$\Lambda\Lambda$ bond energy from the Nijmegen potentials

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The $\Lambda\Lambda$ bond energy $\Delta B_{\Lambda\Lambda}$ in $\Lambda\Lambda$ hypernuclei is obtained from a *G*-matrix calculation which includes the coupling between the $\Lambda\Lambda$, ΞN, and $\Sigma\Sigma$ channels, as well as the effect of Pauli blocking to all orders. The Nijmegen NSC97e model is used as bare baryon-baryon interaction in the strangeness $S=-2$ sector. The $\Lambda\Lambda$-ΞN coupling increases substantially the bond energy with respect to the uncoupled $\Lambda\Lambda$ case. However, the additional incorporation of the $\Sigma\Sigma$ channel, which couples simultaneously to $\Lambda\Lambda$ and ΞN states, has a surprisingly drastic effect and reduces the bond energy down to a value closer to that obtained in an uncoupled calculation. We find that a complete treatment of Pauli blocking reduces the repulsive effect on the bond energy to about half of what was claimed before.

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I. INTRODUCTION

Double-strange Λ hypernuclei are nowadays the best systems to investigate the properties of the $S=-2$ baryon-baryon interaction. Emulsion experiments and subsequent analysis [1–4] have reported the formation of a few $\Lambda\Lambda$ hypernuclei, ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B. From the resulting $\Lambda\Lambda$ binding energies, a quite large $\Lambda\Lambda$ bond energy of around 4 to 5 MeV emerged, contrary to expectations from SU(3) [5]. A series of theoretical works, based either on phenomenological $S=-2$ baryon-baryon interactions or realistic ones, have studied the properties of double-Λ hypernuclei for more than 30 years [6–23].

The recent finding at KEK of a new ${}^{6}_{\Lambda\Lambda}$He candidate having a $\Lambda\Lambda$ bond energy of around 1 MeV [24] has injected a renewed interest on this field. Unless new experiments for the other $\Lambda\Lambda$ hypernuclei also give lower binding energies in the future, it is now an open question to reconcile theoretically the weak attraction found in ${}^{6}_{\Lambda\Lambda}$He with the stronger attraction in the other two heavier systems. Although some progress has been made in Ref. [25], where both short- and long-range correlations were simultaneously treated, further investigations are needed to completely settle this question. Filikhin and Gal [26–28] report Faddeev-Yakubovsky calculations, complementary to those carried out in the earlier work of Ref. [16] but using the new Nijmegen interactions, not finding a simultaneous description of the ${}^{10}_{\Lambda\Lambda}$Be and the new ${}^{6}_{\Lambda\Lambda}$He binding energies. However, the nucleon Pauli blocking effect affecting, through the coupling to ΞN states, the $\Lambda\Lambda$ interaction when the particles are embedded in the nuclear medium has not been considered in most of the earlier nor in these recent works [25–27]. As discussed in detail in Ref. [17], Pauli blocking reduces substantially the additional attraction to the $\Lambda\Lambda$ binding energy induced by the $\Lambda\Lambda \rightarrow \Xi N$ conversion. Recently, an attempt to incorporate the Pauli suppression effect has been made in Ref. [29], where a second order Pauli correcting term is introduced in the intermediate states following the $\Lambda\Lambda \rightarrow \Xi N$ transition. The interaction used in that work is a two-channel $(\Lambda\Lambda, \Xi N)$ Gaussian model, which implicitly includes the $\Sigma\Sigma$ coupling not only in the effective $\Lambda\Lambda$ interaction but also in the $\Lambda\Lambda \rightarrow \Xi N$ transition. In fact, the important role of the coupling to $\Sigma\Sigma$ states has been recently pointed out in Ref. [30] and explicitly worked out for the Nijmegen interactions in the variational calculation of Ref. [31].

The purpose of the present work is to present a careful analysis of the role of coupled channels on the $S=-2$ baryon-baryon interaction in the medium, treating Pauli blocking effects to all orders in all possible transition channels. For practical purposes, most of the recent works [26–29] have used simple parameterizations of the new Nijmegen potentials in terms of the sum of a few Gaussians. In contrast, we start from the original Nijmegen model NSC97e[5], as done also in Ref. [31]. In our approach, we solve the coupled-channel equation for the G-matrix in infinite nuclear matter, and derive from it the $\Lambda\Lambda$ bond energy in finite nuclei. With respect to existing calculations our treatment of the finite system is very simple. This has the practical advantage of permitting us to explore in depth the different effects determining the $\Lambda\Lambda$ bond energy, such as coupled channels or Pauli blocking to all orders.

II. FORMALISM

The $\Lambda\Lambda$ bond energy $\Delta B_{\Lambda\Lambda}$ in $\Lambda\Lambda$ hypernuclei is determined experimentally from the measurement of the binding energies of double- and single-Λ hypernuclei as

$$\Delta B_{\Lambda\Lambda}\left({}^{A}_{\Lambda\Lambda}Z\right) = B_{\Lambda\Lambda}\left({}^{A}_{\Lambda\Lambda}Z\right) - 2B_{\Lambda}\left({}^{A-1}_{\Lambda}Z\right). \quad (1)$$

A reasonable estimation of this quantity when rearrangement effects are small can be obtained from the value of the $\Lambda\Lambda$ *G*-matrix element in a finite hypernucleus

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$$\Delta B_{\Lambda\Lambda}\left({}^{A}_{\Lambda\Lambda}Z\right) \approx -\langle (0s_{1/2})_\Lambda (0s_{1/2})_\Lambda, J=0|G|(0s_{1/2})_\Lambda (0s_{1/2})_\Lambda, J=0\rangle, \quad (2)$$

where the two Λ particles are assumed to be in the lowest single particle state of an appropriate Λ-nucleus mean field potential. We will compute the above matrix element from the infinite nuclear matter one in the following way. First, we construct the $\Lambda\Lambda$ G-matrix in infinite matter by solving the well known Bethe-Goldstone equation which, in partial wave decomposition and using the quantum numbers of the relative and center-of-mass motion, reads

$$\langle Kq'L'S'J(\Lambda\Lambda)|G|KqLSJ(\Lambda\Lambda)\rangle = \langle Kq'L'S'J(\Lambda\Lambda)|V|KqLSJ(\Lambda\Lambda)\rangle + \sum_{B_1B_2L''S''}\int dq''q''^2 \langle Kq'L'S'J(\Lambda\Lambda)|V|Kq''L''S''J(B_1B_2)\rangle \times \frac{Q_{B_1B_2}(K,q'')}{\Omega - \frac{K^2}{2(M_{B_1}+M_{B_2})} - \frac{q''^2(M_{B_1}+M_{B_2})}{2M_{B_1}M_{B_2}} - M_{B_1} - M_{B_2} + i\eta} \times \langle Kq''L''S''J(B_1B_2)|G|KqLSJ(\Lambda\Lambda)\rangle, \quad (3)$$

where the labels B_1B_2 run over $\Lambda\Lambda$, ΞN, and $\Sigma\Sigma$ intermediate states. The starting energy Ω is taken equal to $2M_\Lambda - 2B_\Lambda({}^{A-1}_{\Lambda}Z) - \Delta B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = 2M_\Lambda - B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)$, where the experimental value of $B_{\Lambda\Lambda}$ is taken for each hypernucleus, namely 7.25 MeV for ${}^{6}_{\Lambda\Lambda}$He [24], 17.7 MeV for ${}^{10}_{\Lambda\Lambda}$Be [1] and 27.5 for ${}^{13}_{\Lambda\Lambda}$B [3]. In this way, we are considering the interaction of each Λ particle not only with the nucleons in the nucleus but also with the other Λ particle. The nuclear matter density to be used in the Pauli operator Q is determined, for each hypernucleus, as the average nuclear density felt by the Λ particle in that hypernucleus. This is obtained by weighing the nuclear density at each point with the probability of finding the Λ particle

$$\rho = \int \rho(r)|\Psi_\Lambda(r)|^2 d^3r, \quad (4)$$

where $\rho(r)$ is the nuclear density profile which is conveniently parametrized as

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{a}\right)}, \quad (5)$$

being

$$\rho_0 = \frac{3}{4\pi}\frac{A_N}{R^3}\left(1+\left(\frac{\pi a}{R}\right)^2\right), \quad (6)$$

with $a=0.52$ fm, $R=1.12A_N^{1/3}-0.86A_N^{-1/3}$ fm and A_N the number of nucleons in the hypernucleus. The Λ wave function is obtained by solving the Schrödinger equation using a Woods-Saxon Λ-nucleus potential with parameters $(V_\Lambda, a_\Lambda, R_\Lambda)$ adjusted to reproduce the experimental binding energy of the Λ in the single-Λ hypernucleus. For practical computational purposes, from the resulting Λ root mean square (rms) radius we derive the oscillator parameter, b_Λ, of an equivalent harmonic oscillator wave function which will then be used in obtaining the finite hypernucleus two-body G-matrix elements of Eq. (2) from the nuclear matter ones displayed in Eq. (3).

In the next step, we express the two-body ket state $|(0s_{1/2})_\Lambda(0s_{1/2})_\Lambda, J=0\rangle$, built from the $0s_{1/2}$ states of the equivalent harmonic oscillator potential, in terms of momentum and angular variables $|(k_1,l_1,j_1)_\Lambda(k_2,l_2,j_2)_\Lambda, J=0\rangle$ in the laboratory frame using

$$|(0s_{1/2})_\Lambda(0s_{1/2})_\Lambda, J=0\rangle = \int\int dk_1 dk_2 k_1^2 k_2^2 R_{00}(b_\Lambda k_1) R_{00}(b_\Lambda k_2) \times \left|\left(k_1,0,\tfrac{1}{2}\right)_\Lambda \left(k_2,0,\tfrac{1}{2}\right)_\Lambda, J=0\right\rangle, \quad (7)$$

where $R_{nl}(x)$ is the corresponding harmonic oscillator function.

Finally, we express the two-body state with laboratory coordinates in terms of the states with variables in the relative and center-of-mass system, $|KqLSJ(\Lambda\Lambda)\rangle$, used in the solution of the Bethe–Goldstone equation

$$\left|\left(k_1,0,\tfrac{1}{2}\right)_\Lambda \left(k_2,0,\tfrac{1}{2}\right)_\Lambda, J=0\right\rangle = \int dK K^2 \int dq q^2 \left\langle Kq000(\Lambda\Lambda)\middle|k_1 0\tfrac{1}{2} k_2 0\tfrac{1}{2}, J=0\right\rangle \times |Kq000(\Lambda\Lambda)\rangle, \quad (8)$$

where $\left\langle Kq000(\Lambda\Lambda)\middle|k_1 0\tfrac{1}{2} k_2 0\tfrac{1}{2}, J=0\right\rangle$ are the appropriate transformation coefficients [32,33] from the relative and center-of-mass frame to the laboratory system. We note that the only contribution comes from the partial wave ${}^1S_0$. Transforming the bra state $\langle (0s_{1/2})_\Lambda (0s_{1/2})_\Lambda|$ in a similar way, one can finally evaluate the $\Lambda\Lambda$ bond energy of Eq. (2) in terms of the infinite nuclear matter $\Lambda\Lambda$ G-matrix elements.

In Table I we summarize all the parameters that allow us to determine the relevant Λ and nuclear properties needed in the evaluation of the $\Lambda\Lambda$ bond energy for the three hypernuclei studied in this work, ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B.

TABLE I. Parameters of the Woods-Saxon Λ-nucleus potential (V_Λ, a_Λ, R_Λ), equivalent Λ oscillator parameter (b_Λ), effective nuclear density (ρ) and G-matrix starting energy (Ω) for each ΛΛ hypernucleus.

	${}^{6}_{\Lambda\Lambda}$He	${}^{10}_{\Lambda\Lambda}$Be	${}^{13}_{\Lambda\Lambda}$B
V_Λ[MeV]	28	28	28
a_Λ[fm]	0.59	0.59	0.59
R_Λ[fm]	1.60	2.06	2.67
b_Λ[fm]	2.23	1.89	1.82
ρ[fm^{-3}]	0.277	0.181	0.176
Ω[MeV]	2224.12	2213.67	2204.17

III. RESULTS

The diagonal ${}^1S_0$ ΛΛ G-matrix element for zero center-of-mass momentum and zero relative momentum is shown in Fig. 1 as a function of the nuclear matter density for several starting energy values. As density increases, the G-matrix element loses attraction as a result of Pauli blocking which reduces the available phase space for the intermediate ΞN states. On the other hand, the G-matrix element gains attraction when the starting energy increases, since the coupling to intermediate states is then more efficient. We will return to this behavior when the results of finite hypernuclei are discussed.

Table II displays our results for $\Delta B_{\Lambda\Lambda}$ in ${}^{6}_{\Lambda\Lambda}$He, for various coupled-channel cases. The value of $\Delta B_{\Lambda\Lambda}$ obtained from a calculation that neglects Pauli blocking effects, i.e., directly from the T-matrix, is also displayed with brackets. As expected, incorporating the coupling between the ΛΛ and ΞN, produces a drastic effect over the ΛΛ uncoupled situation, increasing $\Delta B_{\Lambda\Lambda}$ from 0.16 to 0.78 MeV, a value that lies very close to the new experimental datum [24]. We note that, contrary to what it seems to be implied in Ref. [30], the coupling between the ΛΛ and ΞN channels is important even when the interaction is weak, as it is the case of the NSC97e potential used here which produces a scattering length of about −0.5 fm. Actually, in Refs. [17,30] the potential is adjusted for each coupled-channel case to reproduce a common value of the scattering length. Therefore, part of the coupling effect is embedded in the readjusted parameters. We now turn to analyzing the effect of the ΣΣ channel, located more than 150 MeV higher in energy from the ΛΛ and ΞN channels, and which has usually been neglected or taken in an effective way within single-channel (ΛΛ) or two-channel (ΛΛ, ΞN) interaction models. The results shown in Table II reveal, surprisingly, that the role of the ΣΣ channel is very important and reduces substantially the two-channel (ΛΛ, ΞN) value of $\Delta B_{\Lambda\Lambda}$ down to 0.28 MeV, which is closer to the uncoupled single-channel result. Note that the repulsion found for the full coupled-channel G-matrix element around the ΛΛ threshold does not necessarily mean that the ΣΣ channel produces a more repulsive interaction. In fact, the (ΛΛ, ΞN, ΣΣ) Nijmegen model becomes so attractive that it even supports a spurious deeply bound YY state around 1500 MeV below the ΛΛ threshold [31]. However, the size of the G-matrix will not be affected by the presence of this bound state since it lies very far away from the region of energies required by our model. The net effect around the ΛΛ threshold is that the full coupled-channel calculation has a smaller bond energy than the case in which only the ΛΛ and the ΞN channels are retained.

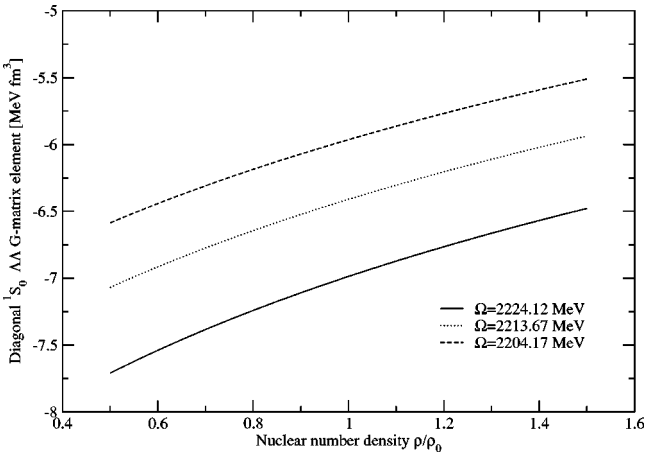

FIG. 1. ${}^1S_0$ diagonal ΛΛ G-matrix element as a function of the nuclear number density in units of ρ_0, with ρ_0=0.17 fm^{-3}.

TABLE II. ΛΛ scattering length and ΛΛ bond energy in ${}^{6}_{\Lambda\Lambda}$He, for various channel couplings. Results within brackets ignore Pauli blocking effects.

	$a_{\Lambda\Lambda}$ [fm]	$\Delta B_{\Lambda\Lambda}$ [MeV]
ΛΛ	−0.25	0.16 (0.16)
ΛΛ, ΞN	−0.84	0.78 (1.02)
ΛΛ, ΞN, ΣΣ	−0.49	0.28 (0.54)

Comparing the results of $\Delta B_{\Lambda\Lambda}$ with those between brackets, which have been obtained from a T-matrix calculation, one observes that Pauli blocking effects (nonexisting in the single channel ΛΛ case) are quite important, especially when the three-channels (ΛΛ, ΞN and ΣΣ) are considered, reducing by half the value of $\Delta B_{\Lambda\Lambda}$. We note that our Pauli unblocked value of 0.54 MeV, obtained for the complete coupled channel calculation using the original Nijmegen potential NSC97e, is reasonably close to results obtained directly in finite hypernuclei but using effective Gaussian parameterizations fitted to the scattering length of the Nijmegen NSC97e interaction, namely $\Delta B_{\Lambda\Lambda}$=0.58 MeV [26] and $\Delta B_{\Lambda\Lambda}$=0.64 MeV [29]. We note that the recent variational calculation using the Nijmegen interactions quotes a slightly larger value of 0.81 MeV [31].

The results of Table II show that a proper treatment of Pauli blocking, neglected in most of the calculations using more sophisticated ways of treating the finite hypernucleus [16,25,26], is needed to draw conclusions on the particular value of $\Delta B_{\Lambda\Lambda}$ predicted by a given interaction. A first attempt to incorporate the Pauli suppression effect within the context of finite hypernuclei has been done recently in Ref. [29], where a Pauli blocking term, correcting the phase space of intermediate ΞN states accessed via $\Lambda\Lambda \rightarrow \Xi N$ conversion up to second order in the effective interaction, is added. The

TABLE III. $\Lambda\Lambda$ bond energy in ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B, for various channel couplings. Units are in MeV.

	${}^{6}_{\Lambda\Lambda}$He	${}^{10}_{\Lambda\Lambda}$Be	${}^{13}_{\Lambda\Lambda}$B
$\Lambda\Lambda$	0.16	0.0046	0.11
$\Lambda\Lambda, \Xi N$	0.78	0.97	0.96
$\Lambda\Lambda, \Xi N, \Sigma\Sigma$	0.28	0.22	0.11
EXP:	$1.01^{+0.38}_{-0.31}$ [24]	4.2 ± 0.4 [1]	4.8 ± 0.7 [3]

$\Lambda\Lambda$ bond energy is then reduced from 0.64 to 0.21 MeV, hence finding a Pauli suppression of 0.43 MeV, which is about twice the size of the reduction we find in the present work, namely $(0.54-0.28)$ MeV$=26$ MeV. The reason for the difference has to be found in higher order terms of the Pauli correction. Indeed, if, in the spirit of the procedure followed in Ref. [29], we truncate the series that defines the T-matrix in terms of the G-matrix, $T=G+G(1/E-Q/E)T$, up to second order in G, then the contribution of the Pauli blocking correcting term, $G(1/E-Q/E)G$, amounts to 0.36 MeV. This is consistent with the value of 0.43 MeV quoted in Ref. [29] which was obtained with a slightly modified effective interaction to fit the new $\Delta B_{\Lambda\Lambda}$ value in ${}^{6}_{\Lambda\Lambda}$He. Moreover, we have checked that the series converges to our T-matrix result and, hence, to our complete Pauli correction of 0.26 MeV. The Pauli correction built directly in the finite nucleus in the full coupled-channel calculation of Ref. [31] is also small and of the order of 0.2 MeV.

The scattering length for each coupled-channel situation is also shown in Table II to illustrate, as in other works [26,30], its correlation with the $\Lambda\Lambda$ bond energy, which increases as the magnitude of the scattering length increases. This correlation is to be expected since $a_{\Lambda\Lambda}$ is proportional to the T-matrix and $\Delta B_{\Lambda\Lambda}$ is proportional to the corresponding medium modified G-matrix. We would also like to point out that the scattering length changes substantially for each of the coupled-channel cases. This is apparently different from the results shown in Refs. [17,30] but, as mentioned before, in these later works the interaction is readjusted in each coupled-channel calculation to reproduce a common value of the scattering length.

Finally, we collect in Table III the results of $\Delta B_{\Lambda\Lambda}$ for the three observed $\Lambda\Lambda$ hypernuclei. The role of coupled channels is qualitatively similar in the three hypernuclei: Coupling the ΞN channel to the $\Lambda\Lambda$ channel increases $\Delta B_{\Lambda\Lambda}$ substantially, while the additional incorporation of the $\Sigma\Sigma$ channel reduces the binding also substantially, bringing the value of the $\Lambda\Lambda$ bond energy closer to the uncoupled result. We also observe that the heavier the nucleus the smaller the binding, contrary to what one would be expecting from the present experimental results. The trend found here is a reflection of the behavior of the ${}^{1}S_{0}$ G-matrix element shown in Fig. 1. Inspecting the nuclear structure parameters for each hypernucleus shown in Table I, we see that the nuclear density for ${}^{6}_{\Lambda\Lambda}$He is the largest, slightly above $1.5\rho_0$, hence this hypernucleus has the strongest Pauli repulsive effect. However, the starting energy Ω is also the largest, which produces a gain in attraction. For the range of densities and starting energies explored by the three $\Lambda\Lambda$ hypernuclear systems studied here, the dependence of the G-matrix on the starting energy is twice more important than that on the density. The net effect is that the largest $\Delta B_{\Lambda\Lambda}$ value is obtained for the lightest system.

IV. CONCLUSIONS

In this work we have obtained the bond energy $\Delta B_{\Lambda\Lambda}$ in several $\Lambda\Lambda$ hypernuclei, following a microscopic approach based on a G-matrix calculation in nuclear matter using, as $S=-2$ interaction, the recent parameterization NSC97e of the Nijmegen group. We have identified the $\Lambda\Lambda$ bond energy with the ${}^{1}S_{0}$ $\Lambda\Lambda$ G-matrix element calculated for values of the nuclear density and starting energy appropriate for each hypernucleus.

Our simplified finite-nucleus treatment has allowed us to explore in depth the effect of the various coupled channels and the importance of Pauli blocking on the intermediate ΞN states, paying a special attention to the role of the $\Sigma\Sigma$ channel usually neglected in the literature. Consistently with other works, we find that the coupling between the ΞN and $\Lambda\Lambda$ channels has a drastic effect, increasing by about 0.6 MeV the calculated $\Delta B_{\Lambda\Lambda}$ in ${}^{6}_{\Lambda\Lambda}$He with respect to a single-channel $\Lambda\Lambda$ calculation. Surprisingly, the additional incorporation of the $\Sigma\Sigma$ channel yields a non-negligible reduction in the binding of 0.4 MeV. It would be interesting to explore the role of the coupling to $\Sigma\Sigma$ states in other three-channel $S=-2$ interactions, such as the Nijmegen hard-core potential F [34]. Unfortunately, our momentum-space method can only handle soft-core interaction models.

We have also explored, within the complete three-channel approach, the effect of Pauli blocking, which is often neglected or considered in a truncated way in previous works. With respect to a T-matrix calculation, our calculated value of $\Delta B_{\Lambda\Lambda}$ in ${}^{6}_{\Lambda\Lambda}$He gets reduced by 0.26 MeV, about half of what was found on the basis of a second order Pauli corrected calculation [29].

Due to our simplified treatment of nuclear structure, we do not expect a quantitative agreement with experimental data for the three hypernuclei studied. However, from the bulk of studies of double-Λ hypernuclei available in the literature, it seems unreasonable to think that a proper finite nucleus calculation which incorporates consistently core-polarization effects, might change the calculated bond energies substantially enough to obtain a simultaneous agreement with the data. In this respect, our results confirm, in accordance with recent cluster calculations [26], the incompatibility between the experimental binding energies of the light double-Λ hypernuclear species. We note, however, that the disagreement would be reduced if the π^- weak decay of the ${}^{10}_{\Lambda\Lambda}$Be ground state was assumed to occur to the first excited state of ${}^{9}_{\Lambda}$Be, as pointed out by Filikhin and Gal [26], hence reducing the bond energy in ${}^{10}_{\Lambda\Lambda}$Be to about 1 MeV. A clarification of the experimental situation, through new experiments and analyses, is certainly needed in order to test the theoretical models and make progress in the field of doubly-strange systems.

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