PHYSICAL REVIEW C VOLUME 7, NUMBER 3 MARCH 1973

Deuteron Interaction Current in Thermal n-p Capture*

M. Gari† and A. H. Huffman
California Institute of Technology, Pasadena, California 91109
(Received 29 September 1972)

The experimental cross section for radiative n-p capture at thermal energies is 10% larger than the theoretical cross section calculated from the impulse terms (free-nucleon isovector magnetic moment). In the present work the pion-exchange contribution is evaluated using the Chew-Low static model amplitude for the photopion process. We demonstrate how the various pieces of the photopion amplitude contribute to the exchange current. The results for Hamada-Johnston, Reid-hard-, and -soft-core potentials are 10.5, 10.9, and 11.5% increases, respectively, to the impulse cross section from pion processes. This work verifies the earlier paper of Riska and Brown demonstrating that the 10% discrepancy can be accounted for as a one-pion-interaction effect.

I. INTRODUCTION

The 10% discrepancy between the experimental cross section for thermal n-p capture and the theoretical cross section calculated from the so-called impulse term has long puzzled physicists. At thermal energies this is an $M1$ transition between the $^1S_0$ scattering state and the $^3S_1$, $^3D_1$ deuteron ground state. The impulse term uses the free-nucleon magnetic moments, of which only the isovector part contributes. The discrepancy was first pointed out by Austern[1] in 1955 and has been repeatedly verified in the literature since then.[2,3] In a recent reevaluation[3] of this effect, Noyes finds a 9.5% discrepancy with the singlet scattering length based on a charge-independent calculation, or a 7.7% discrepancy if charge independence is abandoned. The error owing to other parameters going into the calculation is assigned at 1.5%.

One hopes that this effect would be explainable in terms of meson exchange or interaction currents but up to this year all such calculations have failed to account for the 10%. One of the most recent is that of Adler, Chertok, and Miller[4] who calculated the pion-current diagram (Fig. 1) and found a 3% interaction effect. Calculations based on dispersion relations[5] also fail to account for the discrepancy.

The only suggestion other than meson currents for the discrepancy is that of Breit and Rustgi[6] who pointed out the possibility of a weak transition from the $^3S_1$ scattering state. In the simplest model this is impossible because the long wavelength $M1$ operator has no radial dependence, and the $^3S_1$ wave functions at different energies are orthogonal. However, if the nucleon-nucleon potential is momentum-dependent, the $^3S_1$ wave function may have a small overlap with the deuteron, and Breit and Rustgi have suggested looking for this transition by using a polarized beam-target method. This idea has been further developed by Adler[7] who showed that a contribution from the $^3S_1$ scattering state large enough to account for the 10% discrepancy should cause a significant γ-ray angular dependence in (polarized p) + (polarized n) $\to d+\gamma$, but that such a large contribution was unlikely for other reasons. This experiment has not yet been performed, and we will assume for the rest of this paper that a transition from the $^3S_1$ n-p state is not responsible for the largest part of the 10% discrepancy.

We give the following heuristic argument that this effect should be explainable in terms of one-pion exchange. If the anomalous magnetic moments of the nucleon system, the deuteron-$^1S_0$ n-p state, and the ^{3}H-^{3}He system, are examined, it is found in every case that the anomalous isovector part is large and the anomalous isoscalar part is small. By the "anomalous" magnetic moment of a nucleus like the deuteron, we mean any part which is not explained by wave function admixtures (of the D state in this case). Thus, the magnetic moment of the deuteron $\mu_d=0.857\mu_N$, which is given in terms of the free-nucleon magnetic moments by the formula

$$(\mu_d)_{\text{impulse}}=(\mu_n+\mu_p)-\tfrac{3}{2}P_D(\mu_n+\mu_p-\tfrac{1}{2}), \qquad (1)$$

could be explained by a D-state probability $P_D=0.04$ without any interaction currents. But a larger value $P_D=0.07$ is suggested by the quadrupole moment, photopion production, and other experiments. This value of P_D implies a 2% "anomalous" part to be explained by interaction currents; and indeed, a ρ-π exchange can approximately account for this.[8] This 2% isoscalar effect is to be compared with the 5% "anomalous" isovector effect in the matrix element for the transition between the $^1S_0$ n-p state and the deuteron. Similarly, in the ^{3}H-^{3}He system for a 4% D state and 0% mixed symmetry S state, the anomalous isoscalar moment is less than 1% whereas the anomalous isovector moment is 10%.[9] In the nucleon system the anomalous isoscalar magnetic moment is $\kappa_s = 0.06$ whereas the anomalous isovector magnetic moment is $\kappa_v = 1.85$.

The point is that the one-pion-exchange term is proportional to $(\vec{\tau}_1 \times \vec{\tau}_2)^z$ and therefore contributes only isovector effects. Since the pion is by far the lightest of the mesons, its interaction current will have the largest effect. Thus, by the above arguments we submit that it is really the one-pion-interaction current that is accounting for the bulk of the large isovector effects. For example, in a simple model with a cutoff adjusted to agree with the effective range of pion-nucleon scattering, one pion currents predict $\kappa_v = 1.76$.[10,11] We should have even better hope of accounting for the thermal n-p capture effect since no cutoffs are necessary.

Our approach to calculating the interaction current is somewhat different from previous ones in that instead of calculating all possible diagrams and adding up their effects, we consider the photoproduction of a pion at one nucleon in the Chew-Low static model[12] followed by its absorption at the other nucleon (Fig. 2). We are thus able to show that previous calculations neglected the catastrophic pion current (the Kroll-Ruderman term) which makes a large contribution to the interaction current.[13] In fact, the pion current contribution (Fig. 1) connecting the two S states is actually negative as we will see later.

The challenging 10% discrepancy has been resolved by Riska and Brown.[14] They used the results of Chemtob and Rho,[9] who pioneered the use of the low-energy theorems for calculating interaction currents. Riska and Brown emphasized the importance of including terms connecting to the D state of the deuteron and with the inclusion of resonance effects were able to account for the 10%. Since Gaffney's[15] low-energy photopion theorem essentially reduces to the Chew-Low amplitude in the nonrelativistic limit, our numerical results do not differ greatly from those of Riska and Brown and therefore represent an independent confirmation of their work.

In Sec. II the S matrix for the interaction current owing to photoproduction of a pion at either of the nucleons is developed. At the end of this section, the S matrix for the impulse term is written down. In Sec. III we discuss the terms of the Chew-Low photoproduction amplitude in the static model and show how each term contributes to the interaction current. Finally, in Sec. IV we display and discuss the results of our calculations.

II. KINEMATICS

Let $\Gamma_1(\vec{k}, \vec{q})^i_\alpha$ represent the amplitude for the photoproduction of a pion of momentum $\vec{q}$ by a photon of momentum $\vec{k}$ at nucleon 1. The superscript i is the vector index of the photon, and the subscript α is the isospin index of the pion. (Since time reversal requires equality of amplitudes, it is irrelevant whether we consider the photopro-

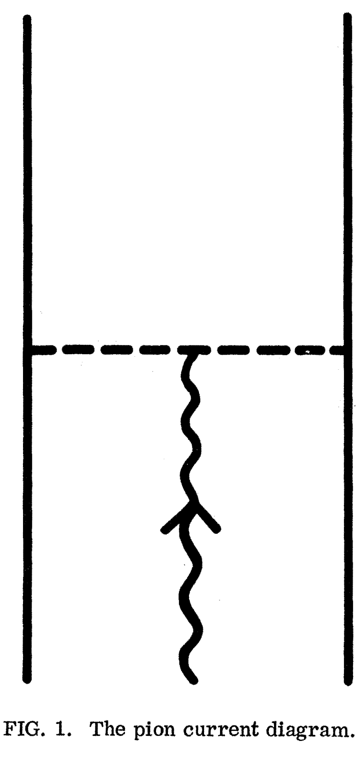

FIG. 1. The pion current diagram.

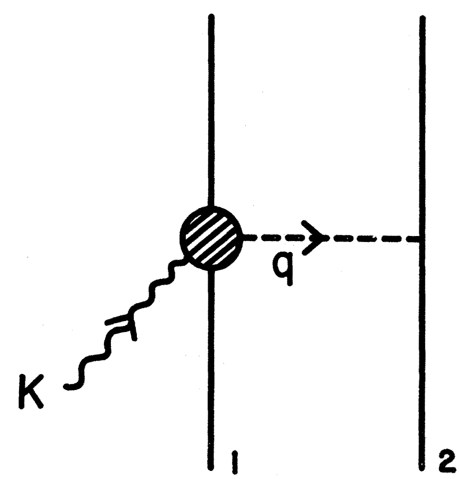

FIG. 2. The photoproduction of a pion at nucleon 1 followed by absorption of the pion at nucleon 2.

duction of a pion contributing to deuteron photodisintegration or pion absorption with emission of a photon at one nucleon contributing to n-p radiative capture.) The amplitude for the p-wave absorption of a pion of momentum $\vec{q}$ at nucleon 2 is

$$-\frac{G}{2m}\vec{\sigma}_2\cdot\vec{q}\,\tau_2^\alpha\,, \tag{2}$$

where

$$\frac{G^2}{4\pi}=14$$

and m is the nucleon mass. Then the S matrix for the process of Fig. 2, the photoproduction of a pion at nucleon 1 followed by absorption of the pion at nucleon 2 is

$$S=\epsilon^i\int d^4r_1\,d^4r_2\,\Psi_f^*(r_1,r_2)\,e^{-ik\cdot r_1}\,\Gamma_1(\vec{k},\vec{q})^i_\alpha\,\frac{ie^{-iq\cdot(r_2-r_1)}}{q^2-\mu^2}\,\frac{d^4q}{(2\pi)^4}\left(-\frac{G}{2m}\right)\vec{\sigma}_2\cdot\vec{q}\,\tau_2^\alpha\Psi_i(r_1,r_2)\,, \tag{3}$$

where the Ψ's are the initial and final two-nucleon wave functions and μ is the pion mass and ϵ^i is the polarization vector of the photon. The next step is to change the variables r_1 and r_2 to the center-of-mass coordinates:

$$R=\tfrac{1}{2}(r_1+r_2)\,, \tag{4}$$

$$r=r_1-r_2\,, \tag{5}$$

$$\Psi_i(r_1,r_2)=e^{-iP_iR}\Phi_i(r)\,, \tag{6}$$

$$\Psi_f(r_1,r_2)=e^{-iP_fR}\Phi_f(r)\,. \tag{7}$$

In the last two equations it has been assumed that the initial and final two-nucleon center of masses are plane waves with momenta P_i and P_f. The change of variables results in the following equation for the S matrix:

$$S=(2\pi)^4\delta^4(P_f-P_i-k)\epsilon^i\int\frac{d^4r\,d^4q}{(2\pi)^4}\,\Phi_f^*(r)e^{ir\cdot(q-k/2)}\Gamma_1(\vec{k},\vec{q})^i_\alpha\left(-i\frac{G}{2m}\right)\frac{\vec{\sigma}_2\cdot\vec{q}}{q^2-\mu^2}\,\tau_2^\alpha\Phi_i(r)\,. \tag{8}$$

The Ψ's of Eq. (3) and Φ's of Eq. (8) are understood to contain the energy dependence of the nonrelativistic wave functions. If this energy dependence is factored out,

$$\Phi(r)=\varphi(\vec{r})\,e^{-iEt}\,, \tag{9}$$

the fourth components of the r and q integrals can be done,[16] setting $q_0=0$. Also let $\vec{q}\to-\vec{q}+\vec{k}/2$, obtaining, finally,

$$S=(2\pi)^4\delta^4(P_f-P_i-k)\left(\frac{-iG}{2m}\right)\epsilon^i\int\frac{d^3q\,d^3r}{(2\pi)^3}\,e^{i\vec{q}\cdot\vec{r}}\varphi_f^*(\vec{r})\Gamma_1(\vec{k},-\vec{q}+\vec{k}/2)^i_\alpha\,\frac{\sigma_2(-\vec{q}+\vec{k}/2)}{(-\vec{q}+\vec{k}/2)^2+\mu^2}\,\tau_2^\alpha\varphi_i(\vec{r})\,. \tag{10}$$

Notice that the c.m. kinematics require that the pion propagator contain the photon momentum, a fact that will be important later. Of course, to this must be added the amplitude for photoproduction at nucleon 2 followed by absorption at nucleon 1. It is easy to see that this second contribution can be obtained from the first by letting $\vec{q}\to-\vec{q}$ and interchanging the labels 1 and 2.

Since $\Gamma_1(\vec{k},\vec{q})^i_\alpha$ is the complete amplitude for photoproduction, the process of Fig. 2, whose S matrix is given by Eq. (10), includes both the interaction contributions of one pion exchange *and* some processes which are already included by the wave functions in the impulse term.[9] Thus, a diagram like Fig. 3, in which the photon interacts with a nucleon and a pion is exchanged later, is already contained in the impulse term. This is because one-pion exchange is part of the potential which generates the initial and final wave functions. Since the left-hand side of Fig. 3 in general contributes to the photoproduction amplitude, one must exclude such pieces from the photoproduction amplitude $\Gamma(\vec{k},\vec{q})^i_\alpha$ in obtaining the S matrix representing interaction pieces only. This question will be discussed below.

Let $\Gamma'(\vec{k},\vec{q})^i_\alpha$ be the photoproduction amplitude with the impulse pieces excluded; then the S matrix for the interaction contribution of one pion exchange is

$$S_{\text{int}}=(2\pi)^4\delta^4(P_f-P_i-k)M_{\text{int}}\,, \tag{11}$$

where M^{int} is the matrix element for the interaction contribution,

$$M_{\text{int}}=\int\frac{d^3q\,d^3r}{(2\pi)^3}\,\varphi_f^*(\vec{r})\,e^{i\vec{q}\cdot\vec{r}}\,\vec{\epsilon}\cdot\vec{J}_{\text{int}}(\vec{k},\vec{q})\varphi_i(\vec{r}) \tag{12}$$

and $\vec{J}_{\rm int}(\vec{k},\vec{q})$ is the current in momentum space, including contributions from both photoproduction at nucleon 1 and photoproduction at nucleon 2,

$$J^i_{\rm int}(\vec{k},\vec{q})=\frac{iG}{2m}\left[\Gamma'_1(\vec{k},-\vec{q}+\vec{k}/2)^i_\alpha\,\frac{\vec{\sigma}_2\cdot(-\vec{q}+\vec{k}/2)}{(-\vec{q}+\vec{k}/2)^2+\mu^2}\,\tau_2^\alpha+\Gamma'_2(\vec{k},\vec{q}+\vec{k}/2)^i_\alpha\,\frac{\vec{\sigma}_1\cdot(\vec{q}+\vec{k}/2)}{(\vec{q}+\vec{k}/2)^2+\mu^2}\,\tau_1^\alpha\right]. \tag{13}$$

However, one must be wary of double counting certain pieces, since the covariant pion propagator represents both a π^+ traveling in one direction and a π^- traveling in the other.

For comparison later we display the impulse matrix element and current,

$$M_{\rm imp}=\int\frac{d^3r}{(2\pi)^3}\,\varphi_f^*(\vec{r})\vec{\epsilon}\cdot\vec{J}_{\rm imp}(\vec{r})\varphi_i(\vec{r}), \tag{14}$$

$$\vec{J}_{\rm imp}(\vec{r})=\frac{e}{2m}\,2\mu_v\,\frac{(\tau_1-\tau_2)^z}{2}\,\vec{k}\times\frac{(\vec{\sigma}_1-\vec{\sigma}_2)}{2}. \tag{15}$$

The impulse current has no radial dependence in the long wavelength approximation as shown above so

$$M_{\rm imp}=\frac{e}{2m}2\mu_v(\vec{\epsilon}\times\vec{k})\cdot\chi_t^{m_d}\,\frac{(\vec{\sigma}_1-\sigma_2)}{2}\,\chi_s\,\mathfrak{M}, \tag{16}$$

where

$$\mathfrak{M}=\int_0^\infty dr\,u_d(r)u_{np}(r), \tag{17}$$

$\mu_v=2.35$, and the deuteron wave function is

$$\varphi_d(\vec{r})=(4\pi)^{-1/2}\frac{1}{r}\left[u_d(r)+\frac{T_2^{lm}(\hat{r})}{\sqrt{8}}\,\sigma_1^l\,\sigma_2^m w_d(r)\right]\chi_t^{m_d}\,\eta_s, \tag{18}$$

where $u_d(r)$ is normalized to be asymptotic to the decaying exponential of the binding energy B,

$$u_d(r)\sim e^{-(mB)^{1/2}r} \tag{19}$$

and T_2 is the second rank tensor

$$T_2^{lm}(\hat{r})=3\hat{r}^l\hat{r}^m-\delta^{lm} \tag{20}$$

and the n-p scattering wave function is

$$\varphi_{np}(r)=(4\pi)^{-1/2}\frac{1}{r}\,u_{np}(r)\chi_s\,\eta_t^0. \tag{21}$$

Here $\chi_t^{m_d}$ and χ_s are the triplet and singlet spin functions, and η_t,η_s the isospin functions. The radial function $u_{np}(r)$ is normalized to be asymptotic to $1-r/a_s$. These deuteron and n-p scattering wave-function normalizations standardize the value of the overlap integral $\mathfrak{M}$ [Eq. (17)] to Refs. 1–4.

III. PHOTOPRODUCTION AMPLITUDE

For the photoproduction amplitude $\Gamma_1(\vec{k},\vec{q})^i_\alpha$, we use the result of the Chew-Low static model[12]

$$\Gamma_1(\vec{k},\vec{q})^i_\alpha=\frac{ieG}{2m}\left[\sigma_1^i-\frac{2\vec{\sigma}_1\cdot(\vec{q}-\vec{k})q^i}{(\vec{q}-\vec{k})^2+\mu^2}\right]\epsilon^{z\alpha\beta}\tau_1^\beta-i\frac{e\mu_v}{4m}\left(\frac{G}{2m}\right)^{-1}\frac{e^{i\delta_3}\sin\delta_3}{q^3}\{2(\vec{q}\times\vec{k})^i-i[(\vec{\sigma}_1\times\vec{q})\times\vec{k}]^i\}(\delta_{z\alpha}-\tfrac{1}{3}\tau_\alpha\tau_z). \tag{22}$$

A very clear derivation of this amplitude and discussion of its terms has been given by Henley and Thirring.[11] The first term, proportional to $\vec{\sigma}_1$, is the catastrophic current[10] (the Kroll-Ruderman term) which produces S-wave pions from $E1$ photons [Fig. 4(a)]. The second term, proportional to $\vec{q}$ is the pion-current term (Fig. 5); the third term, multiplying the phase shift expression, $e^{i\delta_3}\sin\delta_3$ of the 3-3 state, is the rescattering correction of the magnetic terms, equivalent to the 3-3 resonance diagram (Fig. 6).

Before the Chew-Low amplitude [Eq. (22)] can be inserted into the S matrix for the interaction contributions [Eqs. (11)–(13)] one must exclude any pieces which are already included by the wave functions in the impulse term[9] as mentioned above.

Actually, the problem of which pieces are interaction terms and which are included in the impulse term has not been totally solved.[17] In a nonrelativistic context Zachariasen[18] has shown how to

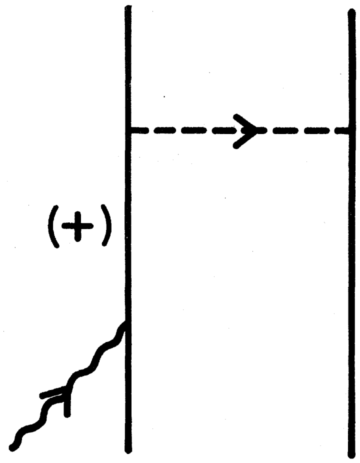

FIG. 3. A diagram which is already included in the impulse term. The (+) indicates that only the positive-energy state of the intermediate nucleon leg is present.

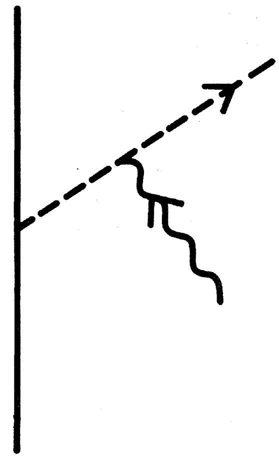

FIG. 5. The pion-current piece of photoproduction.

explicitly separate the interaction terms from the parts that make up the wave functions. He was able to factor out from both sides of the T-matrix operators which generate the initial and final wave functions from the free particle states. To our knowledge no one has carried through a fully relativistic treatment. Thus one cannot be sure that there is not some relativistic correction (to the impulse term) which has been neglected.

We have followed the usual procedure and have used phenomenological wave functions for the deuteron and $^1S_0$ state. This procedure has been criticized by Stichel and Werner[19] as being inconsistent, because wave functions that are really generated out of the various exchanges should be used. They suggest using the one-boson-exchange wave function and all the one-boson interaction currents. Clearly, one would like to have the "true" dynamical description of the strong interaction, but we feel that the use of phenomenological wave functions is justified in the present case since the interaction current of one pion is by far the largest contribution, and the phenomenological wave functions used, Hamada-Johnston[20] and Reid[21] do have the one-pion tail.

Returning to the Chew-Low amplitude and the question of which terms are already included in the wave functions, we recall that the one-pion-exchange potential (OPEP) is calculated from the Feynman graph of one-pion exchange connected to positive-energy nucleon legs. All such graphs with a photon connected to a positive-energy nucleon leg before or after the exchange must be omitted from $\Gamma(k,q)^t_\alpha$. Thus the pion-current diagram (Fig. 5) is definitely an interaction term (Fig. 1). The rescattering diagram (Fig. 6) was derived[11,12] with the assumption that only the 3-3 state is enhanced; it is thus also an interaction term since the OPEP connects only nucleon legs.

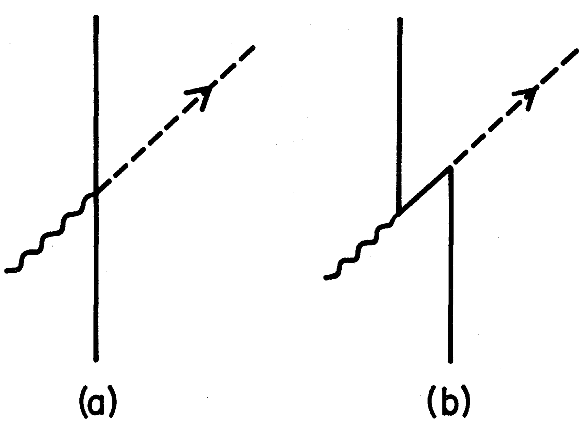

FIG. 4. Equivalence of the catastrophic current (a) to the negative-energy-state contribution in the ps theory (b).

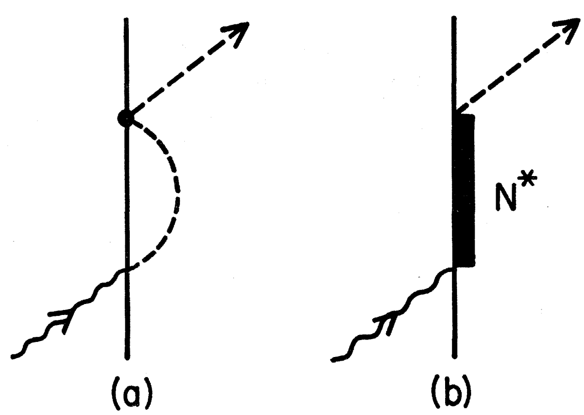

FIG. 6. Equivalence of the rescattering term (a) to the 3-3 resonance contribution (b).

We now examine the catastrophic current more closely. This is just the term that arises from the minimal electromagnetic substitution in the pseudovector (pv) pion-nucleon coupling,

$$\mathcal{L} = i\frac{G}{2m}\bar{N}\gamma^{\mu}\gamma^{5}\tau_{\alpha}N\partial_{\mu}\varphi^{\alpha}. \tag{23}$$

Since the pseudoscalar (ps) pion-nucleon coupling is known to give results equivalent to those of the pv coupling to first order in the coupling constant, this term must also appear in the ps diagrams, and it is found to be the contribution from the diagram in which the intermediate nucleon is in the negative energy state[9] [Fig. 4(b)]. {Of course, in the pv theory there is also a contribution from the negative energy diagram [Fig. 4(b)] but this is of higher order in P/m than the catastrophic current.} Thus, the catastrophic term also contributes to the interaction current because OPEP must connect to positive-energy legs. Therefore, the entire Chew-Low photoproduction amplitude contributes to the interaction current and contains no wave function pieces ($\Gamma = \Gamma'$).

One may wonder how the catastrophic term, which produces S-wave pions from $E1$ photons, will contribute to an $M1$ interaction current. It is because the c.m. kinematics require that the pion propagator contain the photon momentum, that all multipoles appear in the interaction current. To see how this works out we display the catastrophic current in momentum space [Eq. (13)],

$$\vec{J}_{\text{int}}(\text{catastrophic}) = \frac{eG^2}{4m^2}(\vec{\tau}_1\times\vec{\tau}_2)^z\left[\frac{\vec{\sigma}_1\vec{\sigma}_2\cdot(-\vec{q}+\vec{k}/2)}{(-\vec{q}+\vec{k}/2)^2+\mu^2} - \frac{\vec{\sigma}_1\cdot(\vec{q}+\vec{k}/2)\vec{\sigma}_2}{(\vec{q}+\vec{k}/2)^2+\mu^2}\right]. \tag{24}$$

The two terms result from the catastrophic currents at nucleon 1 and 2. The above result is then expanded in orders of the photon momentum $\vec{k}$. The 0th order term in $\vec{k}$ is an $E1$ term,[18] which does not contribute in the present case, a transition between positive-parity states. The first-order term in $\vec{k}$ is proportional to $\vec{\epsilon}\times\vec{k}$ and is the $M1$ contribution needed:

$$J^i_{\text{int}}(\text{catastrophic current to first order in } \vec{k})$$
$$= \frac{eG^2}{4m^2}(\vec{\tau}_1\times\vec{\tau}_2)^z\epsilon^{ijk}k^j\left\{\frac{1}{2}(\vec{\sigma}_1\times\vec{\sigma}_2)^k\left[\frac{1}{q^2+\mu^2} - \frac{2}{3}\frac{q^2}{(q^2+\mu^2)^2}\right] + \frac{1}{6}\frac{q^2}{(q^2+\mu^2)^2}T_2^{kl}(\hat{q})(\vec{\sigma}_1\times\vec{\sigma}_2)^l\right\}. \tag{25}$$

Here we have used the definition of the 2nd rank tensor [Eq. (20)] to separate terms which connect S states to D states from terms which connect S states to S states.

Similarly, the contribution of the pion current (Fig. 1) is given by

$$J^i_{\text{int}}(\text{pion-current to first order in } \vec{k})$$
$$= \frac{eG^2}{4m^2}(\vec{\tau}_1\times\vec{\tau}_2)^z\frac{q^2}{(q^2+\mu^2)^2}\epsilon^{ijk}k^j\left[-\frac{1}{3}(\vec{\sigma}_1\times\vec{\sigma}_2)^k + \frac{1}{6}T_2^{kl}(\hat{q})(\vec{\sigma}_1\times\vec{\sigma}_2)^l\right]. \tag{26}$$

As mentioned in Sec. II, one must be careful not to count this piece twice as a contribution both from nucleon 1 and nucleon 2.

The photoproduction amplitude through the rescattering terms of Eq. (22) is already in the $(\vec{\epsilon}\times\vec{k})M1$ form so we take the lowest-order $\vec{k}$ contribution. We must be careful to include a term for the diagram in which a pion is produced at one nucleon, rescattered from the second nucleon, and absorbed along with the photon [such a diagram interchanges the photon and pion momentum and isospin indices of the rescattering term of Eq. (22)]. The result is

$$J^i_{\text{int}}(\text{rescattering terms}) = (\vec{\tau}_1\times\vec{\tau}_2)^z\epsilon^{ijk}k^j\frac{q^2}{q^2+\mu^2}\left(\frac{e^{i\delta_3}\sin\delta_3}{q^3}\right)\mu_v\frac{1}{3}T_2^{kl}(\hat{q})(\vec{\sigma}_1\times\vec{\sigma}_2)^l. \tag{27}$$

Here we have used the fact that the operators $(\vec{\tau}_1-\vec{\tau}_2)^z(\vec{\sigma}_1-\vec{\sigma}_2)^i$ and $(\vec{\tau}_1\times\vec{\tau}_2)^z(\vec{\sigma}_1\times\vec{\sigma}_2)^i$ have equal matrix elements between the two-nucleon wave functions of the deuteron ($T=0, S=1$) and the n-p scattering state ($T=1, S=0$),

$$\langle d|(\vec{\sigma}_1-\vec{\sigma}_2)^i(\vec{\tau}_1-\vec{\tau}_2)^z|np\rangle = \langle d|(\vec{\sigma}_1\times\vec{\sigma}_2)^i(\vec{\tau}_1\times\vec{\tau}_2)^z|np\rangle. \tag{28}$$

The rescattering terms give only an S-D contribution.

To evaluate the rescattering contribution, a value must be assigned to the expression for the pion-nucleon scattering amplitude in the 3-3 state

$$\frac{e^{i\delta_3}\sin\delta_3}{q^3}. \tag{29}$$

According to Brown, Green, and Gerace,[22] the amplitude should be evaluated at the point

$$\text{energy of pion} = q^0 = 0, \tag{30}$$

since the nucleons in a nucleus have on the average of about 40 MeV of kinetic energy, which is small compared to the pion mass. On the mass shell, therefore, we wish to evaluate expression (29) at the unphysical point $q^0=0$, $|\vec{q}|=i\mu$. Chemtob and Rho[9] have done this by using the Low equation, which is a dispersion relation for the amplitude (29). But, from the equation of the Chew-Low plot,[11]

$$\cot\delta_3 \approx \frac{12\pi q^0}{|q|^3}\,\frac{1-q_0 r_3}{4(G/2m)^2}, \tag{31}$$

one can determine

$$\delta_3(q^0=0, |\vec{q}|=i\mu)=\pi/2 \tag{32}$$

and thus expression (29) has the value $-\mu^{-3}$ at the unphysical point. It is interesting to note that because of the construction of the rescattering terms in the Chew-Low amplitude [Eq. (22)] the interaction current due to the rescattering terms [Eq. (27)] is independent of the pion-nucleon coupling constant.

The interaction currents [Eqs. (25)–(27)] can be transformed into coordinate space by performing the Fourier transform of Eq. (12). Using the fact that the minus and cross operators have equal matrix elements [Eq. (28)] the interaction current can be written in the form

$$\langle d|J^i_{\rm int}(r)|np\rangle = \epsilon^{ijk}k^j\langle d|[G_{\rm I}(r)(\vec{\sigma}_1\times\vec{\sigma}_2)^k + \tfrac{1}{3}G_{\rm II}(r)(\vec{\sigma}_1\times\vec{\sigma}_2)^l T_2^{kl}(\hat{r})](\vec{\tau}_1\times\vec{\tau}_2)^z|np\rangle. \tag{33}$$

We have displayed the results for the various terms in Table I for comparison with other references. Since we have combined the minus and cross terms, our terms $G(r)$ correspond to $\frac{1}{2}[g(r)+h(r)]$ in Chemtob and Rho.[9] All three interaction currents are proportional to $(\vec{\tau}_1\times\vec{\tau}_2)^z$ and have matrix elements only between states of opposite isospin, as mentioned in Sec. I.

The impulse current [Eq. (15)] can be cast into a similar form,

$$\langle d|J^i_{\rm imp}(r)|np\rangle = \epsilon^{ijk}k^j\langle d|F(\vec{\sigma}_1\times\vec{\sigma}_2)^k(\vec{\tau}_1\times\vec{\tau}_2)^z|np\rangle \tag{34}$$

and

$$F=\frac{\mu_v}{2}\,\frac{e}{2m}. \tag{35}$$

IV. RESULTS AND DISCUSSION

For the triplet deuteron and singlet n-p wave functions [Eqs. (18) and (21)] we have numerically integrated Hamada-Johnston[20] (HJ) and Reid[21] hard-(RHC) and soft-core (RSC) potentials. The results for the impulse overlap integral $\mathfrak{M}$ [Eq. (17)] are shown at the top of Tables II–IV. These values are in good agreement with that of Noyes[3] who determined $\mathfrak{M}=3.973\pm0.024$ fm from the Bethe-Longmire approximation. The assigned error is due to the uncertainty in the experimental parameters, primarily the np singlet effective range.

The interaction contributions in Tables II–IV are reported as the ratio of interaction to impulse term in percent,

$$\delta = \frac{\langle d|M_{\rm int}|np\rangle}{\langle d|M_{\rm imp}|np\rangle} = \frac{2}{\mu_v\mathfrak{M}}\left[\tilde{G}_{\rm I} - \frac{\sqrt{2}}{3}\tilde{G}_{\rm II}\right], \tag{36}$$

TABLE I. Terms of the interaction current [Eq. (33)] from the Chew-Low static model.

Current	$G_{\rm I}(r)$	$G_{\rm II}(r)$
Catastrophic (Fig. 4)	$\eta(1+x)\frac{e^{-x}}{x}$	$-\frac{3}{2}\eta(1+x)\frac{e^{-x}}{x}$
Pion (Fig. 5)	$\eta(x-2)\frac{e^{-x}}{x}$	$-\frac{3}{2}\eta(1+x)\frac{e^{-x}}{x}$
Rescattering (Fig. 6)	0	$-\frac{\mu_v}{4\pi}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x}$
$\eta\equiv\frac{1}{12}\frac{e}{2m}\frac{\mu}{m}\frac{G^2}{4\pi}$	$x=\mu r$	
$\mu_v=\frac{\mu_p-\mu_n}{2}$		

TABLE II. Hamada-Johnston potential results. Impulse integral for n-p capture $\mathfrak{M}=4.02$. Interaction contributions to the matrix element for n-p capture in percent.

	S-S	S-D
Catastrophic current	2.95	0.70
Pion current	−1.18	0.70
Rescattering term	0.00	1.94
Total contribution to matrix element = 5.11		
Total contribution to cross section = 10.5		

where

$$\tilde{G}_{\rm I}=\int_0^\infty dr\, u_d(r) G_{\rm I}(r)\, u_{np}(r)\,,$$

$$\tilde{G}_{\rm II}=\int_0^\infty dr\, w_d(r) G_{\rm II}(r)\, u_{np}(r)\,. \tag{37}$$

The total δ for the HJ, RHC, and RSC potentials, respectively, are 10.5, 10.9, and 11.5%. The variation among the results for the three potentials results from the singular behavior of the large S-D rescattering term, which is thus sensitive to the shape of the hard core.

We first remark that we find the pion-current S-S contribution to be negative, contrary to some previous calculations.[4,23,24] It is easy to see from the Chew-Low amplitude [Eq. (22)] that the pion current has the opposite sign of the catastrophic current, but we know of no other way of obtaining the absolute sign than carefully working it through. Padgett, Frank, and Brennan[24] have given a heuristic argument that the pion current (Fig. 5) should augment the nucleon magnetic moment since the sign of the charge of the pion emitted is the same as the sign of the nucleon magnetic moment. Apparently, however, this argument fails when the additional factors for the absorption of the pion at the second nucleon are included (Fig. 1).

The values for the total calculated interaction effect 10.5–11.5% are slightly larger than the experimental value of 9.5%. We have not included the effect of heavier mesons in our calculations, but

TABLE III. Reid hard-core potential results. Impulse integral for n-p capture $\mathfrak{M}=4.04$. Interaction contributions to the matrix element for n-p capture in percent.

	S-S	S-D
Catastrophic current	2.91	0.70
Pion current	−1.15	0.70
Rescattering term	0.00	2.16
Total contribution to matrix element = 5.32		
Total contribution to cross section = 10.9		

TABLE IV. Reid soft-core potential results. Impulse integral for n-p capture $\mathfrak{M}=4.00$. Interaction contributions to the matrix element for n-p capture in percent.

	S-S	S-D
Catastrophic current	3.09	0.72
Pion current	−1.32	0.72
Rescattering terms	0.00	2.40
Total contribution to matrix element = 5.61		
Total contribution to cross section = 11.5		

they are generally found to contribute only very small amounts to interaction effects.[4,9,14,16] Considering the present state of the art and taking into account the lack of knowledge of the precise amount of D state in the deuteron and of how to put the 2π and ρ contributions to the interaction current together, we feel that the agreement is good.

It is interesting to compare the Chew-Low amplitude with the result of the low-energy theorem[15] which gives the photoproduction amplitude correctly to all orders in the strong interaction coupling constant G and to first order in e at the unphysical point $q=k=0$. The low-energy theorem result is important as it permits a consistent treatment of all possible interaction effects of one-pion exchange. We have discussed this point for the weak axial interaction current,[17] but the same arguments are valid for the electromagnetic interaction current. These arguments show that terms of the type considered by Gerstenberger and Nogami[25] in which the magnetic moment of a nucleon is disturbed from its free value by the presence of the second nucleon are effectively included in our calculation. In the nonrelativistic limit the low-energy theorem result reduces to the Chew-Low amplitude with the omission of the rescattering terms if the magnetic terms are neglected. (That is, the Chew-Low amplitude replaces the nucleon magnetic-moment terms of the low-energy theorem result with the

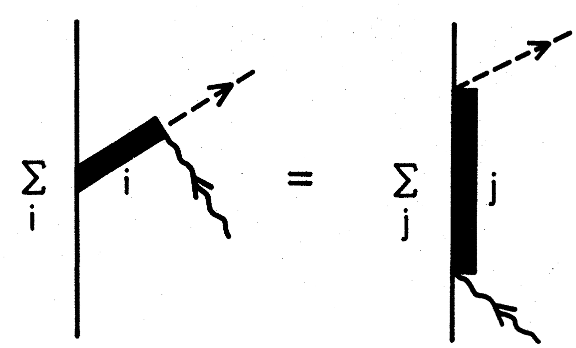

FIG. 7. High-energy duality applied to photopion production.

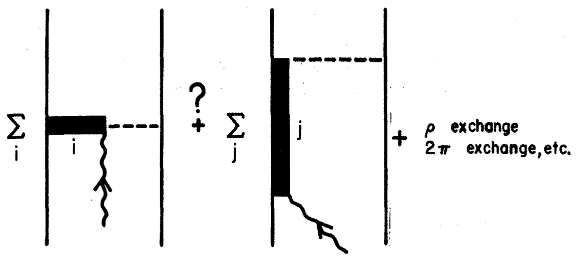

FIG. 8. Possible construction of the complete interaction current.

enhanced 3-3 rescattering contribution.) Thus, the catastrophic and pion pieces of the interaction current are highly reliable, and, indeed, our results for them in Table I agree with those of Chemtob and Rho.[9] The low-energy theorem evaluates the non-Born term, containing the contribution of the 3-3 resonance, to be zero at the unphysical point. Since any method of dealing with the resonance contribution requires an extrapolation away from $q=k=0$, this contribution remains for the present more uncertain than those of the catastrophic and pion current. The Chew-Low model assumes that only the 3-3 state is enhanced and calculates this amplitude in terms of the 3-3 pion-nucleon scattering amplitude. Riska and Brown[14] used the static quark model, but considered their result too large, and adopted the smaller value of Stranahan.[26]

Adler[27] has further noted that, if high-energy duality is true in some sense, it is incorrect to simply add the resonance contributions to the pion contributions. Duality requires that the sum over all t-channel poles [Fig. 7(a)] equal the sum over all s-channel resonances [Fig. 7(b)]. If the pion is absorbed by a second nucleon [Fig. 8] it is easy to see that there may be a problem of double counting in adding the resonance contribution to the pion currents.

In conclusion we have calculated the pion-exchange-current contribution to thermal n-p capture, using the Chew-Low static model to describe the photoproduction of a pion at one of the nucleons. We have seen that the photoproduction amplitude cannot be used in its entirety; not only must the Born terms be omitted, but also the pion-current piece must be used for only one of the nucleons, and to the rescattering piece must be added a term representing rescattering of the pion before photon absorption. Our calculation probably treats the catastrophic and pion current fairly accurately, but a better treatment of the resonance contribution must await advances in the understanding of duality and the amplitudes in physical regions. But we believe on the basis of the present results that the bulk of the long-standing 10% discrepancy in thermal n-p capture is understandable as a one-pion-interaction effect.

ACKNOWLEDGMENTS

The authors would like to thank members of the California Institute of Technology high-energy theory group for many illuminating discussions and one of us (A.H.) would like to thank J. Weneser for originally suggesting this problem.

---

*Work supported in part by the National Science Foundation (GP-28027), in part by Deutsche Forschungsgemeinschaft, and in part by the U. S. Atomic Energy Commission [AT(04-3)-63].

†On leave from Ruhr-Universität, Bochum, West Germany. Present address: State University of New York, Stony Brook, New York 11790.

[1]N. Austern, Phys. Rev. 92, 670 (1953).

[2]N. Austern and E. Rost, Phys. Rev. 117, 1506 (1960).

[3]H. P. Noyes, Nucl. Phys. 74, 508 (1965).

[4]R. J. Adler, B. T. Chertok, and H. C. Miller, Phys. Rev. C 2, 69 (1970).

[5]B. Sakita and C. J. Goebel, Phys. Rev. 127, 1787 (1962); M. H. Skolnick, Phys. Rev. 136, B1493 (1964). Skolnick's claim for agreement with experiment has been disputed by Noyes (Ref. 3).

[6]G. Breit and M. L. Rustgi, Nucl. Phys. A161, 337 (1971).

[7]R. J. Adler, Phys. Rev. C 5, 615 (1972).

[8]R. J. Adler and S. D. Drell, Phys. Rev. Letters 13, 349 (1964).

[9]M. Chemtob and M. Rho, Nucl. Phys. A163, 1 (1971).

[10]S. De Benedetti, *Nuclear Interactions* (Wiley, New York, 1964).

[11]E. M. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill, New York, 1962).

[12]G. F. Chew and F. E. Low, Phys. Rev. 101, 1579 (1956).

[13]A. H. Huffman and M. Gari, Bull. Am. Phys. Soc. 17, 56 (1972).

[14]D. O. Riska and G. E. Brown, Phys. Letters 38B, 193 (1972).

[15]G. W. Gaffney, Phys. Rev. 161, 1599 (1967).

[16]R. J. Adler, Phys. Rev. 169, 1192 (1968).

[17]M. Gari and A. H. Huffman, Astrophys. J. 178, 543 (1972).

[18]F. Zachariasen, Phys. Rev. 101, 371 (1956).

[19]P. Stichel and E. Werner, Nucl. Phys. A145, 257 (1970).

[20]T. Hamada and I. O. Johnston, Nucl. Phys. 34, 382 (1962).

[21]R. V. Reid, Ann. Phys. (N.Y.) 50, 441 (1968).

[22]G. E. Brown, A. M. Green, and W. J. Gerace, Nucl. Phys. A115, 435 (1968).
[23]F. Kaschluhn and K. Lewin, Nucl. Phys. 87, 73 (1966).
[24]D. W. Padgett, W. M. Frank, and J. G. Brennan, Nucl. Phys. 73, 424 (1965).
[25]R. V. Gerstenberger and Y. Nogami, Phys. Rev. Letters 29, 233 (1972).
[26]G. Stranahan, Phys. Rev. 135, B933 (1964).
[27]R. J. Adler, Bull. Am. Phys. Soc. 15, 476 (1970).

PHYSICAL REVIEW C VOLUME 7, NUMBER 3 MARCH 1973

Effect of Isobar Configurations on the Photodisintegration of the Deuteron

H. G. Miller* and H. Arenhövel†
Abteilung für Kernphysik, Max-Planck-Institut für Chemie, Mainz, West Germany
(Received 24 October 1972)

A nonrelativistic calculation of the photodisintegration of the deuteron below pion threshold has been completed in which nucleon polarization effects described by isobar configurations ($N\Delta$ and $\Delta\Delta$) have been included. At lower energies ($E_\gamma^{\text{lab}} \leq 40$ MeV) the inclusion of these isobar configurations leads to an over-all decrease of the total cross section and the proton angular distribution due to renormalization of the normal deuteron components. At higher energies the total cross section is enhanced (up to 5% at $E_\gamma^{\text{lab}} = 120$ MeV) and the proton angular distributions are more forward peaked. These results lead to better agreement with experiment.

It is remarkable that even for the simplest nucleus, the deuteron, discrepancies exist between theory and experiment for the photodisintegration.[1] Recently, it has been shown[2,3] that at threshold nucleon-polarization and meson-exchange-current contributions can account for this discrepancy. We, therefore, have investigated the question: What is the effect of nucleon polarization within the deuteron on the photodisintegration at higher photon energies but below pion-production threshold? We still did not include explicitly meson-exchange-current contributions.

In the present work the nucleon polarization, i.e. excitation of internal degrees of freedom, is described by admixtures of baryon resonances, the so-called isobar configurations,[4] in the two-nucleon system. The large internal excitation energies of the isobar configurations compared to the binding energy of the deuteron and the interaction energy justifies a perturbative treatment in the calculation of these isobar configurations. This has been described in detail by Arenhövel, Danos, and Williams.[5] The inclusion of intrinsic nucleon degrees of freedom affects primarily the short-range region of the nuclear wave function, thereby enhancing the high-momentum components. Such effects, which are in some respect similar to short-range correlations, are of special interest in processes such as electron scattering for high-momentum transfers[6] and photodisintegration of the deuteron since they provide valuable information about the region of small internucleon distances (<1 fm).

In our calculation the normal n-p wave functions for the initial-bound and final-continuum states have been obtained from the Hamada-Johnston potential. At present only the dominant ($N\Delta$) and ($\Delta\Delta$) isobar configurations have been considered. Isospin-selection rule forbids the ($N\Delta$) configuration in the bound-deuteron state. For the inelastic NN-$N\Delta$ and NN-$\Delta\Delta$ interactions, which are needed in the previously mentioned perturbative treatment, we used a one-pion-exchange potential (OPEP). Since we deal with hard-core wave functions it was not necessary to regularize the OPEP. However, we would like to mention that because of the neglect of heavier boson-exchange contributions and off-the-mass-shell effects our results are to some extent qualitative. Some radial wave functions of the normal and isobar configurations are given in Fig. 1 for the bound-deuteron and two continuum states corresponding to incoming-photon lab energies of 40 and 100 MeV. The radial functions of the isobar configurations characteristically peak for small r (~0.7 fm) and fall off rapidly with increasing r. Unlike the normal components they do not vanish inside the hard core. This anomaly, which is not critical, arises from the use of the free propagator of the isobar configuration in the perturbative calculation.

The calculation of the photodisintegration cross section has been performed in close analogy to