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Evolution of the "β excitation" in axially symmetric transitional nuclei

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The evolution of first excited 0^+ states in transitional nuclei of the $N \approx 90$ region is correlated with the structure of the ground band. Iachello's $X(5)$ solution for the Bohr-Hamiltonian for axially symmetric prolate nuclei is generalized to the transition path between $X(5)$ and the rigid-rotor limit using infinite square-well potentials in the quadrupole deformation parameter β with boundaries at $\beta_M > \beta_m \geqslant 0$. Analytical solutions in terms of Bessel functions of first and second kind are derived and describe well the evolutionary trajectory of the studied 0_2^+ states in this mass region.

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A prime example for deformed quantum systems are atomic nuclei [1–5]. As the number of valence nucleons grows, symmetry breaking to a deformed shape can occur. The evolution of the structure of nuclei in these shape-changing regions has traditionally been the most difficult to understand and predict. This situation improved, recently, when Iachello proposed [6,7] analytical solutions of the Bohr-Hamiltonian eigenvalue problem appropriate for the description of nuclei near the critical points of spherical-to-quadrupole deformed shape phase transitions, called $E(5)$ and $X(5)$, with large fluctuations in the deformation parameter β. The evolution of structure was, however, still left to numerical procedures.

In nuclei formed by even numbers of both protons and neutrons there exist intrinsic $J^\pi = 0^+$ excitations, some related to single-particle degrees of freedom, others due to collective excitations. According to the geometrical model of deformed nuclei, the first collectively excited 0^+ state is often considered a "β vibration," an excitation in the space of the intrinsic quadrupole deformation parameter β with small fluctuations around the mean deformation β_0 assumed to be decoupled from the rotational motion. The rotational band of nuclear states, built on top of the β vibration, is called the β band. A considerable effort has been made to identify the β band in deformed nuclei with, up to now, mostly inconclusive results [8–11]. The excitation energy of the "β vibration" either is an explicit model parameter or enters implicitly through the particular choice of the potential.

Generalized models couple rotational motion to intrinsic excitations [4,5] and relate the latter to the structure of the ground band. Some agreement with data on transitional nuclei was observed, e.g., [12,13]. Iachello's $X(5)$ solution represents an analytical solution for this coupling close to the prolate shape phase-transitional point. It predicts the energy of the 0_2^+ state to be 5.65 times the energy of the first excited 2_1^+ state, i.e., $R_{0/2} = E_x(0_2^+)/E_x(2_1^+) = 5.65$. This matches closely the observation in transitional nuclei with neutron numbers $N \approx 90$, and the discussion of 0_2^+ states' energies became an important part of recent nuclear structure assignments, both for the structure of that state and the levels built on top of it and for the transitional character of a nucleus as a whole [14–17]. The nuclear shape phase transition is a topic of high current interest, e.g., [18–29]. While the recent discussions focused on the phase-transitional point, the *evolution* of 0_2^+ states [11] between the benchmarks of nuclear structure has not received much attention.

It is the purpose of this Rapid Communication to focus on a correlation of the relative excitation energy $R_{0/2}$ to the structural observable $R_{4/2} = E_x(4_1^+)/E_x(2_1^+)$ for nuclei in the $N \approx 90$ shape transitional region. We show that the evolution of the empirical correlation is well described in a parameter-free way by an analytical solution of the Bohr-Hamiltonian eigenvalue problem using an infinite square-well potential over a *confined* range of values for the quadrupole deformation parameter β. This solution interpolates between the phase-transitional point and the rigid-rotor limit in an analytic way, and accounts well for energies and $E2$ rates observed in such nuclei.

Consider the Bohr-Hamiltonian,

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_k \frac{Q_k^2}{\sin^2\left(\gamma - \frac{2}{3}\pi k\right)}\right] + V(\beta,\gamma). \quad (1)$$

Here, β, γ are the quadrupole shape parameters. In analogy to Ref. [7] we consider potentials $V(\beta,\gamma) = v(\beta) + u(\gamma)$ for axially symmetric prolate ($\gamma \approx 0^\circ$) nuclei. We adopt here the same treatment of the γ degree of freedom as in Ref. [7] and focus again on the β degree of freedom in the limit of small fluctuations of γ about $\gamma \approx 0^\circ$. The wave functions approximately separate into $\Psi(\beta,\gamma,\theta_i) = \xi_L(\beta)\eta_K(\gamma)\mathcal{D}^L_{M,K}(\theta_i)$, where $\mathcal{D}$ denotes the Wigner functions with θ_i being the Euler angles for the orientation of the intrinsic system and $\eta_K(\gamma)$ from Ref. [7]. Equation (1) yields the "radial" (in the shape parameters) differential equation,

$$-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} - \frac{1}{3\beta^2}L(L+1) + u(\beta)\right]\xi_L(\beta) = E\xi_L(\beta). \quad (2)$$

For prolate axially symmetric solutions *between* $X(5)$ and the rigid rotor, we consider infinite square-well potentials $u(\beta)$ with boundaries at $\beta_M > \beta_m \geqslant 0$. We allow here for $\beta_m \neq 0$, outside of the cases studied before [6,7,22]. The ratio $r_\beta = \beta_m/\beta_M$ uniquely parametrizes the stiffness of the potential and hence the structural evolution from the transitional point ($r_\beta=0$, large fluctuations in β) to the rigid rotor ($r_\beta \to 1$ no fluctuations in β).

Using the new variables $z=\sqrt{E/(\hbar^2/2B)}\beta$ and $\tilde{\xi}[z] = \beta(z)^{3/2}\xi_L[\beta(z)]$, Eq. (2) transforms into the Bessel equation,

$$\tilde{\xi}'' + \frac{\tilde{\xi}'}{z} + \left[1 - \frac{\nu^2}{z^2}\right]\tilde{\xi} = 0, \qquad (3)$$

with solutions that are Bessel functions, $J_\nu(z)$ and $Y_\nu(z)$ of irrational order $\nu = \sqrt{L(L+1)/3 + 9/4}$, where $Y_\nu(z)$ is the Bessel function of second kind. The general solutions are, thus, superpositions of the corresponding Bessel-J and Bessel-Y functions,

$$\tilde{\xi}_\nu(z) \propto J_\nu(z) + \gamma_Y Y_\nu(z). \qquad (4)$$

The choice of infinite square-well potentials imposes the condition that the wave functions in β must vanish outside of the well and, hence, they have nodes at both β_m and β_M, i.e.,

$$\tilde{\xi}_\nu(r_\beta z_M) = \tilde{\xi}_\nu(z_M) = 0 \qquad (5)$$

for $z_M = \sqrt{E/(\hbar^2/2B)}\beta_M$. These two boundary conditions determine the relative amplitude $\gamma_Y = -[J_\nu(r_\beta z_M)/Y_\nu(r_\beta z_M)]$ of the Bessel-Y function and serve as the quantization condition,

$$Q_\nu^{r_\beta}(z_M) = J_\nu(z_M)Y_\nu(r_\beta z_M) - J_\nu(r_\beta z_M)Y_\nu(z_M) = 0. \qquad (6)$$

For each value of $\nu(L)$ and r_β, the appropriate z_M are obtained as the sth zero, $z_{L,s}^{r_\beta}$, of the function $Q_\nu^{r_\beta}(z)$. The quantum number s counts the number of nodes of the wave function in β for $\beta > \beta_m$.

The normalized eigenfunctions of Eq. (2) are

$$\xi_{L,s}(\beta) = c_{L,s}\beta^{-3/2}[J_\nu(z_{L,s}^{r_\beta}\beta/\beta_M) + \gamma_Y Y_\nu(z_{L,s}^{r_\beta}\beta/\beta_M)], \qquad (7)$$

with the eigenvalues

$$E_{L,s} = \frac{\hbar^2}{2B\beta_M^2}(z_{L,s}^{r_\beta})^2 \qquad (8)$$

and the normalization

$$1/c_{L,s}^2 = \int_{\beta_m}^{\beta_M} \beta^4[\xi_{L,s}(\beta)]^2 d\beta. \qquad (9)$$

The parameter $B\beta_M^2$ defines the energy scale.

The full solution to Eq. (1) with our choice of potentials is given by the wave function,

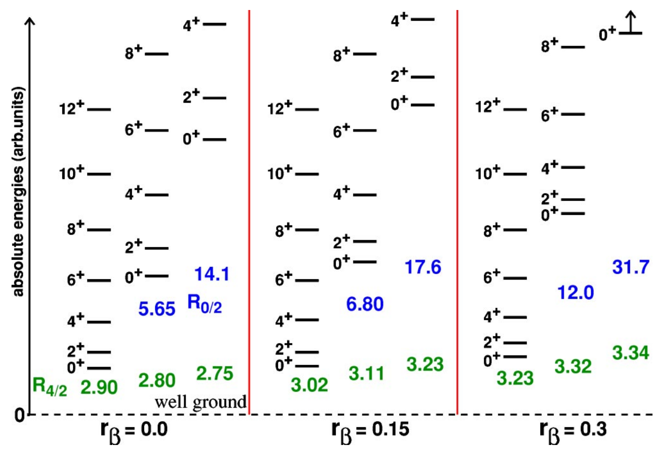

FIG. 1. (Color online) Evolution of energy spectra in the CBS rotor model for three different values of r_β keeping the scale $\hbar^2/2B\beta_M^2$ constant. The structural signatures $R_{4/2}$ (online green) and $R_{0/2}$ (online blue) are given below the bands.

$$|\Psi_{s,n_\gamma,K,L,M}(\beta,\gamma,\theta_i)\rangle = \sqrt{\frac{2L+1}{8\pi}}[\phi_{s,n_\gamma,K,L}(\beta,\gamma)\mathcal{D}^L_{M,K}(\theta_i) + (-)^{L+K}\phi_{s,n_\gamma,-K,L}(\beta,\gamma)\mathcal{D}^L_{M,-K}(\theta_i)], \qquad (10)$$

where $\phi_{s,n_\gamma,K,L}(\beta,\gamma) = \xi_{L,s}(\beta)\eta_K(\gamma)$ with $\xi_{L,s}(\beta)$ from Eq. (7), and $\eta_K(\gamma)$ is the appropriate wave function in the decoupled γ-parameter space [7]. Besides the choice of a finite value of r_β this solution is analogous to the $X(5)$ solution of Iachello [7].

Considering potentials with no structure in β (being β soft) over a range of values *confined* within the variable boundaries of an infinite square well, this approach represents a "confined β-soft" (CBS) rotor model. We denote the 0_2^+ state with $s=2$ as the (first) β *excitation*.

Absolute energies in the CBS rotor model are shown in Fig. 1 for three values of r_β (0, 0.15, 0.3), where $r_\beta=0$ coincides with $X(5)$. Increasing the value of r_β shifts all levels to higher energies since the potential narrows. That energy shift increases with the s quantum number and with decreasing angular momentum. Increasing r_β raises the energies of those wave functions that have larger contributions in the region close to β_m more than others. These are wave functions with many nodes, or those for states with low angular momentum. In the β-soft potential the nucleus gains angular momentum due to centrifugal forces, partially by increasing its rotational moment of inertia rather than its angular velocity, and hence the wave function's center of gravity shifts to larger values of β with increasing L[1], making them less sensitive to the potential at β_m. Therefore, increasing r_β raises the lowest spin states most and shrinks the $2_1^+ - 0_1^+$ energy

[1]Evidence for this model prediction comes from the smooth increase of transitional quadrupole moments along the ground band of a β-soft rotor. The Q_t values for ^{152}Sm, for instance, increase smoothly from 5.9(1) eb for the $2_1^+ \to 0_1^+$ transition to 6.8(3) eb for the $10_1^+ \to 8_1^+$ transition, as can be deduced from Table II.

TABLE I. $R_{4/2}$ values and ground-state band energies in ^{152,154,156}Sm and ^{164}Yb are compared to relevant analytical models, $X(5)$, CBS, and the rigid-rotor model, where those apply. Neither $X(5)$ nor the rigid rotor can competitively describe ^{164}Yb with an $R_{4/2}$ value in the middle between the $X(5)$ and the rotor predictions.

	^{152}Sm			^{154}Sm			^{156}Sm			^{164}Yb	
J	$X(5)$	Expt.	CBS $r_\beta=0.14$	Rotor	Expt.	CBS $r_\beta=0.35$	Rotor	Expt.	CBS $r_\beta=0.43$	Expt.	CBS $r_\beta=0.23$
$R_{4/2}$	2.90	3.01	3.01	3.33	3.26	3.27	3.33	3.29	3.30	3.13	3.13
2_1^+	122	122	122	82	82	82	76	76	76	123	123
4_1^+	354	366	367	273	267	267	253	250	251	386	386
6_1^+	661	707	695	574	544	544	531	517	519	760	753
8_1^+	1033	1125	1093	984	903	901	911	872	873	1223	1202
10_1^+	1465	1609	1554	1503	1333	1325	1391	1307	1306	1753	1723
12_1^+	1954	2149	2077	2131	1826	1810	1973	1819	1810	2330	2315
14_1^+	2499	2736	2660	2869	2323	2353	2656	2401	2379	(2899)	2972

difference by a larger fraction than the $4_1^+-0_1^+$ difference. Consequently, the $R_{4/2}$ value increases with r_β until the rigid-rotor limit with $R_{4/2}=3.33$ is reached for $r_\beta \to 1$, where the gain in angular momentum fully originates in a gain of angular velocity with constant moments of inertia for all values of L. This geometrical description of nuclear level bands neglects, of course, noncollective single-particle degrees of freedom and can, therefore, work only up to the backbending region. Table I demonstrates the good description of ground bands in deformed nuclei with various $R_{4/2}$ values.

For a given spin, the energy shifts increase strongly with s as r_β increases. This is because wave functions with more nodes have their density more evenly spread over the available range in β and, in particular, they have a higher density close to β_m than wave functions with less nodes and the same angular momentum. Therefore, the structural signature $R_{0/2}$ strongly increases with r_β and becomes infinite in the limit $r_\beta \to 1$. Since both structural signatures, $R_{4/2}$ and $R_{0/2}$, depend monotonically on r_β, the CBS rotor model yields a parameter-free relation between them on the structural evolution path between the shape phase-transitional point and the rigid-rotor limit. This relation can be confronted with the data.

We consider the classical region of spherical-to-prolate deformed transitional nuclei, the light rare-earth nuclei around neutron number $N=90$. All nuclei in the first half of the $N=82-126$ neutron major shell, and with at least one known excited 0^+ state, are taken into account. The experimental $R_{0/2}$ values are plotted as a function of $R_{4/2}$ in Fig. 2 and are compared to the model prediction. These $R_{0/2}$ data form a compact trajectory [11] when plotted as a function of $R_{4/2}$, until values of $R_{0/2}>10$ are reached for $R_{4/2}>3.25$. We conclude that the 0_2^+ states in these transitional nuclei with $2.9<R_{4/2}\leqslant 3.2$ have a related collective structure, because they correlate in a unique way to the (collective) structural signature $R_{4/2}$. This remarkable conclusion is model independent.

Second, we note that the shape of the evolutionary path of $R_{0/2}$ data and within about 10% even their absolute values, are surprisingly well predicted as a function of $R_{4/2}$ by the CBS rotor model in a parameter-free way. The β excitation is low lying in transitional nuclei and increases strongly in energy when the rotor limit is approached. For an $R_{4/2}$ value of 3.25 the CBS rotor model predicts an $R_{0/2}$ value of 13.5, i.e., the β excitation at about 1.3 MeV assuming a typical value $E_x(2_1^+)=90-100$ keV for this $R_{4/2}$. It must thus be expected that in good rotors with $R_{4/2}$ values >3.25, the actual 0_2^+ state must be formed by other (e.g., noncollective, pairing, or γ deformation) degrees of freedom because the β excitation is shifted to too high energies. Indeed, the $R_{0/2}$ values for strongly deformed midshell nuclei with $R_{4/2}$ values above 3.25 do not, in general, follow the CBS rotor trajectory and scatter over a large range of values, indicating that their correlation to the $R_{4/2}$ value has been lost. Therefore, β excitations are difficult to observe in good rotors [10], where they are shifted to high energies above the 0_2^+ state. They should, however, be low lying and, thus, easily observable in transitional nuclei.

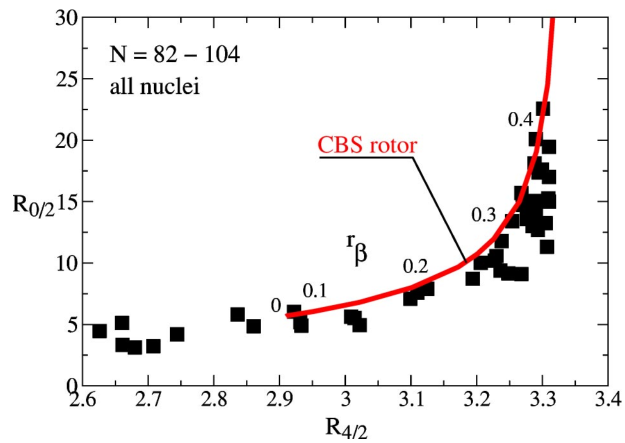

FIG. 2. (Color online) Correlation of $R_{0/2}$ and $R_{4/2}$ values in rare-earth nuclei with neutron numbers around $N=90$. Data [30] for all nuclei in the lower half of the $N=82-126$ neutron major shell with known 0_2^+ state and $R_{4/2}>2.6$ are plotted. The solid curve (online red) corresponds to the parameter-free prediction of the CBS rotor model as a function of r_β.

An important test for the applicability of the CBS rotor to transitional and near rotational nuclei is the comparison of theoretical to experimental $B(E2)$ values. For the calculation of $E2$ transition rates we use the $E2$ operator up to second order in the deformation parameters [31], restricting ourselves to nonzero quadrupole deformation parameters, only.

$$\mathcal{M}_{\mu}^{\rm lab}(E2) = e^{(1)}\alpha_{2\mu} + e^{(2)}\sum_{\nu\nu'} C^{2\mu}_{2\nu 2\nu'}\alpha_{2\nu}\alpha_{2\nu'} + \mathcal{O}(\alpha^3). \tag{11}$$

Since we do not make any assumptions about the charge distribution in the deformed nucleus, the two expansion coefficients, $e^{(1)}$ and $e^{(2)}$, are treated as free parameters to be adjusted to data. Neglecting third- and higher-order terms one obtains

$$\begin{aligned}\mathcal{M}_{\mu}(E2) = e^{(1)}\Bigg[&\beta\cos\gamma D^2_{\mu 0}(\Omega)\\ &+ \beta\sin\gamma\frac{D^2_{\mu 2}(\Omega)+D^2_{\mu -2}(\Omega)}{\sqrt{2}}\Bigg]\\ &- \sqrt{\frac{2}{7}}e^{(2)}\Bigg[\beta^2\cos 2\gamma D^2_{\mu 0}(\Omega)\\ &- \beta^2\sin 2\gamma\frac{D^2_{\mu 2}(\Omega)+D^2_{\mu -2}(\Omega)}{\sqrt{2}}\Bigg],\end{aligned} \tag{12}$$

after transformation to the intrinsic frame and exploiting the properties of the Wigner functions. The terms proportional to $D^2_{\mu 0}$ $[D^2_{\mu 2}+D^2_{\mu -2}]$ describe $\Delta K=0$ $[\Delta K=2]$ transitions. The $E2$ matrix elements are obtained from integration over the Euler angles and over the intrinsic variables β and γ. For separable wave functions the integrations over β and γ are independent.

Since we consider here β excitations with $n_{\gamma}=0$ [7] (and hence $K=0$), only the $\Delta K=0$ part of the $E2$ transition operator is relevant for our discussion. It takes the form

$$\begin{aligned}T_{\mu}^{\Delta K=0}(E2) = e^{(1)}\beta_M\langle\cos\gamma\rangle_{\gamma}\Bigg[&\left(\frac{\beta}{\beta_M}\right)\\ &- \sqrt{\frac{2}{7}}e^{(2)}\beta_M\frac{\langle\cos 2\gamma\rangle_{\gamma}}{\langle\cos\gamma\rangle_{\gamma}}\left(\frac{\beta}{\beta_M}\right)^2\Bigg]D^2_{\mu 0}(\Omega),\end{aligned} \tag{13}$$

$$= e_{\rm eff}[(\beta/\beta_M) + \chi(\beta/\beta_M)^2]D^2_{\mu 0}, \tag{14}$$

with the effective charge $e_{\rm eff}=e^{(1)}\beta_M\langle\cos\gamma\rangle_{\gamma}$ and $\chi=-\sqrt{2/7}e^{(2)}\beta_M\langle\cos 2\gamma\rangle_{\gamma}/\langle\cos\gamma\rangle_{\gamma}$ to be adjusted to data. Note that the integrals over γ are independent of s and L and, hence, they are equal for all transitions between bands with $n_{\gamma}=0$. These constants are absorbed in the parameters $e_{\rm eff}$ and χ of the $E2$ transition operator.

Table II displays $B(E2)$ values for the nuclei ^{152,154}Sm and analytical results for $X(5)$ and the CBS rotor model. The experimental $B(E2)$ values in the ground bands are reproduced by the CBS rotor model within the uncertainties at least up to the 10^+_1 state. Also, most of the $E2$ decays of the β-excitation band members are described with deviations not exceeding 2 W.u. Inclusion of the second-order term in the $E2$ operator allows for a consistent description of the $\Delta K=0$ interband $E2$ matrix element. Consequently, the $\Delta K=0$ interband $E2$ strengths in ^{154}Sm are well predicted from having adjusted the parameter χ in the $E2$ transition operator to the data from ^{152}Sm. This good description of the $E2$ rates results from the analytical solution of a unique Hamiltonian and a simple choice for the $E2$ operator. It does not involve any *ad hoc* assumptions about band mixing [26].

TABLE II. Comparison of $E2$ transition rates in ^{152,154}Sm [32,33] with the $X(5)$ limit and the CBS rotor model for $r_{\beta}=0.14$ and 0.35 from Table I. $\chi=-0.535$ was kept constant in the $E2$ operator. The $B(E2)$ values are in Weisskopf units and scales are adjusted to the $2^+_1\rightarrow 0^+_1$ transition.

Transition	$X(5)$	^{152}Sm	CBS	^{154}Sm	CBS
$2^+_1\rightarrow 0^+_1$	144	144(3)	144	174(5)	174
$4^+_1\rightarrow 2^+_1$	230	209(3)	213	244(6)	251
$6^+_1\rightarrow 4^+_1$	285	245(5)	249	290(8)	281
$8^+_1\rightarrow 6^+_1$	328	285(14)	273	318(17)	300
$10^+_1\rightarrow 8^+_1$	361	320(30)	290	314(16)	314
$2^+_2\rightarrow 0^+_2$	115	107(27)	120		165
$4^+_2\rightarrow 2^+_2$	173	204(38)	172		234
$0^+_2\rightarrow 2^+_1$	90	33(2)	33		8.4
$2^+_2\rightarrow 0^+_1$	3	0.9(1)	0.3	0.94(23)	0.5
$2^+_2\rightarrow 2^+_1$	12	5.5(5)	3		1.4
$2^+_2\rightarrow 4^+_1$	53	19(2)	24	4.1(11)	7.1
$4^+_2\rightarrow 2^+_1$	1.4	0.7(2)	0.1		0.1
$4^+_2\rightarrow 4^+_1$	9	5.4(13)	2		1.1
$4^+_2\rightarrow 6^+_1$	40	4(2)	18	8.2	

Finally, we note that an approximate consideration of the γ-deformation degree of freedom and description of the γ band is possible using the same decoupling approximations, as done by Iachello [7] and recently discussed by Bijker *et al.* [29] for $X(5)$. $E0$ transitions can also be studied in the CBS rotor model. Since the difference of the average 0^+ states' deformation decreases with increasing r_{β}, $E0$ transitions are predicted by the CBS rotor model to be largest for $X(5)$ and decrease when approaching the rigid-rotor limit. For $r_{\beta}<1$, $E0$ transitions between two $K=0$ bands are predicted to decrease with increasing spin. Particle pair transfer strengths cannot be described by the CBS rotor model, because they are outside of a geometrical approach. Finally, we stress that a similar approach will work for an analytical interpolation between $E(5)$ and the deformed γ-soft limit.

In conclusion, we have studied the Bohr-Hamiltonian in the limit of rigid prolate axial symmetry ($\gamma\approx 0°$) with confined β-soft potentials. This CBS rotor model interpolates between $X(5)$ and the rigid-rotor limit, and has been solved analytically in terms of Bessel functions of first and second kind. Full decoupling of β and γ degrees of freedom is assumed for the analytical approximation. The model predicts the evolution of structure for prolate transitional nuclei as a function of one parameter (r_β) and yields a parameter-free correlation of the two structural signatures $R_{0/2}$ and $R_{4/2}$. The relative excitation energies of the first excited 0^+ states of transitional nuclei in the rare-earth mass region correlate to the $R_{4/2}$ ratio. The shape of the correlation path of $R_{0/2}$ as a function of $R_{4/2}$ is well predicted by the model for $2.9 < R_{4/2} < 3.2$. The absolute values of $R_{0/2}$ are overestimated by about 10%. The CBS rotor model well describes ground-band energies, as well as intraband and interband $E2$ rates for transitional nuclei, e.g., ^{152}Sm, analytically with two parameters (r_β, χ) and two scales $(\hbar^2/2B\beta_M^2, e_{\mathrm{eff}})$. Our analysis implies that the 0_2^+ state of axially symmetric transitional nuclei with $R_{4/2} < 3.25$ corresponds to the β excitation of the CBS rotor and that its properties correlate to the variational moment of inertia of the ground band.

Note added in proof. While the present Rapid Communication was under review, another paper has been submitted and published [L. Fortunato and A. Vitturi, J. Phys. G: Nucl. Part. Phys. **30**, 627 (2004); received 18 December 2003] that deals with interesting analytical solutions of the Bohr-Hamiltonian using Coulomb-like and Kratzer-like potentials in β. These authors term their solution a β-soft rotor, too, although their potentials are not β independent. Due to the analogy to the γ independent potentials used earlier by Wilets and Jean [2], known as the γ-soft case, we consider the term β soft more appropriate for the β-independent potentials (confined over a certain range) that are considered in the present paper. It will be interesting to compare the predictive power of both approaches.

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