[28]The definition of the half-shell T matrix element $t(k, k_0)$ is $t(k, k_0) \equiv e^{-i\delta_0(k_0)} t(k, k_0; k_0^2 + i\epsilon)$, where δ_0 is the S-wave phase shift.

[29]M. Baranger, *The Two-Body Force in Nuclei* (Plenum, New York, 1972), p. 367.

[30]J. E. Elias, J. I. Friedman, G. C. Hartmann, H. W. Kendall, P. N. Kirk, M. R. Sogard, and L. P. Van Speybroeck, Phys. Rev. 177, 2075 (1969), and references listed therein.

[31]H. S. Picker, E. F. Redish, and G. J. Stephenson, Jr., Phys. Rev. C 4, 287 (1971).

[32]Electromagnetic process such as deuteron photo- or electrodisintegration or p-$p\gamma$ reactions could possibly yield some information on the $^1S_0$ off-shell T matrix. See, for example, F. Partovi, Ann. Phys. (N.Y.) 27, 79 (1964).

[33]G. E. Brown, A. M. Green, and W. J. Gerace, Nucl. Phys. A115, 435 (1968); C. Pask, Phys. Letters 25B, 78 (1967), estimates a 1.0–1.5-MeV gain in binding for the triton due to three-body forces.

[34]J. Dabrowski, E. Fedoryńsha, P. Haensel, and M. Y. M. Hassan, Phys. Rev. C 4, 1985 (1971), find that possible charge dependence in the N-N interaction can change E_T by several MeV.

PHYSICAL REVIEW C VOLUME 7, NUMBER 1 JANUARY 1973

New Integrodifferential Equation and Integral Constraint for the Correlation Function Between Two Nucleons in Nuclear Matter

M. E. Grypeos
Department of Theoretical Physics, University of Thessaloniki, Thessaloniki, Greece

(Received 4 August 1972)

A nonlinear integrodifferential equation for the correlation function between two nucleons in nuclear matter is obtained. This equation results by writing the (trial) expression of the energy per particle in nuclear matter in a suitable form and applying subsequently the variational principle. The investigation of the behavior of the equation for large internucleon distances r_{12} leads to a new integral constraint on the correlation function.

1. INTRODUCTION

The determination of the nucleon-nucleon correlation function, which is used in the variational or "Jastrow" approach to nuclear matter[1-3] has been a thorny problem for a long time.

In this approach the trial many-body wave function

$$\Psi_N = S_N \prod_{i<j}^{N} f(r_{ij}) \tag{1}$$

is employed for the calculation of the (trial) expression of the energy per particle E/N.

In expression (1), S_N is a Slater determinant in which the orbital parts of the single-particle wave functions are plane waves and f the nucleon-nucleon correlation function. This should be chosen to be zero inside and at the hard-core radius c of the nucleon-nucleon potential and to approach unity sufficiently rapidly for large internucleon distances.

The E/N is written in the form of a cluster expansion

$$\frac{E}{N} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \cdots \tag{2}$$

in which only the first few terms are retained. The expansion is often truncated at $\mathcal{E}_2$ and the energy per particle depends upon the correlation function only through the second term, which is proportional to the constant density ρ of the (infinite) nuclear matter. The first term is just the Fermi energy:

$$\mathcal{E}_1 = \frac{3}{5}\frac{\hbar^2 K_F^2}{2m}. \tag{3}$$

The problem of determining f has been faced in two ways:

In the first, a suitable analytic form is assumed for the f in which there are certain parameters and the E/N is minimized with respect to them. It was realized, however, that the correlation function has to be restricted in order to avoid the so called "Emery difficulty,"[3,4] which is due to the absence of normalization in the above-mentioned truncated cluster expansion of E/N. Various restrictions have been used. The early types of them have been suggested in Ref. 3. The conditions which were employed recently are of integral form, and one of the parameters in the expression of the f is fixed by the requirement that the considered condition is satisfied. It should be noted, however, that it is not clear which of the various conditions is the appropriate one to be used, and consequently there is a degree of arbitrariness in imposing the one condition or the other.

In the second way of determining f, a functional variation is applied to the truncated cluster expansion. This approach meets also with difficulties, because the Euler equation does not have the proper asymptotic behavior, and as a result f does not tend to unity sufficiently rapidly for large distances.

In order to avoid the existing difficulties, Jastrow had the ingenious idea of suggesting the imposition of an *"ad hoc"* integral constraint when the variational principle is applied to the truncated expansion. This has been further pursued recently by Schäfer and Schütte.[5] Similar work has also been done, for hypernuclear matter, by Grypeos, Kok, and Ali.[6] Although this approach is quite attractive, the introduction of the arbitrary constraint in the variation is not fully satisfactory.

In this paper we attempt to solve the existing problem by trying to apply the variational principle in a different way, in which the missing normalization becomes apparent. The present investigation is somehow similar tc that we have developed for other systems of physical interest, namely, the impure nuclear matter[7] and the many-particle Bose system.[8] The experience from the study of these systems[7-10] has been quite useful. The problem of nuclear matter, however, is much more difficult, owing to the antisymmetry of the wave function. Note that in the case of impure nuclear matter (e.g. of a Λ particle in nuclear matter) we are interested in the determination of the Λ-nucleon correlation function and no difficulties of this sort arise because of the nonidentity of the two particles.

In the next section we derive a new nonlinear integrodifferential equation for the nucleon-nucleon correlation function. This is the Euler equation of the suitably posed variational problem we mentioned previously.

In Sec. 3 we study the behavior of the integrodifferential equation for large internucleon distances r_{12} and we derive an integral constraint on the correlation function, which is suggested by the requirement of the proper asymptotic behavior of this equation. The final section is devoted to a discussion.

2. DERIVATION OF A NONLINEAR INTEGRODIFFERENTIAL EQUATION FOR THE NUCLEON-NUCLEON CORRELATION FUNCTION

The system which we consider consists of N fermions (nucleons) enclosed in a volume Ω. This system is supposed to be in its ground state and also that N is very large, as well as Ω, such that the density is constant:

$$\frac{N}{\Omega}=\rho . \tag{4}$$

The Hamiltonian of the system is

$$\hat{H}=-\frac{\hbar^2}{2m}\sum_{K=1}^{N}\nabla_K{}^2+\sum_{l<n}^{N}V(r_{ln}), \tag{5}$$

where V_{ln} are the (short-range) interparticle potentials, which are assumed to have a hard core of radius c. If there is a soft core instead, then $c\to 0$. In such a case the correlation function is expected to be suppressed for small r_{ln}.

The expectation value of the Hamiltonian with respect to trial many-body wave function (1) is

$$\langle\hat{H}\rangle=\frac{\int S_N^*\prod_{i<j}^{N}f(r_{ij})\left(-(\hbar^2/2m)\sum_{K=1}^{N}\nabla_K{}^2+\sum_{l<n}^{N}V(r_{ln})\right)S_N\prod_{i<j}^{N}f(r_{ij})d\vec{r}_1\cdots d\vec{r}_N}{\int S_N^*S_N\prod_{i<j}^{N}f^2(r_{ij})d\vec{r}_1\cdots d\vec{r}_N}. \tag{6}$$

As was shown by Aviles,[2] we can obtain from the above formula the following (trial) expression for the energy per particle in nuclear matter:

$$\frac{E}{N}=\frac{\langle\hat{H}\rangle}{N}=\frac{3}{5}\frac{\hbar^2K_F{}^2}{2m}+\frac{\rho}{2}\int\left[\left(\frac{\hbar^2}{2m}[(\nabla f)^2-f\nabla^2 f]+V(r_{12})f^2\right)G_F(r_{12})-\frac{\hbar^2}{2m}\tfrac{1}{2}\vec{\nabla}f^2\cdot\vec{F}(\vec{r}_{12})\right]d\vec{r}_{12}, \tag{7}$$

where the radial distribution functions $G_F(r_{12})$ and $\vec{F}(\vec{r}_{12})$ are defined as follows:

$$G_F(r_{12})=\frac{N(N-1)}{\rho^2 f^2(r_{12})}\frac{\int\prod_{i<j}^{N}f^2(r_{ij})S_N^*S_N d\vec{r}_3\cdots d\vec{r}_N}{\int\Psi_N^*\Psi_N d\vec{r}_1\cdots d\vec{r}_N}, \tag{8}$$

$$\vec{F}(\vec{r}_{12})=\frac{N(N-1)}{\rho^2 f^2(r_{12})}\frac{\int\prod_{i<j}^{N} f^2(r_{ij})\vec{\nabla}_1(S_N^*S_N)d\vec{r}_3\cdots d\vec{r}_N}{\int \Psi_N^*\Psi_N d\vec{r}_1\cdots d\vec{r}_N}. \tag{9}$$

The cluster expansions of these functions are well known[2]:

$$G_F(r_{12})=D_{12}+\rho\int[h_{13}h_{23}D_{123}+2h_{13}(D_{123}-D_{13}D_{12})]d\vec{r}_3+\cdots, \tag{10}$$

$$\vec{F}(\vec{r}_{12})=\vec{\nabla}_1 D_{12}+\rho\int[h_{13}h_{23}\vec{\nabla}_1 D_{123}+2h_{13}(\vec{\nabla}_1 D_{123}-D_{13}\vec{\nabla}_1 D_{12})]d\vec{r}_3+\cdots, \tag{11}$$

where

$$D_{12}=1-\frac{1}{s}l^2(K_F r_{12}), \quad l(K_F r_{12})=\frac{3j_1(K_F r_{12})}{K_F r_{12}}, \tag{12}$$

$$D_{123}=1-\frac{1}{s}[l^2(K_F r_{12})+l^2(K_F r_{13})+l^2(K_F r_{23})]+\frac{2}{s^2}l(K_F r_{12})l(K_F r_{13})l(K_F r_{23}), \tag{13}$$

and

$$h_{ij}=f^2(r_{ij})-1, \quad i,j=1,2,3. \tag{14}$$

For nuclear matter in which we are particularly interested, the degeneracy of the single-particle levels is $s=4$. We write the radial distribution functions G_F and $\vec{F}$ in the following forms [which are easily obtained from expressions (8) and (9)]:

$$G_F(r_{12})=\frac{N-1}{\rho}\frac{G_F(r_{12})}{\int f^2(r_{12})G_F(r_{12})d\vec{r}_{12}}, \tag{15}$$

$$\vec{F}(\vec{r}_{12})=\frac{N-1}{\rho}\frac{\vec{F}(\vec{r}_{12})}{\int f^2(r_{12})G_F(r_{12})d\vec{r}_{12}} \tag{16}$$

and we substitute into expression (7):

$$\frac{E}{N}=\frac{3}{5}\frac{\hbar^2 K_F{}^2}{2m}+\frac{(N-1)\frac{1}{2}\rho\int\left[\left(\frac{\hbar^2}{2m}[(\nabla f)^2-f\nabla^2 f]+V(r_{12})f^2\right)G_F(r_{12})-\frac{\hbar^2}{2m}\frac{1}{2}\vec{\nabla}f^2\cdot\vec{F}(\vec{r}_{12})\right]d\vec{r}_{12}}{\rho\int f^2(r_{12})G_F(r_{12})d\vec{r}_{12}}. \tag{17}$$

By writing the E/N in this form we can allow for the normalization. Variation of (17) is equivalent (in the limit $N\to\infty$, $\Omega\to\infty$, such that $N/\Omega=\rho$) to variation of (7) provided that

$$\rho\int[f^2G_F(r_{12})-1]d\vec{r}_{12}=\text{finite constant}. \tag{18}$$

We should note that this constraint has its origin to the denominator and the factor $(N-1)$ in expression (17), which appear because we have written the functions G_F and $\vec{F}$ in the form (15) and (16), respectively.

It is observed that condition (18) is a sort of "general constraint" in the sense that it merely, expresses the fact (in a concrete manner, however, which is very important for the structure of the equation to be obtained and its asymptotic behavior) that the correlation function must be of "finite range." The precise value of the integral in (18) is not yet known. The "specific constraint" on the correlation function will be derived in the next section.

The variational problem we have to solve is therefore the following:

$$\delta\frac{4\pi\rho}{2}\int_c^\infty \mathcal{L}(r_{12},f,f',f'')dr_{12}=0, \tag{19}$$

where $\mathcal{L}$ is given by

$$\mathcal{L}=\mathcal{L}_1+\lambda\mathcal{L}_2 \tag{20}$$

with

$$\mathcal{L}_1=\left(\left\{\frac{\hbar^2}{2m}\left[\left(\frac{df}{dr_{12}}\right)^2-f\left(\frac{d^2f}{dr_{12}^2}+\frac{2}{r_{12}}\frac{df}{dr_{12}}\right)\right]+V(r_{12})f^2\right\}G_F(r_{12})-\frac{\hbar^2}{2m}f\frac{df}{dr_{12}}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\right)r_{12}^2 \tag{21}$$

and

$$\mathcal{L}_2=[f^2G_F(r_{12})-1]r_{12}^2 . \tag{22}$$

The Euler equation of the variational problem is

$$\frac{\partial\mathcal{L}}{\partial f}-\frac{d}{dr_{12}}\frac{\partial\mathcal{L}}{\partial f'}+\frac{d^2}{dr_{12}^2}\frac{\partial\mathcal{L}}{\partial f''}=0 . \tag{23}$$

By taking into account the dependence (through integral expressions) of the distribution functions G_F and $\vec{F}$ on the correlation function we obtain after some algebra the following equation:

$$\begin{aligned}\left(-\frac{\hbar^2}{m}\right)&\left[r_{12}^2G_F(r_{12})\frac{d^2f}{dr_{12}^2}+\left(2r_{12}G_F(r_{12})+r_{12}^2\frac{dG_F(r_{12})}{dr_{12}}\right)\frac{df}{dr_{12}}\right]\\ &+\left(r_{12}^2[V(r_{12})+\lambda]G_F(r_{12})+\frac{\hbar^2}{2m}r_{12}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\right.\\ &\left.+\left(-\frac{\hbar^2}{2m}\right)r_{12}\left\{\frac{dG_F(r_{12})}{dr_{12}}+\tfrac{1}{2}r_{12}\left[\frac{d^2G_F(r_{12})}{dr_{12}^2}-\frac{d}{dr_{12}}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\right]\right\}\right)f\\ &+r_{12}^2\left\{\left(\frac{\hbar^2}{2m}\right)\left[\left(\frac{df}{dr_{12}}\right)^2-f\left(\frac{d^2f}{dr_{12}^2}+\frac{2}{r_{12}}\frac{df}{dr_{12}}\right)\right]+[V(r_{12})+\lambda]f^2\right\}\frac{1}{2}\frac{\partial G_F(r_{12})}{\partial f}\\ &+r_{12}^2\left(-\frac{\hbar^2}{2m}\right)f\frac{df}{dr_{12}}\frac{1}{2}\frac{\partial}{\partial f}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)=0 .\end{aligned} \tag{24}$$

In this equation $\partial/\partial f$ denotes partial derivative with respect to f.

We see that the Euler equation is a nonlinear integrodifferential equation. This has to be solved with boundary conditions

$$f(c)=0 , \quad f(\infty)=1 . \tag{25}$$

We may note in passing that (24) has similar structure to the Euler equation for a many-particle Bose system. In the case of such a system the terms arising from the $\vec{F}$ do not appear and also instead of G_F we have the function G, defined by

$$G(r_{12})=\frac{N(N-1)}{\rho^2f^2(r_{12})}\frac{\int\prod_{i<j}^{N}f^2(r_{ij})d\vec{r}_3\cdots d\vec{r}_N}{\int\prod_{i<j}^{N}f^2(r_{ij})d\vec{r}_1\cdots d\vec{r}_N} . \tag{26}$$

In Ref. 7 an approximate expression for $G(r_{12})$ has been used.

We give finally the equation which results if we make the transformation

$$f(r_{12})=1-\frac{c}{r_{12}}u(r_{12}) . \tag{27}$$

This equation is:

$$
\begin{aligned}
&\frac{\hbar^2}{m} c r_{12}\left\{\left(G_F(r_{12})+\frac{1}{4}\,\frac{\partial G_F(r_{12})}{\partial f}\right)\frac{d^2u}{dr_{12}^{\,2}}+\left[\frac{dG_F(r_{12})}{dr_{12}}+\frac{1}{4}\,\frac{\partial}{\partial f}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\right]\frac{du}{dr_{12}}\right\}\\
&\quad+c\left(\frac{\hbar^2}{2m}\left\{\tfrac{1}{2}r_{12}\left[\frac{d^2G_F(r_{12})}{dr_{12}^{\,2}}-\frac{d}{dr_{12}}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\right]\right.\right.\\
&\qquad\left.\left.-\frac{dG_F(r_{12})}{dr_{12}}-\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})-\frac{1}{2}\,\frac{\partial}{\partial f}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\right\}-r_{12}[V(r_{12})+\lambda]\left(G_F(r_{12})+\frac{\partial G_F(r_{12})}{\partial f}\right)\right)u\\
&=\left(-r_{12}^{\,2}[V(r_{12})+\lambda]\left(G_F(r_{12})+\frac{1}{2}\,\frac{\partial G_F(r_{12})}{\partial f}\right)\right.\\
&\qquad\left.-\frac{\hbar^2}{2m}r_{12}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)+\frac{\hbar^2}{2m}r_{12}\left\{\frac{dG_F(r_{12})}{dr_{12}}+\tfrac{1}{2}r_{12}\left[\frac{d^2G_F(r_{12})}{dr_{12}^{\,2}}-\frac{d}{dr_{12}}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\right]\right\}\right)\\
&\quad-[V(r_{12})+\lambda]\frac{c^2}{2}\,\frac{\partial G_F(r_{12})}{\partial f}u^2+\frac{\hbar^2c^2}{2m}\left\{\frac{1}{2}\,\frac{\partial G_F(r_{12})}{\partial f}\left[u\frac{d^2u}{dr_{12}^{\,2}}-\left(\frac{u}{r_{12}}-\frac{du}{dr_{12}}\right)^2\right]-\frac{1}{2}\,\frac{\partial}{\partial f}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)\left(\frac{u}{r_{12}}-\frac{du}{dr_{12}}\right)u\right\}.
\end{aligned}
\tag{28}
$$

The boundary conditions for the new correlation function u are

$$u(c)=1\,,\quad u(\infty)=0\,. \tag{29}$$

3. INTEGRAL CONSTRAINT ON THE CORRELATION FUNCTION

It is interesting to investigate the behavior of the integrodifferential equation for large values of r_{12}. As we shall see this investigation will lead to the "specific" constraint on the correlation function, we mentioned in the previous section.

Let us consider first the simplest case, namely, that the cluster expansions for G_F and $\vec{F}$ are truncated at their first term. That is

$$G_F=D_{12}\,,\quad \vec{F}=\vec{\nabla}_1 D_{12}\,. \tag{30}$$

Therefore we have

$$\frac{\partial G_F}{\partial f}=0\,,\quad \frac{\partial\vec{F}}{\partial f}=0\,. \tag{31}$$

Taking into account expression (12) and that for large r_{12},

$$l(K_F r_{12})\approx 0\,, \tag{32}$$

we see that in the region of large distances r_{12}, G_F tends to unity and $\vec{F}$ to zero. The potential $V(r_{12})$ tends also to zero. Therefore, beyond a certain very large distance d, the equation takes the form

$$\frac{\hbar^2}{m}\,\frac{d^2u}{dr_{12}^{\,2}}-\lambda u=\frac{r_{12}}{c}(-\lambda)\,,\quad d<r_{12}<\infty\,. \tag{33}$$

It is obvious that the boundary condition at infinity $[u(\infty)=0]$ cannot be satisfied, since the right-hand side of the above equation becomes infinitely large when $r_{12}\to\infty$. Even if $\lambda=0$ [which means that constraint (18) is not taken into account] this condition cannot be satisfied. In such a case the solution of (33) is $c_1r_{12}+c_2$ $(d<r_{12}<\infty)$ and even if we choose $c_1=0$, u becomes a constant and therefore f behaves for large r_{12} as $1-cc_2/r_{12}$, with the effect that the integral in (18) has not a finite value, as we expect.

Let us now truncate the cluster expansions for G_F and $\vec{F}$ at the next term (proportional to ρ). That is we include the three-body terms but we neglect the higher terms, indicated by dots in formulas (10) and (11). We see that the distribution function G_F can be written in the form

$$G_F(r_{12})=1+G_F^{(0)}(r_{12})\,, \tag{34}$$

where

$$G_F^{(0)}(r_{12}) = -\frac{1}{s}l_{12}^2 + \rho\int h_{13}h_{23}\left(1-\frac{1}{s}(l_{12}^2+l_{13}^2+l_{23}^2)+\frac{2}{s^2}l_{12}l_{13}l_{23}\right)d\vec{r}_{13} + 2\rho\int h_{13}\left(-\frac{1}{s}l_{23}^2+\frac{2}{s^2}l_{12}l_{13}l_{23}-\frac{1}{s^2}l_{13}^2l_{12}^2\right)d\vec{r}_{13}\,. \tag{35}$$

[We have put $l(K_F r_{ij}) = l_{ij},\ i,j=1,2,3.$]

Taking into account expression (32) and that the integrals in expression (35) are supposed to be calculated with a function $h=f^2-1=(c^2/r_{12}^2)u^2-(2c/r_{12})u$ approaching zero sufficiently rapidly for large r_{12}, we see that for $d<r_{12}<\infty$

$$G_F^{(0)}(r_{12}) \approx 0 \tag{36}$$

and therefore G_F becomes again close to unity for large r_{12}. Obviously the functions dG_F/dr_{12} and d^2G_F/dr_{12}^2 should tend quite rapidly to zero. The examination of the asymptotic behavior of the other functions in the equation, namely,

$$\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12}),\quad \frac{\partial G_F}{\partial f},\quad \frac{\partial}{\partial f}\left(\frac{\vec{r}_{12}}{r_{12}}\cdot\vec{F}(\vec{r}_{12})\right)$$

can be done in a similar way. Among these functions the $\partial G_F/\partial f$ requires some special attention and we shall deal with it in detail.

The expression for $\partial G_F/\partial f$ can be easily obtained from (10) (truncated at the three-body terms):

$$\frac{\partial G_F}{\partial f} = \rho\int[4f_{13}h_{23}D_{123}+4f_{13}(D_{123}-D_{13}D_{12})]d\vec{r}_3\,. \tag{37}$$

We write

$$f=1+f^{(0)}\quad\left(\text{that is } f^{(0)}=-\frac{c}{r}u\right), \tag{38}$$

where $f^{(0)}$ tends sufficiently rapidly to zero for large distances. In this way we obtain the following expression for $\partial G_F/\partial f$:

$$\frac{\partial G}{\partial f} = K + \left(\frac{\partial G}{\partial f}\right)^{(0)}, \tag{39}$$

where

$$K = 4\rho\int(D_{12}-1)d\vec{r}_{12} + 4\rho\int(f^2-1)D_{12}d\vec{r}_{12}\,. \tag{40}$$

Note that we have put, in the above definite integrals, r_{12} instead of r_{23}. The function $(\partial G_F/\partial f)^{(0)}$ is given by

$$\left(\frac{\partial G}{\partial f}\right)^{(0)} = 4\rho\int h_{23}\left(-\frac{1}{s}l_{12}^2-\frac{1}{s}l_{13}^2+\frac{2}{s^2}l_{12}l_{13}l_{23}\right)d\vec{r}_3 + 4\rho\int f_{13}^{(0)}h_{23}\left(D_{23}-\frac{1}{s}l_{12}^2-\frac{1}{s}l_{13}^2+\frac{2}{s^2}l_{12}l_{13}l_{23}\right)d\vec{r}_3$$
$$+4\rho\int f_{13}^{(0)}\left(-\frac{1}{s}l_{23}^2\right)d\vec{r}_3 + 4\rho\int(1+f_{13}^{(0)})\left(\frac{2}{s^2}l_{12}l_{13}l_{23}-\frac{1}{s^2}l_{13}^2l_{12}^2\right)d\vec{r}_3\,. \tag{41}$$

It is easily seen from this expression that for large r_{12}

$$\left(\frac{\partial G}{\partial f}\right)^{(0)} \approx 0\,. \tag{42}$$

Taking into account all the above remarks we see that while the left-hand side of (28) [having also divided both sides of (28) by cr_{12}] can approach zero as r_{12} becomes very large, there is a term surviving in the right-hand side of the equation. Unless it is required that this term be zero, the existence of a solution u, approaching zero for large r_{12} is not possible. This term is

$$\frac{r_{12}}{c}[-\lambda(1+\tfrac{1}{2}K)]\,. \tag{43}$$

Therefore we must have $K=-2$. That is

$$4\rho\int(D_{12}-1)d\vec{r}_{12} + 4\rho\int[f^2(r_{12})-1)D_{12}d\vec{r}_{12} = -2\,. \tag{44}$$

This condition may be written more compactly as follows:

$$2\rho\int[1-f^2(r_{12})D_{12}]d\vec{r}_{12}=1\,. \tag{45}$$

We may remark that the very important term $r_{12}/c[-\lambda(1+\frac{1}{2}K)]$, as well as the other equally important terms in the left-hand side of the equation (28) which become asymptotically $-\lambda(1+K)$ [note that we have divided both sides of (28) by cr_{12}] owe their origin to constraint (18) (since otherwise λ should be zero), and to the inclusion of the three-body terms in the cluster expansion of G_F. If these terms had not been included then $K=0$ and the terms (43) could not become zero, since $\lambda\neq 0$. It is also clear that in order (28) has a solution which tends to zero for $r_{12}\to\infty$, the value of λ must be negative.

If four-body and higher terms are included in the cluster expansions, the examination of the asymptotic behavior of the equation may proceed in a similar way. This, however, becomes very complicated.

We finally point out that condition (45) is a "specific" constraint on the correlation function in contrast to (18), which merely requires the value of the integral $\rho\int[f^2G_F(r_{12})-1]d\vec{r}_{12}$ to be any finite number. The examination of the asymptotic behavior of (28) led us to "specific" constraint (45) and therefore the value of the Lagrange multiplier λ should be chosen in such a way that this condition is satisfied. Then the value of the integral in (18) is also specified.

4. DISCUSSION

According to the results of the previous sections the nucleon-nucleon correlation function is obtained by solving numerically the nonlinear equation (24) with boundary conditions (25) [or equivalently (28) with boundary conditions (29)], choosing the value of λ in the prescribed way.

It might be argued that due to the complexity of the problem such an approach is not practical. It should be noted, however, that at least an approximate solution may be obtained if we linearize first this equation and follow techniques analogous to those used for the impure nuclear matter problem.[11] The pure nuclear matter problem we are discussing is of course more complicated, but no fundamental difficulty should arise in obtaining such an approximate solution, provided that a computer with very high speed and large memory is available.

On the other hand the analysis of Sec. 3 has led to a new integral constraint on the correlation function. This constraint could therefore be used when a correlation function of given analytical shape is assumed, as for example[12-14]

$$f(r_{12})=\begin{cases}0\,, & 0\leq r_{12}\leq c\\ (1-e^{-\mu_1(r_{12}-c)})(1+\nu e^{-\mu_2(r_{12}-c)})\,, & c\leq r_{12}<\infty\,.\end{cases} \tag{46}$$

The value of ν should then be chosen to be one of the roots of the second-order equation

$$A\nu^2+B\nu+C=0 \tag{47}$$

to which condition (45) is equivalent for a correlation function of the type (46). The A, B, and C are given integral expressions. The other parameters (μ_1, μ_2, and K_F) are determined by minimizing the trial expression of the energy per particle with respect to them.

It is clear that in the above described approach at least two parameters should appear in the correlation function, so that the one is fixed by condition (45) while the other is left for the minimization of E/N. Therefore one-parameter correlation functions, such as

$$f(r_{12})=\begin{cases}0\,, & 0\leq r_{12}\leq c\\ 1-e^{-\mu(r_{12}-c)}\,, & c\leq r_{12}<\infty\,,\end{cases} \tag{48}$$

which has been frequently used in the past, are not appropriate.

It might also be advisable to take $\mu_1=\mu_2$ in the expression of the correlation function (46) if the value of the so-called "healing integral" (or "wound integral"),

$$H=\rho\int(1-f_{12})^2D_{12}d\vec{r}_{12}\,, \tag{49}$$

is large, since this would indicate that the magnitude of the omitted higher terms is not sufficiently small.

It should be also noted that the constraint we have obtained on the basis of the theoretical arguments developed in Sec. 3 differs from two other well-known conditions (given in the form of inte-

TABLE I. Comparison of the constraints on the correlation function for various physical systems.

Physical system	Many-particle Bose system	Impure nuclear matter (hypernuclear matter)	Nuclear matter
Constraint on the correlation function	$2\rho\int(1-f_{B_1B_2}{}^2)\,d\vec{r}_{B_1B_2}=1$	$\rho\int(1-f_{12}{}^2D_{12})\,d\vec{r}_{12}=1$	$2\rho\int(1-f_{12}{}^2D_{12})d\vec{r}_{12}=1$

gral constraints), which have been used in nuclear matter calculations, but have not been obtained in a way similar to that we have exhibited, namely:
(a) The (first order) "normalization condition":

$$\rho \int (1 - f_{12}{}^2 D_{12}) d\vec{r}_{12} = 1 \, ; \qquad (50)$$

(b) The "orthogonality condition," which arises from the imposition of an "average Pauli principle restriction":

$$\rho \int (1 - f_{12}) D_{12} d\vec{r}_{12} = 0 \, . \qquad (51)$$

These conditions have been discussed recently by Clark and Ristig.[15] The first[16, 17] is formally closer to ours, though there is a difference by a factor of 2. The second[12-15] is imposed "in a desire to make Jastrow theory look as much like Brueckner theory as possible."[15]

Finally it seems appropriate to make a comparison of integral constraint (45) with the corresponding constraints for the other systems, that is of the hypernuclear matter and the many-particle Bose system, we have studied in a rather similar way.[7, 8]

It is seen from Table I that in the case of the many-boson system the constraint differs from (45) in certain aspects. This constraint goes over to (45) if $f_{B_1B_2} \to f_{12}$ and the antisymmetry is neglected (that is $D_{12} = 1$). Obviously this is expected.

Pertaining to the case of hypernuclear matter the difference is due to the lack of a factor of 2 in the constraint for this system. This may well be understood by making a comparative study of the derivations of the constraints in the two cases. In the ordinary nuclear matter the pair of nucleons (1, 2), which we treat explicitly in the variational treatment "is coupled" to the third nucleon by <u>*two*</u> functions $h = f^2 - 1$, while in the case of hypernuclear matter the pair $(1, \Lambda)$ is coupled to nucleon 2 by only <u>*one*</u> h function. Note also that in the latter case the constraint is again on the nucleon-nucleon correlation function and not on the Λ-nucleon one, for which the integrodifferential equation was derived.

In conclusion we would like to emphasize the significance of the appropriate treatment of the energy denominator and the inclusion of the three-body terms in the cluster expansions.

---

[1]R. Jastrow, Phys. Rev. <u>98</u>, 1479 (1955).
[2]J. B. Aviles, Ann. Phys. (N.Y.) <u>5</u>, 251 (1958).
[3]V. J. Emery, Nucl. Phys. <u>6</u>, 585 (1958).
[4]J. S. Bell and E. J. Squires, Advan. Phys. <u>10</u>, 211 (1961).
[5]K. Schäfer and G. Schütte, Nucl. Phys. <u>A183</u>, 1 (1972).
[6]M. E. Grypeos, L. P. Kok, and S. Ali, Nucl. Phys. <u>B4</u>, 335 (1968).
[7]M. E. Grypeos, Nuovo Cimento <u>6</u>, 245 (1971).
[8]M. E. Grypeos, International Centre for Theoretical Physics Report No. 71/73 (to be published).
[9]P. Westhaus, Ph.D. thesis, 1966 (unpublished).
[10]M. E. Grypeos, Nuovo Cimento Letters <u>4</u>, 973 (1970).
[11]M. E. Grypeos, Nuovo Cimento Letters <u>3</u>, 129 (1972).
[12]D. A. Chakkalakal, Ph.D. thesis, 1968 (unpublished).
[13]S. O. Bäckman, D. A. Chakkalakal, and J. W. Clark, Nucl. Phys. <u>A130</u>, 635 (1969).
[14]C. W. Wong, Phys. Rev. C <u>3</u>, 1058 (1971).
[15]J. W. Clark and M. L. Ristig, Subsidiary Conditions on Nuclear Many Body Theories, Abo Academy report (to be published).
[16]G. Müeller and J. W. Clark, Nucl. Phys. <u>B7</u>, 227 (1968).
[17]M. E. Grypeos and E. Mavrommatis, The Pair Distribution Function and the Conditions Imposed in Jastrow Method for Nuclear Matter, Nuclear Research Centre "Democritos," 1972 (to be published).