

## Generalized parton distributions of ${}^3\text{He}$

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(Received 7 April 2004; published 20 July 2004)

A realistic microscopic calculation of the unpolarized quark generalized parton distribution (GPD)  $H_q^3$  of the  ${}^3\text{He}$  nucleus is presented. In impulse approximation,  $H_q^3$  is obtained as a convolution between the GPD of the internal nucleon and the non-diagonal spectral function, describing properly Fermi motion and binding effects. The proposed scheme is valid at low values of  $\Delta^2$ , the momentum transfer to the target, the most relevant kinematical region for the coherent channel of hard exclusive processes. The obtained formula has the correct forward limit, corresponding to the standard deep inelastic nuclear parton distributions, and first moment, giving the charge form factor of  ${}^3\text{He}$ . Nuclear effects, evaluated by a modern realistic potential, are found to be larger than in the forward case. In particular, they increase with increasing the momentum transfer when the asymmetry of the process is kept fixed, and they increase with the asymmetry at fixed momentum transfer. Another relevant feature of the obtained results is that the nuclear GPD cannot be factorized into a  $\Delta^2$ -dependent and a  $\Delta^2$ -independent term, as suggested in prescriptions proposed for finite nuclei. The size of nuclear effects reaches 8% even in the most important part of the kinematical range under scrutiny. The relevance of the obtained results to study the feasibility of experiments is addressed.

DOI: 10.1103/PhysRevC.70.015205

PACS number(s): 13.40.-f, 13.60.Hb, 21.45.+v

### I. INTRODUCTION

Generalized parton distributions (GPDs) [1] parametrize the nonperturbative hadron structure in hard exclusive processes (for comprehensive reviews, see, e.g., [2–5]). The measurement of GPDs would provide information which is usually encoded in both the elastic form factors and the usual parton distribution functions (PDFs) and, at the same time, it would represent a unique way to access several crucial features of the nucleon [6,7], such as its angular momentum content [7] and its structure in the transverse plane [8]. According to a factorization theorem derived in QCD [9], GPDs enter the long-distance dominated part of exclusive lepton deep inelastic scattering (DIS) off hadrons. In particular, deeply virtual compton scattering (DVCS), i.e., the process  $eH \rightarrow e'H'\gamma$  when  $Q^2 \gg m_H^2$ , is one of the most promising to access GPDs (here and in the following,  $Q^2$  is the momentum transfer between the leptons  $e$  and  $e'$ , and  $\Delta^2$  the one between the hadrons  $H$  and  $H'$ ) [6,7,10]. Therefore, relevant experimental efforts to measure GPDs by means of DVCS off hadrons are likely to take place in the next few years. As a matter of fact, a few DVCS data have already been published [11,12].

Recently, the issue of measuring GPDs for nuclei has been addressed. The first paper on this subject [13], concerning the deuteron, contained already the crucial observation that the knowledge of GPDs would permit the investigation of the short light-like distance structure of nuclei, and thus the interplay of nucleon and Parton degrees of freedom in the nuclear wave function. In standard DIS off a nucleus with four-momentum  $P_A$  and  $A$  nucleons of mass  $M$ , this information can be accessed in the region where  $Ax_{Bj} \approx Q^2/2M\nu > 1$ , being  $x_{Bj} = Q^2/(2P_A \cdot q)$  and  $\nu$  the energy transfer in the laboratory system. In this region measurements are very difficult, because of vanishing cross-sections. As explained in [13], the same physics can be accessed in

DVCS at much lower values of  $x_{Bj}$ . The usefulness of nuclear GPDs has been stressed also for finite nuclei in Ref. 14, where it has been shown that they could provide us with peculiar information about the spatial distribution of energy, momentum and forces experienced by quarks and gluons inside hadrons. Since then, DVCS has been extensively discussed for nuclear targets. Impulse approximation (IA) calculations, supposed to give the bulk of nuclear effects at  $0.05 \leq Ax_{Bj} \leq 0.7$ , have been performed for the deuteron [15] and for spinless nuclei [16]. For nuclei of any spin, estimates of GPDs have been provided and prescriptions for nuclear effects have been proposed in [17]. Recently, an analysis of nuclear DVCS has been performed beyond IA, with estimates of shadowing effects and therefore involving large light-like distances and correlations in nuclei [18]. It was found that these effects are sizable up to  $Ax_{Bj} \approx 0.1$ , i.e., up to larger values of  $Ax_{Bj}$  with respect to normal DIS. Besides, the possibility of measuring DVCS at an electron-ion-collider has also been established in [18].

The study of GPDs for  ${}^3\text{He}$  is interesting for many aspects. In fact,  ${}^3\text{He}$  is a well known nucleus, for which realistic studies are possible, so that conventional nuclear effects can be safely calculated. Strong deviations from the predicted behavior could therefore be ascribed to exotic effects, such as the ones of non-nucleonic degrees of freedom, not included in a realistic wave function. Besides,  ${}^3\text{He}$  is extensively used as an effective neutron target. In fact, the properties of the free neutron are being investigated through experiments with nuclei, whose data are analyzed taking nuclear effects properly into account. Recently, it has been shown that unpolarized DIS off three body systems can provide relevant information on PDFs at large  $x_{Bj}$  [19–22], while it has been known for a long time that its particular spin structure suggests the use of  ${}^3\text{He}$  as an effective polarized neutron target [23–26]. Polarized  ${}^3\text{He}$  will therefore be the first candidate for experiments aimed at the study of spin-

dependent GPDs in the free neutron, to unveil details of its angular momentum content.

In this paper, an impulse approximation (IA) calculation of the quark unpolarized GPD  $H_q^3$  of  $^3\text{He}$  is presented. A convolution formula is derived and afterwards numerically evaluated using a realistic non-diagonal spectral function, so that Fermi motion and binding effects are rigorously estimated. The proposed scheme is valid for  $\Delta^2 \ll Q^2, M^2$  and despite this it permits one to calculate GPDs in the kinematical range relevant to the coherent, no break-up channel of deep exclusive processes off  $^3\text{He}$ . In fact, the latter channel is the most interesting one for its theoretical implications, but it can be hardly observed at large  $\Delta^2$ , due to the vanishing cross section [18]. The nuclear GPDs obtained here are therefore a prerequisite for any calculation of observables in coherent DVCS off  $^3\text{He}$ , although they can be compared neither with existing data, nor with forthcoming ones. A detailed analysis of DVCS off  $^3\text{He}$ , with estimates of observables, such as cross-sections or spin asymmetries, is in progress and will follow in a separate publication. Thus, the main result of this investigation is not the size and shape of the obtained  $H_q^3$  for  $^3\text{He}$ , but the size and nature of nuclear effects on it. This will permit one to test directly, for the  $^3\text{He}$  target at least, the accuracy of prescriptions which have been proposed to estimate nuclear GPDs [17], providing a useful tool for the planning of future experiments and for their correct interpretation.

The paper is organized as follows. The notation used is introduced in Sec. II, together with the derivation of the main result, a convolution formula obtained from an IA analysis. Such a formula is used to evaluate the numerical results shown in Sec. III and thoroughly discussed in Sec. IV, where the most interesting outcome of the present study is to be found. Eventually, conclusions are drawn in Sec. V.

## II. FORMALISM

The formalism introduced in Ref. [2] is adopted. One has to think to a spin 1/2 hadron target, with initial (final) momentum and helicity  $P(P')$  and  $s(s')$ , respectively. The GPDs  $H_q(x, \xi, \Delta^2)$  and  $E_q(x, \xi, \Delta^2)$  are defined through

$$\begin{aligned} F_{s',s}^q(x, \xi, \Delta^2) &= \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' s' | \bar{\psi}_q \left( -\frac{\lambda n}{2} \right) \not{n} \psi_q \left( \frac{\lambda n}{2} \right) | P s \rangle \\ &= H_q(x, \xi, \Delta^2) \frac{1}{2} \bar{U}(P', s') \not{n} U(P, s) \\ &\quad + E_q(x, \xi, \Delta^2) \frac{1}{2} \bar{U}(P', s') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} U(P, s), \end{aligned} \quad (1)$$

where  $\Delta = P' - P$  is the four-momentum transfer to the hadron,  $\psi_q$  is the quark field and  $M$  is the hadron mass. It is convenient to work in a system of coordinates where the photon four-momentum,  $q^\mu = (q_0, \mathbf{q})$ , and  $\bar{P} = (P + P')/2$  are collinear along  $z$ . The  $\xi$  variable in the arguments of the GPDs is the so-called ‘‘skewedness,’’ parametrizing the asymmetry of the process. It is defined by

$$\xi = -\frac{n \cdot \Delta}{2} = -\frac{\Delta^+}{2\bar{P}^+} = \frac{x_{Bj}}{2 - x_{Bj}} + \mathcal{O}\left(\frac{\Delta^2}{Q^2}\right), \quad (2)$$

where  $n$  is a light-like four-vector satisfying the condition  $n \cdot \bar{P} = 1$ . One should notice that the variable  $\xi$  is completely fixed by the external lepton kinematics. Here and in the following, use is made of the definition  $a^\pm = (a^0 \pm a^3)/\sqrt{2}$ . As explained in [7], GPDs describe the amplitude for finding a quark with momentum fraction  $x + \xi$  (in the infinite momentum frame) in a hadron with momentum  $(1 + \xi)\bar{P}$  and replacing it back into the nucleon with a momentum transfer  $\Delta$ . Besides, when the quark longitudinal momentum fraction  $x$  of the average nucleon momentum  $\bar{P}$  is less than  $-\xi$ , GPDs describe antiquarks; when it is larger than  $\xi$ , they describe quarks; when it is between  $-\xi$  and  $\xi$ , they describe  $q\bar{q}$  pairs. One should keep in mind that, in addition to the variables  $x$ ,  $\xi$  and  $\Delta^2$  explicitly shown, GPDs depend, as the standard PDFs, on the momentum scale  $Q^2$  at which they are measured or calculated. Such a dependence will not be discussed in the present paper and, for an easy presentation, it will be omitted. The values of  $\xi$  which are possible for a given value of  $\Delta^2$  are

$$0 \leq \xi \leq \sqrt{-\Delta^2}/\sqrt{4M^2 - \Delta^2}. \quad (3)$$

The well known natural constraints of  $H_q(x, \xi, \Delta^2)$  are

(i) the so-called ‘‘forward’’ or ‘‘diagonal’’ limit,  $P' = P$ , i.e.,  $\Delta^2 = \xi = 0$ , where one recovers the usual PDFs

$$H_q(x, 0, 0) = q(x); \quad (4)$$

(ii) the integration over  $x$ , yielding the contribution of the quark of flavor  $q$  to the Dirac form factor (f.f.) of the target:

$$\int dx H_q(x, \xi, \Delta^2) = F_1^q(\Delta^2); \quad (5)$$

(iii) the polynomiality property [2], involving higher moments of GPDs, according to which the  $x$ -integrals of  $x^n H^q$  and of  $x^n E^q$  are polynomials in  $\xi$  of order  $n+1$ .

In [27,28], an expression for  $H_q(x, \xi, \Delta^2)$  of a given hadron target, for small values of  $\xi^2$ , has been obtained from the definition Eq. (1).

It reads

$$\begin{aligned} H_q(x, \xi, \Delta^2) &= \int d\mathbf{k} \delta\left(\frac{k^+}{\bar{P}^+} - (x + \xi)\right) \\ &\quad \times \left[ \frac{1}{(2\pi)^3 V k_0} \sum_\lambda \langle P' | b_{q,\lambda}^\dagger(k^+ + \Delta^+, \mathbf{k}_\perp \right. \\ &\quad \left. + \Delta_\perp) b_{q,\lambda}(k^+, \mathbf{k}_\perp) | P \rangle \right] + \mathcal{O}(\xi^2), \end{aligned} \quad (6)$$

where states and creation and annihilation operators are normalized as follows:

$$\langle P' | P \rangle = (2\pi)^3 \delta(P'^+ - P^+) \delta(\mathbf{P}'_\perp - \mathbf{P}_\perp) \quad (7)$$

and

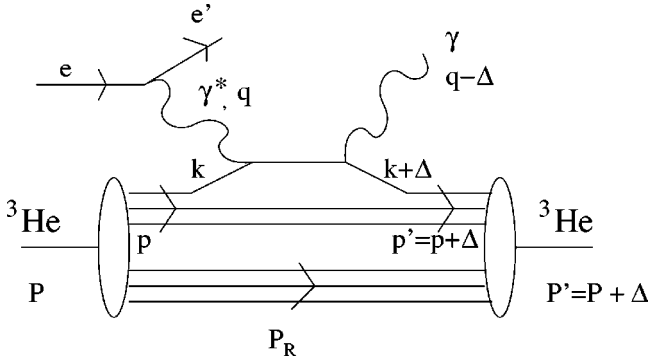


FIG. 1. The handbag contribution to the coherent DVCS process off  ${}^3\text{He}$ , in the present approach.

$$\{b(k^+, \mathbf{k}_\perp), b^\dagger(k'^+, \mathbf{k}'_\perp)\} = (2\pi)^3 \delta(k'^+ - k^+) \delta(\mathbf{k}'_\perp - \mathbf{k}_\perp), \quad (8)$$

respectively.

All the steps leading from Eq. (1) to Eq. (6) are fully described in Ref. [28]. They include initially the use of light-cone spinors and fields, neglecting terms of order  $O(\xi^2)$  on the right-hand side of Eq. (1), and later a transition to the expression (6), where the same quantities are normalized according to Eqs. (7) and (8).

The case of a spin 1/2 hadron with three composite constituents has been discussed in [28]. In that paper, the GPDs of the proton have been studied, assuming that it is made of complex constituent quarks. Here, the approach will be extended to  ${}^3\text{He}$ . The GPD  $H_q^3$  of  ${}^3\text{He}$  will be obtained in IA as a convolution between the non-diagonal spectral function of the internal nucleons, and the GPD  $H_q^N$  of the nucleons themselves.

The scenario is depicted in Fig. 1 for the special case of coherent DVCS, in the handbag approximation. One parton with momentum  $k$ , belonging to a given nucleon of momentum  $p$  in the nucleus, interacts with the probe and it is afterwards reabsorbed, with momentum  $k+\Delta$ , by the same nucleon, without further re-scattering with the recoiling system of momentum  $P_R$ . Finally, the interacting nucleon with momentum  $p+\Delta$  is reabsorbed back into the nucleus. The analysis suggested here is quite similar to the usual IA approach to DIS off nuclei [29,30].

In the class of frames discussed above, and in addition to the kinematical variables,  $x$  and  $\xi$ , already defined, one needs a few more to describe the process. In particular,  $x'$  and  $\xi'$ , for the ‘‘internal’’ target, i.e., the nucleon, have to be introduced. The latter quantities can be obtained defining the ‘‘+’’ components of the momentum  $k$  and  $k+\Delta$  of the struck parton before and after the interaction, with respect to  $\bar{P}^+$  and  $\bar{P}^+ = \frac{1}{2}(p+p')^+$ :

$$k^+ = (x + \xi)\bar{P}^+ = (x' + \xi')\bar{P}^+, \quad (9)$$

$$(k + \Delta)^+ = (x - \xi)\bar{P}^+ = (x' - \xi')\bar{P}^+. \quad (10)$$

From the above expressions,  $\xi'$  and  $x'$  are immediately obtained as

$$\xi' = -\frac{\Delta^+}{2\bar{P}^+}, \quad (11)$$

$$x' = \frac{\xi'}{\xi}x \quad (12)$$

and, since  $\xi = -\Delta^+/(2\bar{P}^+)$ , if  $\tilde{z} = p^+/P^+$ , one also has

$$\xi' = \frac{\xi}{\tilde{z}(1 + \xi) - \xi}. \quad (13)$$

These expressions have been already found and used in [13,16,17,28].

In order to derive a convolution formula in IA for  $H_q^3$ , the procedure used for standard DIS will be adopted [29,30]. First, in Eq. (6), two complete sets of states, corresponding to the interacting nucleon and to the recoiling two-body system, are properly inserted to the left- and right-hand sides of the quark operator:

$$\begin{aligned} H_q^3(x, \xi, \Delta^2) &= \langle P'S | \sum_{P'_R, S'_R, p', s'} \{ |P'_R S'_R\rangle |p' s'\rangle \} \{ \langle P'_R S'_R | \langle p' s' | \} \} \\ &\times \int \frac{dk}{(2\pi)^3} \frac{k^+}{k_0} \delta\left(\frac{k^+}{\bar{P}^+} - (x + \xi)\right) \frac{1}{V} \\ &\times \sum_{q, \lambda} b_{q, \lambda}^\dagger(k^+ + \Delta^+, \mathbf{k}_\perp + \Delta_\perp) b_{q, \lambda}(k^+, \mathbf{k}_\perp) \\ &\times \sum_{P_R, S_R, p, s} \{ |P_R S_R\rangle |ps\rangle \} \{ \langle P_R S_R | \langle ps | \} |PS\rangle; \quad (14) \end{aligned}$$

second, since eventually use has to be made of NR nuclear wave functions, a state  $|\mathbf{p}\rangle$  has to be normalized in a NR manner:

$$\langle \mathbf{p} | \mathbf{p} \rangle = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}), \quad (15)$$

so that in Eq. (14) one has to perform the substitution

$$|p\rangle \rightarrow \sqrt{\frac{p^+}{p_0^+}} |\mathbf{p}\rangle. \quad (16)$$

Since, using IA in the intrinsic frame of  ${}^3\text{He}$  one has, for the NR states,

$$\{ \langle \mathbf{P}_R S_R | \langle \mathbf{p} s | \} | \mathbf{P} S \rangle = \langle \mathbf{P}_R S_R, \mathbf{p} s | \mathbf{P} S \rangle (2\pi)^3 \delta^3(\mathbf{P} - \mathbf{P}_R - \mathbf{p}) \delta_{S, S_R s},$$

a convolution formula is readily obtained from Eq. (14):

$$\begin{aligned}
H_q^3(x, \xi, \Delta^2) &= \sum_N \int dE \int d\mathbf{p} \sqrt{\frac{p^+(p+\Delta)^+}{p_0(p+\Delta)_0}} P_N^3(\mathbf{p}, \mathbf{p} + \Delta, E) \\
&\times \frac{\xi'}{\xi} H_q^N(x', \xi', \Delta^2) + \mathcal{O}(\xi^2) \quad (17) \\
&= \sum_N \int dE \int d\mathbf{p} [P_N^3(\mathbf{p}, \mathbf{p} + \Delta, E) \\
&+ \mathcal{O}(\mathbf{p}^2/M^2, \Delta^2/M^2)] \frac{\xi'}{\xi} H_q^N(x', \xi', \Delta^2) + \mathcal{O}(\xi^2). \quad (18)
\end{aligned}$$

In the above equations, the kinetic energies of the residual

$$\begin{aligned}
H_q^N(x', \xi', \Delta^2) &= \int d\mathbf{k} \delta\left(\frac{k^+}{\bar{p}^+} - (x' + \xi')\right) \frac{1}{(2\pi)^3 V k_0} \sum_{\lambda} \langle p' | b_{q,\lambda}^\dagger(k^+ + \Delta^+, \mathbf{k}_\perp + \Delta_\perp) b_{q,\lambda}(k^+, \mathbf{k}_\perp) | p \rangle \\
&= \frac{\xi'}{\xi} \int d\mathbf{k} \delta\left(\frac{k^+}{\bar{p}^+} - (x + \xi)\right) \frac{1}{(2\pi)^3 V k_0} \sum_{\lambda} \langle p' | b_{q,\lambda}^\dagger(k^+ + \Delta^+, \mathbf{k}_\perp + \Delta_\perp) b_{q,\lambda}(k^+, \mathbf{k}_\perp) | p \rangle \quad (20)
\end{aligned}$$

is, according to Eq. (6), the GPD of the bound nucleon  $N$  up to terms of order  $\mathcal{O}(\xi^2)$ , and in the above equation use has been made of Eqs. (11) and (12).

The delta function in Eq. (18) defines  $E$ , the so-called removal energy, in terms of  $E_{\min} = |E_{3\text{He}}| - |E_{2\text{H}}| = 5.5$  MeV and  $E_R^*$ , the excitation energy of the two-body recoiling system. The main quantity appearing in the definition Eq. (19) is the overlap integral

$$\langle \mathbf{P}M | \mathbf{P}_R S_R, \mathbf{p}S \rangle = \int d\mathbf{y} e^{i\mathbf{p}\cdot\mathbf{y}} \langle \chi^s, \Psi_R^{S_R}(\mathbf{x}) | \Psi_3^M(\mathbf{x}, \mathbf{y}) \rangle, \quad (21)$$

between the eigenfunction  $\Psi_3^M$  of the ground state of  ${}^3\text{He}$ , with eigenvalue  $E_{3\text{He}}$  and third component of the total angular momentum  $M$ , and the eigenfunction  $\Psi_R^{S_R}$ , with eigenvalue  $E_R = E_2 + E_R^*$  of the state  $R$  of the intrinsic Hamiltonian pertaining to the system of two interacting nucleons [31]. Since the set of the states  $R$  also includes continuum states of the recoiling system, the summation over  $R$  involves the deuteron channel and the integral over the continuum states.

Concerning Eqs (18)–(21), two comments are in order.

The first concerns the accuracy of the actual calculations which will be presented. In the following, a NR spectral function will be used to evaluate Eq. (18), so that the accuracy of the calculation is of order  $\mathcal{O}(\mathbf{p}^2/M^2, \Delta^2/M^2)$ . In fact, if use is made of NR wave functions to calculate the nondiagonal spectral function, the result holds for  $\mathbf{p}^2, (\mathbf{p} + \Delta)^2 \ll M^2$ . The same constraint can be written  $\mathbf{p}^2, \Delta^2 \ll M^2$ . While the first of these conditions is the usual one for the NR treatment of nuclei, the second forces one to use Eq. (18) only at low values of  $\Delta^2$ , for which the accuracy is good

nuclear system and of the recoiling nucleus have been neglected, and  $P_N^3(\mathbf{p}, \mathbf{p} + \Delta, E)$  is the one-body off-diagonal spectral function for the nucleon  $N$  in  ${}^3\text{He}$ :

$$\begin{aligned}
P_N^3(\mathbf{p}, \mathbf{p} + \Delta, E) &= \frac{1}{(2\pi)^3} \frac{1}{2} \sum_M \sum_{R,s} \langle \mathbf{P}'M | (\mathbf{P} - \mathbf{p}) S_R, (\mathbf{p} + \Delta) s \rangle \\
&\times \langle (\mathbf{P} - \mathbf{p}) S_R, \mathbf{p}S | \mathbf{P}M \rangle \delta(E - E_{\min} - E_R^*). \quad (19)
\end{aligned}$$

Besides, the quantity

enough. As will be explained, the interest of the present calculation is indeed to investigate nuclear effects at low values of  $\Delta^2$ .

Second, from the study of forward DIS off nuclei, it is known that, to properly estimate nuclear effects, in going from a covariant formalism to one where use can be made of the usual nuclear wave functions, one has to keep the correct normalization [29,32]. This procedure leads to the appearance of the “flux factor,” represented in Eq. (17) by the expression  $\sqrt{p^+(p+\Delta)^+/p_0(p+\Delta)_0}$  (which gives, in forward DIS, the usual  $p^+/p_0$  term). This factor gives one at order  $\mathcal{O}(\mathbf{p}^2/M^2, \Delta^2/M^2)$ , which is the accuracy of the present analysis, and it will not be included in the actual calculation. In other words, if one takes into account that the intrinsic light-front frame and the usual rest frame of  ${}^3\text{He}$ , where the wave functions are evaluated, are not the same frame, the flux factor should be introduced [29,32]; anyway, in the NR approach used here, the two frames do not differ. This is an effect of the used approximations, which have to be relaxed if one wants to be predictive in more general processes at higher momentum transfer. In fact, in that case the accuracy of Eq. (18) is not good enough anymore and the calculation, in order to be consistent, has to be performed taking into account relativistic corrections. Work is being done presently in this direction, and the discussion of this important point will be included in a following paper, describing a calculation performed in a light-front framework. One final remark about the flux-factor: to neglect the corresponding quantity for the quark inside the internal nucleon, i.e., the term  $k^+/k_0$  in the GPD of the bound nucleon, Eq. (20), would be a rough approximation, with respect to the nuclear case, because the motion of quarks in the nucleon is relativistic, even at the

constituent level. In any case in the present calculation, as described in the following section, the GPD for the bound quark will not be evaluated, while an available model for it will be used. The main emphasis of the present approach, as already said, is not on the absolute values of the results, but in the nuclear effects, which can be estimated by taking any reasonable form for the internal GPD.

Equation (18) can be written in the form

$$\begin{aligned} H_q^3(x, \xi, \Delta^2) &= \sum_N \int dE \int d\mathbf{p} P_N^3(\mathbf{p}, \mathbf{p} + \mathbf{\Delta}) \frac{\xi'}{\xi} H_q^N(x', \xi', \Delta^2) \\ &= \sum_N \int_x^1 \frac{dz}{z} \int dE \int d\mathbf{p} P_N^3(\mathbf{p}, \mathbf{p} + \mathbf{\Delta}) \\ &\quad \times \delta\left(z - \frac{\xi}{\xi'}\right) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right). \end{aligned} \quad (22)$$

Taking into account that

$$z - \frac{\xi}{\xi'} = z - [\bar{z}(1 + \xi) - \xi] = z + \xi - \frac{p^+}{P^+}(1 + \xi) = z + \xi - \frac{p^+}{\bar{P}^+}, \quad (23)$$

Eq. (22) can also be written in the form

$$H_q^3(x, \xi, \Delta^2) = \sum_N \int_x^1 \frac{dz}{z} h_N^3(z, \xi, \Delta^2) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right), \quad (24)$$

where

$$h_N^3(z, \xi, \Delta^2) = \int dE \int d\mathbf{p} P_N^3(\mathbf{p}, \mathbf{p} + \mathbf{\Delta}) \delta\left(z + \xi - \frac{p^+}{\bar{P}^+}\right). \quad (25)$$

One should notice that Eqs. (24) and (25) or, which is the same, Eq. (18), fulfill constraint (i)–(iii) previously listed.

The constraint (i), i.e., the forward limit of GPDs, is certainly verified. In fact, by taking the forward limit ( $\Delta^2 \rightarrow 0, \xi \rightarrow 0$ ) of Eq. (24), one gets the expression which is usually found, for the parton distribution  $q_3(x)$ , in the IA analysis of unpolarized DIS off  ${}^3\text{He}$  [20,29,30]:

$$q_3(x) = H_q^3(x, 0, 0) = \sum_N \int_x^1 \frac{dz}{z} f_N^3(z) q_N\left(\frac{x}{z}\right). \quad (26)$$

In the latter equation,

$$f_N^3(z) = h_N^3(z, 0, 0) = \int dE \int d\mathbf{p} P_N^3(\mathbf{p}, E) \delta\left(z - \frac{p^+}{\bar{P}^+}\right) \quad (27)$$

is the light-cone momentum distribution of the nucleon  $N$  in the nucleus,  $q_N(x) = H_q^N(x, 0, 0)$  is the distribution of the quark of flavor  $q$  in the nucleon  $N$  and  $P_N^3(p, E)$ , the  $\Delta^2 \rightarrow 0$  limit of Eq. (22), is the one body spectral function.

Constraint (ii), i.e., the  $x$  integral of the GPD  $H_q$ , is also naturally fulfilled. In fact, by  $x$  integrating Eq. (24), one easily obtains

$$\begin{aligned} \int dx H_q^3(x, \xi, \Delta^2) &= \sum_N \int dx \int \frac{dz}{z} h_N^3(z, \xi, \Delta^2) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right) \\ &= \sum_N \int dx' H_q^N(x', \xi', \Delta^2) \int dz h_N^3(z, \xi, \Delta^2) \\ &= \sum_N F_q^N(\Delta^2) F_N^3(\Delta^2) = F_q^3(\Delta^2). \end{aligned} \quad (28)$$

In the equation above,  $F_q^3(\Delta^2)$  is the contribution, of the quark of flavor  $q$ , to the nuclear f.f.;  $F_q^N(\Delta^2)$  is the contribution, of the quark of flavor  $q$ , to the nucleon  $N$  f.f.;  $F_N^3(\Delta^2)$  is the so-called  ${}^3\text{He}$  ‘‘pointlike f.f.’’, which would represent the contribution of the nucleon  $N$  to the f.f. of  ${}^3\text{He}$  if  $N$  were point-like.  $F_N^3(\Delta^2)$  is given, in the present approximation, by

$$F_N^3(\Delta^2) = \int dE \int d\mathbf{p} P_N^3(\mathbf{p}, \mathbf{p} + \mathbf{\Delta}, E) = \int dz h_N^3(z, \xi, \Delta^2). \quad (29)$$

Eventually the polynomiality, condition (iii), is formally fulfilled by Eq. (18), although one should always remember that it is a result of order  $\mathcal{O}(\xi^2)$ , so that high moments cannot be really checked.

Summarizing this section, we have derived in IA a convolution formula for the GPD  $H_q^3$  of  ${}^3\text{He}$ , Eq. (18) [or, which is equivalent, Eq. (24)], at order  $\mathcal{O}(p^2/M^2, \Delta^2/M^2, \xi^2)$ , in terms of a non-diagonal nuclear one-body spectral function, Eq. (19), and of the GPD  $H_q^N$  of the internal nucleon.

### III. NUMERICAL RESULTS

$H_q^3(x, \xi, \Delta^2)$ , Eq. (18), has been evaluated in the nuclear Breit Frame.

The nondiagonal spectral function Eq. (19), appearing in Eq. (18), has been calculated along the lines of Ref. [33], by means of the overlap Eq. (21), which exactly includes the final state interactions in the two nucleon recoiling system, the only plane wave being that describing the relative motion between the knocked-out nucleon and the two-body system [31]. The realistic wave functions  $\Psi_3^M$  and  $\Psi_R^{SR}$  in Eq. (21) have been evaluated using the AV18 interaction [34] and taking into account the Coulomb repulsion of protons in  ${}^3\text{He}$ . In particular  $\Psi_3^M$  has been developed along the lines of Ref. [35]. The same overlaps have been already used in Ref. [20].

The other ingredient in Eq. (18), i.e., the nucleon GPD  $H_q^N$ , has been modeled in agreement with the double distribution representation [36], as described in [37]. For reader convenience, the explicit form of  $H_q^N$  is listed below [3,36]:

$$H_q^N(x, \xi, \Delta^2) = \int_{-1}^1 d\bar{x} \int_{-1+|\bar{x}|}^{1-|\bar{x}|} \delta(\bar{x} + \xi\alpha - x) \tilde{\Phi}_q(\bar{x}, \alpha, \Delta^2) d\alpha. \quad (30)$$

With some care, the expression above can be integrated over  $\bar{x}$  and the result is explicitly given in Ref. [3].

In [36], a factorized ansatz is suggested for the DD's:

$$\tilde{\Phi}_q(\tilde{x}, \alpha, \Delta^2) = h_q(\tilde{x}, \alpha, \Delta^2) \Phi_q(\tilde{x}) F_q(\Delta^2), \quad (31)$$

with the  $\alpha$  dependent term,  $h_q(\tilde{x}, \alpha, \Delta^2)$ , which has the character of a mesonic amplitude, fulfilling the relation

$$\int_{-1+|\tilde{x}|}^{1-|\tilde{x}|} h_q(\tilde{x}, \alpha, \Delta^2) d\alpha = 1. \quad (32)$$

Besides, in Eq. (31)  $\Phi_q(\tilde{x})$  represents the forward density and, eventually,  $F_q(\Delta^2)$  the contribution of the quark of flavor  $q$  to the nucleon form factor. Equation (30) fulfills the crucial constraints of GPDs, i.e., the forward limit, the first-moment and the polynomiality condition. One needs now to model the three functions appearing in Eq. (31). For the amplitude  $h_q$ , use will be made of one of the simple normalized forms suggested in [36], on the bases of the symmetry properties of DD's:

$$h_q^{(1)}(\tilde{x}, \alpha) = \frac{3(1-\tilde{x})^2 - \alpha^2}{4(1-\tilde{x})^3}. \quad (33)$$

For the forward distribution  $\Phi_q(\tilde{x})$ , the general simple form

$$\Phi_q(\tilde{x}) = \frac{\Gamma(5-a)}{6\Gamma(1-a)} \tilde{x}^{-a} (1-\tilde{x})^3, \quad (34)$$

with  $a=0.5$ , has been taken.

Concerning the model used for the internal nucleon  $H_q^N$ , two caveats are in order.

First of all, Eq. (30), with the choices Eqs. (33) and (34), corresponds to the *NS*, valence quark contribution proposed in [37], which is symmetric in the variable  $x$ . I stress again that the main point of the present study is not to produce realistic estimates for observables, but to investigate and discuss nuclear effects, which do not depend on the form of any well-behaved internal GPD, whose general structure is safely simulated by Eqs. (30)–(34). For the moment being, only the *NS* part of  $H_q^N$  is therefore calculated; if one wanted to estimate the DVCS cross-sections, also the quark singlet and the gluon contributions would be certainly necessary. As already said, a detailed study of DVCS on  ${}^3\text{He}$  is in progress and will be shown elsewhere.

Second, to take advantage of the used parametrization, Eqs. (30)–(34), of the nucleonic GPD, one should work in a symmetric frame for the nucleon, such as the Breit frame. Actually, the present calculation is performed in the nuclear Breit frame, which does not coincide with the Breit frame of the internal nucleon. For this reason, one should perform a boost to properly work in the nuclear Breit frame. Anyway, as it is pointed out in Ref. [16], any NR evaluation of nuclear GPDs, as a result of the NR reduction, is intrinsically frame dependent and it is valid only with accuracy  $\mathcal{O}(p^2/M^2)$ , like Eq. (18). In this calculation therefore, the effect of using a different reference frame for the nucleon is not taken into account, since in any case it would be an effect of order  $\mathcal{O}(p^2/M^2)$ , which is the overall accuracy of the present work [cf. Eq. (18)]. In the above procedure, therefore, no further approximations are introduced in addition to the ones leading

to Eq. (18). To overcome such a problem, one could use a light-front approach [38], as it is done for the GPDs of the deuteron in [13,15].

Eventually, the  $F_q(\Delta^2)$  term in Eq. (31), i.e., the contribution of the quark of flavor  $q$  to the nucleon form factor, has been obtained from the experimental values of the proton,  $F_1^p$ , and of the neutron,  $F_1^n$ , Dirac form factors. For the  $u$  and  $d$  flavors, neglecting the effect of the strange quarks, one has

$$F_u(\Delta^2) = \frac{1}{2}(2F_1^p(\Delta^2) + F_1^n(\Delta^2)),$$

$$F_d(\Delta^2) = 2F_1^n(\Delta^2) + F_1^p(\Delta^2). \quad (35)$$

The contributions of the flavors  $u$  and  $d$  to the proton and neutron f.f. are therefore

$$F_u^p(\Delta^2) = \frac{4}{3}F_u(\Delta^2),$$

$$F_d^p = -\frac{1}{3}F_d(\Delta^2) \quad (36)$$

and

$$F_u^n(\Delta^2) = \frac{2}{3}F_d(\Delta^2),$$

$$F_d^n(\Delta^2) = -\frac{2}{3}F_u(\Delta^2), \quad (37)$$

respectively.

For the numerical calculations, use has been made of the parametrization of the nucleon Dirac f.f. given in Ref. [39].

Now the ingredients of the calculation have been completely described, so that numerical results can be presented. First of all, the forward limit of  $H_q^3$ , Eq. (26), will be discussed, together with its  $x$  integral, Eq. (28).

In Fig. 2, it is shown the forward limit of the ratio

$$R_q(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{2H_q^p(x, \xi, \Delta^2) + H_q^n(x, \xi, \Delta^2)}, \quad (38)$$

i.e., the quantity

$$R_q(x, 0, 0) = \frac{H_q^3(x, 0, 0)}{2H_q^p(x, 0, 0) + H_q^n(x, 0, 0)} = \frac{q^3(x)}{2q^p(x) + q^n(x)} \quad (39)$$

for the flavor  $q=u, d$ , as a function of  $x_3=3x$ . In the above equation, the numerator is given by Eq. (26), while the denominator clearly represents the distribution of the quarks of flavor  $q$  in  ${}^3\text{He}$  if nuclear effects are completely disregarded, i.e., the interacting quarks are assumed to belong to free nucleons at rest. The behavior which is found is typically *EMC*-like, as it is usually obtained in IA studies of DIS on nuclei [20,29,30], so that, in the forward limit, well-known results are recovered. One should notice that, had the structure functions ratio, instead of the PDFs one, been shown, the ratio would be 1 for  $x=0$ , due to the normalization of the spectral function in Eq. (27). It is also useful, for later convenience, to realize that nuclear effects for the  $d$  flavor are a bit larger than those which are found for the  $u$  flavor. This is due to the fact that the forward  $d$  distribution is more sensitive than the  $u$  one to the neutron light-cone distribution, Eq. (27), which is different from that of the proton. In fact, the

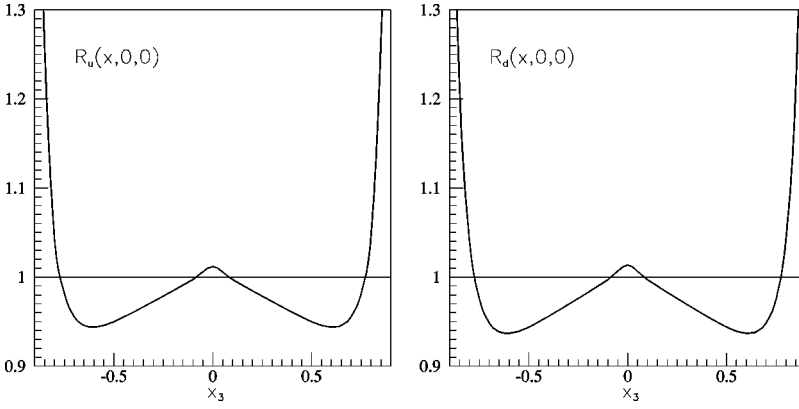


FIG. 2. The ratio Eq. (39) as a function of  $x_3$ , for the flavor  $u$  (left panel) and  $d$  (right panel).

average momentum of the neutron in  ${}^3\text{He}$  is larger than the one of the proton [30,31].

In Fig. 3, the quantity

$$\frac{1}{2} \sum_q \int dx H_q^3(x, \xi, \Delta^2) = \frac{1}{2} \sum_q \sum_N \int dx \int \frac{dz}{z} h_N^3(z, \xi, \Delta^2) \times H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right) \quad (40)$$

is shown. According to Eq. (28), in the present approximation, this should give, up to contributions of heavier quarks,

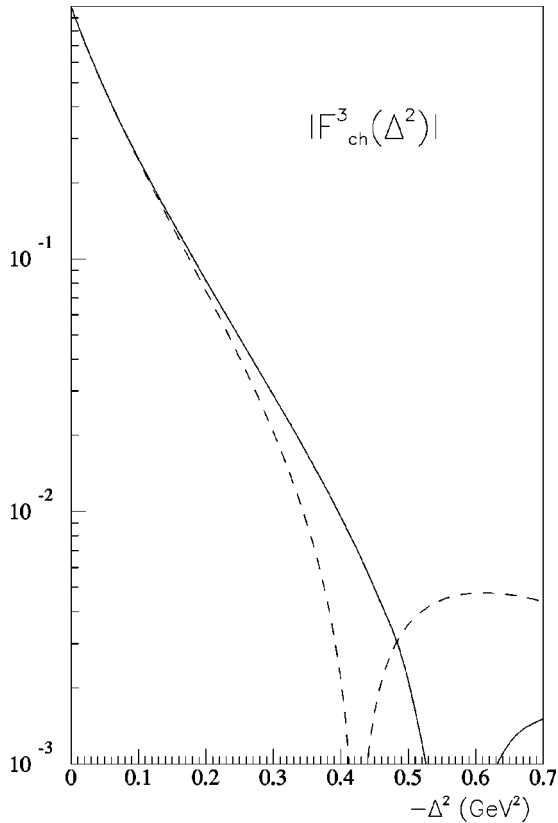


FIG. 3. The charge f.f. of  ${}^3\text{He}$ , calculated by means of Eq. (41) (full line), is compared with a parametrization of data [40] (dashed line), in the region of low- $\Delta^2$ , relevant to the present study.

the charge form factor  $F_{ch}^3(\Delta^2)$  of  ${}^3\text{He}$  (the usual normalization  $F_{ch}^3(0)=1$  is chosen):

$$F_{ch}^3(\Delta^2) = \frac{1}{2} [F_u^3(\Delta^2) + F_d^3(\Delta^2)]. \quad (41)$$

In the figure, the calculated integral is compared with experimental data of  $|F_{ch}^3(\Delta^2)|$  [40]. It is found that the present approach reproduces well the data up to a momentum transfer  $-\Delta^2=0.25 \text{ GeV}^2$ , which is enough for the aim of this calculation. In fact, the region of higher momentum transfer is not considered here, being phenomenologically not relevant for the calculation of GPDs entering coherent DVCS. The full curve in the left panel of Fig. 3 is very similar to the one-body contribution to the  ${}^3\text{He}$  charge f.f. shown in [41], as it must be, due to the IA used here. The agreement with data in the relevant kinematical region,  $-\Delta^2 \leq 0.25 \text{ GeV}^2$ , confirms that the inclusion of two-body currents is not required in the present calculation.

As an illustration, the result of the evaluation of  $H_u^3(x, \xi, \Delta^2)$  and  $H_d^3(x, \xi, \Delta^2)$  by means of Eq. (18) is shown in Figs. 4 and 5, for  $\Delta^2=-0.15 \text{ GeV}^2$  and  $\Delta^2=-0.25 \text{ GeV}^2$ , respectively, as a function of  $x_3$  and  $\xi_3=3\xi$ . The GPDs are shown for the  $\xi_3$  range allowed by Eq. (3) and in the  $x_3 \geq 0$  region. The  $x_3 \leq 0$  one, being symmetric to the latter, is not interesting and not shown.

The quality and size of the nuclear effects are discussed in the next section.

#### IV. DISCUSSION OF NUCLEAR EFFECTS

The full result for the GPD  $H_q^3$ , Eq. (18), shown in Figs. 4 and 5, will now be compared with a prescription based on the assumptions that nuclear effects are completely neglected and the global  $\Delta^2$  dependence can be described by the f.f. of  ${}^3\text{He}$ :

$$H_q^{3,(0)}(x, \xi, \Delta^2) = 2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2), \quad (42)$$

where the quantity

$$H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi) F_q^3(\Delta^2) \quad (43)$$

represents the flavor  $q$  effective GPD of the bound nucleon  $N=n, p$  in  ${}^3\text{He}$ . Its  $x$  and  $\xi$  dependencies, given by the function  $\tilde{H}_q^N(x, \xi)$ , is the same as the GPD of the free nucleon  $N$  (represented in this calculation by Eq. (30)), while its  $\Delta^2$

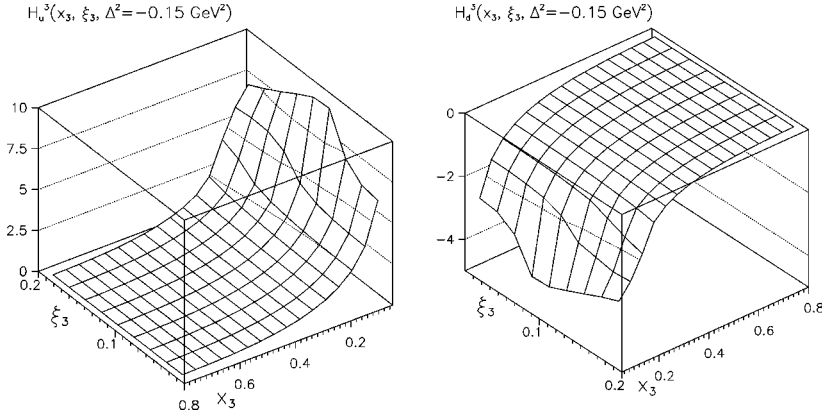


FIG. 4. In the left panel, for the  $\xi_3$  values which are allowed at  $\Delta^2 = -0.15 \text{ GeV}^2$  according to Eq. (3),  $H_u^3(x_3, \xi_3, \Delta^2)$ , evaluated using Eq. (24), is shown for  $0.05 \leq x_3 \leq 0.8$ . The symmetric part at  $x_3 \leq 0$  is not presented. In the right panel, the same is shown, for the flavor  $d$ .

dependence is governed by the contribution of the quark of flavor  $q$  to the  $^3\text{He}$  f.f.,  $F_q^3(\Delta^2)$ .

The effect of Fermi motion and binding can be shown through the ratio

$$R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{H_q^{3(0)}(x, \xi, \Delta^2)}, \quad (44)$$

i.e., the ratio of the full result, Eq. (18), to the approximation Eq. (42). The latter is evaluated by means of the nucleon GPDs used as input in the calculation, and taking

$$F_u^3(\Delta^2) = \frac{10}{3} F_{ch}^3(\Delta^2), \quad (45)$$

$$F_d^3(\Delta^2) = -\frac{4}{3} F_{ch}^3(\Delta^2), \quad (46)$$

where  $F_{ch}^3(\Delta^2)$  is the f.f. which is calculated within the present approach, by means of Eq. (28). The coefficients  $10/3$  and  $-4/3$  are simply chosen assuming that the contribution of the valence quarks of a given flavor to the f.f. of  $^3\text{He}$  is proportional to their charge. One should remember that the normalization of the f.f. has been chosen in Eq. (41).

The choice of calculating the ratio Eq. (44) to show nuclear effects is a very natural one. As a matter of fact, the forward limit of the ratio Eq. (44) is the same of the ratio Eq. (38), yielding the EMC-like ratio for the parton distribution  $q$  and, if  $^3\text{He}$  were made of free nucleon at rest, the ratio Eq. (44) would be one. This latter fact can be immediately real-

ized by observing that the prescription Eq. (42) is exactly obtained by placing  $z=1$ , i.e., no Fermi motion effects and no convolution, into Eq. (18). In the present situation the ratio Eq. (38) cannot be used to show nuclear effects, since it does not contain the  $\Delta^2$  dependence coming from the nuclear structure, i.e., the one provided by the non-diagonal spectral function in the present approach [cf. Eq. (18)].

One should note that the prescription suggested in Ref. [17] for finite nuclei in the valence quark sector, basically assuming that the nucleus is a system of almost free nucleons with approximately the same momenta, has the same  $\Delta^2$  dependence of the prescription Eq. (42).

Results are presented in Figs. 6 and 7, where the ratio Eq. (44) is shown for  $\Delta^2 = -0.15 \text{ GeV}^2$  and  $\Delta^2 = -0.25 \text{ GeV}^2$ , respectively, as a function of  $x_3$ , for three different values of  $\xi_3$ , for the flavors  $u$  and  $d$ .

Some general trends of the results are apparent:

(i) Nuclear effects, for  $x_3 \leq 0.7$ , are as large as 15% at most. Larger effects at higher  $x_3$  values are actually related with the vanishing denominator in Eq. (44).

(ii) Fermi motion and binding have their main effect for  $x_3 \leq 0.3$ , at variance with what happens in the forward limit (cf. Fig. 2).

(iii) At fixed  $\Delta^2$ , nuclear effects increase with increasing  $\xi$ , for  $x_3 \leq 0.3$ .

(iv) At fixed  $\xi$ , nuclear effects increase with increasing  $\Delta^2$ , for  $x_3 \leq 0.3$ .

(v) Nuclear effects for the  $d$  flavor are larger than for the  $u$  flavor.

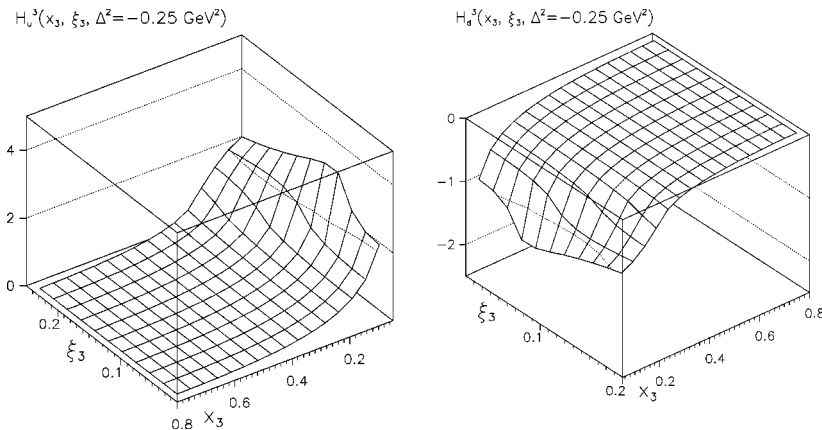


FIG. 5. The same as in Fig. 4, at  $\Delta^2 = -0.25 \text{ GeV}^2$ .



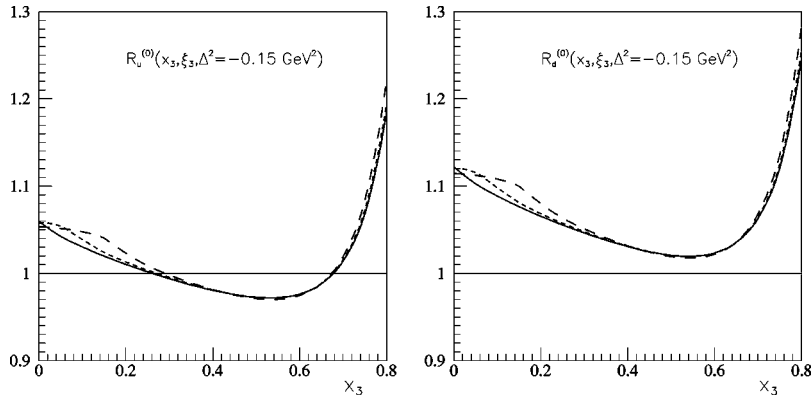


FIG. 6. In the left panel, the ratio Eq. (44) is shown, for the  $u$  flavor and  $\Delta^2 = -0.15 \text{ GeV}^2$ , as a function of  $x_3$ . The full line has been calculated for  $\xi_3 = 0$ , the dashed line for  $\xi_3 = 0.1$  and the long-dashed one for  $\xi_3 = 0.2$ . The symmetric part at  $x_3 \leq 0$  is not presented. In the right panel, the same is shown, for the flavor  $d$ .

The behavior described above can be explained as follows. As already said in Sec. II, in IA and in the forward limit, at  $x_3 = 0$  one basically recovers the spectral function normalization and no nuclear effects, so that the ratio Eq. (38) slightly differs from one. This is not true of course in the present case, due to nuclear effects hidden not only in the  $x'$  dependence, but also in the  $\xi'$  one, according to its definition, Eq. (13). Moreover, even if  $x_3 = \xi_3 = 0$ , in the present situation the ratio Eq. (44) does not give the spectral function normalization as in the forward case, because of the  $\Delta^2$  dependence. One source of such dependence is that, in the approximation Eq. (42), it is assumed, through Eqs. (45) and (46) that the quarks  $u$  and  $d$ , belonging to the protons or to the neutron in  ${}^3\text{He}$ , contribute to the charge f.f. in the same way, being the contribution proportional to their charge only. Actually, the effect of Fermi motion and binding is stronger for the quarks belonging to the neutron, having the latter a larger average momentum with respect to the proton [30,31]. This can be seen noticing that the pointlike f.f., Eq. (29), for the proton, shows a stronger  $\Delta^2$ -dependence, with respect to the neutron one, the difference being 17% (23%) at  $\Delta^2 = -0.15 \text{ GeV}^2$  ( $\Delta^2 = -0.25 \text{ GeV}^2$ ). The prescription given by Eqs. (45) and (46) could be correct only if the pointlike f.f. had a similar  $\Delta^2$  dependence. Besides, nuclear effects studied by means of the ratio Eq. (44) at fixed  $x$  and  $\xi$  depend on  $\Delta^2$ , showing clearly that such a dependence cannot be factorized, i.e., the nuclear GPD cannot be written as the product of a  $\Delta^2$  dependent and a  $\Delta^2$  independent term, confirming what has been found for the deuteron case in Ref. [15]. One should notice that, if factorization were valid, Figs. 6 and 7 would be equal. This fact clearly indicates that a model based on the

assumption of factorization, such as the one of Ref. [17], is not motivated and cannot be used to parametrize nuclear GPDs for estimates of DVCS cross sections and asymmetries for light nuclei.

The fact that nuclear effects are larger for the  $d$  distribution is also easily explained in terms of the different contribution of the spectral functions for the protons and the neutron, the latter being more important for the GPDs of the  $d$  rather than for the ones of the  $u$  flavor. This has already been shown in the forward case in Fig. 2, and shortly commented in the previous section. When the  $\Delta^2$ -dependence is studied, the effect found in the forward case increases, being impossible, as explained above, to really obtain the  $u$  and  $d$  contributions to the  ${}^3\text{He}$  f.f. by means of a simple charge rescaling, as it is done in Eqs. (45) and (46). In particular, the approximation Eq. (46) for the  $d$  flavor is worse than the approximation Eq. (45) for the  $u$  flavor, the disagreement increasing with increasing  $\Delta^2$ . One finds that  $F_u^3(\Delta^2)$  given by Eq. (45) differs from the one calculated through Eq. (28) by 4% (6%) at  $\Delta^2 = -0.15 \text{ GeV}^2$  ( $-0.25 \text{ GeV}^2$ ), while for the  $d$  flavor the disagreement reaches 9% (13%) for the same value of the momentum transfer. This flavor dependence is to be expected for any target with isospin different from zero.

As already said, the calculation of DVCS observables is beyond the scope of the present paper and will be presented elsewhere. Anyway, a first rough estimate of nuclear effects on DVCS observables can be already sketched. In fact it is known that the point  $x = \xi$  gives the bulk of the contribution to hard exclusive processes, since at leading order in QCD the amplitude for DVCS and for meson electroproduction just involve GPDs at this point, which is therefore also the

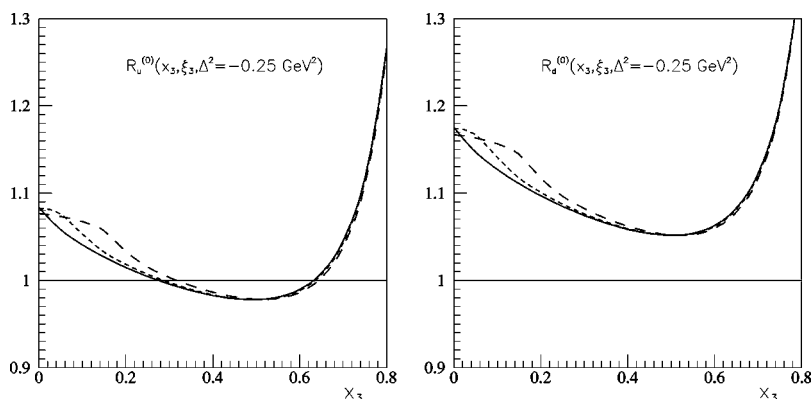


FIG. 7. The same as in Fig. 6, at  $\Delta^2 = -0.25 \text{ GeV}^2$ .

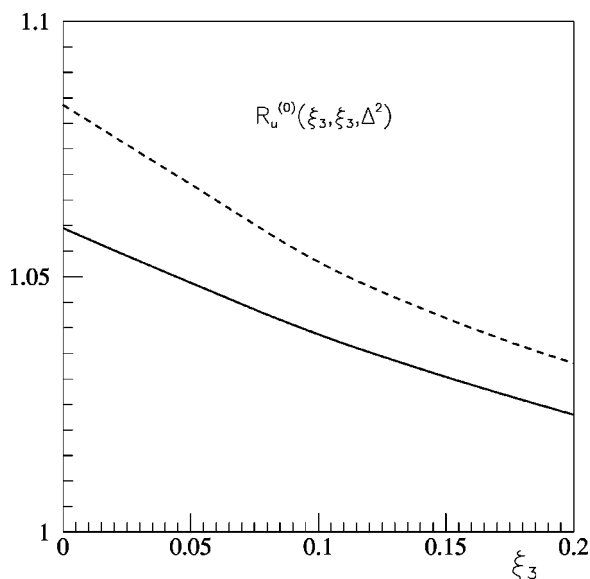


FIG. 8. The ratio Eq. (44) for the  $u$  flavor, for  $x_3 = \xi_3$ , as a function of  $\xi_3$ , at  $\Delta^2 = -0.15 \text{ GeV}^2$  (full line), and at  $\Delta^2 = -0.25 \text{ GeV}^2$  (dashed line).

easiest region to access experimentally. In order to figure out how Fermi motion and binding affect the “slice”  $H_q^3(\xi, \xi, \Delta^2)$ , the ratio (44) for  $x = \xi$ , is shown in Fig. 8, as a function of  $\xi_3 = x_3$ , for the flavor  $u$ . It can be seen that even in this crucial region nuclear effects are systematically underestimated by the approximation Eq. (42), up to a maximum of 8% for the flavor  $u$ .

The issue of applying the obtained GPDs to calculate DVCS off  $^3\text{He}$ , to estimate cross-sections and to establish the feasibility of experiments, as it has been done already for the deuteron target in Ref. [15], is in progress and will be presented elsewhere. In particular, the contributions of the incoherent break-up channel, and of the shadowing effects, beyond IA, at  $\xi \approx x_{Bj}/2 \leq 0.05$ , have to be evaluated. Besides, the study of polarized GPDs will be very interesting, due to the peculiar spin structure of  $^3\text{He}$  and its implications for the study of the angular momentum of the free neutron.

## V. CONCLUSIONS

In this paper, a realistic microscopic calculation of the unpolarized quark GPD  $H_q^3(x, \xi, \Delta^2)$  for  $^3\text{He}$  is presented. In an impulse approximation framework, a convolution formula has been obtained where Fermi motion and binding effects are properly taken into account through an off-diagonal spectral function. The range of validity of the scheme is  $|\Delta^2| \leq 0.25 \text{ GeV}^2$ , the most relevant for the coherent channel of hard exclusive processes. The forward limit and the  $x$ -integral of the obtained  $H_q^3(x, \xi, \Delta^2)$  are formally and numerically in agreement with the theoretical and experimental knowledge of the  $^3\text{He}$  nucleus.

Nuclear effects are found to be larger than in the forward case and to increase with  $\Delta^2$  at fixed  $\xi$ , and with  $\xi$  at fixed  $\Delta^2$ . In particular the latter  $\Delta^2$  dependence does not simply factorize, in agreement with previous findings for the deuteron target and at variance with prescriptions proposed for finite nuclei. Moreover, nuclear effects are found to be larger for the  $d$  flavor than for the  $u$  one, being the target non isoscalar. In particular, it has been shown that nuclear effects are as large as 8%, even at the crucial point  $x = \xi$ .

The shown results represent a prerequisite for any calculation of DVCS observables. The evaluation of these quantities, to establish the feasibility of experiments, is in progress and will be presented in a later paper, together with the contributions of the incoherent, break-up channel, and of shadowing effects, beyond IA, at small values of  $\xi \approx x_{Bj}/2$ . The present analysis is also a first step toward a complete study of the GPDs for  $^3\text{He}$ , which could provide, in the polarized case, with an important tool to unveil details of the angular momentum structure of the free neutron.

## ACKNOWLEDGMENTS

I would like to thank C. Ciofi degli Atti, L. P. Kaptari, E. Pace and G. Salmè for many discussions about the nuclear three body systems, during our collaboration in the past years. I am also grateful to V. Vento for constant encouragement. This work is supported in part by MIUR through the funds COFIN03.

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