to one's convenience. For small A, the wave function does not seem to be so nearly factorable and solving Eq. (27) should lead to an improvement in both the energy and the wave function.

In summary, we point out that a continuous class of transformations (for generating a state which is an eigenstate of the total momentum from a state which is not) have been presented. Of this class, the transformation which minimizes the energy is the transformation characterized by the function G which satisfied Eq. (27). This transformation yields the variationally "best" intrinsic wave function $(\hat{G}|\Psi\rangle$. Thus quantities which may be sensitive to how the center of mass is treated[4,5] should be calculated using the transformation characterized by $\hat{G}$ as shown above.

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Renormalization of the Axial-Vector Coupling Constant in Nuclear β Decay*

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The isoscalar magnetic moments and Gamow-Teller β decay of the systems $A=11$, 13, 15, 17, 19, and 21 are analyzed in an attempt to extract, in a largely model-independent way, the effective nuclear renormalization of the axial coupling constant: $g_A \rightarrow g_{Ae}$. We find: $g_{Ae}/g_A = 0.920 \pm 0.047$.

INTRODUCTION

Conservation of the vector current, which, for the purposes of this paper, we take to be absolute, assures us, in effect, that the Fermi β decay of nucleons bound into a nucleus is identical in intrinsic strength with that for nucleons in the free state: The strong interactions that clothe the bare nucleon in the free state and that, additionally, bind nucleons together into complex systems do not renormalize the effective intrinsic vector coupling constant; we do not have to ask how the overall nuclear Fermi β-decay strength is divided between baryonic and mesonic components, since an exact compensation exists and the differing mesonic currents as between one nucleus and another merely rearrange the seat of the vector β decay as between baryons and mesons but do not change its total strength. The situation with axial decay is quite otherwise; the axial current is not conserved; the strong interactions renormalize the axial coupling constant of the bare nucleon so that, for free nucleons, $|g_A| \approx 1.23|g_V|$ and must effect an additional renormalization when they bind nucleons together into nuclei. The intrinsic Gamow-Teller β-decay strength is therefore not constant from nucleus to nucleus nor from transition to transition within a single nuclear system nor can it be associated in a unique way with baryons and with mesons; the effect of meson currents on the axial decay, the division of the strength between single nucleon and many-body terms and so on must be expected to vary with every circumstance but without the overriding compensation between the various sources that brings such simplicity to the vector case. In short, even if we were in possession of perfect nuclear (nucleonic) wave functions we should not expect the rate of Gamow-Teller β decay to be given by the conventional expression incorporating the free-nucleon axial coupling constant g_A. We could, given such perfect wave functions, extract, from each transition, an effective axial coupling constant g_{Ae} but we should not, even then, expect the g_{Ae} to be constant from transition to transition within a single nuclear system nor simply to change slowly and smoothly from one nucleus to the next because, as remarked above, axial β decay is not a purely single-nucleon affair. We might, however, reasonably hope that, in sufficiently large nuclei, the many effects contributing to the axial decay might boil down to an acceptable representation in terms of a g_{Ae} almost constant from transition to transition and changing little from A to $A+1$. But even this is only a reasonable hope. It would obviously be of very great interest to know the degree to which $g_{Ae} \neq g_A$ and to see whether the differences, if any, resemble those expected from ideas about the renormalization mechanisms.

Unfortunately we cannot simply extract g_{Ae} from experimental data because we are far from having available perfect nuclear wave functions. It is possible that our present theoretical knowledge of the wave functions of the $A=3$ system is good enough but that cannot yet be assured us and, in any case, $A=3$ can surely not be a "sufficiently large" nucleus in the sense of the previous paragraph. Intensive experimental and theoretical studies of tritium β decay indeed suggest that the Gamow-Teller component of that decay is slightly accelerated over expectation based on the use of g_A but we shall not review this special situation here. Returning to heavier nuclei, we reiterate our lack of confidence in theoretical Gamow-Teller matrix elements if, as is presumably the case, accuracy of a few percent is to be aimed at. The situation is obviously least gloomy for "good single-particle" systems such as $A=17$ but even there the effects of configuration mixing may not yet be wholly understood and, in any case, g_{Ae} extracted from a single transition could be misleading for the reasons just discussed.

Since, on the one hand, we must be able to study a suite of neighboring A systems in order to iron out the fluctuations to be expected in g_{Ae} while, on the other hand, we cannot have adequate confidence in the theoretical matrix elements, it seems that our quest must fail. We argue here that this is not so and that we may, in fact, proceed towards g_{Ae} in a largely empirical way without major reliance on theoretical wave functions at all.

β DECAY OF MIRROR NUCLEI

Consider β decay between the ground states of $T=\frac{1}{2}$ mirror nuclei such as $^{17}F \to {}^{17}O$ etc. Such decay has two components, namely, a (superallowed) Fermi component, whose strength may be confidently and accurately estimated using conserved vector current, and a Gamow-Teller component in which we are interested. In detail, we have

$$ft=\frac{6152\pm 10}{1+R_e{}^2(J+1/J)\langle\tau_3\sigma_3\rangle^2}\ \text{sec}. \qquad (1)$$

ft is the ft value incorporating the "outer" radiative correction[1]; J is the spin of the ground state of the mirror pair; $\langle\tau_3\sigma_3\rangle$ is evaluated in the ground state of either member of the mirror pair.[2] $R_e=|g_{Ae}/g_V|$ which we hope to compare with $R=|g_A/g_V|=1.226\pm 0.011$[3,4] for the free nucleon so that R_e/R is our measure of the effective additional *nuclear* renormalization of the axial coupling constant (if it should eventually turn out to make sense to attempt to define such a quantity). The constant 6152 ± 10 sec derives from pure vector nuclear β decay using the two best-studied ($T=1$) cases of $^{26}Al^m$($ft=3073\pm 5$ sec)[3] and ^{14}O ($ft=3081\pm 11$ sec)[5] which we combine to give 3076 ± 5 sec and then double as appropriate for our $T=\frac{1}{2}$ cases.[6]

So from mirror β decay we may extract $R_e|\langle\tau_3\sigma_3\rangle|$ and hence the desired R_e if only $\langle\tau_3\sigma_3\rangle$ were known reliably. We now address ourselves to this problem.

MAGNETIC MOMENTS OF MIRROR NUCLEI

Consider the conventional shell-model expression for the magnetic moment μ:

$$2\mu=\langle(1-\tau_3)(l_3+\sigma_3\mu_p)+(1+\tau_3)\sigma_3\mu_n\rangle, \qquad (2)$$

where μ_p and μ_n are the free-space nucleon moments.[7] This expression is not complete because of the mesonic renormalization effects that operate upon μ_p and μ_n, and also upon the orbital g factors, within nuclear matter in much the same way as they operate upon g_A in our own problem. These mesonic exchange effects are of quantitatively unknown magnitude but they may well amount to several percent and therefore seriously limit the use that may be made of expression (2). The mesonic exchange effects are however very largely absent (Sachs theorem) from the *isoscalar* magnetic moment:

$$2\mu_0=\langle l_3\rangle+(\mu_p+\mu_n)\langle\sigma_3\rangle. \qquad (3)$$

Noting further that

$$J=\langle l_3\rangle+\tfrac{1}{2}\langle\sigma_3\rangle,$$

we have

$$2\mu_0 = J = (\mu_p + \mu_n - \tfrac{1}{2})\langle\sigma_3\rangle . \qquad (4)$$

Now

$$2\mu_0 = \mu_+ + \mu_- ,$$

where μ_+ and μ_- are the magnetic moments of the two members of the mirror pair so that if both of these moments are known it seems that we know $\langle\sigma_3\rangle$ in a purely empirical and model-independent way. This is, however, not strictly true because the Sachs theorem is not completely valid: There is a small but important isoscalar exchange moment μ_{x0} that must be added (×2) to the right-hand side of expression (3) and to which we return in earnest shortly. (We ignore the presumably very small isoscalar modification to the intrinsic nucleon moments themselves.)

TABLE I. Experimental data used in the analysis. $R_e|\langle\tau_3\sigma_3\rangle|$ is derived from the ft value for the β decay using expression (1) of the text. $\langle\sigma_3\rangle_\mu$ is derived from the isoscalar magnet moment using expression (4) of the text.

A	J	$R_e\,\vert\langle\tau_3\sigma_3\rangle\vert$ [a]	$\langle\sigma_3\rangle_\mu$ [b]
11	$\frac{3}{2}$	0.5747 ± 0.0040	0.5914 ± 0.0026
13	$\frac{1}{2}$	0.3235 ± 0.0024	-0.3153 ± 0.0009
15	$\frac{1}{2}$	0.3655 ± 0.0017	-0.1689 ± 0.0021
17	$\frac{5}{2}$	$1.096\ \ \pm0.0043$	0.8656 ± 0.0032
19	$\frac{1}{2}$	0.9216 ± 0.0021	0.6358 ± 0.0026
21	$\frac{3}{2}$	$0.560\ \ \pm0.015$	0.5909 ± 0.0003

[a] All the masses involved in the compilation of the ft values are taken from Ref. 10 with the exception of that of ^{15}O which is from data given in Ref. 11 which yield a positron maximum kinetic energy of 1731.8 ± 0.7 keV. All decays have been corrected for K capture following standard procedures. For $A=11$, 13, 15, and 17 no allowed β^+ branch in competition with the ground-state branch that we are concerned with is possible. For $A=19$ ^{19}Ne could enjoy an allowed decay to the $J^\pi=\frac{3}{2}^+$ state at 1554 keV but a $\log ft$ of 4.0 would give a branch of only 0.11% so this possibility has been ignored. For $A=21$ correction has been made for the $(2.3\pm0.2)\%$ β^+ branch to the first excited state of ^{21}Ne following Ref. 12. For the ^{11}C lifetime we have used $t_{1/2}=20.40\pm0.04$ min from Ref. 13 which agrees with the $t_{1/2}=20.39\pm0.06$ min recommended in Ref. 14. For the ^{13}N and ^{15}O lifetimes we have followed the recommendations of Ref. 15. For ^{17}F we have used $t_{1/2}=64.50\pm0.25$ sec from Ref. 16. For ^{19}Ne we have followed the recommendation of Ref. 17. For ^{21}Na we have followed the recommendation of Ref. 18.

[b] For all magnetic moments except that of ^{11}C we have used the recommended values of Ref. 19 with errors taken from the sources quoted there. For ^{11}C we have used $\mu=-(0.964\pm0.001)\mu_N$ from Ref. 20.

With this reservation about μ_{x0} we now know $\langle\sigma_3\rangle$ experimentally. But what we need is $\langle\tau_3\sigma_3\rangle$.[8] At this point we notice that $\langle\sigma_3\rangle$ and $\langle\tau_3\sigma_3\rangle$ are connected by $\langle(1\pm\tau_3)\sigma_3\rangle$. This latter quantity would be zero in the extreme naive shell model (the positive or negative sign being taken as the proton-rich member of the mirror pair is of odd or even Z, respectively) and so we might hope that it would remain small in fact, i.e., for realistic independent-particle model wave functions. If this should turn out to be correct we should gain $\langle\tau_3\sigma_3\rangle$ and hence the desired R_e from the experimental $\langle\sigma_3\rangle$ with only slight reliance on theory. In point of fact the hope that $\langle(1\pm\tau_3)\sigma_3\rangle$ be small is justified as we shall see shortly but before discussing this we turn to the experimental situation.[9]

EXPERIMENTAL SITUATION

As remarked in the Introduction, g_{Ae} is a caricature of a more complex situation but we might hope to extract some meaningful mean value if we could derive information from a series of neighboring A values. At present the only such suite of mirror nuclei for which the isoscalar magnetic moments and accurate β-decay information are available is $A=11$, 13, 15, 17, 19, and 21. Fortunately, for these nuclei, excellent wave functions are also available which, although not good enough to give us $\langle\tau_3\sigma_3\rangle$ directly with high accuracy, we may use to derive the hopefully small $\langle(1\pm\tau_3)\sigma_3\rangle$. The experimental situation is given in Table I[10-20] where we label as $\langle\sigma_3\rangle_\mu$ the value of $\langle\sigma_3\rangle$ extracted by applying expression (4) to μ_0 and J. Errors are not given for $\langle\sigma_3\rangle_\mu$ (they merely reflect those given for μ_0) nor shall we later refer to those given for $R_e|\langle\tau_3\sigma_3\rangle|$. This is because the uncertainties residing in the later stages of our analysis are sufficiently greater than the experimental errors in $\langle\sigma_3\rangle_\mu$ and $R_e|\langle\tau_3\sigma_3\rangle|$ for us to regard the latter quantities as essentially exact.

EXTRACTION OF $R_{e\mu}$

The theoretical $\langle(1\pm\tau_3)\sigma_3\rangle$ that we use come from the following wave functions and were supplied by Dr. J. Millener and Dr. D. Strottman:

A = 11 and 13. The Cohen-Kurath $(1p)^{A-4}$ wave functions in their (8–16) 2BME form.[21]

A = 15, 17, and 19. For $A=15$: $1p^{-1}+1p^{-3}(2s,1d)^2$; for $A=17$ and 19: $(2s,1d)^{A-16}+(1p)^{-2}(2s,1d)^{A-14}$. The $1p$-shell matrix elements are those of Cohen-Kurath as above; the $(2s,1d)$-shell matrix elements are those of Kuo-Brown,[22] and the particle-hole matrix elements those of Hsieh, Lee, and Chen-Tsai.[23] The $1p$-shell single-particle energies come from Cohen-Kurath and the $(2s,1d)$-shell single-particle energies from ^{17}O; the $1p$-

$(2s, 1d)$ splitting is given by fitting known particle-hole states in $A=16$ and 18. The $2s, 1d$ states were selected by using SU_3 to include >99% of the wave function resulting from the complete related $(2s, 1d)$ diagonalization.

A = 21. $(2s, 1d)^5$ using Kuo[24] matrix elements with the basis truncated according to the prescription of Akiyama, Arima, and Sebe.[25] (Omission of pair excitation from the $1p$ shell is unlikely to be serious: Its omission from $A=15$, 17, and 19 nowhere brings as great a change as 4% in $\langle\tau_3\sigma_3\rangle_\mu$.) Table II shows these theoretical $\langle(1\pm\tau_3)\sigma_3\rangle, \langle\sigma_3\rangle_\mu$ taken from Table I, the deduced $\langle\tau_3\sigma_3\rangle_\mu$ and also $R_{e\mu}|\langle\tau_3\sigma_3\rangle_\mu|$. We see that indeed, as hoped,

$$\langle(1\pm\tau_3)\sigma_3\rangle \ll \langle\sigma_3\rangle_\mu \,;$$

the former quantity averages only 5.1% of the latter. This we take to assure us that $\langle\tau_3\sigma_3\rangle_\mu$ cannot contain gross errors on account of the intervention of theory and that we may regard it as an almost-experimental quantity. However, we see that $R_{e\mu}$ shows wide variations: It is evident that the isoscalar exchange moment μ_{x0} must be playing a significant role. Since we know almost nothing about μ_{x0} we must proceed empirically, taking such guidance as may be available.

ISOSCALAR EXCHANGE MOMENT

It was long ago remarked by Jensen and Mayer[26] that expression (2) is invalidated by velocity-dependent forces. They took the shell-model one-particle model Hamiltonian with a spin-orbit term and restored gauge invariance through the conventional minimal substitution:

$$\vec{p} \to \vec{p} - (e/c)\vec{A} \qquad (5)$$

and showed that this implies an extra magnetic moment for a proton (nothing for a neutron) of

$$\mu_{so} = \mp \xi \frac{2j+1}{2j+2}\mu_N, \qquad (6)$$

$\mp$ being for $j=l\pm\frac{1}{2}$, where $\xi\approx 0.25$. This leads to the expectation $\mu_{x0}\approx\mp 0.1\mu_N$.

It is clear, however, that the Jensen-Mayer estimate cannot be relied upon quantitatively. For example, as pointed out by Blin-Stoyle,[27] one should not wait until the model Hamiltonian is reached before imposing gauge invariance but rather do this at the earlier stage of the two-body NN interaction that underlies the model Hamiltonian. There have been two attempts to do this, one by Philpott (reported by Blin-Stoyle)[27] and one by Chemtob[28] again using the minimal substitution of expression (5). Philpott's results imply $\mu_{x0}\approx\mp 0.05\mu_N$; those of Chemtob $\mu_{x0}\approx\mp 0.01\mu_N$; both may be represented roughly by expressions of the form and having the sign of that deriving from the naive one-particle shell model of Jensen-Mayer viz expression (6).

We see that although there is agreement as to the sign of μ_{x0} in relation to $j=l\pm\frac{1}{2}$ there is no agreement as to magnitude and it is clear from the computations of Chemtob that, in any case, substantial fluctuations in magnitude might be expected from case to case.

Presumably what is really needed is more fundamental than summing the effects of the phenomenological two-body force, namely, an explicit calculation in terms of the mesons that mediate the nuclear forces (and that generate the velocity dependence): For example, a ρ meson emitted by one nucleon, transforming into a pion through interaction with the electromagnetic field, the pion then being absorbed by another nucleon, generates an isoscalar moment. Such computations are not yet available for nuclei as heavy as those we deal with here although they are beginning for the lightest systems.

ALLOWANCE FOR μ_{x0}: R_e

We clearly cannot make any *a priori* allowance for μ_{x0} but we have tried three reasonable and independent approaches, of a statistical nature, to the problem.

Approach No. 1. Our nuclei cover several shells such that the signs expected for μ_{x0}, following the computations reported in the previous section, change from case to case; magnitudes must also be expected to change from case to case. Under these circumstances the most likely expectation for $\sum\mu_{x0}$, the summation being extended over the six cases, is zero. We, therefore, for each A value listed in Tables I and II, ask what value

TABLE II. Extraction of $R_{e\mu}=R_e|\langle\tau_3\sigma_3\rangle|/|\langle\tau_3\sigma_3\rangle_\mu|$ (which has no allowance for the isoscalar exchange magnetic moment μ_{x0}) from the experimental $\langle\sigma_3\rangle_\mu$ taken from Table I and the theoretical $\langle(1\pm\tau_3)\sigma_3\rangle$ taken from the wave functions discussed in the text. (The sign in the latter quantity is positive or negative according as the proton-rich member of the mirror pair is of odd or even Z, respectively.)

A	$\langle\sigma_3\rangle_\mu$	$\langle(1\pm\tau_3)\sigma_3\rangle$	$\langle\tau_3\sigma_3\rangle_\mu$	$R_{e\mu}$
11	0.591	0.030	0.561	1.024
13	−0.315	−0.044	0.271	1.194
15	−0.169	−0.004	−0.165	2.215
17	0.866	−0.018	−0.884	1.240
19	0.636	0.010	0.626	1.472
21	0.591	0.036	−0.555	1.009

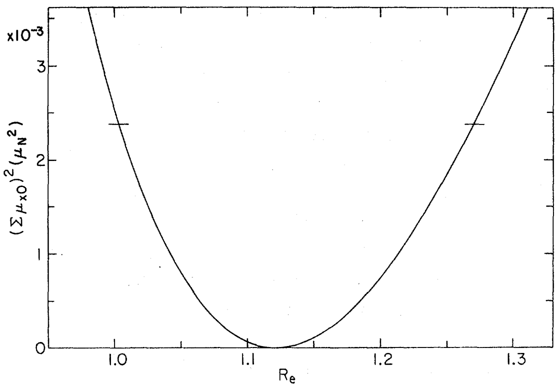

FIG. 1. $(\sum\mu_{x0})^2$ versus R_e from approach No. 1. The horizontal bars are at $(\sum\mu_{x0})^2=2.4\times 10^{-3}\mu_N^2$ and indicate the error range of R_e.

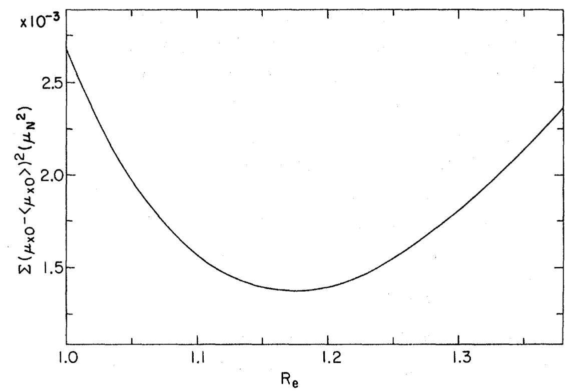

FIG. 2. $\sum(\mu_{x0}-\langle\mu_{x0}\rangle)^2$ versus R_e from approach No. 2.

$\langle\tau_3\sigma_3\rangle$ would have to take in order to give a particular value of R_e on the basis of the known $R_e|\langle\tau_3\sigma_3\rangle|$; this then implies a certain value of μ_{x0} for each A as a function of the presumed R_e and therefore a certain value for $\sum\mu_{x0}$. We then take as the "best value" of R_e that which gives $\sum\mu_{x0}=0$. The standard theory of random flights then associates with this R_e a variance $5\langle\mu_{x0}^2\rangle$ in $\sum\mu_{x0}$. Figure 1 shows $(\sum\mu_{x0})^2$ as a function of R_e. $5\langle\mu_{x0}^2\rangle$ when $\sum\mu_{x0}=0$ is $2.4\times10^{-3}\mu_N^2$ which, as may be seen from Fig. 1 leads to our first estimate:

$$R_{e1}=1.12\pm0.13$$

the actual μ_{x0} values at $\sum\mu_{x0}=0$ are given in Table III; they will be discussed later but since this is the only one of our three methods of analysis in which the analysis itself fully determines the *signs* of the μ_{x0} we should note with satisfaction that five out of the six signs generated by approach No. 1 are in fact in agreement with expectation based on the naive jj-shell model using the results reported in the previous section [expression (6)]. The exception is $A=11$ which is certainly not a good jj-coupling nucleus and so for which the $j=l\pm\frac{1}{2}$ expectation as to the sign of μ_{x0} cannot necessarily be expected to hold; we also note that the magnitude $|\mu_{x0}|$ is small in this case.

Approach No. 2. Although we must expect the magnitudes of $|\mu_{x0}|$ to scatter, they will scatter about some mean value and we may take as most satisfactory a situation that minimizes that scatter, due regard being taken for the sign of μ_{x0}. We therefore compute, as a function of a presumed R_e, the value of μ_{x0} required for each A value to give that R_e. We then count positively those μ_{x0} that have the sign expected for them under the naive shell model (as indicated in Table III) and negatively those that have the opposite sign, to establish $\langle\mu_{x0}\rangle$. This enables us to construct $\sum(\mu_{x0}-\langle\mu_{x0}\rangle)^2$ which we then minimize to determine R_{e2}.[29] The error in R_{e2} we determine by associating a normalized χ^2 of unity with the minimum in $\sum(\mu_{x0}-\langle\mu_{x0}\rangle)^2$. Figure 2 gives the result of this exercise from which we derive

$$R_{e2}=1.17\pm0.10\,.$$

Table III gives the resultant values of μ_{x0} associated with the minimum in Fig. 2.

Approach No. 3. We accept the suggestion of the naive shell model as to the sign of μ_{x0} for each A value, again ignore the weak j dependence possibly

TABLE III. Estimates of the isoscalar exchange magnetic moment μ_{x0} according to the three approaches discussed in the text.

A	Naive shell model sign		$\mu_{x0}\mu_N$	
		Approach No. 1	Approach No. 2	Approach No. 3
11	−	+0.009	+0.013	−0.021
13	+	+0.003	+0.001	+0.021
15	+	+0.031	+0.028	+0.021
17	−	−0.018	−0.010	−0.021
19	−	−0.037	−0.030	−0.021
21	+	+0.012	+0.016	+0.021

expected for its magnitude, assuming the same $|\mu_{x0}|$ for all A, then, for each A value, derive R_e as a function of $|\mu_{x0}|$ and so $\langle R_e \rangle$ as a function of $|\mu_{x0}|$. We take as the best value of $|\mu_{x0}|$ that which minimizes the value of $\sum (R_e - \langle R_e \rangle)^2$; R_{e3} is $\langle R_e \rangle$ at this minimum and the associated error is that derived in the usual way from the scatter of the individual R_e. Figure 3 shows $\sum (R_e - \langle R_e \rangle)^2$ versus $|\mu_{x0}|$ and we find

$$R_{e3} = 1.102 \pm 0.085 .$$

Table III shows μ_{x0} resulting from this approach.

As may be seen from Table III the magnitude of μ_{x0} as deduced from these statistical approaches is about $0.02\mu_N$, and its sign, where freely determined, as already remarked, is usually in accord with the naive jj expectation. $|\mu_{x0}| \approx 0.02\mu_N$ is also entirely reasonable in view of the wide range up to $0.1\mu_N$ expected from the various theoretical estimates referred to earlier. These facts, and the agreement between the R_{e1}, R_{e2}, R_{e3} of the different independent approaches, gives some confidence in the possible meaningfulness of the R_e values and we average the three of them to gain our final result

$$R_e = 1.128 \pm 0.058 ,$$

or

$$g_{Ae}/g_A = 0.920 \pm 0.047$$

for the range of nuclei in question.[30]

DISCUSSION

We must firstly emphasize that our result is statistical both in that the critical isoscalar exchange moment has been handled statistically and also in that g_{Ae} is a statistical concept that can

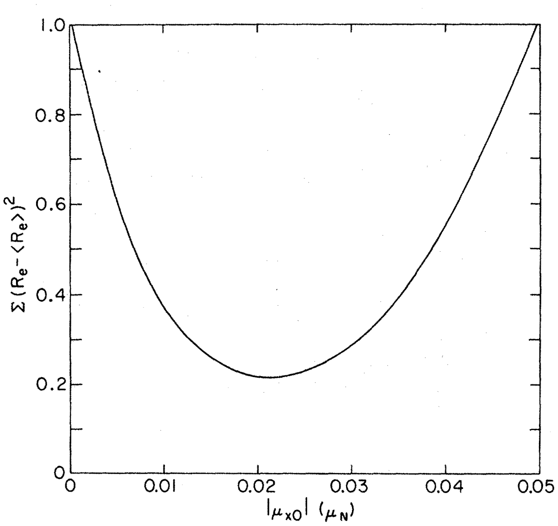

FIG. 3. $\sum (R_e - \langle R_e \rangle)^2$ versus $|\mu_{x0}|$ from approach No. 3.

only take weight if it is derived by averaging over a sufficiently large number of transitions in sufficiently large nuclei; there is nothing to assure us that our cases are sufficiently large in either sense. But it seems clear that, whatever its meaning, g_{Ae} may well be significantly less than g_A: It seems quite likely that the axial β decay of complex nuclei is significantly slower than we should expect if nucleons bound into the nucleus simply decayed as individuals with the same strength that they would dispose of in the free state.

A semiquantitative attempt has been made by Ericson[31] to calculate this effect. In this attempt the pion-nucleus interaction is linked to axial β decay. The sum of all pionic vertices between the ground and excited states of a nucleus is connected to an integral over the total pion-nuclear cross section in a dispersion relation. The Goldberger-Treiman relation then transforms this sum into a sum over the corresponding Gamow-Teller matrix elements in a nuclear Adler-Weisberger sum rule. Thus, with the neglect of certain processes likely to be of major importance only in the very lightest nuclei, hopefully outside our present range, we may use empirical pion-nucleus scattering data to predict the sum over all Gamow-Teller transitions leading from a given state. This does not tell us what will be the g_{Ae} effective for any *individual* transition from that state but we may tentatively follow our earlier conjecture that, with sufficient averaging in sufficiently large nuclei, the mean g_{Ae} will follow the sum rule. The prediction is A-dependent because the nuclear renormalization is connected with the shadowing effect of the inner nucleons of the nucleus against pions by those towards the surface. However, the expected A dependence in our range of A is weak (a change of only ± 0.006 in g_{Ae}/g_A about its mean value) and the prediction averaged over $A = 11$, 13, 15, 17, 19, and 21 is $g_{Ae}/g_A = 0.932$ against our "experimental" $g_{Ae}/g_A = 0.920 \pm 0.047$.

We conclude, cautiously, that it seems quite likely that g_A is indeed effectively renormalized inside nuclear matter as we should qualitatively expect and that that renormalization is of about the expected magnitude, its sense being to slow down the decay. Any sharpening of our tentative conclusions must await better theoretical guidance as to μ_{x0} – it would be illuminating to evaluate this directly using a realistic NN force and wave functions such as we have used for $\langle (1 \pm \tau_3)\sigma_3 \rangle$ – and also reliable estimates of the relativistic effects. It will also be important to understand relativistic effects on g_{Ae}.

Note added in proof: Very recent work of Arima and his colleagues has shown the importance of core polarization (single-particle excitation through many oscillator spacings). This modifies the theoretical expectation values; however, since the core-polarization term has the same form and effect as the isoscalar-exchange magnetic moment the conclusions about g_{Ae} are not sensibly changed.

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[1]D. H. Wilkinson and B. E. F. Macefield, Nucl. Phys. A158, 110 (1970).

[2]Everywhere in this article, notations such as $\langle\tau_3\sigma_3\rangle$ understand summation over all nucleons of the nucleus and evaluation in the substate $M=J$ (we take $\tau_3=+1$ for a neutron).

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[4]The g_A etc. as we use them are defined as including the effects of the "inner" radiative correction (Ref. 1).

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[6]All ft values quoted in this note, in addition to incorporating the "outer" radiative correction, are corrected for all finite nuclear size effects, screening, and K capture but not for explicitly second-forbidden effects (which are negligible for our present purposes).

[7]We shall ignore relativistic effects not so much because they may not be important as because they have not yet been evaluated with reliability. They might easily effect a few percent change to our conclusions; their sense is to lower our final R_e.

[8]From now on we will take it that $\langle\tau_3\sigma_3\rangle$ etc. are evaluated for the proton-rich member of the mirror pair.

[9]There have already been several discussions of the magnetic moments and Gamow-Teller matrix elements of mirror nuclei in which the above expressions have appeared: R. Leonardi and M. Rosa-Clot, Phys. Rev. Letters 21, 377 (1968); R. Leonardi and M. Rosa-Clot, Lettere Nuovo Cimento 1, 829 (1969); K. Sugimoto, Phys. Rev. 182, 1051 (1969). However, these discussions have based themselves on the assumption that $g_{Ae}=g_A$ and have then used the Gamow-Teller strength to *derive* $\langle\tau_3\sigma_3\rangle$; they are therefore orthogonal to the present discussion which has as its precise aim the determination of g_{Ae} without reference to g_A.

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[29]In defining this scatter we do not consider it profitable to do other than consider the *sign* expected for μ_{x0} under the naive shell model. The $(2j+1)/(2j+2)$ dependence is weak and its introduction would be unjustifiable.

[30]The entire motivation of the approach to R_e of the present article is to get away from full reliance on $\langle\tau_3\sigma_3\rangle$ values computed directly from wave functions themselves. However, the direct approach is presumably least unreliable in the immediate vicinity of an LS-closed shell, i.e., for cases $A=15$ and 17 of the present study. For those cases the $\langle\tau_3\sigma_3\rangle$ of the wave functions used here to provide the $\langle(1\pm\tau_3)\sigma_3\rangle$ give, together with the $R_e|\langle\tau_3\sigma_3\rangle|$ of Table I: $R_e=1.108\pm0.005$ ($A=15$); $R_e=1.103\pm0.004$ (A=17). The agreement between the $R_e\approx1.11$ of these direct results and the $R_e\approx1.13$ of our indirect "model-independent" approach is to be noted.

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