# New semiempirical formula for exotic cluster decay

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A new semiempirical formula, with only three parameters, is proposed for cluster decay half-lives. The parameters of the formula are obtained by making a least squares fit to the available experimental cluster-decay data. The calculated half-lives are compared with the results of the earlier proposed model-independent scaling law and the empirically fitted analytical super-asymmetric fission model (ASAFM), showing more closeness to the ASAFM results.

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In a radioactive decay series, the end product is reached not only via the emission of $\alpha$ and $\beta$ particles but also directly via the emission of heavy nuclei (clusters), such as carbon, oxygen, fluorine, neon, magnesium, and silicon. Such a process is known as cluster decay, first proposed theoretically in 1980 by Săndulescu, Poenaru, and Greiner [1], before its experimental realization in 1984 by Rose and Jones [2]. In this decay process a parent nucleus $(A,Z)$ breaks into two fragments, viz., the emitted cluster $(A_2,Z_2)$ (heavier than the $\alpha$ particle but lighter than the lightest fission fragment observed so far), and the associated daughter $(A_1,Z_1)$ with $A=A_1+A_2$, $Z=Z_1+Z_2$.

The cluster decay process has been studied extensively using different theoretical models with different realistic nuclear interaction potentials. In general, two kinds of models are used. In one kind, the $\alpha$ particle as well as the heavy cluster(s) is assumed to be preborn (with different probabilities) in a parent nucleus, before they could penetrate the barrier with the available $Q$ value (the Gamow-like barrier penetration). These models are called the preformed cluster models [3–7]. In the other kind of model called the unified fission model [8–14], only Gamow's idea of barrier penetration is used without considering the cluster(s) being or not being preformed in the parent nucleus. In this paper, we attempt to give a model-independent, semiempirical formula for studying the above-mentioned process of exotic cluster decay.

Geiger and Nuttall [15] were the first who proposed a semiempirical law connecting the $\alpha$-decay half-life and its $Q$ value. Now, a large number of $\alpha$ decays are observed and several attempts have been made to give a universal formula [16–18], including also the heavy cluster decays. These formulas vary among themselves mainly in the number of parameters. One such scaling law proposed recently by Horoi *et al.* [16] is

$$\log_{10}T_{1/2} = (a_1\mu^x + b_1)\left[\frac{(Z_1Z_2)^y}{\sqrt{Q}} - 7\right] + (a_2\mu^x + b_2). \quad (1)$$

Here, the half-life $T_{1/2}$ is expressed in seconds and Q in MeV. The $\mu=mA_1A_2/(A_1+A_2)$ is the reduced mass, with m as the nucleon mass, and the six parameters $a_1=9.1$, $b_1=-10.2$, $a_2=7.39$, $b_2=-23.2$, $x=0.416$, and $y=0.613$ are obtained by fitting 119 $\alpha$ decays and 11 cluster decays from various even-even parents. Another analytical formula used extensively is the analytical superasymmetric fission model (ASAFM) of Poenaru *et al.* [8] where the constants of the model are fitted to some 379 $\alpha$ emitters and the $^{14}$C decay of $^{223}$Ra. The new semiempirical formula proposed in this work is based on three parameters only, which are least squares fitted to the cluster-decay data alone.

The new formula for cluster decay half-lives is based on the following three simple experimental facts:

(i) It is known from experiments that the cluster decay half-lives increase with the size of the clusters. Hence, the empirical formula should contain terms showing a direct dependence on the mass and charge numbers of the cluster.

(ii) Since the same cluster is emitted by different parents, the formula should contain dependence on the mass and charge asymmetries

$$\eta = \frac{A_1 - A_2}{A}; \quad \eta_z = \frac{Z_1 - Z_2}{Z}, \quad (2)$$

respectively.

(iii) Since the $\alpha$ decay and cluster decay are physically similar processes, the $Q$ dependence is taken to be the same as in Geiger-Nuttall law for $\alpha$ decay, i.e., $\log_{10}T_{1/2} \propto Q^{-1/2}$.

Combining the above three results, we get

$$\log_{10}T_{1/2}^{AZ} = \frac{aA_2\eta + bZ_2\eta_z}{\sqrt{Q}} + c. \quad (3)$$

The constants $a=10.603$, $b=78.027$, and $c=-80.669$ are obtained by making a least squares fit of the available experimental half-lives, in seconds, for exotic cluster decays, with an rms deviation $d_{\rm rms}$(=0.89 s) defined as

 

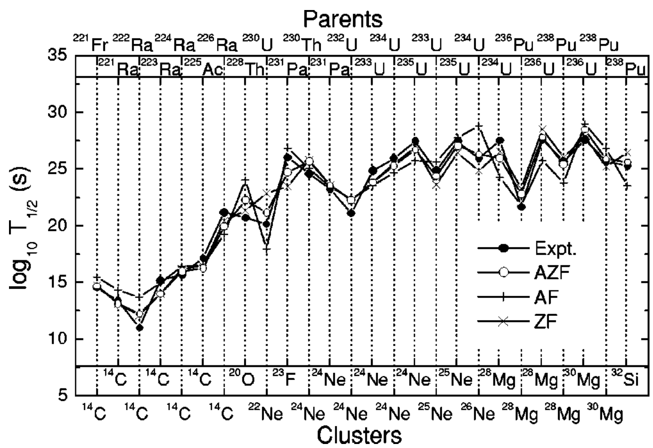


FIG. 1. $\log_{10}T_{1/2}$ (s) for different clusters emitted from various radioactive parents, calculated by using the AZ formula (AZF) and compared with experimental data. Also, the results of calculations for AF ($b=0$) and ZF ($a=0$) truncations of AZF are shown for comparisons.

$$d_{\mathrm{rms}} = \sum_{i=1}^{n} \left[ \frac{y_i - f(x_i, a_j)}{\sigma} \right]^2.$$

The function $f(x_i, a_j)$ is the right-hand side of Eq. (3), $n$ the number of measurements, and $y_i$ the experimentally observed $\log_{10}T_{1/2}$ values [19]. The variance

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (y_i - \overline{y})^2,$$

giving the standard deviation $\sigma$, with

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

as the arithmetic average of the experimentally measured quantities. Equation (3) is referred to here as the AZ formula (AZF). Note that the dependence on both $\eta$ and $\eta_Z$ must be included in the AZ formula since these are separately measurable quantities, and the correlation between them is known to be weak [20–22]. This is also evident when we consider the $\eta$ and $\eta_Z$ dependence separately. The two special cases of the AZ formula, i.e., $b=0$ or $a=0$ in Eq. (3), are expected to give reasonably good results, though poorer than the results of AZF. These truncated expressions are referred to as the A formula (AZ) and the Z formula (ZF), whose constants are also obtained directly by the least squares fit to data. These constants are: $a=30.568$, $c=-51.348$ for AF and $b=112.197$, $c=-89.025$ for ZF, with rms deviations $d_{\mathrm{rms}}=1.652$ s and 1.112 s, respectively. Apparently, the rms deviations for truncated expressions are larger, and hence the fits are poorer, compared to AZ formula.

Figure 1 and Table I give $\log_{10}T_{1/2}$ values calculated by using the AZ formula for different clusters emitted from various radioactive parents and these results are compared with the experimental data. In Fig. 1, we have also plotted the results of our calculations for AF ($b=0$) and ZF ($a=0$) versions of the AZ formula. It is evident that the AZF fit to the data is better than the AF and ZF fits, as expected from our discussion above.

TABLE I. The $\log_{10}T_{1/2}$ (s) of decay half-lives of different clusters emitted from various radioactive nuclei, calculated by using the semiempirical AZ formula and compared with the scaling law [16] and the ASAFM [8] calculations and the experimental data [19]. The experimental $Q$ values (in MeV) are also given.

| Parent | Cluster | $Q^{\mathrm{Expt.}}$ | $\log_{10}T_{1/2}$ (s) | | | |
|---|---|---|---|---|---|---|
| | | (MeV) | AZF | Ref. [16] | Ref. [8] | Expt. [19] |
| $^{221}$Fr | $^{14}$C | 31.28 | 14.54 | 13.56 | 15.00 | 14.52 |
| $^{221}$Ra | $^{14}$C | 32.39 | 12.96 | 12.28 | 13.80 | 13.39 |
| $^{222}$Ra | $^{14}$C | 33.05 | 11.98 | 11.00 | 12.60 | 11.01 |
| $^{223}$Ra | $^{14}$C | 31.85 | 13.81 | 13.38 | 14.80 | 15.20 |
| $^{224}$Ra | $^{14}$C | 30.54 | 15.94 | 16.13 | 17.40 | 15.68 |
| $^{225}$Ac | $^{14}$C | 30.48 | 16.20 | 17.26 | 18.50 | 17.16 |
| $^{226}$Ra | $^{14}$C | 28.21 | 20.08 | 21.50 | 22.40 | 21.19 |
| $^{228}$Th | $^{20}$O | 44.72 | 21.90 | 21.20 | 22.40 | 20.72 |
| $^{230}$U | $^{22}$Ne | 61.59 | 21.78 | 19.28 | 20.50 | 20.14 |
| $^{231}$Pa | $^{23}$F | 51.84 | 24.30 | 23.85 | 24.80 | 26.02 |
| $^{230}$Th | $^{24}$Ne | 57.78 | 25.77 | 23.88 | 24.90 | 24.61 |
| $^{231}$Pa | $^{24}$Ne | 60.42 | 23.62 | 21.30 | 22.00 | 23.23 |
| $^{232}$U | $^{24}$Ne | 62.31 | 22.24 | 19.94 | 20.40 | 21.08 |
| $^{233}$U | $^{24}$Ne | 60.5 | 23.87 | 22.53 | 23.10 | 24.83 |
| $^{234}$U | $^{24}$Ne | 58.84 | 25.42 | 25.01 | 25.70 | 25.92 |
| $^{235}$U | $^{24}$Ne | 57.36 | 26.87 | 27.31 | 28.10 | 27.42 |
| $^{233}$U | $^{25}$Ne | 60.75 | 24.15 | 22.98 | 23.30 | 24.83 |
| $^{235}$U | $^{25}$Ne | 57.83 | 26.94 | 27.49 | 28.10 | 27.42 |
| $^{234}$U | $^{26}$Ne | 59.47 | 25.85 | 25.75 | 26.20 | 25.92 |
| $^{234}$U | $^{28}$Mg | 74.13 | 26.24 | 24.74 | 24.60 | 27.54 |
| $^{236}$Pu | $^{28}$Mg | 79.67 | 23.01 | 20.83 | 19.80 | 21.67 |
| $^{236}$U | $^{28}$Mg | 71.69 | 28.18 | 27.98 | - | 27.58 |
| $^{238}$Pu | $^{28}$Mg | 75.93 | 25.70 | 25.39 | 24.80 | 25.70 |
| $^{236}$U | $^{30}$Mg | 72.51 | 28.36 | 28.36 | - | 27.58 |
| $^{238}$Pu | $^{30}$Mg | 77.03 | 25.71 | 25.41 | 24.40 | 25.70 |
| $^{238}$Pu | $^{32}$Si | 91.21 | 25.99 | 25.68 | 23.70 | 25.27 |

In Table I, we have also added the results of two other empirical model calculations [8,16]. Furthermore, the comparison between the experiments and the AZ formula, along with the results of scaling law [16] and ASAFM [8], are also displayed in Fig. 2 for the illustrative cases of $^{14}$C and $^{24}$Ne cluster decays. As expected, ASAFM fits the $^{14}$C cluster data nearly exactly since these data are used in fitting the constants of this model. Apparently, however, our semiempirical AZ formula is overall much closer to experiment, as compared to the other two formulas, the scaling law [16] and ASAFM [8].

Finally, in order to see the predictive power of the proposed AZ formula, we calculate the decay half-lives of some clusters from parents where the ASAFM model calculations predict their decay rates within the present limits of the experimental methods. One such region is the emission of $^{8}$Be, $^{12}$C, and $^{16}$O clusters from Hg parents [8]. Figure 3 presents such a calculation, where the results of our semiempirical

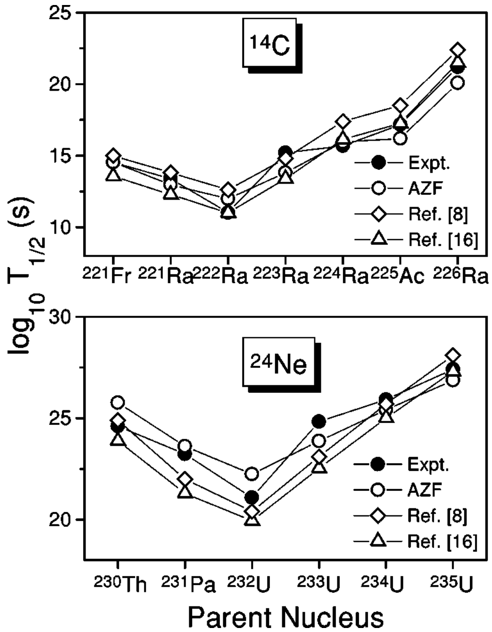


FIG. 2. The experimental data on $\log_{10}T_{1/2}$ (s) for the emission of $^{14}$C and $^{24}$Ne clusters from different radioactive parents, compared with the results of calculations using AZ formula (AZF) proposed here, the scaling law [16] and the ASAFM [8].

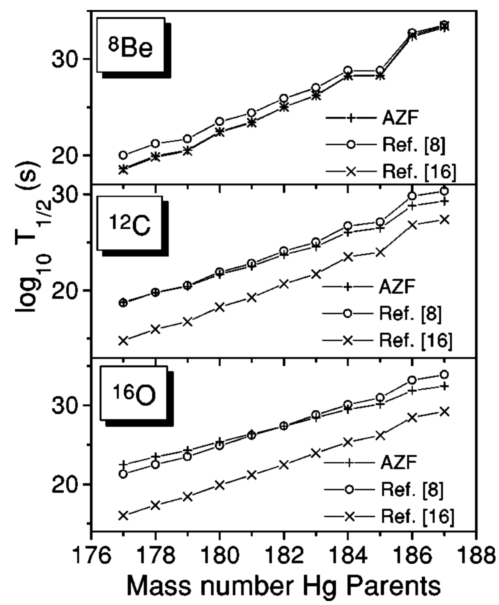


FIG. 3. The calculated $\log_{10}T_{1/2}$ (s) values for the emission of $^{8}$Be, $^{12}$C, and $^{16}$O clusters from different Hg parents using AZ-formula (AZF), compared with the calculations of ASAFM [8] and the scaling law [16].

AZF are compared with the results of ASAFM [8] and the scaling law [16]. It is evident that, in general, the predictions of our AZF are closer to the ASAFM results, the predictions of scaling law [16] lying mostly lower. The resulting good agreement suggests that the AZ formula could be used to make predictions of cluster decay half-lives for the guidance of new experiments, similarly to that of Poenaru *et al.* [8].

In this paper, we have proposed a model-independent three parameter formula for calculating the half-lives of cluster decays of nuclei. The evolution of the formula is based on three simple experimental observations about the characteristics of exotic cluster decays. The inputs of the formula are simply the mass and charge numbers of the parent and cluster, along with the $Q$ value of decay. The predictions of the proposed formula are comparable to the model-independent scaling law [16] as well as the empirical ASAFM [8] formulation.

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