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Rethinking the properties of the quark-gluon plasma at $T_c < T < 4T_c$

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We argue that although at asymptotically high temperatures the quark-gluon plasma (QGP) in bulk behaves as a gas of weakly interacting quasiparticles (modulo long-range magnetism), it is different at temperatures up to few times the critical temperature T_c. In particular, we emphasize the role of several *zero binding lines* on the phase diagrams, below which $\bar{c}c$, light quarks, gg as well as exotic gq, qq bound states exist. Near these lines quasiparticle rescattering is enhanced dramatically, possibly explaining why heavy-ion collisions at the relativistic heavy-ion collider exhibit robust collective phenomena. Although in QCD the coupling constant in the QGP phase reaches only values $g \sim 1$, using Maldacena duality one may study these phenomena in the strong coupling limit $g^2 N_c \ll 1$ in conformal gauge theories at finite temperature [e.g., $\mathcal{N}=4$ supersymmetric Yang-Mills (SUSY YM)]. Trapped atoms is another system which allows us to study strongly interacting matter, in the limit of infinite scattering length.

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Soon after the discovery of QCD, it was found that at high temperature T the color charge is *screened* [1] (rather than antiscreened in the vacuum), and the corresponding phase of matter was named the quark-gluon plasma (QGP). Also, it has been shown, both analytically and on the lattice, that at high temperature, bulk quarks and gluons exist in it as relatively free propagating quasiparticles, modulo color-magnetic effects that are known to be nonperturbative [2]. Although explicit perturbative series for thermodynamical quantities were found to be badly divergent, it was still hoped that some kind of resummation will make the weak-coupling quasiparticle picture work, as the screening would keep the effective coupling weak anywhere at $T > T_c$. The earliest suggested QGP signal was a disappearance of familiar hadronic peaks—ρ, ω, ϕ mesons—in the dilepton spectra [3]. Moreover, even small-size deeply bound $\bar{c}c$ states, $\eta_c, J/\psi$, were expected to melt at $T \approx T_c$ [4,5].

However, in wide region $T < 4T_c$, the interaction is strong enough, causing bound states of quasiparticles. More specifically, those are limited by the *zero binding lines* on the phase diagram [see Fig. 1(a)].

Discussions of particular hadronic states in QGP have been done many times before, see, e.g., application of the sum rules [6] or instantons [7]. There are also important lattice results. It has been shown [9] that (time-direction) correlators at $T > T_c$ display significant deviations from free behavior, in quantitative agreement with predictions in [7]. Their analysis (by the minimal entropy method) has suggested existence of light quark resonances above T_c. Most definite are recent studies [8] which found (contrary to earlier expectations) that the lowest charmonium states remain bound up to $T < T_{\bar{c}c} \approx (1.6-2)T_c$.

The bound states of $\bar{q}q$ can only be colorless mesons (the octet channel is repulsive), but in QGP there can be *colored* bound states. Quite famous are quark Cooper pairs qq which drive the color superconductivity at sufficiently high density and low T, but pairs themselves should exist outside this region as well. Gluons can form a number of states with attraction, and there can also be gq hybrids. A generic reason why we think all of them exist is that at T close to T_c, all quasiparticles are very heavy.

Starting with quarks, let us briefly discuss their quantum numbers. Ignoring current quark masses and instanton effects, we note that in QGP the chiralities (L,R) are good quantum numbers. Furthermore, there are two different modes of quarks depending on whether helicity is the same as chirality or not (these states are called "particle" and "plasmino," respectively). All in all there are $4N_f^2$ states, connected by continuity to the pseudoscalar, scalar, vector, and axial vector nonets in the vacuum [see Fig. 1(b)].

Gluons in QGP have not two but three polarizations, with a longitudinal "plasmon" mode. Furthermore, for gg one can have not only color singlet (a continuation of the vacuum glueballs) but also octet binary composites [10]. The exotic states qq, qg, or octet gg have smaller Casimirs and need stronger couplings, so they are melted first.

Motivation. Before we go into (model-dependent) descriptions of some such states, let us point out that our motivation comes primarily from experimentally observed robust collective effects at the relativistic heavy-ion collides (RHIC), known as radial and elliptic flows, indicating a surprisingly low viscosity of QGP.

Transport properties of QGP were so far studied mostly perturbatively, in powers of weak coupling. This approach predicted large mean-free path, $T\, l_{\rm mfp} \sim 1/g^4 \ln(1/g) \gg 1$. Similar pQCD-inspired ideas have led to the pessimistic expectation that the RHIC project in Brookhaven National Laboratory would produce a firework of multiple minijets rather than QGP.

However, already the very first RHIC run, in the summer of 2000, has shown spectacular collective phenomena known as radial and elliptic flows. The spectra of about 99% of all kinds of secondaries (except the high-p_t tail) are accurately described by ideal hydrodynamics [11], based on the opposite—zero mean-free path—regime. Further studies of partonic cascades [12] and viscosity corrections [13] have confirmed that one can only explain these data by increasing pQCD cross sections by large factors, $\sim$50 or so. It implies

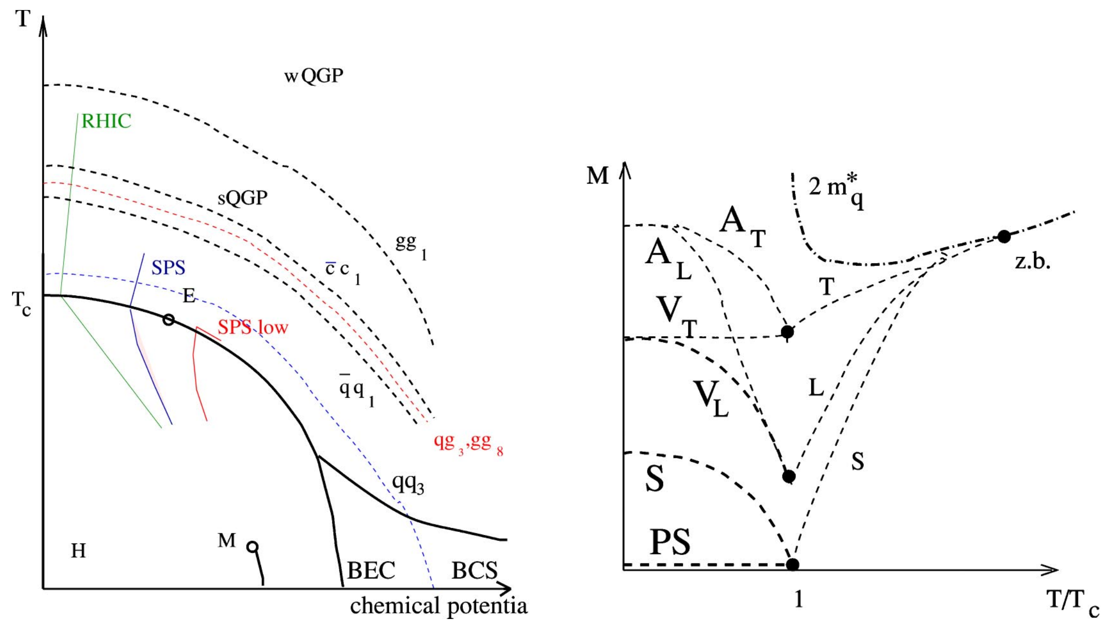

FIG. 1. (Color online) (a) Schematic position of several zero binding lines on the QCD phase diagram temperature T —baryonic chemical potential. (b) Motion of scalar (S), psuedoscalar (PS), vector (A) and axial (A) hadronic states with temperature. Note that V and A states at nonzero momentum have different transverse (T) and longitudinal (L) modes. Black dot marked z.b. is the zero binding point for all states with orbital momentum $l=0$.

that QGP at the RHIC is by far the *most perfect liquid known*, with a viscosity-to-entropy ratio $\eta/s\sim 0.1$ never seen before. The strong coupling limit of plasmalike phases of gauge theories has been recently studied in $\mathcal{N}=4$ supersymmetric gauge theory: and indeed small viscosity was found [14].

Our main idea is that a sequence of loosely bound states [indicated by the black dot marked "z.b." in Fig. 1(b)] is of crucial importance for quasiparticle rescattering, since there its cross section is strongly enhanced. Huge cross sections induced by low-lying resonances are well known throughout all parts of physics. The Breit-Wigner cross section (modulo the obvious spin factors depending on the channel) is

$$\sigma(k) \sim \frac{4\pi}{k^2}\frac{\Gamma_i^2/4}{(E-E_r)^2+\Gamma_t^2/4}. \tag{1}$$

For $E-E_r\approx 0$ the in- and total widths approximately cancel: the resulting "unitarity limited" scattering is determined by the quasiparticle wavelengths, which can be very large. We conjecture that this phenomenon significantly contributes to viscosity reduction.

The issue of $\overline{c}c$ binding can be addressed using the usual nonrelativistic Schrodinger equation, which for standard radial wave function $\chi(r)=\psi/r$ has the usual form, with the reduced mass $m=m_c/2$ and an effective potential including T-induced screening of the charge [1], which we will write in a traditional Debye form [15] $V=-[4\alpha_s(r)/3r]\exp(-M_D r)$. We have chosen the ($T$-dependent) inverse screening mass to be our unit of length, so that the equation to be solved reads

$$\frac{d^2\chi}{dx^2}+\left[\kappa^2+\frac{4m_c}{3M_D}\frac{\alpha_s(x)}{x}e^{-x}\right]\chi=0, \tag{2}$$

with $\kappa^2=m_c E/M_D^2$. Furthermore, at a zero binding point, $\kappa=0$. So, if the coupling constant α_s would not run and be just a constant, all pertinent parameters appear in a single combination. Solving the equation, one finds the zero binding condition to be

$$\frac{4m_c}{3M_D}\alpha_s = 1.68. \tag{3}$$

For example, using $4/3\alpha_s=0.471$ and $m_c=1.32$ GeV, as Karsch *et al.* [5] did long ago, one finds a restriction on the screening mass $M_D<M_D^{crit}=0.37$ GeV. Lattice measurements of the screening masses for the near-critical QGP [16,17] found that (not very close to T_c) $M_D/T\approx(2.25\pm.25)$ in a relevant range of T. If so, the condition (3) is satisfied marginally if at all, and these authors concluded that charmonium s-wave states $\eta_c, J/\psi$ cannot exist inside the QGP phase.

A loophole in this argument is the assumption that the gauge coupling is a constant, with the *same* value as in the in-vacuum charmonium potential. However, in-vacuum potential includes a large confining linear term, absent at $T>T_c$. The true form of $V_{eff}(r)$ remains unknown. One of us [18] suggested that at $T>T_c$ nothing prevents the QCD coupling from running at larger distances to larger values until it is stopped at the screening mass scale. Very close to the critical point, where the screening mass is very small, the coupling can reach $\alpha_s\sim 1$ and presumably be limited by some nonperturbative phenomena.

Lattice data on the potential itself exist, see, e.g., rather detailed work for SU(2) gauge theory [17]. In principle, performing a multiparameter fit, one can extract from it all parameters in question, the screening mass, the static quark mass renormalization $[2\Delta M=V(r\to\infty)]$, and the running coupling $\alpha_s(r)$: Unfortunately, the results of such a comprehensive fit are not yet available to us.

Because of that we use a simple model-dependent potential, in which the charge continues to run till it is limited by $\alpha_s < 1$. Such potential with the running coupling does indeed provide a more liberal condition for charmonium binding, which is $M_D < 0.62$ GeV. This translates into charmonium zero binding point at

$$T < T_{\bar{c}c} \approx 1.6\ T_c, \tag{4}$$

which agrees well with recent lattice measurements [8]: we thus conclude that our model-dependent potential has passed its first test.

The binding of light $\bar{q}q$ states. Chiral symmetry for massless quarks excludes the usual mass from being developed, and L,R-handed quarks propagate independently. Nevertheless, propagating quasiparticles, if the QGP have dispersion curves with the nonzero "chiral" or "thermal" mass, are defined as the energy of the mode at zero momentum $M_q = \omega(\vec{p}=0)$. Perturbatively it is $M_q = gT/\sqrt{6}$ to the lowest order [19], the same for both fermionic modes: (i) with the same chirality and helicity, the dispersion curve at small p is $\omega = M_q + p/3 + p^2/3M_q + \cdots$; (ii) with the opposite chirality and helicity the mode is often called a "plasmino," its dispersion curve has a shallow minimum at $p = 0.17gT$ with the energy $E_{\min} = 0.38\ gT$ slightly below M_q. For a general analysis of these modes, see [20].

Lattice data on the quasiparticle dispersion curves are rather sketchy, obtained in a Coulomb gauge [21]. They can be described by $\omega^2 = p^2 + M^2$, with the following values (at $T = 1.5T_c$)

$$\frac{m_q}{T} = 3.9 \pm 0.2, \quad \frac{m_g}{T} = 3.4 \pm 0.3. \tag{5}$$

Note that at such T (which is not very close to T_c) the quark and gluon masses are still quite large, while their ratio is very different from the weak coupling prediction $1/\sqrt{6}$. At $T \sim 3T_c$ and higher they are somewhat reduced toward the perturbative values, which, however, are not reached till presumably very high T.

The effective equation of motion suitable for discussion of the bound-state problem can be obtained by standard substitution of the covariant derivatives in the place of momentum and frequency, provided the dispersion law is known. So if the dispersion curve can be parameterized as $\omega = M_q + p^2/2M' + \ldots$, it has the form of the nonrelativistic Schrodinger equation, in general with two different constants M, M'. Both for weak coupling and lattice data, such approximations seem to be accurate within several percents.

Addressing the issue of binding, we first note that if all effective masses grow linearly with T, including the screening mass, the explicit T dependence drops out of Eq. (3) to the exception of the logarithmic dependence in T left out in $\alpha_s \sim 1/\ln(T/\Lambda_{QCD})$. This is why the region of "strongly coupled QGP" turns out to be relatively substantial. For a qualitative estimate, let us set the coupling to its maximum, $\alpha_s = 1$. The combination of constants is $(4 \times 3.9\ T)/(3 \times 2.25\ T) = 2.3$, larger than the critical value (3), so one should expect the occurrence of (strong) Coulomb bound states. Although the (plasmon) gluon modes are somewhat lighter than quarks in Eq. (5), their Coulomb interaction has a larger coefficient due to a different Casimir operator for the adjoint representation, 3 instead of 4/3. As a result, the effective combination in the potential is $3m_g\alpha_s/M_D$, which is about twice larger than for quarks, and thus the gluons are bound even stronger (modulo collisional broadening).

Solving the equations, one can make a quantitative analysis, using the same potential as above. We found that the highest temperature T, at which light quark states are Coulomb bound, is somehow lower than that of charmonium,

$$T_{\bar{q}q} \approx 1.45\ T_c \approx 250\ \text{MeV}, \tag{6}$$

while the s-wave gg gluonium states remain bound till higher temperatures $T_{gg} \approx 4\ T_c$, used in this paper as the upper limit on the QGP with bound states.

How reliable is our approach, based on screened effective potential? Obviously even the best in-matter potential does not include all many-body effects. However one may think that the large size of states at near-zero binding provides additional stability, averaging out local perturbations. This is known to be true for large-size Cooper pairs in superconductors, or "excitons" [22] in semiconductors and insulators. Depending on a number of parameters, including the density of excitons and temperature, the system exhibits various phases, ranging from an ideal gas of excitons to a liquid or plasma, or even a Bose-condensed gas. On its way from a gas to a liquid, clustering with three- and four-body states plays an important role. Although one cannot directly relate these two problems (quarks and gluons have N_c and $(N_c^2 - 1)$ colors, respectively, while particles and holes have simply charges $\pm e$), one may think that in the QGP at $T \sim T_c$ some of these phenomena may well be there.

More generally, strongly coupled many-body problems, with very large (infinite) binary scattering length, can be studied in at least two other settings: (i) a gas of neutrons, with $a = 18$ fm due to a virtual level; (ii) trapped atomic Li^6, Li^7 in which the scattering length can be tuned by Feshbach resonances. Its equation of state and transport properties are of great interest.

Discussion and Outlook. The main idea of this work is the existence of zero binding or termination lines for many binary bound states of quasiparticles: we expect those lines to play a crucial role in defining transport properties of QGP. With the additional model assumption (that at $T > T_c$ the gauge coupling is allowed to run to about 1), we estimated where this line is for charmonium, light quarks, and colorless gg states. Needless to say, those calculations can be made more accurate as soon as lattice results about quasiparticles and their interaction be improved.

Although we focused on zero binding, more generally one may ask how all these states are evolved from $T = T_c$ and how they are related to known hadrons at $T = 0$. Some mesons are rather deeply bound in this region: so both relativistic and nonperturbative effects become important, we study this region elsewhere [23].

In QCD there are limits on the coupling, reducing T from its large value toward T_c, we may reach $\alpha_s \sim 1$, but as soon as the lowest σ, π states hit zero there is a phase transition into a hadronic phase. This is, however, impossible for conformal gauge theories (CFTs), such as $\mathcal{N}=4$ supersymmetric gauge theory, and thus the gauge coupling is allowed to become supercritical or even large $\lambda \equiv g^2 N_c \gg 1$. Maldacena's AdS/CFT duality [24] has opened a way to study this strong coupling limit using classical gravity. At finite T it was recently actively discussed, for Debye screening [25], bulk thermodynamics [26] and kinetics [14]. Although the thermodynamical quantities are only modified by an overall factor of $3/4$ in comparison to the black-body limit, kinetics is changed dramatically. In our separate paper [27] we show that in this regime the matter is made of very deeply bound binary composites, in which the supercritical Coulomb can be balanced by centrifugal force. Specific towers of such bound states can be considered as a continuation of Fig. 1 to the left, toward stronger and stronger coupling.

Note added: After the paper was submitted, a number of spectacular experimental discoveries with trapped Li^6 atoms were made. If one uses the Feshbach resonance to increase the scattering length, it was indeed found possible to reach the hydrodynamical expansion regime, evidenced by the elliptic flow [28] very similar to that seen at RHIC. Furthermore, it was found [29] that an adiabatic crossing through the resonance converts nearly all atoms into very loosely bound (but remarkably stable) "Cooper pairs," which can also Bose condense [30] if the temperature is low enough. Since in heavy-ion collisions the system also crosses the zero binding lines adiabatically, various bound pairs of quarks and gluons should also be generated this way.

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