn sum rule are kept constant; this latter is always set at 0.25);

(2) the following finite nuclei properties: binding energies and charge radii of ^{40,48}Ca, ^{56}Ni, and ^{208}Pb together with the binding energy of ^{132}Sn;

(3) the spin-orbit splitting of the neutron $3p$ shell in ^{208}Pb;

(4) the surface energy, calculated in the ETF approximation and fitted to the value of the SkM* force in order to obtain good mean-field properties at large deformations (especially a good fission barrier of the ^{240}Pu nucleus).

Furthermore, the parameter x_2 is fixed to -1 in order to ensure the stability of the fully polarized neutron matter in a simple but tractable way [22]. Unlike the case of the SLy4 force, the equation of state of neutron matter is checked but not fitted in order to have a large enough variational space of parameters when the nuclear incompressibility and the symmetry energy are varied. The forces which have been built have K_∞ equal to 230, 240, and 250 MeV, whereas J is varied between 26 and 40 MeV. In Fig. 2 we show the accuracy of the present forces in reproducing the ground-state observables (binding energies and charge radii).

Motivated by the comparison with Ref. [11], we have also built another set of forces with a similar protocol, but with the density-dependent term ϱ^α having the same exponent $\alpha=0.3563$ as the force SK255 introduced in Ref. [11]. The forces of this set have K_∞ equal to 250, 260, and 270 MeV, while J is varied between 28 and 42 MeV. Figure 3 gives an idea of the accuracy of this set, in the same way as for the previous one.

The nuclear matter properties associated to all the new forces introduced in this paper are summarized in Tables I and II. By looking at the values of the effective mass, one can recognize the well-known correlation between K_∞, α, and m^*/m (see, e.g., Fig. 2 of Ref. [21]). In the last column of Tables I and II we provide the values of the χ^2 per point associated with the forces. It can be noted that in most cases the present interactions have the same quality of those introduced in Ref. [21] (see Fig. 5 of that work). As far as the comparison with the work of other authors is concerned, we should call that the meaning of the χ^2 values must be judged in relation to the quantities chosen for the fit. Since there is not a universal protocol to determine an effective nucleon-nucleon interaction, in different cases the values of the χ^2 can vary simply because of a markedly different choice of the reference observables. Therefore, the values in the tables are a useful tool but should be taken with the proper caution. Figures 2 and 3 may be more illustrative as the reader is able to compare with corresponding values, e.g., in Refs. [8,11].

IV. RESULTS AND DISCUSSION

Using these new Skyrme interactions, the ISGMR centroid energies $E_{-1}=\sqrt{m_1/m_{-1}}$ in ^{208}Pb have been calculated in a fully self-consistent manner. The energy-weighted sum rule m_1 is obtained from the well-known double commutator expectation value, while the inverse energy-weighted sum rule m_{-1} is extracted by means of a constrained HF (CHF) calculation [23]. Adding to the Hamiltonian a term $\lambda\hat{M}$, where $\hat{M}$

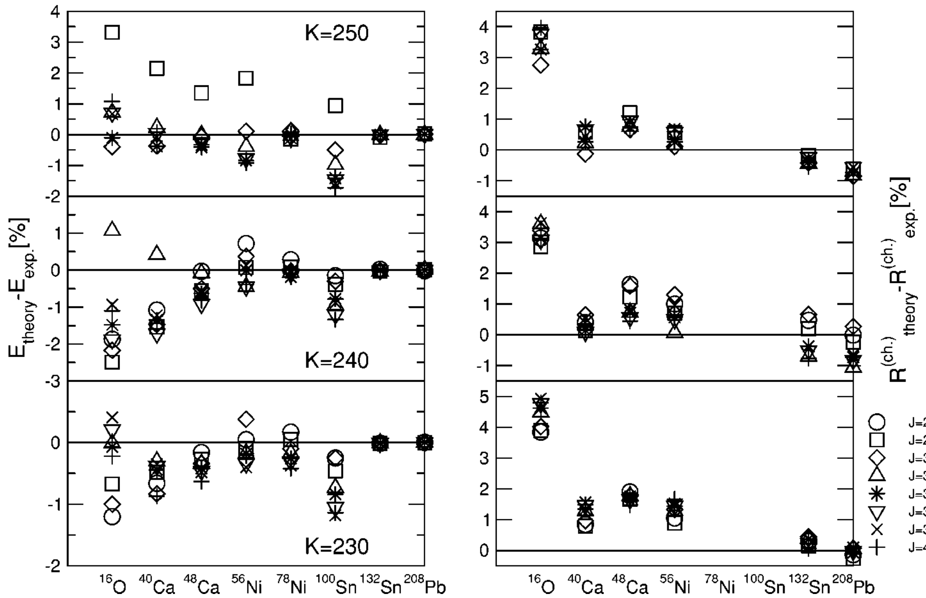

FIG. 2. Difference between experimental binding energies (left) or experimental charge radii (right) with the predictions of forces characterized by $\alpha=1/6$ and by different values of K_∞ and J, for typical spherical nuclei. Note that the binding energies of ^{40,48}Ca, ^{56}Ni, ^{132}Sn, and ^{208}Pb, as well as the charge radii of ^{40,48}Ca, ^{56}Ni, and ^{208}Pb are used in the fit of the force parameters.

is in this case the monopole operator $\Sigma_{i=1}^{A} r_i^2$ and $\lambda > 0$ (to avoid an Hamiltonian without lowest bound), the value of m_{-1} can be extracted in two different ways, that is,

$$m_{-1} = -\frac{1}{2}\frac{\delta\langle M\rangle_0}{\delta\lambda} = \frac{1}{2}\frac{\delta^2\langle H\rangle_0}{\delta\lambda^2}. \tag{11}$$

By varying the steps in λ and by comparing the outcome of these two different expressions, numerical tests concerning the accuracy of m_{-1} can be performed. We have come to the conclusion that this quantity can be determined with an accuracy of ±3% or better. This is definitely more reliable than the result of usual RPA calculations made using a basis expansion, since the convergence of m_{-1} with the basis size can be quite slow. Moreover, as already discussed in Sec. I, the Skyrme RPA calculations of the ISGMR performed so far lack full self-consistency since part of the residual interaction (the two-body Coulomb and two-body spin-orbit terms) are dropped. This has been shown to lead to a systematic error in the monopole centroid energies [5].

In Figs. 4 and 5 we show the results for the monopole energy E_{-1} obtained with the present interactions, as a function of the associated values of K_∞ and J. Figure 4 refers to

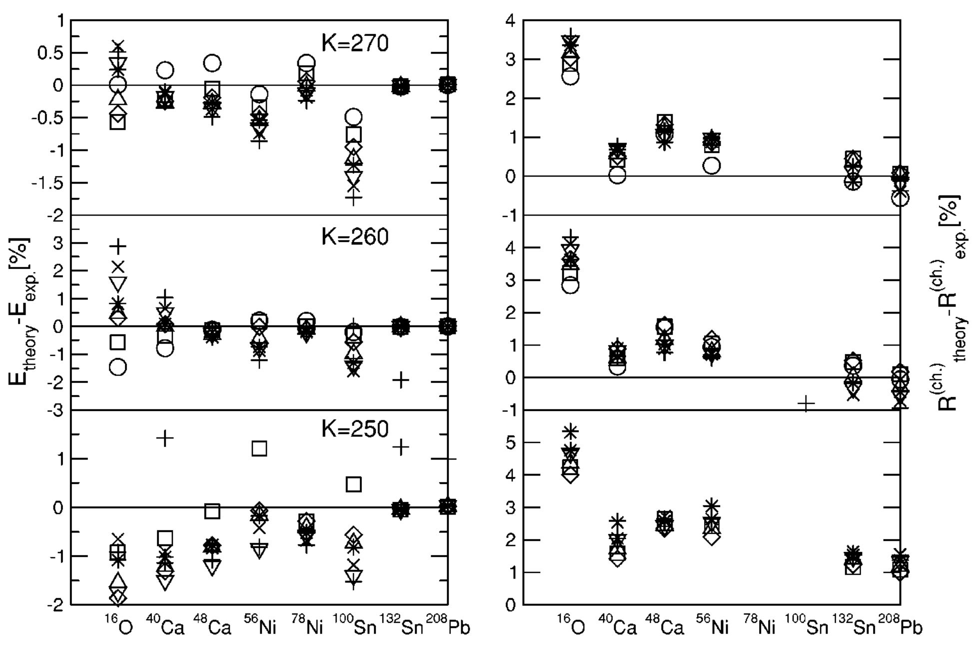

FIG. 3. Same as Fig. 2 in the case of the forces with $\alpha=0.3563$.

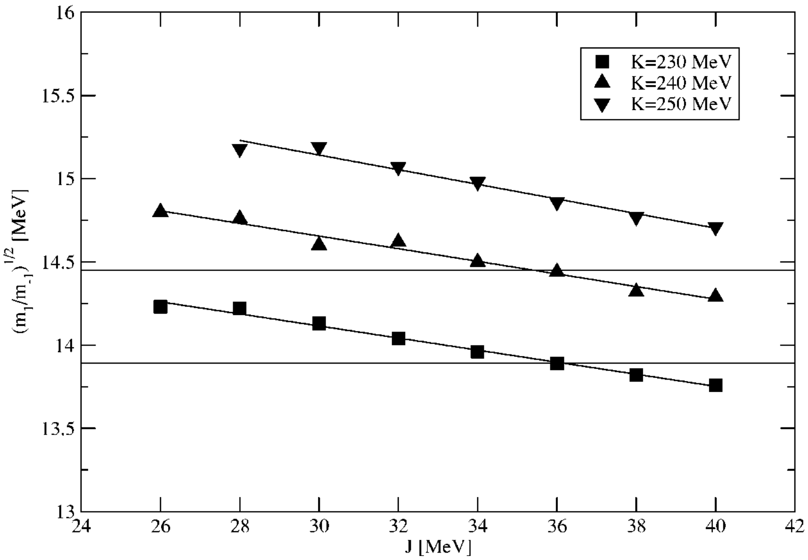

FIG. 4. The ^{208}Pb ISGMR centroid energy E_{-1} calculated with the Skyrme parameter sets with $\alpha=1/6$, as a function of J. The different symbols correspond to the values of K_∞ (see inset). Lines are numerical fits and are simply intended to guide the eye. The area delimited by the two horizontal lines correspond to the experimental value.

the forces with $\alpha=1/6$, whereas Fig. 5 is for those with $\alpha=0.3563$. In the case of the forces with $\alpha=0.3563$, we have used the same procedure of Ref. [11], that is, we have neglected the Coulomb exchange and we have omitted the c.m. correction in the HF variation, by subtracting it afterwards from the total energy, in the harmonic-oscillator approximation $\left(E_{\text{c.m.}}=\frac{3}{4}41A^{-1/3}\right)$. We have checked that this lowers the monopole energy by about 150 keV. The straight lines in the figures are linear fits of the CHF results corresponding to the different symbols, whereas the experimental range for the monopole energy [6] is delimited by the horizontal lines. These figures can be compared with Fig. 5 (upper panel) of Ref. [8]. The results for the monopole energy are, as expected, much less sensitive to J than to K_∞. By varying K_∞ by 10 MeV, i.e., by about 4%, the monopole energy changes by 0.5 MeV. In order to obtain the same change, J should be varied from 26 to 40 MeV, which is about 50%. The RMF results show qualitatively the same pattern.

We have to stress that the existence of a definite, yet not strong, dependence on J is in agreement with the discussion in Sec. II, where the role of K_{sym} as one of the crucial parameters governing the monopole energy has been emphasized. It is clear from Figs. 4 and 5, and from Tables I and II, that the monopole energies do depend on the parameter K_{sym}, associated with the density dependence of the symmetry energy. With increasing K_{sym}, the monopole energy increases (a change of about 300 MeV in K_{sym}, produces a variation of 0.5 MeV in the monopole energy). On the other hand, in the forces we have built, there is a strong correlation between K_{sym} and J, essentially independent of K_∞ but not of α. In fact, according to the numbers displayed in Tables I and II, the modification of the exponent α in the Skyrme functional, allows us to change the value of K_{sym} keeping fixed the values of K_∞ and J. According to the argument developed at the end of Sec. II, this should allow us to change the value of K_∞ extracted from the experimental ISGMR data.

By considering only the set with $\alpha=1/6$, we confirm the previous result of Ref. [5] that $K_\infty=230-240$ MeV is the preferred value for the nuclear incompressibility. This is not fully compatible with the RMF result. In fact, extrapolating from Fig. 4, one can see that a hypothetic Skyrme parametrization having that associated value of K_∞, would reproduce the experimental monopole energy only with an unrealistic

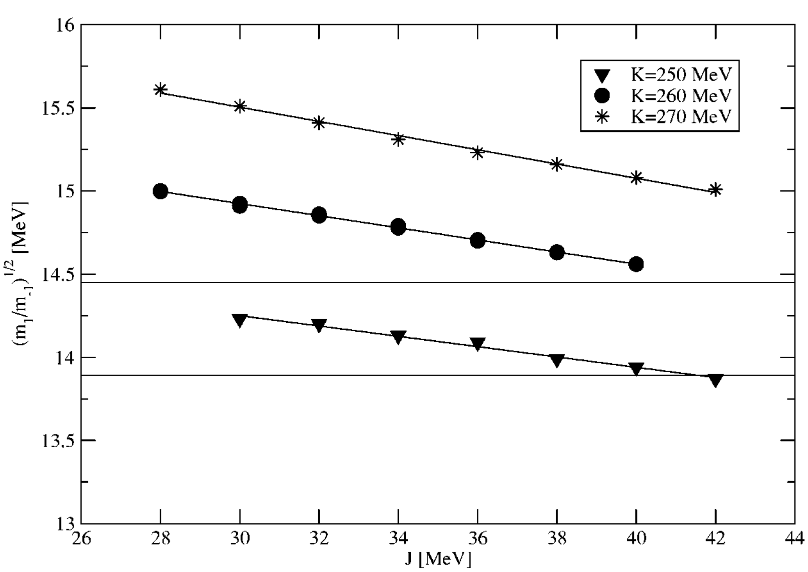

FIG. 5. Same as Fig. 4, for the Skyrme parameter sets with $\alpha=0.3563$.

value of J above 50 MeV. On the other hand, the set of forces with $\alpha = 0.3563$ allows us to extract a value of K_∞ around 250 MeV, in agreement with the outcome of the RMF calculations.

We should at this point stress that the more negative values of K_{sym} which characterize the forces with $\alpha = 0.3563$ cannot, alone, explain the extraction of a larger K_∞. The forces which better reproduce the experimental monopole energy are those with (α, K_∞, J) given either by (1/6, 230 MeV, 28 MeV) or by (0.3563 250 MeV, 30 MeV). We can apply Eq. (10) to the case of these two forces, by inserting the values given in the Tables I and II and by taking into account that $c \sim -1$ for $K_\infty = 230$ MeV but ~ -1.1 for $K_\infty = 250$ MeV. This gives for the two forces, respectively,

$$K_A = 154.16 = 230 - 38.82 - 10.23 - 26.79 \ (\text{MeV})$$

and

$$K_A = 160.64 = 250 - 46.41 - 15.18 - 27.77 \ (\text{MeV}),$$

where the four numbers on the r.h.s. correspond, respectively, to the volume, surface, symmetry, and Coulomb contributions. It is clear that most of the gain of 20 MeV in K_∞ comes from the increase of the surface term (8 MeV), and the more negative K_{sym} which, multiplied by the tiny value of δ^2 for ^{208}Pb, contributes 5 MeV. A small contribution of 1 MeV results from the increase of K_{Coul}. Finally, Eq. (10) does not consider that, in the calculations done by employing the forces with $\alpha = 0.3563$, the Coulomb exchange and center-of-mass corrections are neglected. As mentioned above, this lowers the ISGMR energies by about 150 keV and hence K_A by about 5 MeV. This brings the two results for K_A rather close to each other.

V. CONCLUSION

Until recently, the extraction of the nuclear incompressibility from the monopole data was plagued by a marked model dependence: the Skyrme energy functionals seemed to point to 210–220 MeV, the Gogny functionals to 235 MeV, and the relativistic functionals to 250–270 MeV. It has been shown in Ref. [5] that the result of the Skyrme functionals is, in fact, consistent with that of Gogny, i.e., 235 MeV using the ^{208}Pb data. The previous value of 210–220 MeV was derived using non-fully-self-consistent calculations, neglecting the residual Coulomb and spin-orbit interactions. The discrepancy between the nonrelativistic value of 235 MeV and the relativistic prediction remained, since relativistic calculations confirmed the lower bound of about 250 MeV.

The work of Agrawal *et al.* [11] suggests that it is possible to build Skyrme forces that fit nuclear ground states and lead to the correct monopole energy, with $K_\infty = 255$ MeV. In the present work we systematically explore the conditions that lead to such different results for K_∞ within the Skyrme framework.

To this aim, we build classes of Skyrme forces which span a wide range of values for K_∞ and for the symmetry energy at saturation J. All these forces reproduce the ground-state observables with good accuracy. We use them to calculate the monopole energy in ^{208}Pb, defined as $E_{-1} = \sqrt{m_1/m_{-1}}$. We stress again that we can obtain this quantity without any lack of self-consistency, and with a numerical error that is not larger than the experimental uncertainity.

A first class of forces are built using the SLy4 protocol and have a density dependence characterized by the exponent $\alpha = 1/6$. With these forces, a value of K_∞ around 230–240 MeV is obtained, confirming the previous results of [5]. To obtain the correct monopole energy with larger values of K_∞ would require an unrealistically large value of J, since E_{ISGMR} increases with K_∞ and decreases with J. We understand this latter dependence as a consequence of the direct relation between the K_∞ and the *density dependence* of the symmetry energy $S(\varrho)$, and of the unavoidable correlation between $S(\varrho)$ and J.

To solve the discrepancy with the result of Agrawal *et al.*, we have built a second class of forces which have the density dependence $\alpha = 0.3563$. Using this class of forces we can arrive at K_∞ between 250 and 260 MeV. Actually, we can reproduce very accurately the results of Ref. [11], if we use the same approximations made in that work, namely, if we neglect the Coulomb exchange and c.m. corrections in the HF mean field. This shows that the differences between Ref. [11] and our work in the detailed protocol used to determine the forces, are unimportant. We have observed that the differences between the results of the two classes of Skyrme forces built in the present paper, come both from the surface and symmetry contributions, as a consequence of the change in the exponent α, and from the neglect of Coulomb exchange and c.m. corrections, which affect the monopole energy by about 150 keV and, therefore, K_∞ by about 5 MeV.

In conclusion, within the nonrelativistic framework there is not a unique relation between the value of K_∞ associated with an effective force and the monopole energy predicted by that force. Bona fide Skyrme forces can either predict 230–240 MeV for K_∞ or arrive at 250 MeV if a different density dependence is adopted and if one excludes some terms from the energy functional. This latter procedure, although it may mimic the relativistic case, is not conceptually well justified.

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