Lifetime measurements of microsecond isomers in the $N=48$ nuclei ^{88}Zr and ^{90}Mo using recoil-isomer tagging

A. Chakraborty, Krishichayan, S. S. Ghugre, R. Goswami, S. Mukhopadhyay, N. S. Pattabiraman, S. Ray, and A. K. Sinha
Inter University Consortium for DAE Facilities, Calcutta Center, Sector III/LB-8, Bidhan Nagar, Kolkata 700098, India

S. Sarkar
Department of Physics, The University of Burdwan, Burdwan 713104, India

P. V. Madhusudhana Rao
Department of Nuclear Physics, Andhra University, Waltair 530003, India

U. Garg
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA

S. K. Basu
Variable Energy Cyclotron Center, Sector-I/AF, Bidhan Nagar, Kolkata 700064, India

L. Chaturvedi, A. Dhal, and R. K. Sinha
Department of Physics, Banaras Hindu University, Varanasi 221005, India

I. M. Govil
Department of Physics, Panjab University, Chandigarh 160014, India

M. B. Chatterjee and M. Saha Sarkar
Saha Institute of Nuclear Physics, Sector-I/AF, Bidhan Nagar, Kolkata 700064, India

R. K. Bhowmik, A. Jhingan, N. Madhavan, S. Muralithar, S. Nath, R. P. Singh, and P. Sugathan
Nuclear Science Center, Aruna Asaf Ali Marg, New Delhi 110067, India
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The recoil-isomer tagging technique has been used to study the isomeric states of nuclei in the vicinity of the $N=50$ shell closure. The nuclei of interest were separated from the projectilelike nuclei and other evaporation residues and transported to the focal plane of a recoil separator. The decay of the tagged isomer was studied at the focal plane using a high-purity Ge detector. The prompt transitions feeding the isomer were detected in an array of Clover Ge detectors placed around the target. The measured half-lives of the 8^+ isomers in ^{88}Zr and ^{90}Mo are $1.41^{(+0.12)}_{(-0.09)}$ μs and $1.17^{(+0.19)}_{(-0.07)}$ μs, respectively. The corresponding transition probabilities are in reasonable agreement with the predictions of shell model calculations. The systematics of effective charges (e_p and e_n) for $N=48$ nuclei are qualitatively understood in these calculations.

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I. INTRODUCTION

Microsecond isomers at moderate spins ($J\sim 6-8\,\hbar$) [1] are a general feature of nuclei in and around the neutron $N=50$ shell closure. These isomers occur because of large spin differences in the configuration of the initial and final states or a reduction in transition energies as one approaches the highest spin possible for the given seniority multiplet. Although traditionally the $p_{1/2}, g_{9/2}$ valence space has been considered sufficient to explain the observed experimental information in the $A\sim 90$ region, a recent investigation on ^{92}Mo [2] established the need for a larger configuration space to interpret even the moderate spin states. The transition matrix elements from the measured lifetimes provide a stringent test for the nuclear model and the model space employed. However, several previous lifetime measurements [3,4] investigating isomers in this mass region are not consistent. For example, De Boer *et al.* [3] investigated the decay of the 8^+ isomeric level in ^{90}Mo using delayed coincidences. The isomeric level decays via the 810-, 1054-, 948-keV positive-parity cascade as well as the 263-, 547-keV negative-parity cascade (Fig. 1). The 810-keV ($6^+\rightarrow 4^+$) transition displayed a 1.0-μs half-life, which was consistent with other contemporary measurements [5]. However, the 1054-keV ($4^+\rightarrow 2^+$) and 948-keV ($2^+\rightarrow 0^+$) transitions also indicated a considerably long-lived component with $T_{1/2}=1.8(2)$ μs. This was attributed to a possible isomer in the negative-parity cascade, the location of which could not be determined from the existing data. Similar discrepancies also exist in the reported lifetime of the 8^+ isomer in ^{88}Zr. Ishihara *et al.* [5] have reported a value of 1.75 ± 0.20 μs, while Hausser *et al.* reported a value of 1.32 ± 0.025 μs [4].

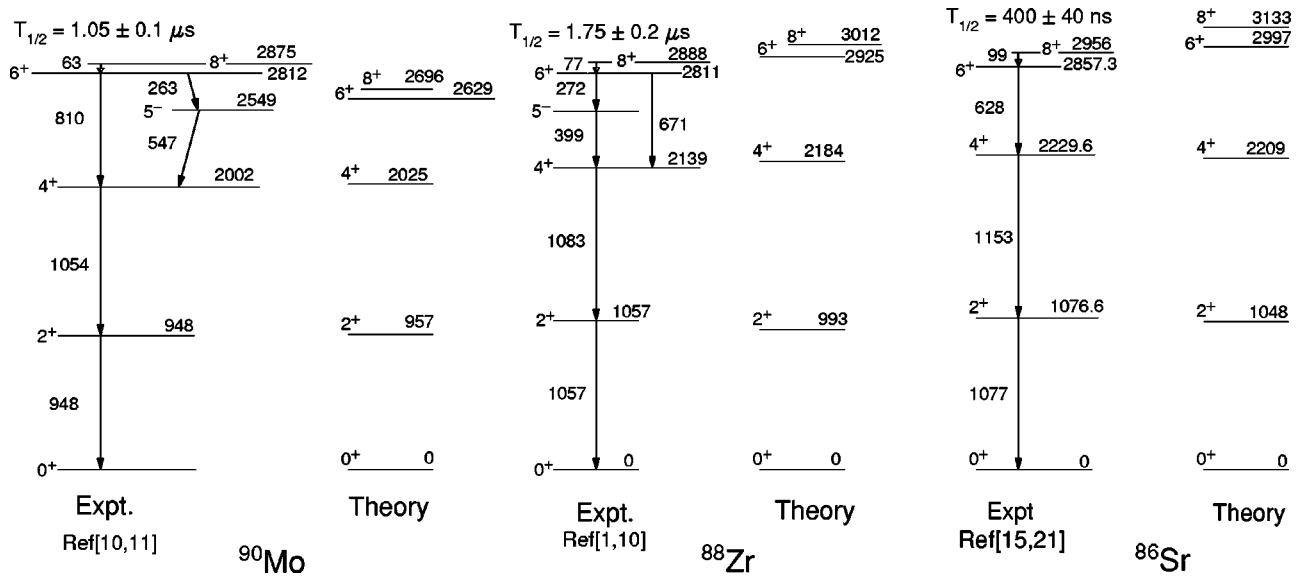

FIG. 1. Partial level schemes illustrating the decay of the 8^+ isomeric level in ^{90}Mo, ^{88}Zr, and ^{86}Sr. The quoted half-lives are from the earlier measurements. The experimental excitation energies are compared with our shell model calculations (see text for details).

It is of interest, therefore, to perform lifetime measurements for these μs isomeric levels with an improved technique. It would also be interesting to search for new isomers in these nuclei and develop the level structure above these isomers. The measurements require selecting these specific states from the much more abundant prompt states populated in a typical heavy-ion-induced fusion evaporation reaction and transporting the recoiling nuclei to a low-background region where the decay of the isomer may be studied. In recoil-isomer tagging, the isomeric levels are selected and separated from the beamlike particles and the evaporation residues by a recoil separator, allowing its decay to be measured in a relatively low-background environment at its focal plane. The flight path through a recoil separator, such as the Heavy Ion Reaction Analyzer (HIRA) [6] at the 15 UD Pelletron Facility at the Nuclear Science Center (NSC), New Delhi is typically about 1 μs. This technique is, thus, limited to isomers having half-lives of the order of a few microseconds. The decay of the isomers in this mass region are low multiplicity events. Hence one needs to employ only a few (or even a single) high-purity Ge (HPGe) detectors at the focal plane. At the same time an array of γ detectors at the target position may be used to record the prompt γ decays feeding this isomer. The advantage of this technique for the study of isomers is that it uniquely allows the investigation of the correlations between the prompt decays feeding the isomer and the delayed events so that both the feeding and decay properties of the isomer may be studied at the same time.

II. EXPERIMENTAL DETAILS

The experimental arrangement comprised an eight-element Clover detector array, the Indian National Gamma Array (INGA), coupled with the HIRA facility at NSC. The prompt γ rays were recorded using INGA at the target position. The recoiling nuclei of interest were separated in flight from the background of primary beam and other reaction products by HIRA. The separated nuclei pass through a multiwire proportional counter (MWPC) at the focal plane and were implanted into the perspex flange. A HPGe detector was placed in close proximity of the end flange of the focal plane of HIRA to detect the delayed γ transitions originating from the decay of the isomeric level. The ^{90}Mo and ^{88}Zr nuclei, where microsecond isomers are reported at moderate spin $J\sim 8\hbar$, were populated using the ^{63}Cu (^{31}P,*xnyp*) reaction at an incident energy of 120 MeV. The self-supporting ^{63}Cu target had a thickness of about 760 μg/cm^2. The ^{31}P beam was provided by the 15 UD Pelletron facility at NSC. Data were recorded with the condition that either (a) INGA recorded a γ-γ coincidence or (b) a single γ ray was detected by INGA in coincidence with a recoiling nucleus at the focal plane. The coincidence between a recoil signal and an accepted master signal was used to start a TAC, which was then stopped by the HPGe (at the focal plane) signal, which detected the γ rays following the decay of the isomeric level. The range of this recoil-delayed γ-TAC was initially set to 50 μs, but was subsequently set to 10 μs when it became clear that there were no isomers present in the 5–50 μs range. This ensured a coverage of time interval which approximately corresponded to about 5 to 6 half-lives for the isomeric levels (assuming a typical half-life of about 2 μs for these states). This TAC along with the HPGe data were recorded in addition to the conventional prompt γ-γ coincidence list mode data. In all about 600 million prompt γ-γ coincident events and about two million prompt-delay γ-γ coincident events were recorded. The γ spectra at the focal plane are illustrated in Fig. 2. Figure 2(a) depicts the raw γ-ray spectrum without any mass selection. Figures 2(b) and 2(c) correspond to the mass 90 and 88 gated HPGe spectra, respectively. The contamination from the 1054-keV transition belonging to ^{90}Mo is minimized in the 1057-keV transition belonging to ^{88}Zr using the mass gating. Figure 3 il-

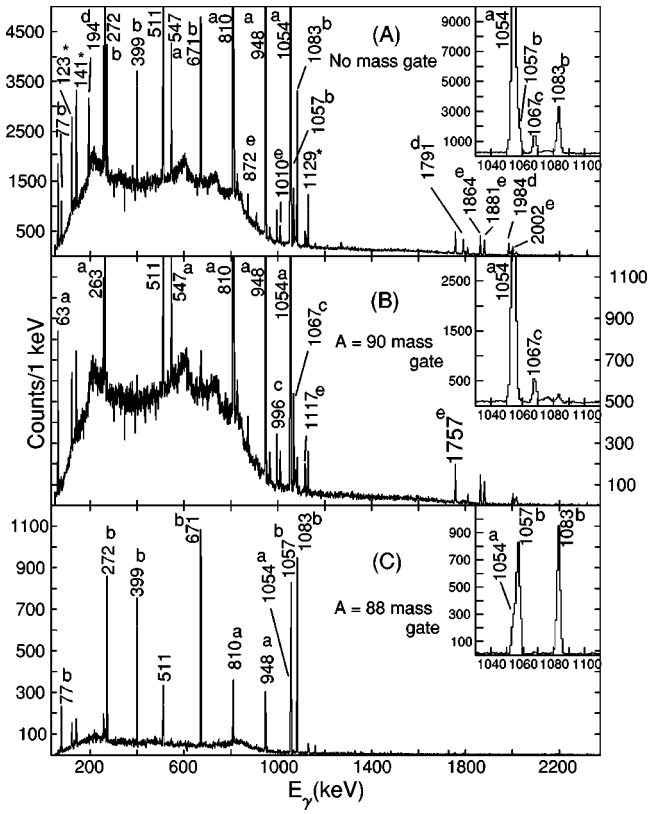

FIG. 2. Spectra of γ rays obtained from the focal plane HPGe detector. The peaks labeled as "a," "b," "c," and "d" belong to ^{90}Mo, ^{88}Zr, ^{90}Nb, and ^{91}Nb, respectively. The peaks labeled as "e" and "*" correspond to contaminants and due to β-decay, respectively. The inset highlights the advantage of mass gating.

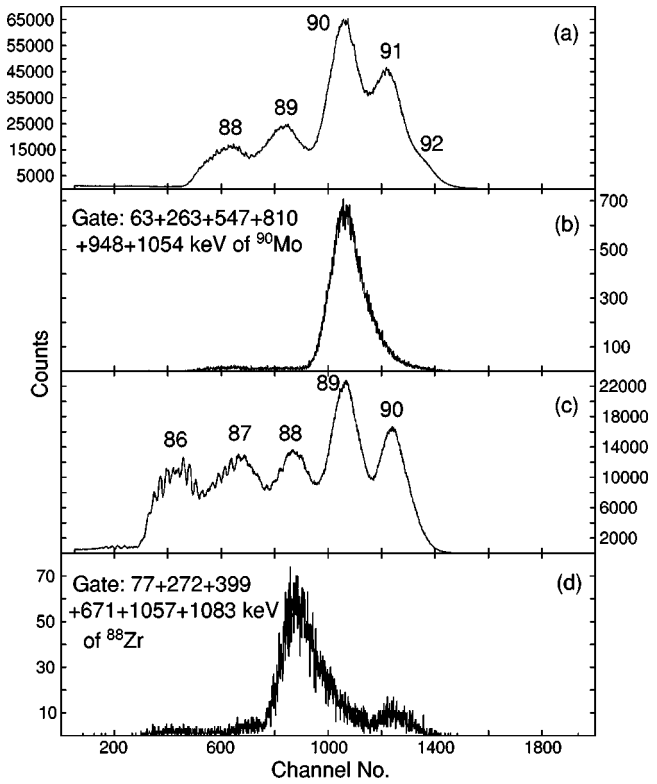

FIG. 3. Mass spectra as recorded by the MWPC at the focal plane of HIRA; (a) inclusive mass spectrum when HIRA was tuned for $A=90$ as the central mass, (b) the corresponding mass spectrum gated by the delayed transitions, belonging to ^{90}Mo, (c) inclusive mass spectrum for $A=88$ as the central mass, (d) the corresponding mass spectrum gated by the delayed transitions, as indicated, belonging to ^{88}Zr.

lustrates the corresponding γ gated mass spectra. This information was useful to ascertain the contamination, if any, from neighboring nuclei in the observed delayed γ events.

III. DATA ANALYSIS

The data were analyzed by using the Linux-based data analysis programs IUCSORT [7] and RADWARE [8]. The list mode data were sorted into several two-dimensional histograms. To obtain the half-lives of the isomeric levels a γ-time matrix was formed. One axis of this matrix corresponded to the recoil-delayed γ-TAC, and the other axis contained the corresponding γ energy as recorded by the HPGe detector at the focal plane. As mentioned above, this TAC had a range of 10 μs. As seen from Fig. 2, transitions originating from the decay of the known μs isomers in several $A\sim90$ and $A\sim88$ nuclei have been identified. γ-ray spectra were then obtained with successive 0.5- and 1.0-μs-wide time slices (with and without the mass selection) on the recoil-delayed γ-TAC. The time calibration was performed using a precision RF oscillator that generated pulses at an interval of 162 ns. The zero time corresponded to the arrival of the isomeric nucleus at the focal plane, obtained from the anode signal from the MWPC. The representative time spectra and decay curves obtained for the 810-, 1054-, 948- and 63-, 263-, 547-keV γ rays following the decay of the isomer at $J^\pi=8^+$ in ^{90}Mo are shown in Figs. 4 and 5. As seen from the figures, the estimated half-lives are identical within the error limits. In Figs. 4, 5(a), and 6(a), we have plotted the recoil-delayed γ-TAC spectra by putting gates on the specific γ rays (as mentioned in the figures) in the focal plane HPGe spectrum. Representative decay curves for the delayed γ rays are illustrated in Figs. 5(b)–5(d) and 6(b)–6(d). As mentioned above these were obtained after integrating over successive 0.5-μs [Fig. 5(c)] and 1-μs [Figs. 5(b), 5(d), and 6(b)–6(d)] wide slices on the TAC spectra. Another matrix containing the prompt (feeding γ rays detected at the target position) and the delayed (γ rays detected at the focal plane originating from the decay of the isomer) events was also formed. The analysis of these data has helped to establish the feeding and the decay pattern of the isomeric level. Further, the level scheme above the isomer could be established from the prompt γ-γ events at the target. These data helped us develop the level scheme for ^{90}Nb, above the known $J^\pi=11^-$ isomer [9], and those results will be presented in a forthcoming paper.

IV. EXPERIMENTAL RESULTS

Hausser *et al.* [4] have performed magnetic moment measurements for $N=48$, 49 nuclei. They have reported a value

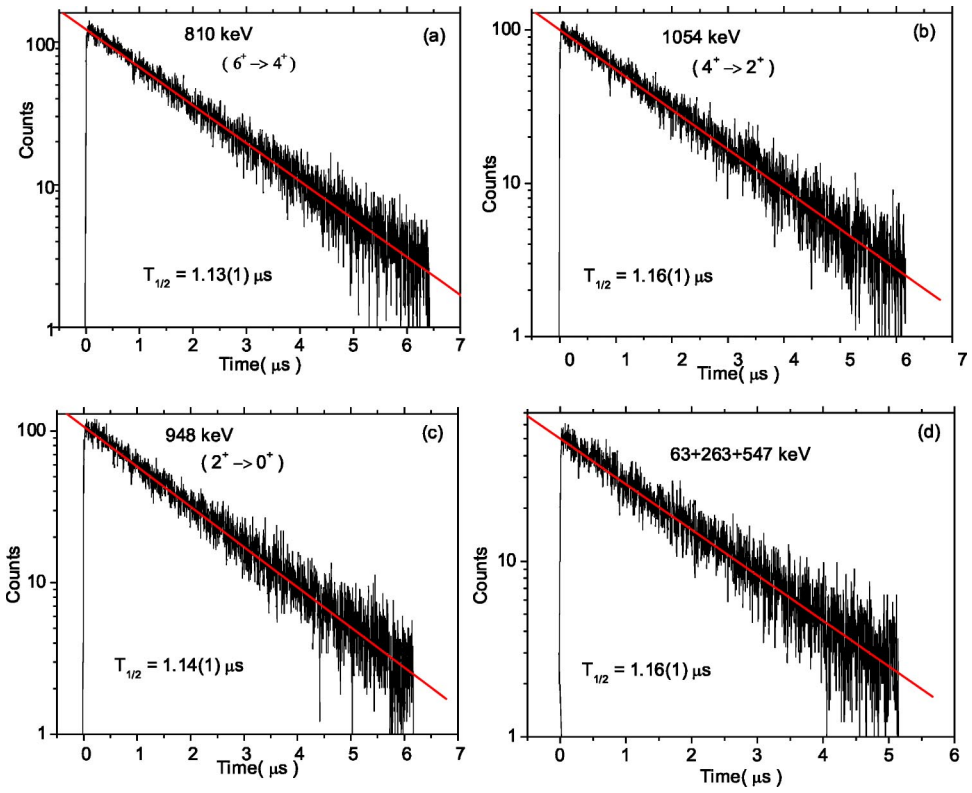

FIG. 4. (Color online) Representative time spectra for the 810-keV ($6^+ \rightarrow 4^+$), 1054-keV ($4^+ \rightarrow 2^+$), 948-keV ($2^+ \rightarrow 0^+$) transitions and also for the sum gate of 63- ($8^+ \rightarrow 6^+$), 263- ($6^+ \rightarrow 5^-$), and 547- ($5^- \rightarrow 4^+$) keV transitions belonging to ^{90}Mo. A fit to the time spectra yields a half-life of $1.17^{(+0.19)}_{(-0.07)}$ μs for the 8^+ isomeric level (see text for details).

of 1.14 ± 0.05 μs for the lifetime of 8^+ isomeric level in ^{90}Mo. As mentioned earlier, this value is not in agreement with the reported value of Ishihara *et al.* [5] and De Boer *et al.* [3]. Further, De Boer *et al.* had indicated the presence of a negative parity isomer in this nucleus.

The γ rays that dominate the delayed spectrum at the focal plane were 263-, 547-, 810-, 948- and 1054-keV transitions, which are associated with the decay of the 8^+ isomeric level in ^{90}Mo. Besides using mass selection, the prompt-delayed matrix was also used to confirm that these

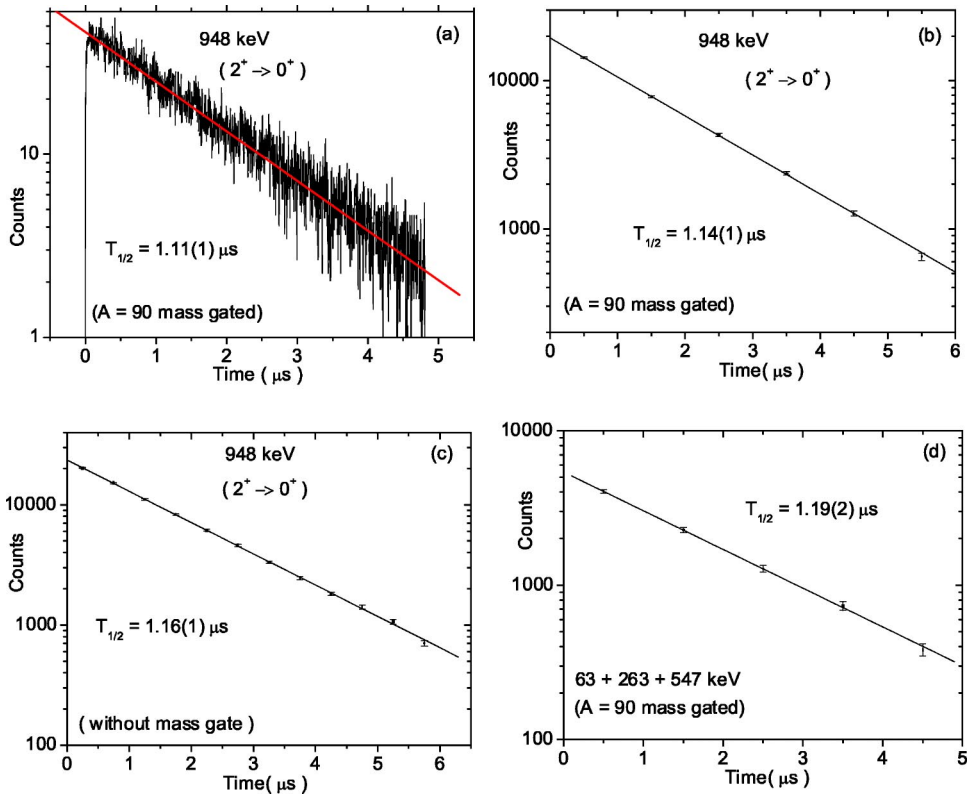

FIG. 5. (Color online) Decay curves for the sum gate of 63-, 263-, and 547-keV γ rays and 948-keV transition of ^{90}Mo, following the decay of the 8^+ isomeric level. The zero of time corresponds to the arrival of the isomer at the focal plane. The corresponding data without mass gating are illustrated in Fig. 4. In (b,c,d) the data points correspond to the area under the photopeak for the corresponding time window.

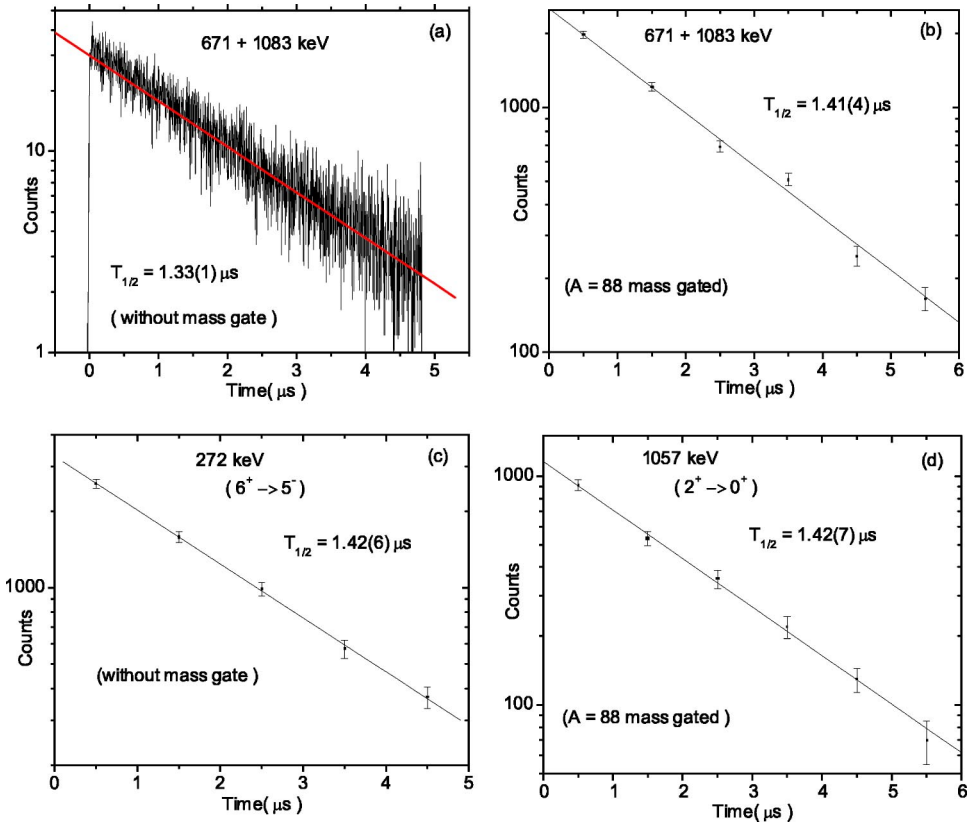

FIG. 6. (Color online) Decay curves for transitions deexciting the 8^+ isomeric level in ^{88}Zr. A half-life of $1.41^{(+0.12)}_{(-0.09)}$ μs for the 8^+ isomeric level is obtained from this measurement.

strong transitions originated from the decay of this isomer; it has been possible to identify all the known transitions up to spin of $J=22\hbar$ and excitation energy of $E_x\sim 10$ MeV [10] in prompt spectra gated with the delayed transitions. An exactly similar decay pattern is exhibited by all γ rays originating from the decay of the 8^+ isomer. This implies the same half-life associated with all the transitions, including the 263-keV $(6^+\rightarrow 5^-)$ and 547-keV $(5^-\rightarrow 4^+)$ transitions belonging to the negative-parity cascade. The 948- and 1054-keV transitions do not indicate any additional delayed component as is evident from their decay curves (Fig. 4) and the half-life obtained for these transitions is identical to that obtained for their 810-keV in-band partner.

The half-life for the 8^+ isomeric level in ^{90}Mo as obtained from the present measurements is $1.17^{(+0.19)}_{(-0.07)}$ μs. This is the average of the lifetimes obtained for various delayed transitions and incorporating the analyses done with and without mass gating for 0.5-μs- and 1-μs-wide time slices and also from the time spectra of the corresponding delayed transitions. The quoted error indicates the range of individual values obtained from the aforementioned data sets. No additional isomers in this time range have been observed in our measurements for this nucleus. In particular, no evidence was found for the negative-parity isomer conjectured about by De Boer *et al.* [3]. However, the possibility of a non-yrast isomeric state cannot be ruled out since such states are not populated with any significant strength in (HI,*xnyp*)-type reactions. Our measurements confirm the value reported by Hausser *et al.* [4].

The 8^+ isomeric level in ^{88}Zr was also populated in the present investigation. This level deexcites via the 77-, 272-, 399-, 671-, 1057-, and the 1083-keV transitions (Fig. 1). The decay curves for these transitions are illustrated in Fig. 6. The value for $T_{1/2}$ extracted for this isomer is $1.41^{(+0.12)}_{(-0.09)}$ μs, whereas the earlier reported values are 1.75 ± 0.20 μs [1,5] and 1.32 ± 0.025 μs [4]. Our value is, again, in close agreement with the value (1.32 ± 0.025 μs) obtained by Hausser *et al.* [4].

V. SHELL MODEL CALCULATIONS

Extensive shell model calculations for $N\sim 50$ and $Z\sim 38$ nuclei have been reported in previous works [11,12]. The calculations by Brown *et al.* [11] employed the $p_{1/2}$, $g_{9/2}$ model space. The investigations of Ji and Wildenthal [13] and Pattabiraman *et al.* [2] have demonstrated the need for the inclusion of $f_{5/2}$ and $p_{3/2}$ orbitals for adequate description of the observed states even up to $J\sim 12\hbar$ in these nuclei. Rudolph *et al.* [12] have presented detailed shell model calculations in these nuclei using the small $\pi(p_{1/2},g_{9/2})$ valence space as well as the enlarged space including the $\pi(p_{3/2},f_{5/2})$ orbitals. However, considerable deviations exist between the predicted and the observed lifetimes for these microsecond isomers.

We have performed detailed shell model calculations for the $N=48$ ^{86}Sr, ^{88}Zr, and ^{90}Mo nuclei to explore the systematic behavior, if any, of the configurations associated with these isomers. We have performed shell model calculations using the code OXBASH [14] and with the use of the model space comprised of the $\pi(1f_{5/2},2p_{3/2},2p_{1/2},1g_{9/2})$ and $\nu(2p_{1/2},1g_{9/2},1g_{7/2},2d_{5/2},3s_{1/2})$ orbitals with ^{66}Ni as the inert core. We have used the GWBXB set of (1+2)-body ma-

TABLE I. Experimental vs theoretical γ ray transition strengths for 8^+ isomers in $N=48$ isotones.

Nucleus	$J_i \rightarrow J_f$	$\tau(J_i)$	E_γ (keV)	Branching ratio (%)	$B(E2)_{expt}$ (e^2 fm^4)	$B(E2)_{theory}$ (e^2 fm^4)	e_p^{eff}	e_n^{eff}
^{86}Sr	$8^+ \rightarrow 6^+$	577 ± 58 ns[a]	98.7[b]	100	73 ± 7	74.2	1.5	0.65
^{88}Zr	$8^+ \rightarrow 6^+$	$2.03^{(+0.18)}_{(-0.13)}$ μs[c]	77[c]	100	$38.15^{(+2.61)}_{(-3.10)}$	38.85	1.5	0.75
^{90}Mo	$8^+ \rightarrow 6^+$	$1.69^{(+0.27)}_{(-0.10)}$ μs[c]	63.1[c]	100	$65.58^{(+4.12)}_{(-8.98)}$	63.6	1.5	0.85

[a]Reference [17].
[b]Reference [18].
[c]Present experiment.

trix elements. These calculations have been performed in a truncated model space; details of the truncation procedure are given in Ref. [15].

Comparisons between the experimental and theoretical excitation energies for the positive-parity yrast states up to $J=8^+$ in these nuclei are presented in Fig. 1. As seen from the figure the agreement between the two is quite satisfactory. The results of the shell model calculations could be summarized as follows: (a) The energy spacing between the isomeric 8^+ level and the 6^+ level is well reproduced. (b) The ground state of these nuclei is fragmented with significant contribution ($\sim$15%) from $(p_{3/2})^{-n}$ configurations. (c) Although the $\pi(p_{1/2}, g_{9/2})^n$ configurations dominate ($\sim$80%) both the 8^+ and 6^+ levels, the wave function has a contribution of $\sim$10% from the $\pi[(f_{5/2})^6, (p_{3/2})^{-2}]$ configurations. (d) The ground state is dominated primarily by seniority 0 configurations ($\sim$85%), with individual contributions from seniority 2 and 4 configurations being less than 2%. (e) The 8^+ isomeric level and the 6^+ level in ^{90}Mo are dominated by seniority 0 configuration ($\sim$90%) while the individual contribution from seniority 2 and 4 was $\sim$2%. (f) Similar results were also observed for the 8^+ and 6^+ levels in ^{88}Zr. (g) In ^{86}Sr, the 8^+ level is dominated by the seniority 0 configuration ($\sim$84%), with the $(p_{3/2})^{-1} \otimes (g_{9/2})^1$ seniority 2 configuration contributing ($\sim$10%) to the wave function. (h) The 6^+ level has a contribution of about 18% from the seniority 2 $(p_{3/2})^{-1} \otimes (g_{9/2})^1$ configuration. Table I summarizes the experimental and theoretical transition probabilities for these nuclei. As seen from the table, the agreement between the experimental and theoretical $B(E2)$'s is excellent with the derived effective charges (the comparison has been done with respect to the central values). As it happens, just a single set of effective charges, viz., $e_p=1.5e$ and $e_n=0.75e$ can reproduce fairly well the experimental $B(E2)$ values (within the error limits) for the three aforementioned $N=48$ isotones. A value of $e_p=(1.2-1.7)e$ also results in a reasonable fit to the experimentally observed $B(E2)$ values. This also preserves the systematic behavior in that as one increases the proton number from Sr to Mo the required effective neutron charge increases by $\sim 0.05e$ (for a given value of e_p lying between $1.2-1.7e$). This set of effective charges, within this model space, can bring theoretical $B(E2)$ values in good agreement with the experimental results. Unlike the case in the $N=50$ isotones [16], we find that the proton effective charge is almost insensitive to the increase in proton number for $N=48$ isotones.

VI. CONCLUSIONS

The recoil-isomer tagging technique has been employed to obtain the half-lives of a few μs isomers in the $N=48$ nuclei, ^{88}Zr and ^{90}Mo. The extracted lifetimes are $1.41^{(+0.12)}_{(-0.09)}$ and $1.17^{(+0.19)}_{(-0.07)}$ μs, respectively, for the ^{88}Zr and ^{90}Mo. These values are in agreement with the values previously reported by Hausser *et al.* from their magnetic moment measurements. The effective charges (e_p, e_n) obtained by fitting the theoretical $B(E2)$ values to the observed experimental $B(E2)$ shows that both e_p and e_n are influenced substantially by the occupation of the $\pi(p_{1/2})$ and $\nu(g_{9/2})$ orbitals as well as the $\pi(p_{3/2}, f_{5/2} \rightarrow p_{1/2})$ excitations.

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