

## Spin observables and the determination of the parity of $\Theta^+$ in photoproduction reaction

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Spin observables in the photoproduction of the  $\Theta^+$  are explored for the purpose of determining the parity of the  $\Theta^+$ . Based on reflection symmetry in the scattering plane, we show that certain spin observables in the photoproduction of the  $\Theta^+$  can be related directly to its parity. We also show that measurements of both the target nucleon asymmetry and the  $\Theta^+$  polarization may be useful in determining the parity of  $\Theta^+$  in a model-independent way. Furthermore, we show that no combination of spin observables involving only the polarization of the photon and/or nucleon in the initial state can determine the parity of  $\Theta^+$  unambiguously.

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The pentaquark  $\Theta^+$  was predicted by Diakonov *et al.* [1] in 1997, in the chiral soliton model, as the lowest member of an antidecuplet of baryons. The recent discovery of this truly exotic baryon [2–8] has triggered an intensive investigation aimed at a determination of its basic properties. Currently, the available data does not allow for the determination of either its spin or its parity. Moreover, theoretical predictions of these quantum numbers, and especially the parity, are largely controversial. For example, the quenched lattice QCD calculations [9] identified the spin 1/2  $\Theta^+$  as an isoscalar negative-parity state (see, however, a recent quenched lattice QCD calculation with exact chiral symmetry [10], where a positive parity is predicted for  $\Theta^+$ ). Also, QCD sum-rule calculations [11] predict a spin 1/2 negative-parity state. In contrast, chiral/Skyrme soliton models [1,12] and many other models [13] predict a spin 1/2 positive-parity isoscalar state. There also exist theoretical studies that explore the possibility of determining the quantum numbers, and especially, the parity of  $\Theta^+(1540)$  experimentally [14–17]. In particular, spin observables, such as photon asymmetry and spin-correlation functions, are shown to be very sensitive to the parity of  $\Theta^+$  [15–17]. However, all these analyses rely on the particular model(s) used. A number of authors have also carried out model-independent analyses aimed at an unambiguous determination of the  $\Theta^+$  parity in both hadronic- [18] and electromagnetic- [19] induced reactions.

In the present work, we perform a model-independent analysis of the  $\gamma N \rightarrow \bar{K}\Theta^+$  reaction and show that certain spin observables can be related directly to the parity of  $\Theta^+$ . We also show that measurements of both the target nucleon asymmetry and the  $\Theta^+$  polarization may be useful in determining the parity of  $\Theta^+$ , unambiguously. Furthermore, we show that no combination of spin observables involving only the polarization of the photon and/or nucleon in the initial state can pin down the parity of  $\Theta^+$  in a completely model-independent way. To obtain these results, we first derive the most general spin structure of the reaction amplitude for both the positive and negative parity  $\Theta^+$ . Here, we extract the spin structure of the reaction amplitude following the method used in Ref. [20], which is based on its partial-wave expansion. The method is quite general and, in principle, can be applied to any reaction process in a systematic way. Usually, the structure of a reaction amplitude is derived, based solely

on symmetry principles; the advantage of the present method is that it yields the coefficients multiplying each spin operator in terms of the partial-wave matrix elements. Details of the derivation will be reported elsewhere. In what follows, we consider the  $\Theta^+$  to be a spin 1/2 baryon. Hereafter, the superscript  $\pm$  on any quantity (other than  $\Theta$ ) stands for the positive (+) or negative (–) parity of  $\Theta^+$ .

For a positive parity  $\Theta^+$ , the reaction amplitude takes the form<sup>1</sup>

$$\hat{M}^+ = F_1 \vec{\sigma} \cdot \vec{\epsilon} + iF_2 \vec{\epsilon} \cdot \vec{n} + F_3 \vec{\sigma} \cdot \hat{k} \vec{\epsilon} \cdot \hat{q} + F_4 \vec{\sigma} \cdot \hat{q} \vec{\epsilon} \cdot \hat{q}, \quad (1)$$

where  $\hat{k}$  and  $\hat{q}$  are unit vectors in the direction of the relative momenta before and after the collision, respectively, and  $\vec{n} \equiv \hat{k} \times \hat{q}$ ;  $\vec{\epsilon}$  stands for the polarization of the incident photon. The coefficients  $F_j$  are linear combinations of the partial-wave matrix elements multiplied by spherical harmonics and weighted with geometrical factors. As such, they are functions of the energy of the system and scattering angle  $\cos(\theta) \equiv \hat{k} \cdot \hat{q}$  only;  $\theta$  is the scattering angle of the kaon relative to the incident photon beam direction,  $\hat{k}$ . The explicit expressions for these coefficients will be given elsewhere. It should be noted that the spin structure given in Eq. (1) is equivalent to that of Ref. [22].

Similarly, for a negative parity  $\Theta^+$ , we obtain

$$\begin{aligned} \hat{M}^- = & iG_1 \vec{\epsilon} \cdot \hat{q} + G_2 \vec{\sigma} \cdot (\vec{\epsilon} \times \hat{q}) + G_3 \vec{\sigma} \cdot (\vec{\epsilon} \times \hat{k}) + G_4 \vec{\sigma} \cdot \hat{k} \vec{\epsilon} \cdot \vec{n} \\ & + G_5 (\vec{\sigma} \cdot \hat{q} \vec{\epsilon} \cdot \vec{n} + \vec{\sigma} \cdot \vec{n} \vec{\epsilon} \cdot \hat{q}). \end{aligned} \quad (2)$$

Quite recently, Zhao and Al-Khalili [15] have also given the spin structure of the reaction amplitude for the case of negative parity  $\Theta^+$ . The structure given above is equivalent to that of Eq. (18) in Ref. [15], except for the term  $\vec{\sigma} \cdot \vec{n} \vec{\epsilon} \cdot \hat{q}$  in Eq. (2), which has not been included in Ref. [15] on the grounds that it is a higher-order contribution. However, this term and

<sup>1</sup>Actually, there is an issue of the parity (more precisely the relative parity) of the kaon not being known. For a recent discussion, see Ref. [21]. Throughout this work, we assume the kaon to be a pseudoscalar meson. If the parity of the kaon happens to be positive, all the results in this work referred to be for positive parity  $\Theta^+$  should be interchanged with those for negative parity  $\Theta^+$ .

the  $\vec{\sigma} \cdot \vec{q} \vec{\epsilon} \cdot \vec{n}$  term contribute with the same coefficient  $G_5 (=iC_4)$  [23].

In what follows,  $\vec{\epsilon}_\perp \equiv \hat{y}$  and  $\vec{\epsilon}_\parallel \equiv \hat{x}$  denote the photon polarization perpendicular and parallel to the reaction plane ( $xz$  plane), respectively. Recall that the reaction plane is defined as the plane containing the vectors  $\vec{k}$  (in the  $+z$  direction) and  $\vec{q}$ , and that  $\vec{k} \times \vec{q}$  is along the  $+y$  direction. Then, from Eq. (1),

$$\begin{aligned}\hat{M}^{+\perp} &= \alpha_y \sigma_y + i\alpha_0 \sin(\theta), \\ \hat{M}^{+\parallel} &= \alpha_x \sigma_x + \alpha_z \sin(\theta) \sigma_z,\end{aligned}\quad (3)$$

where

$$\begin{aligned}\alpha_0 &\equiv F_2, \quad \alpha_x \equiv F_1 + F_4 \sin^2(\theta), \quad \alpha_y \equiv F_1, \\ \alpha_z &\equiv F_3 + F_4 \cos(\theta).\end{aligned}\quad (4)$$

Similarly, from Eq. (2),

$$\begin{aligned}\hat{M}^{-\perp} &= \beta_x \sigma_x + \beta_z \sin(\theta) \sigma_z, \\ \hat{M}^{-\parallel} &= \beta_y \sigma_y + i\beta_0 \sin(\theta),\end{aligned}\quad (5)$$

where

$$\beta_0 \equiv G_1, \quad \beta_x \equiv G_5 \sin^2(\theta) + G_3 + G_2 \cos(\theta),$$

$$\beta_y \equiv G_5 \sin^2(\theta) - G_3 - G_2 \cos(\theta), \quad \beta_z \equiv G_5 \cos(\theta) + G_4 - G_2.\quad (6)$$

Equations (3) and (5) exhibit an interesting feature, in that the Pauli spin structure of  $\hat{M}^{+\perp}$  is the same as that of  $\hat{M}^{-\parallel}$ , while the structure of  $\hat{M}^{+\parallel}$  is the same as that of  $\hat{M}^{-\perp}$ . This is a consequence of reflection symmetry. Ultimately, we will exploit this feature to construct spin observables which can determine the parity of  $\Theta^+$  unambiguously.

We first consider the spin observables involving only the polarization of the photon and/or nucleon in the initial state, for they are more easily measured than observables involving the spin of  $\Theta^+$ . For a given photon polarization  $\vec{\epsilon}_\lambda$ , and target nucleon spin in the  $i$  direction ( $i=x, y, z$ ), the corresponding spin-correlation coefficient  $A_i^\lambda$  can be expressed as

$$\sigma^\lambda A_i^\lambda = \frac{1}{2} \text{Tr}[\hat{M}^\lambda \sigma_i \hat{M}^{\lambda\dagger}],\quad (7)$$

where  $\hat{M}^\lambda \equiv \sum_{m=0}^3 M_m^\lambda \sigma_m$ , with  $\sigma_0=1$ ,  $\sigma_1=\sigma_x$ , etc., denotes any of the  $\hat{M}^\lambda$  (parity index  $\pm$  suppressed) given in Eqs. (3) and (5). The coefficients  $M_m^\lambda$  can be read off from these equations.  $\sigma^\lambda \equiv \sum_{m=0}^3 |M_m^\lambda|^2$  is the cross section with the polarization of the photon  $\vec{\epsilon}_\lambda$  incident on an unpolarized target. Carrying out the trace in Eq. (7) yields

$$\sigma^\lambda A_i^\lambda = 2 \text{Re}[M_0^\lambda M_i^{\lambda*}] + 2 \text{Im}[M_j^\lambda M_k^{\lambda*}],\quad (8)$$

where the subscripts  $(i, j, k)$  run cyclically, i.e. (1,2,3), (2,3,1), (3,1,2). In terms of individual cross sections  $A_i^\lambda$  may be written as

$$A_i^\lambda = \frac{\sigma_i^\lambda(+)-\sigma_i^\lambda(-)}{\sigma_i^\lambda(+)+\sigma_i^\lambda(-)},\quad (9)$$

where  $\sigma_i^\lambda(+/-)$  denotes the cross section when photons with polarization  $\vec{\epsilon}_\lambda$  are incident on a target nucleon with spin in the (positive/negative)  $i$  direction.

Similarly, the target nucleon asymmetry,  $A_i$ , obtained by using an unpolarized photon beam on a target nucleon polarized in the  $i$  direction, is given by

$$\begin{aligned}\sigma_u A_i &= \frac{1}{2} \text{Tr}[\hat{M} \sigma_i \hat{M}^\dagger] \\ &= \sum_\lambda [2 \text{Re}[M_0^\lambda M_i^{\lambda*}] + 2 \text{Im}[M_j^\lambda M_k^{\lambda*}]] = \sum_\lambda \sigma^\lambda A_i^\lambda,\end{aligned}\quad (10)$$

where  $\sigma_u \equiv \sum_\lambda \sigma^\lambda$  denotes the completely unpolarized cross section; again, the subscripts  $(i, j, k)$  run cyclically. In the above equation, the first equality in the second row follows from Eqs. (3) and (5). In terms of individual cross sections,  $A_i$  may be written as

$$A_i = \frac{\sigma_i(+)-\sigma_i(-)}{\sigma_i(+)+\sigma_i(-)},\quad (11)$$

where  $\sigma_i(+/-)$  denotes the cross section when unpolarized photons are incident on a target nucleon with spin in the (positive/negative)  $i$  direction.

We also consider the (linear) photon asymmetry given by

$$\Sigma \equiv \frac{\sigma^\perp - \sigma^\parallel}{\sigma^\perp + \sigma^\parallel}.\quad (12)$$

Using Eqs. (3) and (5) we find no *model-independent* way of relating the spin observables in Eqs. (8), (10), and (12), which are associated with only a polarized beam and/or target, to the parity of the  $\Theta^+$ . Here, what could happen at best is that, by constraining the kinematics of the reaction, one of these observables might exhibit a markedly different angular dependence for the two choices of the parity of  $\Theta^+$ . Note that when the coefficients  $F_j$  and  $G_j$  in Eqs. (1) and (2) are expanded in partial waves, their angular dependences become explicit. It could also happen that, by constraining the kinematics, one of these observables vanishes for one of the choices of the parity of  $\Theta^+$ . If this is the case, and the corresponding measurement yields a nonvanishing value, we would know the parity of  $\Theta^+$ . We have investigated these possibilities by restricting the reaction to near-threshold kinematics and considering only  $S$  and  $P$  waves in the final state. In this case, the coefficients  $F_4$  and  $G_5$  in Eqs. (1) and (2) vanish, for they only contain partial waves higher than the  $P$  wave in the final state.<sup>2</sup> Unfortunately, none of these three spin observables was found to exhibit the features described above.

The above considerations exhaust the spin observables involving only polarization of the photon and/or nucleon in the

<sup>2</sup> $S$  waves contribute only to the coefficients  $F_1$  and  $G_3$  in Eqs. (1) and (2).

initial state and show that these observables are unable to determine the parity of  $\Theta^+$  in a model-independent way.

We now turn our attention to spin observables which also involve the measurement of the polarization of  $\Theta^+$ . These observables are particularly suited to the use of Bohr's theorem [24]. This theorem is a consequence of the invariance of the transition amplitude under rotation and parity inversion and, in particular, reflection symmetry in the scattering plane, and takes the form [25]

$$\pi_{fi} = (-)^{M_f - M_i}. \quad (13)$$

$\pi_{fi}$  denotes the product of the total intrinsic parity of the initial and final states and  $M_{(fi)}$  denotes the sum of the spin projections in the (final/initial) state along an axis normal to the scattering plane, i.e., the  $y$  (or  $\vec{k} \times \vec{q}$ ) axis. Equation (13) must be satisfied by all parity-allowed transitions. For example, in the present case, if the parity of  $\Theta^+$  is positive, then we must have  $(-)^{M_f - M_i} = +1$ , while if it is negative,  $(-)^{M_f - M_i} = -1$ .

We now exploit the reflection symmetry as manifested in Eq. (13), and consider the (linear) photon asymmetry in conjunction with the polarization transferred from the target nucleon to the  $\Theta^+$ , which is given by

$$\Sigma_{yy}(i, j) \equiv \frac{\sigma_y^\perp(i, j) - \sigma_y^\parallel(i, j)}{\sigma_y^\perp(i, j) + \sigma_y^\parallel(i, j)}, \quad (14)$$

where  $\sigma_y^{(\perp/\parallel)}(i, j)$  stands for the cross section for the photon polarization  $\vec{\epsilon}_{(\perp/\parallel)}$  and the spin orientation  $i$  ( $j$ ) of the nucleon ( $\Theta^+$ ) [up/down as  $i, j = +/-$ ] along the  $y$  axis. As mentioned above, it is easily verified from Eq. (13) that, for the positive-parity case, only spin-aligned transitions ( $i=j$ ) contribute to  $\sigma_y^\perp(i, j)$ , while only the spin antialigned transitions ( $i \neq j$ ) contribute to  $\sigma_y^\parallel(i, j)$ . It follows from Eq. (13) that this feature is just reversed in the case of a negative parity  $\Theta^+$ . [Equations (3) and (5) are consistent with these results as they should be.] As a consequence,

$$\Sigma_{yy}(j, j) = -\Sigma_{yy}(j, -j) = \pi_\Theta, \quad (15)$$

where  $\pi_\Theta$  stands for the parity of  $\Theta^+$ . This result is completely model independent and holds for any kinematic condition.<sup>3</sup> It should be emphasized that the result in Eq. (15) is based on the assumption that the  $\Theta^+$  is a spin-1/2 particle. If the spin of  $\Theta^+$  is regarded as unknown, Eq. (15) takes the more general form  $\Sigma_{yy}(M_f - M_i \text{ even}) = -\Sigma_{yy}(M_f - M_i \text{ odd}) = \pi_\Theta$ . Therefore, it is clear that  $\Sigma_{yy}(M_f - M_i)$  measures di-

<sup>3</sup>These same model (and kinematic) independent considerations can be used to relate the parity of the  $\theta^+$  to spin observables in other reactions. For example, in the  $pp \rightarrow \Sigma^+ \theta^+$  reaction,  $\pi_\theta$  can be determined directly from

$$\frac{\sigma_y(+, +, +, +) - \sigma_y(+, -, +, +)}{\sigma_y(+, +, +, +) + \sigma_y(+, -, +, +)} = \pi_\theta,$$

where  $\sigma_y(ij, kl)$  denotes the cross section with the spin orientations  $i$  and  $j$  of the initial two protons (up/down as  $i, j = +/-$ ) along the  $y$  axis and the spin orientations  $k$  and  $l$  along the  $y$  axis of the outgoing  $\Sigma^+$  and  $\theta^+$ , respectively.

rectly the parity of the  $\Theta^+$  for an arbitrary spin.

Another quantity which is related directly to the parity of  $\Theta^+$  is the spin-transfer coefficient induced by a linearly polarized photon beam,  $K_{ij}^\lambda$ , which is given by

$$\begin{aligned} \sigma^\lambda K_{ij}^\lambda &= \frac{1}{2} \text{Tr}[\hat{M}^\lambda \sigma_i \hat{M}^{\lambda\dagger} \sigma_j] \\ &= (2|M_0^\lambda|^2 - \sigma^\lambda) \delta_{ij} + 2 \text{Re}[M_i^\lambda M_j^{\lambda*}] + 2\epsilon_{ijk} \text{Im}[M_k^\lambda M_0^{\lambda*}], \end{aligned} \quad (16)$$

where  $\epsilon_{ijk}$  denotes the Levi-Civita antisymmetric tensor and  $(i, j, k)$  may take any of the values (1, 2, 3). The diagonal terms reduce to

$$\sigma^\lambda K_{jj}^\lambda = |M_0^\lambda|^2 + |M_j^\lambda|^2 - \sum_{k \neq j} |M_k^\lambda|^2. \quad (17)$$

In terms of the individual cross sections,  $K_{ij}^\lambda$  may be written as

$$K_{ij}^\lambda = \frac{[\sigma_j^\lambda(+, +) + \sigma_j^\lambda(-, -)] - [\sigma_j^\lambda(+, -) + \sigma_j^\lambda(-, +)]}{[\sigma_j^\lambda(+, +) + \sigma_j^\lambda(-, -)] + [\sigma_j^\lambda(+, -) + \sigma_j^\lambda(-, +)]}, \quad (18)$$

where, as before,  $\sigma_j^\lambda(+, -)$ , for example, corresponds to the cross section induced by a photon beam with polarization  $\vec{\epsilon}_\lambda$  on a target nucleon spin in the positive(+)  $j$  direction and leading to the outgoing  $\Theta^+$  spin in the negative(-)  $j$  direction. Given the spin structure of the amplitude, Eq. (18) is often helpful in determining the characteristics of  $K_{ij}^\lambda$ . Exploiting the structure of the amplitudes given in Eqs. (3) and (5), it is straightforward to obtain

$$K_{yy}^\perp = \pi_\Theta, \quad K_{yy}^\parallel = -\pi_\Theta, \quad (19)$$

which are also model-independent results and hold for any kinematic condition. It is also immediate that Eq. (17), together with Eqs. (3) and (5), yields  $K_{xx}^\parallel = \pi_\Theta$  in collinear kinematics or near threshold.<sup>4</sup> Apart from a minus sign, this result corresponds to one of the results obtained recently in Ref. [19], i.e. {Eq. (8) in [19]}.

An alternative way to determine the parity of  $\Theta^+$  is in terms of the spin-transfer coefficient, using an unpolarized photon beam defined, similar to Eq. (16), by

$$\begin{aligned} \sigma_u K_{ij} &= \frac{1}{2} \text{Tr}[\hat{M} \sigma_i \hat{M}^\dagger \sigma_j] \\ &= (2|M_0|^2 - \sigma_u) \delta_{ij} + 2 \text{Re}[M_i M_j^*] + 2\epsilon_{ijk} \text{Im}[M_k M_0^*], \end{aligned} \quad (20)$$

where  $(i, j, k)$  may take any of the values (1, 2, 3) as in Eq. (16). The diagonal terms reduce to

<sup>4</sup>Note that in this kinematic condition, only the terms  $\alpha_{(x/y)}$  and  $\beta_{(x/y)}$  are nonvanishing in Eqs. (3) and (5), for all the coefficients in Eqs. (1) and (2) vanish except  $F_1$  and  $G_3$ .

$$\sigma_u K_{jj} = |M_0|^2 + |M_j|^2 - \sum_{k \neq j} |M_k|^2, \quad (21)$$

with  $|M_i|^2 = \sum_\lambda |M_i^\lambda|^2$ . One can then immediately relate  $K_{yy}$  to the (linear) photon asymmetry given by Eq. (12),

$$K_{yy} = \pi_\Theta \Sigma, \quad (22)$$

which shows that by measuring both the spin-transfer coefficient and photon asymmetry, one can determine the parity of  $\Theta^+$  unambiguously. This relation also holds for any kinematic condition, and it was pointed out recently by Rekaló and Tomasi-Gustafsson [19] as a possible method to pin down the parity of  $\Theta^+$ .

Another way of determining the parity of  $\Theta^+$  is by measuring two double-polarization observables, namely, the spin-correlation coefficient,  $A_i^\lambda$ , given by Eq. (7), and the polarization,  $P_i^\lambda$ , of the outgoing  $\Theta^+$  in the  $i$  direction, induced by a photon beam with polarization  $\vec{\epsilon}_\lambda$ . The latter is given by

$$\sigma^\lambda P_i^\lambda = \frac{1}{2} \text{Tr}[\hat{M}^\lambda \hat{M}^{\lambda\dagger} \sigma_i] = 2 \text{Re}[M_0^\lambda M_i^{\lambda*}] - 2 \text{Im}[M_j^\lambda M_k^{\lambda*}], \quad (23)$$

where the subscripts ( $i, j, k$ ) run cyclically. An analogous relationship to that given by Eq. (9) also holds for  $P_i^\lambda$ , except that the argument (+/-) of  $\sigma_i^\lambda$  now refers to the spin orientation of the outgoing  $\Theta^+$  along the  $i$  direction. Exploiting the feature exhibited in Eqs. (3) and (5), it is straightforward to obtain

$$A_y^\perp = \pi_\Theta P_y^\perp, \quad A_y^\parallel = -\pi_\Theta P_y^\parallel. \quad (24)$$

These results are again completely model independent and hold for any kinematic condition.

Now, the two results in Eq. (24) may be combined to yield

$$\sigma^\perp A_y^\perp - \sigma^\parallel A_y^\parallel = \pi_\Theta \sigma_u P_y, \quad (25)$$

where  $P_i$  denotes the polarization of the outgoing  $\Theta^+$  in the  $i$  direction, induced by an unpolarized photon beam incident on an unpolarized target nucleon; it is given by

$$\begin{aligned} \sigma_u P_i &= \frac{1}{2} \text{Tr}[\hat{M} \hat{M}^\dagger \sigma_i] \\ &= \sum_\lambda (2 \text{Re}[M_0^\lambda M_i^{\lambda*}] - 2 \text{Im}[M_j^\lambda M_k^{\lambda*}]) = \sum_\lambda \sigma^\lambda P_i^\lambda, \end{aligned} \quad (26)$$

where, again, the subscripts ( $i, j, k$ ) run cyclically. The first equality in the second row follows from Eqs. (3) and (5). Note that in Eq. (25), the right-hand side of the equality involves the single polarization observable,  $P_y$ , which may

be easier to measure than the corresponding double polarization observable,  $P_y^\lambda$ .

Yet, another possibility of determining the parity of  $\Theta^+$  is to measure two single polarization observables, namely, the target nucleon asymmetry,  $A_i$ , given by Eq. (10), and the polarization of the outgoing  $\Theta^+$ ,  $P_i$ , given by Eq. (26). Using Eqs. (3) and (5), we now form appropriate combinations of them, giving

$$A_y^+ - P_y^+ = 0,$$

$$A_y^- - P_y^- = 4 \text{Im}[\beta_z \beta_x^*] \sin(\theta) / \sigma_u, \quad (27)$$

for positive and negative parity  $\Theta^+$ , respectively. Again, the above results are completely model independent and hold for any kinematic condition. However, unlike Eqs. (15), (19), (22), (24), and (25), the distinction between the positive and negative parity  $\Theta^+$  is made by exclusion: if the measurement of  $A_y - P_y$  yields a nonvanishing value, the parity of  $\Theta^+$  must be negative. Nothing can be said about its parity, however, if the measurement yields a null value.

Obviously, measurements of any of the spin observables discussed in this work (which can determine the parity of  $\Theta^+$ ) pose an enormous experimental challenge, for they require measuring the spin of  $\Theta^+$  through its decay products  $K+N$ , in addition to the spin of the target nucleon and/or photon. Furthermore, one also needs to consider the background contribution, which may potentially hinder the interpretation of the required measurements, especially if the parity of  $\Theta^+$  happens to be negative [16].

In summary, based on reflection symmetry in the scattering plane as encoded either in Bohr's theorem [Eq. (13)] or in the explicit forms of the scattering amplitudes [Eqs. (3) and (5)], we have demonstrated that some spin observables in  $\Theta^+$  photoproduction can be related directly to the parity of  $\Theta^+$ . In particular, Eqs. (15), (19), (22), (24), and (25) offer ways of providing a model-independent determination of the parity of  $\Theta^+$ . Also, we have shown that measurements of the target nucleon asymmetry and the  $\Theta^+$  polarization induced using an unpolarized photon beam [Eq. (27)] may be useful in determining the parity of  $\Theta^+$  in a model-independent way. Furthermore, we have also shown that, in this reaction, no spin observables involving only the polarization of the photon and/or nucleon in the initial state can determine the parity of  $\Theta^+$  unambiguously. Finally, because of its generality, Bohr's theorem [Eq. (13)] may, of course, be used in a similar way to analyze other reactions induced by photons or other probes.

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