

High-lying single-particle modes, chaos, correlational entropy, and doubling phase transitionChavdar Stoyanov¹ and Vladimir Zelevinsky²¹*Institute for Nuclear Research and Nuclear Energy, Tzarigradsko Chaussee 72, 1784 Sofia, Bulgaria*²*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-1321, USA*

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Highly excited single-particle states in nuclei are coupled with the excitations of a more complex character, first of all with collective phononlike modes of the core. In the framework of the quasiparticle-phonon model, we consider the structure of resulting complex configurations, using the $1k_{17/2}$ orbital in ^{209}Pb as an example. Although, on the level of one- and two-phonon admixtures, the fully chaotic Gaussian orthogonal ensemble regime is not reached, the eigenstates of the model carry a significant degree of complexity that can be quantified with the aid of correlational invariant entropy. With artificially enhanced particle-core coupling, the system undergoes the doubling phase transition with the quasiparticle strength concentrated in two repelling peaks. This phase transition is clearly detected by correlational entropy.

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I. INTRODUCTION

It is now firmly established that many-body quantum dynamics acquire typical features of quantum chaos in the region of high level density. The interrelation between many-body physics and quantum chaos was studied, in particular, in the framework of the nuclear shell model with realistic and random interactions, see Refs. [1–4], and references therein. A description of a stable many-body system starts with the mean field and quasiparticles, determined by the symmetry of the mean field. As excitation energy and level density increase, residual interactions convert stationary states of independent quasiparticles into exceedingly complex superpositions of many basis states. The many-body structure at this stage should be described in statistical terms, and in the limiting case of extreme chaos it locally approaches the predictions of the Gaussian orthogonal ensemble (GOE) of random matrices.

The complexity of stationary many-body states can be quantified with the aid of such characteristics as information entropy or inverse participation ratio [2,3,5,6] of generic wave functions. These quantities smoothly change with the excitation energy, revealing strong mixing of original states and a loss of simple quantum numbers, typical features of quantum chaos. However, such measures of complexity are unavoidably associated with a starting basis, which in these cases is determined by the mean field. They actually provide the localization length of a complex state in a chosen basis with no information on the correlations still present in the wave function.

Different characteristics can be used in order to probe the sensitivity of the system to external perturbations. In classical cases, such sensitivity is known to be the main property of dynamical chaos. A special entropylike quantity, called *invariant correlational entropy* (ICE), was suggested in Ref. [7] as a measure of complexity related to the response of a system to a random noise included in the Hamiltonian. This quantity is, by construction, invariant with respect to the basis transformations. It also reflects the correlations and phase relationships between the components of the wave functions

as far as they are revealed in the response of the system to a perturbation. Taking the strength of the interaction as a control parameter, one can clearly see the quantum phase transition as it was demonstrated in the interacting boson model [8] and in the evolution of pairing in the shell model [9].

The problem of the interaction of a quasiparticle with an even-even core is of considerable interest for nuclear physics. It was quite well studied for low-lying excited states where the interactions of the quasiparticle predominantly with quadrupole and octupole surface vibrations are important. At higher excitation energies, a large number of other nuclear modes influence the damping of the quasiparticle motion. The mixing of the simple mode with the states of the next levels of complexity leads to the fragmentation of the single-particle strength over a wide domain of excitation energy—the single-particle state obtains a spreading width.

It was shown in Ref. [10] that the spreading occurs in two stages. At the first stage of fragmentation, the single-particle state is spread over several doorway states. At the second step, the doorway states are spread through the mixing with many complex excitations related to other degrees of freedom. The limiting case of the self-consistent hierarchy of multistep spreading was discussed in Ref. [11].

The highly excited single-particle mode has been experimentally studied via one-nucleon transfer reactions [10]. For example, broad structures located around a 10 MeV excitation energy have been observed in the reaction $^{208}\text{Pb}(\alpha, ^3\text{He})^{209}\text{Pb}$. It was shown [12] that the structures are connected with the excitation of high-spin orbitals $1j_{13/2}$, $2h_{11/2}$, and $1k_{17/2}$. The fragmented wave functions based on these states contain many components. The related complexity of the wave function was analyzed in Ref. [13]. It was concluded that the damping of a simple mode cannot be unambiguously associated with the random contribution of more complex components to the structure of the states.

The goal of the presented study is to introduce a new measure of the complex structure of highly excited states. The paper is organized as follows. In Sec. II the formalism of correlation entropy is introduced. In Sec. III the main fea-

tures of the used model for the structure of excited states are presented. The complexity of the excited states and the connection to the ICE is discussed in Sec. IV. The doubling phase transition in the regime of strong coupling in odd nuclei is demonstrated in Sec. V. Finally, in Sec. VI, the conclusions are drawn.

II. CORRELATIONAL ENTROPY

To study the sensitivity of the excited states to variation of external parameters, we use the ICE [7]. The ICE method presumes that Hamiltonian $H(\lambda)$ of a system depends on a random parameter λ . The parameter λ (“noise”) is considered as a member of an ensemble characterized by the normalized distribution function $\mathcal{P}(\lambda)$,

$$\int d\lambda \mathcal{P}(\lambda) = 1. \quad (1)$$

In an arbitrary primary basis $|k\rangle$, we follow the evolution of any stationary state $|\alpha; \lambda\rangle$ as a function of λ . At a given value of λ , the state can be decomposed as

$$|\alpha; \lambda\rangle = \sum_k C_k^\alpha(\lambda) |k\rangle. \quad (2)$$

The ICE is defined as

$$S^\alpha = -\text{Tr}\{\varrho^\alpha \ln(\varrho^\alpha)\}, \quad (3)$$

where ϱ^α is the density matrix of the state $|\alpha\rangle$ averaged over the noise ensemble. In the basis $|k\rangle$ prior to the averaging, we first construct this matrix for a given value of λ as

$$\varrho_{kk'}^\alpha(\lambda) = C_k^\alpha(\lambda) C_{k'}^{\alpha*}(\lambda), \quad (4)$$

and then average over the ensemble,

$$\varrho_{kk'}^\alpha = \int d\lambda \mathcal{P}(\lambda) \varrho_{kk'}^\alpha(\lambda). \quad (5)$$

While the density matrix ϱ^α , Eq. (4), of a pure state, being a projector onto the state $|\alpha\rangle$ and having correspondingly only one nonzero eigenvalue equal to one, leads to zero entropy S^α , the averaged density matrix (5) has its eigenvalues between 0 and 1 and produces nonzero correlational entropy.

In contrast to the information (Shannon) entropy I^α of the same state $|\alpha\rangle$, conventionally used for quantifying the complexity,

$$I^\alpha = -\sum_k |C_k^{(\alpha)}|^2 \ln(|C_k^{(\alpha)}|^2), \quad (6)$$

the ICE is basis-independent von Neumann entropy that reflects the correlations between the wave-function components that are subject to fluctuations determined by the parameter λ . The value S^α for a given state typically increases with the complexity of the state and reaches the maximum at the point where the change of the parameter around some average value implies the most radical change of the structure of the system. Such a point (in fact, a region in finite systems) can be identified with the quantum phase transition or crossover, and in the vicinity of this point, the structural

fluctuations of the wave functions are strongly enhanced. With further change of the average value of the noise parameter, the ICE, as a rule, goes down. This means that the stronger interaction has established a new order with greater rigidity with respect to fluctuations of parameters. In this way, the sharp boundaries between the different symmetry classes were confirmed [8] in the interacting boson models, and the critical strengths of isovector and isoscalar pairing in the sd shell model were established [9].

III. THE MODEL OF EXCITED STATES IN ODD NUCLEI

We adopted the quasiparticle-phonon model (QPM) by Soloviev [14] to describe the properties of highly excited states in odd spherical nuclei. According to the model, the Hamiltonian of the system of an odd number $(A+1)$ of particles has the form

$$H = h + H_{\text{core}} + H_{\text{coupl}}. \quad (7)$$

The first term, h , describes the motion of a quasiparticle in a mean-field potential U created by the even-even core,

$$h = -\frac{1}{2m} \nabla^2 + U. \quad (8)$$

The core Hamiltonian is a sum of single-particle Hamiltonians h_i and two-body residual interactions $V_{i,j}$,

$$H_{\text{core}} = \sum_{i=1}^A h_i + \sum_{i<j}^A V_{i,j}. \quad (9)$$

The last term in Eq. (7) is a sum of interactions between the odd quasiparticle and particles in the core,

$$H_{\text{coupl}} = \sum_{i=1}^A V_{0,i}. \quad (10)$$

In the spirit of the QPM, the Hamiltonian H_{core} is treated in the random-phase approximation (RPA), i.e., the particle-hole configurations are built with the subsequent RPA diagonalization.

The properties of the $(A+1)$ -nucleus can be described in terms of the quasiparticle states $\alpha_a^\dagger|0\rangle$, quasiparticle-plus-phonon states $[\alpha_a^\dagger \otimes Q_\nu^\dagger]|0\rangle$, and quasiparticle-plus-two-phonon states $[\alpha_a^\dagger \otimes [Q_\mu^\dagger \otimes Q_\nu^\dagger]]|0\rangle$, where all combinations have the same total spin and parity quantum numbers $J^\pi M$. Here, α_a^\dagger is the quasiparticle creation operator with shell-model quantum numbers $a \equiv (n, l, j, m)$, whereas $Q_\nu^\dagger \equiv Q_{\lambda\mu}^\dagger$ denotes the phonon creation operator with the angular momentum λ , projection μ , and RPA root number i .

The following wave functions describe, in the QPM, the ground and excited states of the odd nucleus with angular momentum J and projection M :

$$\Psi_{JM}^\alpha = C_J^\alpha \left\{ \alpha_{JM}^\dagger + \sum_{av} D_a^\nu(J\alpha) [\alpha_a^\dagger Q_\nu^\dagger]_{JM} + \sum_{avv'} F_a^{\nu\nu'}(I; J\alpha) [\alpha_a^\dagger [Q_\nu^\dagger Q_{\nu'}^\dagger]_{I}]_{JM} \right\} \Psi_0. \quad (11)$$

The wave function Ψ_0 represents the ground state of the

neighboring even-even nucleus (also the quasiparticle and phonon vacuum), and α stands for the number within a sequence of states of given J^π . The coefficients C_J^α , $D_a^\nu(J\alpha)$, and $F_a^{\nu\nu'}(I;J\alpha)$ are the quasiparticle, quasiparticle + phonon and quasiparticle + two phonons amplitudes, respectively, for the state α . The norm of the wave function (11) reads

$$\langle \Psi_{JM}^{\alpha*} | \Psi_{JM}^\alpha \rangle = (C_J^\alpha)^2 \left\{ 1 + \sum_{av} [D_a^\nu(J\alpha)]^2 + 2 \sum_{avv'} [F_a^{\nu\nu'}(I;J\alpha)]^2 \right\}. \quad (12)$$

The Hamiltonian (7) contains several parameters. The mean field (8) was chosen to be the Woods-Saxon potential. The residual interaction (9) in the particle-hole channel was taken in a separable form with interaction strengths in each mode considered as adjusted parameters. The quasiparticle-phonon coupling term (10) does not contain any additional freedom. The core excitations with all spins and natural parity, 1^- , 2^+ , 3^- , 4^+ , 5^- , 6^+ , 7^- , and 8^+ , were included in the calculations. For each angular momentum and parity, the RPA states of up to a 20 MeV excitation energy were taken into account. The large phonon basis is necessary for a correct description of the single-particle strength distribution in a broad range of excitation energy.

The used set of parameters have been successfully applied to describe the properties of low-lying as well as highly excited states in ^{209}Pb , see for example Ref. [15]. For studying the ICE, we selected the single-particle $1k_{17/2}$ state. The properties of this state are studied in detail in Ref. [15]. This orbital is quasibound in the Woods-Saxon potential being located at 4.88 MeV, a much higher energy than the Fermi level of ^{209}Pb . Because of its high energy, the state is surrounded by many quasiparticle-plus-phonon and quasiparticle-plus-two-phonon states.

IV. COMPLEXITY OF STATES AND CORRELATIONAL ENTROPY

At a high excitation energy, the level density is large and it is convenient to calculate the single-particle strength distribution by means of the strength function [10] using an averaging Lorentzian function. The distribution of the single-particle strength [coefficient C^2 of Eq. (11)] of the $1k_{17/2}$ state is shown in Fig. 1. The number of quasiparticle-plus-phonon components included in the wave function (11) is 420, while the number of quasiparticle-plus-two-phonon components is 1116. The Lorentzian smoothing parameter was chosen to be 0.2 MeV.

It is seen from Fig. 1 that the single-particle strength is spread over a broad interval of excitation energies. The largest fraction, 81% of the strength, is concentrated between 5 and 13 MeV excitation energy, and the state acquires the large spreading width in this domain, $\Gamma^\downarrow = 1.5$ MeV (Γ^\downarrow is calculated as the second moment of the distribution). It was shown [10,16] that the main mechanism leading to the spreading is the interaction of the particle with the vibrations of the even core.

In the case of ^{209}Pb , the lowest core excitation is in the 3_1^- state. The component $[1j_{15/2} \otimes 3_1^-]_{17/2^+}$ dominates in the

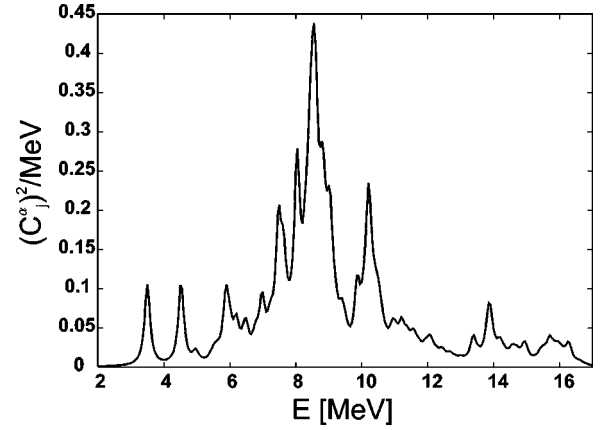


FIG. 1. The distribution of the single-particle strength of the state $1k_{17/2}$ in ^{209}Pb .

structure of the lowest two excited states, where 8% of the single-particle strength is concentrated. The component $[2g_{9/2} \otimes 4_1^+]_{17/2^+}$ also contributes to the structure of these excited states. The contribution of the components, where the particle is coupled to 2_1^+ , 4_1^+ , and 6_1^+ phonons, is more important at the higher part of the distribution (above 13 MeV). The coupling of particles with 5^- phonons is significant in the domain of the main peak, where the coupling with the 3_1^- phonon is also important [15]. The 7^- and 8^+ phonons mainly contribute at excitations around 10 MeV [15]. More complex quasiparticle-plus-two-phonon components influence predominantly the secondary fragmentation of quasiparticle-plus-phonon components.

The correlation entropy (3) connected with the excited states was calculated using two types of the distribution function, the Lorentzian function,

$$\mathcal{P}(\lambda) = \frac{1}{2\pi} \frac{\Delta}{\lambda^2 + \Delta^2/4}, \quad (13)$$

and normal (Gaussian) distribution,

$$\mathcal{P}(\lambda) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\lambda^2}{2\sigma^2}\right). \quad (14)$$

The value of the parameters used in the calculations was $\Delta = 0.8$ and $\sigma = 0.5$. For this choice of parameters, both distributions have the same maximum value.

The calculation was done as follows. For the values of $\lambda = \lambda_c$, the matrix elements of H_{coupl} , Eq. (10), are multiplied by the corresponding value of $\mathcal{P}(\lambda_c)$. The new matrix elements are used to calculate the eigenvalues and eigenvectors of the Hamiltonian (7). The elements of the density matrix $\rho_{kk'}^\alpha(\lambda_c)$ are calculated according to the Eq. (4) and averaged, Eq. (5). In the case of the Lorentzian distribution, the value of λ_c is changed in the interval $[-3\Delta, 3\Delta]$, while in the case of the normal distribution function, λ_c is taken from the interval $[-2.5\sigma, 2.5\sigma]$.

To estimate quantitatively the absolute value of the ICE, we also use the summed correlational entropy of N states,

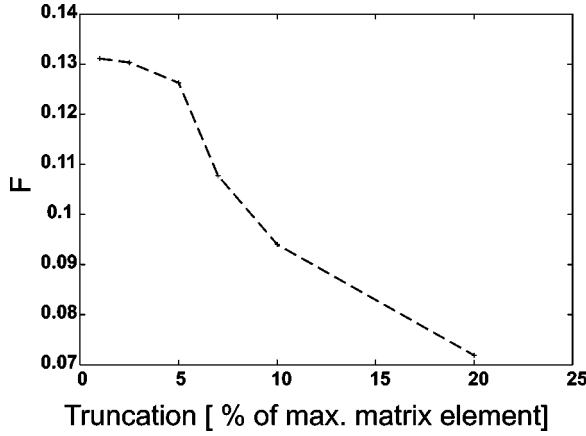


FIG. 2. The dependence of correlation entropy on the truncation of the quasiparticle-plus-phonon basis.

$$F(N) = \sum_{\alpha=1}^N S^{\alpha}, \quad (15)$$

where α labels the eigenstates in the sector with given quantum numbers. The normalized value of $F(N)$ is defined as

$$F = \frac{1}{N} F(N). \quad (16)$$

The wave function (11) includes many components. Because of the different structure of the components, the values of the matrix elements of H_{coupl} reveal large fluctuations. There are many small matrix elements and the corresponding components are weakly correlated. Their contribution to the spreading process has to be negligible. The ICE gives the opportunity to construct the appropriate basis including only the most important components. The dependence of the ICE on the dimension of the basis is presented in Fig. 2. The basis is truncated according to the criterion connected with the maximum matrix element. All components having the matrix element of the coupling with the original single-particle state less than a certain fraction of the maximum matrix element, are not included in the calculation. Here, only the one-phonon components $[\alpha_a^{\dagger} \otimes Q_{\nu}^{\dagger}] \langle 0 \rangle$ of the wave function (11) are taken into account. It is seen that the normalized entropy, Eq. (16), reveals saturation for the small values of the truncation parameter. The components, whose matrix element is less than 5% of the maximum one, contribute weakly to F . The number of such components is large, but their influence on the spreading process is negligible.

The results for the ICE found with the two distribution functions (13) and (14) are compared in Fig. 3. Figure 3(a) displays the correlation entropy of the obtained states in the case of the normal distribution, Eq. (14). There is a background of small values of the ICE due to the large amount of weakly interacting states. The greater values of entropy are located in the vicinity of the peaks of the single-particle strength distribution pointing to the regions where the proximity of many strongly coupled states leads to an enhanced sensitivity of the wave functions. One can see the tendency

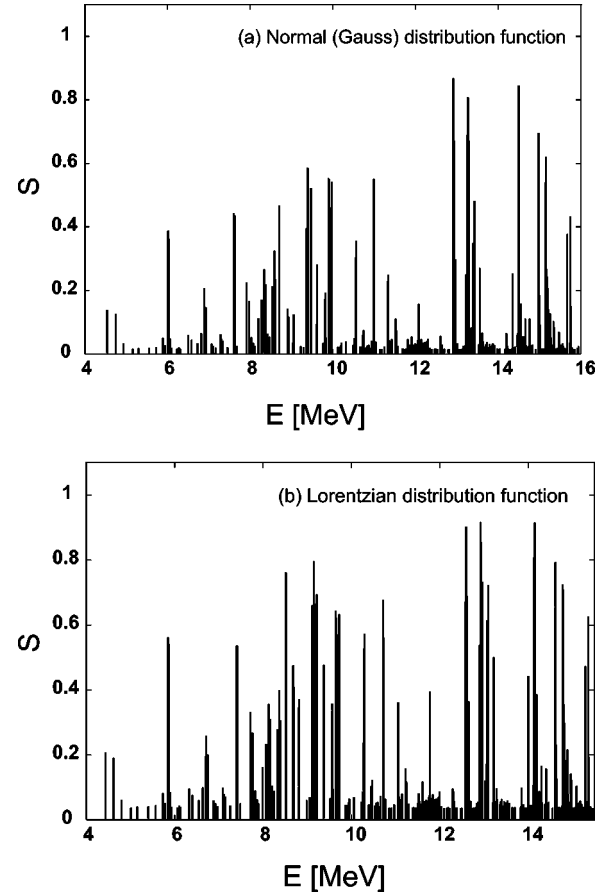


FIG. 3. The correlation entropy of the excited states $17/2^+$ in ^{209}Pb . (a) The normal (Gaussian) distribution function is used in the calculation for the upper panel, and (b) the Lorentzian distribution is used for the lower panel. The wave function includes only quasiparticle-plus-phonon components.

to the enlargement of entropy when the excitation energy increases and a larger density of the states at a higher excitation energy enters the game.

The calculation of the ICE with the Lorentzian distribution function shown in Fig. 3(b) leads to results similar to those in Fig. 3(a). The regions including the enhanced density of states with large entropy appear at the same excitation energy. In the case of the normal distribution the value of $F(N)$, Eq. (15), is 37.50, while for the Lorentzian it is 54.44, mainly because the Lorentzian has the long tails. In spite of this difference in the value of $F(N)$, the main features of both distributions of the ICE are similar being determined mostly by the interaction and density of states, and only marginally by the type of the noise distribution function.

To test the sensitivity of the results to the parameters of the model, we introduce a new parameter k . Multiplying the matrix elements of H_{coupl} [Eq. (10)] by this number, we vary the overall strength of the particle-core coupling. The value of $k=1$ corresponds to the realistic strength of H_{coupl} , chosen according to the initially fitted parameters. This case is presented in Fig. 3. Only the components $[\alpha_a^{\dagger} \otimes Q_{\nu}^{\dagger}] \langle 0 \rangle$ of the wave function (11) are taken into account.

The absolute value of entropy $F(N)$, Eq. (15), depends on the value of k , i.e., $F(N) \Rightarrow F(N, k)$; the function $F(N, k)$ is

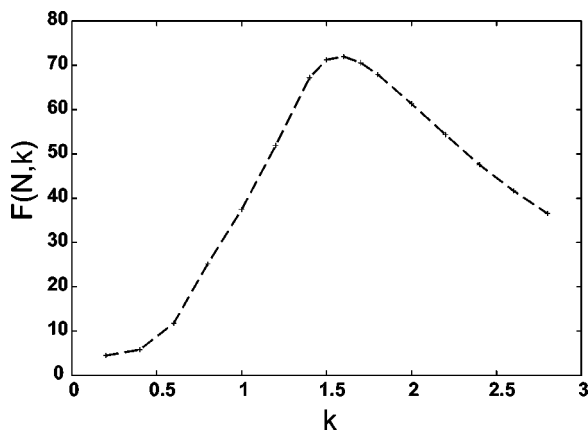


FIG. 4. The dependence of the integral correlation entropy on the strength of particle-core coupling. Only the components $[\alpha_a^\dagger \otimes Q_\nu^\dagger]|0\rangle$ of the wave function (11) are taken into account.

shown in Fig. 4 revealing a pronounced maximum at $k_c = 1.6$. The maximum of $F(N,k)$ at $k=k_c$ can be identified with a quantum phase transition smeared, as expected, in a finite system. The single-particle state interacts strongly with the sea of more complex excitations, and its contribution to the structure of any individual excited state is strongly reduced. But it has to be pointed out that even for the case of maximum entropy, $k=1.6$, the single-particle component preserves its dominance in the main peaks of the strength distribution. When the more complex quasiparticle-plus-two-phonon components are included in the wave function (11), the correlation entropy rapidly increases. The value of normalized entropy [Eq. (16)] for the case presented in Fig. 1 is larger than the maximum value shown in Fig. 2 by a factor close to 6. The additional correlations are mainly due to the coupling of quasiparticle-plus-one- and quasiparticle-plus-two-phonon components. Because of this hierarchy of couplings, the distribution of the single-particle strength is not affected too much by the new terms in the wave function (11).

For small values of k , the mixing of states is suppressed and the ICE decreases rapidly, see Fig. 4. This case corresponds to the weak particle-core coupling and relatively simple wave function. The strength of single-particle states is distributed in a narrow vicinity of its unperturbed energy. The nearest-neighbor level spacing distribution for $k=0.2$ is very close to the Poisson distribution. In the case where only quasiparticle-plus-phonon components are included in the wave function (11), even at $k=1$ the correlations are quite large, see Fig. 4. This influences the level spacing distribution of $17/2^+$ states. The nearest-neighbor spacing distribution of $17/2^+$ excitations can be fit by the Brody distribution [1] with the Brody parameter equal to 0.4. At this stage, the single-particle component is not completely smeared and chaotically distributed over the excitations of the system. Clear remnants of the simple excitation seen in the complicated wave functions are similar to the phenomenon of scars [17] existing in simple quantum systems.

The nearest-neighbor spacing distribution of $17/2^+$ excitations for the case when the quasiparticle-plus-two-phonons components are included in the wave function (11) is shown

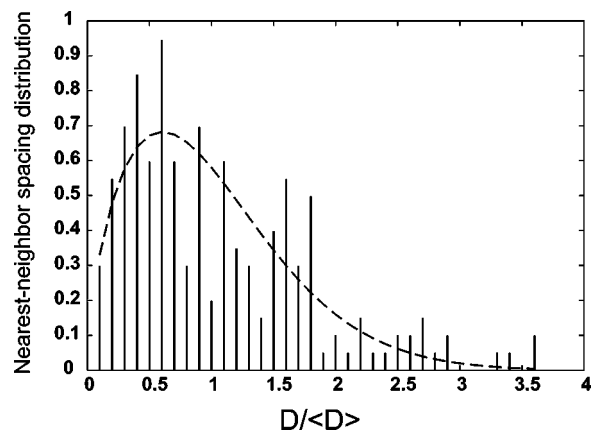


FIG. 5. The nearest-neighbor spacing distribution of $17/2^+$ states in ^{209}Pb . The calculated values are fit by the Brody distribution with the Brody parameter equal to 0.6.

in Fig. 5. The distribution is calculated for the region of the main single-particle strength, 5–13 MeV, where the density of the states is not changed much. As seen from Fig. 5, the distribution can be described by the Brody distribution with the parameter equal to 0.6. The higher degree of chaoticity indicates the importance of more complex components and their influence on the damping process. At the first stage, when the single-particle states interact only with quasiparticle-plus-one-phonon components, the regularities induced by the mean field, such as the structure of the level density, are not completely destroyed. At the next stage, when the interaction with the components of the next level of complexity is switched on, these regularities are partly smeared, but the wave function (11) is still relatively simple. One can recall old results [23], where the coupling of an unpaired particle with the collective monopole and quadrupole modes was considered in detail (later, these results were applied [24] to the spreading width of giant resonances due to their mixing with low-lying shape vibrations). In exactly solvable models with a particle attached to a single level it was shown [23] that the main effect of the particle-phonon coupling is in creating a coherent state of the phonon field. The chaotic elements should be associated with the mixing between various quasiparticle levels and coupling with different phonon modes.

More multiphonon components are to be taken into account for reaching the full complexity. As the degree of complexity grows, the wave function (11) approaches the realistic wave function necessary to describe the complex structure of high-lying excited states. It has to be pointed out that a few quasiparticle components are weakly influenced by the growth of complexity. By this reason, even a rather simple wave function, truncated, for example, by the quasiparticle and quasiparticle-plus-phonon components, can be used to describe the transition probabilities connecting high-lying and low-lying excited states. For this purpose, a simple practical procedure of averaging high-lying single-particle strength can be used [25].

V. DOUBLING PHASE TRANSITION

For large values of k , the particle-core coupling becomes very strong. The single-particle strength is split into two

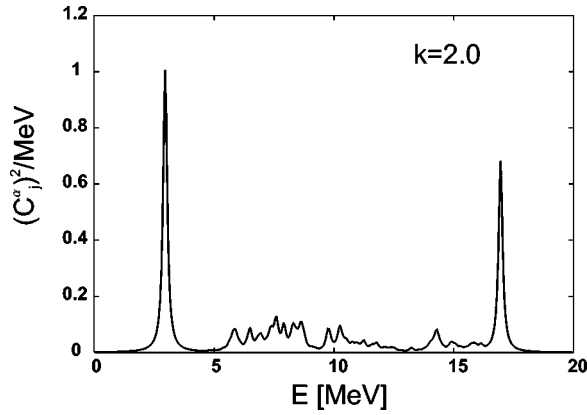


FIG. 6. The distribution of the single-particle strength of the state $1k_{17/2}$ in ^{209}Pb for a large value of particle-core coupling after the doubling phase transition. The value of k is $k=2.0$.

main pieces repelled to low and high excitation energies. This effect is clearly seen for the value of $k=1.6$ (the maximal value of ICE). The strength distribution for $k=2.0$, where the doubling effect is fully pronounced, is shown in Fig. 6. The single-particle strength corresponding to the low and high energy peaks is 31% and 21%, respectively. The rest of the strength is distributed in between the peaks.

To identify the physical nature of the quantum phase transition that occurs at an enhanced particle-phonon coupling, we can analyze the behavior of the strength function of the original single-particle state coupled with states of a more complex character. Such an analysis was first presented in Ref. [18] in application to giant resonances. Our formulation is, in fact, a particular limiting case of a generic problem [19] of a “bright” state coupled to the background that contains a hierarchy of states of increasing complexity. In this consideration, the bright state $|0\rangle$, in our case the bare single-particle state with unperturbed energy E_0 , is mixed with the background states $|\nu\rangle$, that contain, apart from the quasiparticle, the phonon excitations of the core with unperturbed energies ϵ_ν . The eigenstates $|\alpha\rangle$ that emerge from this mixing are complicated superpositions

$$|\alpha\rangle = C_0^\alpha |0\rangle + \sum_\nu C_\nu^\alpha |\nu\rangle. \quad (17)$$

Their energies E_α are the roots of the secular equation

$$X(E) \equiv E - E_0 - \sum_\nu \frac{|V_\nu|^2}{E - \epsilon_\nu} = 0, \quad (18)$$

where V_ν are the matrix elements of coupling of the bright state with the background states. The fragmentation of the strength over the eigenstates $|\alpha\rangle$ is given by

$$|C_0^\alpha|^2 = \left(\frac{dX}{dE} \right)_{E=E_\alpha} = \left[1 + \sum_\nu \frac{|V_\nu|^2}{(E_\alpha - \epsilon_\nu)^2} \right]^{-1}. \quad (19)$$

The strength function of the bright state is found as

$$\mathcal{F}(E) = \sum_\alpha |C_0^\alpha|^2 \delta(E - E_\alpha). \quad (20)$$

If the quantities V^2 , the average value $\langle |V_\nu|^2 \rangle$ of coupling matrix elements squared, and D , the mean-level spacing in the background, can be considered to be weakly fluctuating from one state $|\nu\rangle$ to another, the result of the mixing is determined by the ratio of the “standard” spreading width,

$$\Gamma_s = 2\pi \frac{V^2}{D}, \quad (21)$$

to the energy range $a \approx ND$, where N is a number of effectively interacting background states. Typically, one has a Breit-Wigner strength function [19] with the width (21) for $\Gamma_s \ll a$; the further evolution as a function of increasing coupling leads to the Gaussian shape with the width increasing towards and beyond a [18,20–22]. In the transitional region, the dependence of the strength function upon the coupling intensity V changes from the quadratic to the linear [18,20]. Finally, at even stronger coupling, Eqs. (18) and (19) predict a phase transition to the new situation when the strength is accumulated in two peaks on both sides of the centroid (see Fig. 2 in Ref. [18]). In the extreme limit, the peaks are pushed out of the region of the background states,

$$E \approx E_0 \pm \left[\sum_\nu |V_\nu|^2 \right]^{1/2}, \quad (22)$$

and the strength of the original state, according to Eq. (19), is evenly divided between the peaks.

The physical mechanism of this *doubling phase transition* (known also in quantum optics) is the following. In the regime of strong coupling, the background states of the same symmetry turn out to be intensely interacting among themselves through the bright state. Effectively, this interaction is close to the factorized one built as a product of the matrix elements to and from the bright state. The factorized coupling leads to the formation of the second collective state as a coherent superposition of the background states. This state accumulates a significant strength, and the repulsion between the two collective states leads to the two-peak pattern. The doubling pattern of a similar nature (transfer of the quasiparticle strength to the so-called Hubbard bands) was also found in the Mott metal-insulator phase transition [26]. We see that the doubling phase transition is adequately identified by the maximum of correlational entropy.

VI. CONCLUSION

In the presented study, the recently suggested measure of the complexity was tested for the wave function of a highly excited quasiparticle state in a system where the fermionic quasiparticle strongly interacts with bosonic collective excitations of the core. The new quantity—invariant correlational entropy—was used to estimate the growth of complexity as a result of the admixture of many new components to the wave function of a quasiparticle.

In contrast to information entropy, which displays the degree of complexity of individual states with respect to a certain reference basis, the ICE is basis independent, reflecting mostly the sensitivity of a given state to the external noise. It was shown that the ICE depends mainly on the interaction

strength and density of the background levels, but less on the distribution function used as a noise generator.

We have also calculated the neighboring level spacing distribution as a conventional indicator of quantum chaos. It is shown that this distribution is correlated with ICE, moving in the direction of the Wigner distribution characteristic for the Gaussian orthogonal ensemble (but not reaching this limit), parallel to the increase of the value of ICE. Although the model wave function truncated on the level of the quasiparticle-plus-two-phonon components is not sufficiently chaotic to entirely manifest the complicated structure of high-lying excited states, it can be used to describe the distribution of a few quasiparticle components. These components, in fact, play a role similar to the scars in quantum chaos.

As a function of the overall strength of quasiparticle-phonon interaction, the ICE increases and reaches a pronounced maximum at the coupling strength equal to 1.6 of its realistic value. In this region, the system would undergo the quantum doubling transition when the original single-particle excitation forms its own counterpart built of more complicated states with the same quantum numbers, and the two peaks repel each other and share the original strength. The ICE properly reflects this transformation.

The question remains whether such a phenomenon, known in quantum optics and condensed matter physics, could be observed in real nuclear spectra. Our estimates show that in near-magic nuclei, where the single-particle

modes are well studied, the coupling of quasiparticles with collective modes is strong enough to make the resulting wave functions sufficiently complex, but it is still weaker than would be needed for the doubling transition. Especially interesting would be a search in nuclei with soft vibrational modes, near the instability of a spherical shape, where the quasiparticles strongly interact with collective degrees of freedom. Another manifestation of analogous physics can be seen in low-lying “pigmy”-branches of giant resonances, which borrow the strength from the main peak and become more pronounced in loosely bound nuclei [27]. Here, the driving force is the strong interaction with and through the continuum.

Similar calculations have been also performed for the excited neutron deep-hole orbital $1h_{11/2}$ in ^{207}Pb and high-lying proton orbital $1i_{13/2}$ in ^{145}Eu . The obtained results are in agreement with the above conclusions, confirming the generic features of underlying physics.

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