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Effective Interaction in Nuclei

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An equivalent potential is derived by best fitting the Sussex matrix elements. The derived equivalent potential is compared with the equivalent Yale potential obtained in a similar manner and with central effective potentials in widespread current use. Remarks are made about the general nature of the effective nuclear force.

I. INTRODUCTION

In any nuclear structure calculation the choice of the interaction is crucial. Recently there has been much emphasis on calculating the various nuclear properties using the realistic interactions such as Hamada-Johnston, Yale, and Tabakin. The first two of these have a hard core. This makes the perturbation treatment more difficult starting from the independent particle wave functions. In actual practice, one uses the reaction matrix G instead. The calculation of the G matrix from these potentials is possible only within certain approximations,[1,2] and the convergence of the perturbation expansion should also be taken into account.[3] This difficulty of the infinitely repulsive hard core may be avoided by the use of a separable nonlocal potential e.g., the Tabakin potential. The principal drawback of such a potential is that it does not approach the one-pion-exchange potential at large distances. The Sussex group[4] has derived the relative matrix elements of the nucleon-nucleon interaction directly from the observed nucleon-nucleon phase shifts. Thus the usual intermediate step of first deriving the potential from the scattering data is eliminated. This has the special advantage that the deficiencies in one's treatment of the many-body problem are not hidden by the adjustment of the free parameters of the interaction to fit some particular properties. Various properties of nuclear systems, such as deuteron properties,[5] spin-orbit splittings,[6] binding energies, rms radii, single-particle energies, etc.,[7] have been calculated using the Sussex matrix elements and the results are in good agreement with experiment and with other realistic interactions.

On the other hand, there are numerous nuclear structure calculations using the phenomenological effective interactions, generally central potentials of smooth shape. In fact, before the separation technique of Moszkowski and Scott[8] became available for use with the hard-core problem of realistic interactions, the calculations were possible only for nonsingular effective interactions so that perturbation theory could be used directly. In these, the effective interaction is taken as a suitable combination of the conventional exchange forces such as Winger, Majorana, Bartlett, and Heisenberg. The parameters of the effective interaction are adjusted to best reproduce the energy levels and other properties of the nuclei considered. This approach has been very useful and has given us much information about the nature of the effective nuclear force. However, the effective interaction is dependent on the configuration space chosen and includes renormalizations from the admixture of the various configurations. Thus, there is always a danger that the description of the relevant states might not be correct and that, a wrong configuration having been chosen, the good agreement might have been forced by adjusting the various parameters of the interaction. This has been very strongly pointed out by Cohen, Lawson, and Soper[9] in their calculations on the "pseu-

donium" nuclei.

In an effort to learn about the nature of the effective force in light and intermediate nuclei, we have here attempted to simulate the Sussex interaction by an equivalent potential for the nuclear spectroscopic calculations. To facilitate comparison with other effective potentials in common use, mostly central in form, the equivalent potential is chosen to be a central force plus a Wigner-type force contributing only in states off diagonal in relative angular momentum ($l' = l \pm 2$). The latter term is for the simulation of the tensor force. Other authors, for example Bethe,[10] have also used the central interaction for the simulation of the tensor forces. This choice reduces the computational difficulties and facilitates comparison with other phenomenological central potentials. A similar analysis for the simulation of the Yale potential for nuclear spectroscopic calculations was reported earlier[11] and was reasonably successful.

In Sec. II, we give the form of the equivalent potential chosen. Here we also give the method for determining the various parameters and their values. Section III describes the method of calculation of the low-lying energy levels of the nuclei and also gives the results of such calculations. In Sec. IV we discuss the results. Comparison is made between the results obtained with the derived equivalent potential and with the Sussex interaction. The derived equivalent potential is also compared with the other phenomenological potentials in order to get some information about the nature of the effective nuclear force.

II. EQUIVALENT POTENTIAL

A. Nature of the Equivalent Effective Interaction

The Sussex group[4] has tabulated the relative matrix elements of the interaction separately for each channel, both for the uncoupled and coupled ones. The relative matrix elements with oscillator sizes $b = 1.4$ through 2.6 ($b = \sqrt{\hbar/m\omega}$) are given. The equivalent phenomenological potential is also chosen to have two parts, one corresponding to the uncoupled channels and the other to the coupled ones. We choose the following form for the equivalent potential

$$H^{\text{eqt}} = \sum_{ll'} H^c(l = l') + \sum_{ll'} H^{\text{od}}(l' = l \pm 2) . \tag{1}$$

The sum is over those values of l and l' which are needed for the calculation of the particular two-body matrix elements. H^c is taken as a local central potential comprising the usual exchange forces

$$H^c = V_c(W + B P^\sigma + M P^r - H P^\tau) f(r) , \tag{2a}$$

where W, B, M, and H are the strengths of the Wigner, Bartlett, Majorana, and Heisenberg exchange forces, respectively, the corresponding operators being unity, P^σ, P^r, and P^τ. H^{od} is chosen as

$$H^{\text{od}} = V_{\text{od}} g(r) P^{\text{od}} S_{12} , \tag{2b}$$

where the operator P^{od} projects out only those states which differ in relative angular momentum by units of two viz. $l' = l \pm 2$. S_{12} is the usual tensor operator.

As mentioned earlier, the choice of this particular form of the equivalent potential is made to facilitate the comparison with effective phenomenological potentials, mostly central in form, in widespread current use.

Any radial form for the interactions can be used. We have used the Gaussian form

$$\begin{aligned} f(r) &= \exp(-r^2/r_c^2) , \\ g(r) &= \exp(-r^2/r_{\text{od}}^2) . \end{aligned} \tag{2c}$$

Another conventional set of constants used[12] to define a nuclear interaction are A_{TS}, where T is the isospin and S is the spin. We have, then

$$\begin{aligned} H^c &= V_c(A_{01} + A_{00} + A_{11} + A_{10}) f(r) , \\ &= (V_{01} + V_{00} + V_{11} + V_{10}) f(r) , \end{aligned} \tag{3}$$

where V_{TS} is the strength of the force operating in state with isospin T and spin S. The relation between the parameters of Eqs. (2) and (3) is

$$\begin{aligned} A_{01} &= W + B + M + H , \\ A_{00} &= W - B - M + H , \\ A_{10} &= W - B + M - H , \\ A_{11} &= W + B - M - H . \end{aligned} \tag{4}$$

TABLE I. Parameters of the equivalent Sussex interaction.

Relative state	TS	Potential Strength (MeV)	Potential Range (Gaussian) (fm)
$^1S_0$	10	−26.45	1.87
$^1D_2$	10	−21.96	1.87
$^3S_1$	01	−33.47	1.87
^3D (central)	01	−27.52	1.87
$^1P_1$	00	72.43	1.35
^3P (central)	11	−1.49	1.87
$^3S_1$-$^3D_1$	01	−101.92 [a]	2.18
$^3P_2$-$^3F_2$	11	20.41 [a]	2.18

[a] This force operates only in the states off diagonal in orbital angular momentum.

B. Evaluation of the Parameters of the Equivalent Potential

The Sussex matrix elements are derived directly from the observed nucleon-nucleon phase shifts essentially by the distorted-wave Born approximation method. Harmonic-oscillator wave functions are used as the basis for convenience. The relative matrix elements for each channel are calculated and tabulated separately. The parameters of the equivalent potential are determined such that they best reproduce the Sussex matrix elements for each channel. This is done by a least-squares-fit program. The parameters of the derived equivalent potential are given in Table I.

III. CALCULATIONS AND RESULTS

The low-lying energy levels of ^{18}F obtained with the equivalent potential are compared with those from the Sussex interaction in Fig. 1. Nuclear spectroscopic calculations are done in the spirit of the conventional shell model in the j-j coupling scheme. Harmonic-oscillator wave functions are used for the basis. The size parameter of the harmonic-oscillator wave function is so chosen that it reproduces the mean square radius $\langle r^2\rangle^{1/2}$ from the electron scattering experiments of the nucleus under study. The details of the calculations of the two-body matrix elements are given in our earlier paper.[11] The experimental single-particle energies are used for both the interactions, the Sussex and the equivalent. This has the advantage that it treats some of the interactions to all orders of perturbation theory.[1]

In Table II we give the two-body matrix elements of the derived equivalent potential and that of the Sussex interaction in the p-f shell region. The over-all agreement between the two is satisfactory. We also give the eigenvectors of selected $T=1$ and $T=0$ states for ^{18}F in Tables III and IV, respectively.

IV. DISCUSSION

A. Comparison of the Equivalent Potential with the Sussex Interaction

The low-lying energy levels of ^{18}F obtained with the equivalent potential are compared with those from the Sussex interaction in Fig. 1. They have rms deviation of 314 keV. The large deviations

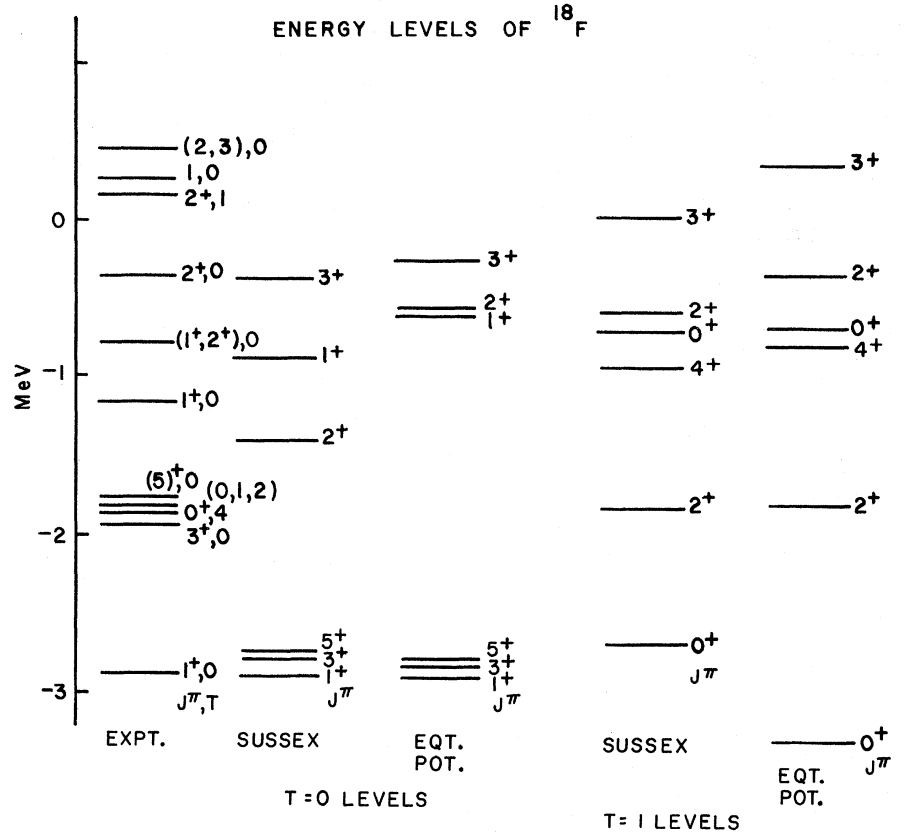

FIG. 1. Comparison of the energy levels of ^{18}F for the equivalent potential with those for the Sussex interaction.

TABLE II. Comparison of the two-body matrix elements calculated from the equivalent with that of Sussex group ($\gamma = 3.24$).

Matrix element	J, T	Equivalent potential	Sussex
$\langle 0f_{7/2}^2 \mid V \mid 0f_{7/2}^2 \rangle$	1, 0	0.217	0.332
	3, 0	−0.176	−0.039
	5, 0	−0.602	−0.596
	7, 0	−2.331	−2.218
	0, 1	−1.942	−0.604
	2, 1	−0.886	−0.881
	4, 1	−0.432	−0.408
	6, 1	−0.296	−0.231
$\langle 0f_{7/2}^2 \mid V \mid 1p_{3/2}^2 \rangle$	3, 0	+0.080	−0.052
	4, 0	−0.326	−0.334
	5, 0	−0.126	−0.110
	6, 0	−1.14	−1.07
	2, 1	−0.587	−0.423
	3, 1	−0.122	−0.181
	4, 1	−0.213	−0.167
	5, 1	−0.02	−0.125
$\langle 1p_{3/2}^2 \mid V \mid 1p_{3/2}^2 \rangle$	1, 0	−0.266	−0.305
	3, 0	−1.142	−1.697
	0, 1	−1.780	−1.061
	2, 1	−0.569	−0.629

are observed for the levels ($J = 0_1^+$, $T = 1$) and ($2_1^+, 0$); the ($0_1^+, 1$) level is more strongly bound in the equivalent potential case whereas the ($2_1^+, 0$) level is more repulsive. The subscript "1" denotes the lowest energy level with these quantum numbers. Parallel effects were observed in a similar analysis[11] for the simulation of the Yale interaction made earlier. If these levels are excluded, the rms deviation becomes 221 keV only.

The difference in the $T = 1$ level position arises mainly from the averaging of the ^3P states in the Sussex interaction by an equivalent central force. Whereas the $^3P_0$ and $^3P_2$ states in the Sussex interaction are attractive and the $^3P_1$ state is repulsive, the equivalent interaction is attractive in all three states. If the equivalent potential in the triplet-odd state is made more repulsive, the ($0_1^+, 1$) state in the equivalent potential comes closer to that in the Sussex interaction, but the ($2^+, 1$) states become less tightly bound. For the same reason the ($3^+, 1$) level is slightly more repulsive in the equivalent interaction than in the Sussex interaction.

For $T = 0$ states we find that the equivalent potential is more repulsive for the ($2^+, 0$) level. The difference arises again because we average the interaction in ^3D states by a central force. Whereas in the Sussex interaction the $^3D_1$ state is repulsive and the $^3D_2$ and $^3D_3$ states attractive, the equivalent potential is attractive in all three states. If the triplet-even potential is made more attractive, the ($2^+, 0$) level will come down but the ($3_1^+, 0$) level comes below that in the Sussex case. For example, in ^{18}F if the triplet-even strength (^3D state) is changed from −27.5 to −36 MeV, the ($2^+, 0$) level comes down from −0.61 to −0.74 MeV, but the ($3_1^+, 0$) level goes down from −2.84 to 3.12 MeV (below the 1^+, 0 level). In the Sussex interaction the ($1_1^+, 0$) level is lower, in agreement with experiment.

The relative contributions of the relative matrix elements of the ^3P and ^3D states for some selected two-body matrix elements of ^{18}F are given in Tables V and VI.

TABLE III. Comparison of the eigenvectors of the selected $T = 1$ states of ^{18}F of the Sussex equivalent interaction with those of the Sussex interaction. The first 1 state is denoted by 1_1 etc. The states $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ are denoted by 5, 1, and 3, respectively. The first row for each state corresponds to the Sussex interaction and the second row to the Sussex equivalent.

J^π, T	Energy	55	11	53	51	33	31
$0_1^+, 1$	−2.71	0.881	0.419			0.220	
	−3.15	0.906	0.388			0.169	
$0_2^+, 1$	−0.49	−0.416	0.907			−0.062	
	−0.63	−0.391	0.920			−0.015	
$2_1^+, 1$	−1.64	0.750	0.621	−0.095		0.089	−0.186
	−1.68	0.718	0.661	−0.118		0.074	−0.167
$2_2^+, 1$	−0.28	0.650	−0.753	−0.004		−0.009	0.105
	−0.26	0.684	−0.725	−0.012		0.008	0.086
$3_1^+, 1$	0.30				1.000		0.025
	0.61				1.000		0.000
$4_1^+, 1$	−0.69	−0.971		0.238			
	−0.70	−0.975		0.220			

TABLE IV. Comparison of the eigenvectors of some $T=0$ states of ^{18}F. The first row for each state corresponds to the Sussex interaction and the second row to the Sussex equivalent. For explanation see Table III.

J^π, T	Energy	55	11	53	51	33	31
$1_1^+, 0$	−2.77	−0.615	−0.580	−0.527	−0.032	0.083	
	−2.89	−0.586	−0.628	−0.503	−0.059	0.083	
$1_2^+, 0$	−0.96	0.589	−0.781	0.148	0.078	−0.126	
	−0.52	−0.630	0.740	−0.183	0.096	0.109	
$2_1^+, 0$	−1.25			0.417	−0.827		0.378
	−0.58			0.327	−0.878		0.351
$3_1^+, 0$	−2.65	0.551		0.173	0.816	−0.017	
	−2.46	0.559		0.810	0.178	−0.012	
$3_2^+, 0$	−0.35	0.809		0.130	−0.573	0.012	
	0.01	0.814		0.065	−0.577	−0.029	

B. Equivalent Potential

The ranges of the interactions in different states (Table I) have been kept equal to 1.87 fm except for the $^1P_1$ state where a shorter range of 1.35 fm has been found to be necessary. Also the potential is repulsive in this state consistent with the accepted view.[13] The potential in the ^3P state is very weak but attractive. For the Yale equivalent potential[11] also, we have found it to be weaker than the force in other states. This is clear from Table VII where the various potentials in common use are listed. (Note that the derived S-state potentials are different from the derived D-state potentials in both singlet and triplet states.) This result conforms to other realistic interactions, e.g. the Reid potential.[14] This shows that the potential is not purely central. This state dependence may result from the velocity dependence or nonlocality of the interaction.

Finally, as is the common belief,[13] the tensor force in the triplet-odd state is weak and repulsive in contrast to the stronger attractive triplet-even state force. Also, the tensor force has longer range than the central potential.

C. Comparison with Other Phenomenological Interactions

The parameters of the various interactions are given in Table VII. We first compare the equivalent Sussex potential with the equivalent potential obtained by a similar analysis[11] from the Yale potential. In the equivalent interaction from the Sussex interaction the triplet-odd force is much weaker than the equivalent potential from the Yale interaction. Whereas the singlet-even force is almost similar in the two cases, the equivalent potential

TABLE V. The relative contribution of the relative matrix elements of the ^3P state for some selected ($T=1$) two-body matrix elements of ^{18}F. The numbers give the two-body matrix elements with each relative matrix elements assumed unity. Only relative matrix elements diagonal in the radial quantum number are required. Contributions larger than 0.001 only are mentioned. The states $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ are denoted by 5, 1, and 3, respectively.

Configuration	State: $J\backslash n=0$	$^3P_0$ 1		$^3P_1$ 0	1	$^3P_2$ 0	1
5555	0			0.200	0.200		
	2			0.08	0.056	0.180	0.170
	4			0.151		0.250	
5551	2			0.085	−0.05	−0.085	−0.050
5151	2			0.092		0.108	0.015
	3			0.170		0.250	0.150
5511	0				−0.132		
1111	0				0.000		
5353	1	0.167	0.167	0.125	0.125	0.208	0.208
	2			0.148	0.063	0.232	0.189
	3			0.021		0.313	
	4			0.038		0.0625	
3333	0			0.301	0.300		
	2			0.232	0.061	0.130	0.018

TABLE VI. The relative contribution of the relative matrix elements of the ^3D state for some selected ($T=0$) two-body matrix elements of ^{18}F. For explanation see Table V.

Configuration	State: $J \backslash n$	$^3D_1$ 0	$^3D_1$ 1	$^3D_2$ 0	$^3D_2$ 1	$^3D_2$ 0	$^3D_2$ 1
5555	1	0.037	0.013	0.093		0.003	
	3	0.024		0.060		0.316	0.051
	5	0.000		0.000		0.250	
5551	3	−0.015		−0.083		−0.189	0.124
5151	2			0.044	0.117	0.062	0.000
	3			0.111	0.000	0.281	0.292
5511	1	0.026	−0.132			−0.026	
1111	1	0.111		0.185		0.259	
5353	1	0.162	0.023	0.013		0.075	
	2	0.156		0.306	0.083	0.178	
	3	0.011		0.094		0.167	0.031
	4			0.208		0.042	
3333	1	0.173	0.047	0.190		0.004	
	3	0.171		0.027		0.019	0.001

from the Sussex interaction is only slightly more attractive. The Sussex equivalent interaction has a shorter range but a larger strength than the equivalent Yale potential. For example, the $^1P_1$ force strength and range are 72.43 MeV and 1.35 fm for the equivalent Sussex potential. The corresponding quantities for the equivalent Yale interaction are 43.31 MeV and 1.73 fm. Looking at the relative matrix elements of the two interactions for this state, one sees that the Yale potential is slightly more repulsive on balance. The triplet-even force in the equivalent Yale potential is only slightly more attractive than the equivalent Sussex interaction. It should be mentioned that the Sussex relative matrix elements do not include the second-order terms, whereas the Yale relative matrix elements include them. The second-order terms are particularly important for the triplet-even force.

Next we compare the derived Sussex and Yale equivalent potentials with different central potentials in widespread use. The parameters of the

TABLE VII. Comparison of the parameters of the different normalized nucleon-nucleon central potentials in widespread current use. The normalization used is $V_{10}=-40$ MeV and $W+M+B+H=1$.

Interactions	V_{00} (MeV)	V_{01} (MeV)	V_{11} (MeV)	W	M	B	H	$\frac{B-M}{B+M}$
Serber	0	−40	0	0.5	0.5	0	0	−1
Rosenfeld	72	−24	14	−0.13	0.93	0.46	−0.26	−0.58
Soper	16	−20	−4	0.30	0.43	0.27	0	−1
Elliott and Flowers [a]	−28.57	−57.14	14.86	0.485	0.365	−0.125	0.265	−1.92
Inoue *et al.* [b]	0	−60.67	46.67	0.32	0.57	−0.07	0.18	−1.43
Ferrel and Visscher [c]	−3	−25	13	0.55	−0.75	0.31	0.01	−2.41
True [d]	0	52.00	0	0.44	0.44	0.06	0.06	−0.77
Schmittroth [e]								
(i) For p shell [f]	18.77	−46.67	17.11	0.27	0.66	0.04	0.03	−0.89
(ii) For sd shell [g]	−151.97	−68.82	21.89	0.87	−0.08	−0.53	0.74	0.74
Gillet, Green and Sanderson [h]								
(i) "CAL"	26	−20	6	0.35	1.15	0.00	−0.50	−1
(ii) "COP"	−24	−24	25	0.66	0.68	−0.68	0.34	0
Clark and Elliott [j]	52	−40	8	0.125	0.875	0.275	−0.275	−0.52
Yale equivalent [k]	50.46	−52.44	−19.73	0.29	0.59	0.39	−0.27	−0.2
Present	112.56 [l]	−50.62	−2.25	−0.1	0.99	0.62	−0.51	−0.23

[a] Reference 15.
[b] Reference 16.
[c] Reference 17.
[d] Reference 18.
[e] Reference 19.
[f] Reference 20.
[g] Reference 21.
[h] Reference 22.
[j] Reference 23.
[k] Reference 11.
[l] Range is 1.35 fm whereas in other states it is 1.87 fm.

various interactions are given in Table VII. The choice of the interactions is such that the effective potentials derived by different considerations are represented. Thus, besides the most commonly used potentials, such as those of Elliott and Flowers,[15] Inoue *et al.*,[16] and Ferrel and Visscher[17] obtained for *sd*-shell region and that of True[18] for $A=14$, Schmittroth's potentials[19] are derived by a least-squares fit with the two-body matrix elements obtained by a Talmi fit made by Arima *et al.*[20] for the *sd*-shell region and by Cohen and Kurath[21] for *p*-shell nuclei. The potentials of Gillet, Green, and Sanderson[22] were originally used for ^{12}C and ^{16}O but have been found satisfactory for ^{40}Ca and ^{208}Pb also. The interaction of Clark and Elliott[23] has been obtained by best fitting the observed spectra for a wide range of nuclei.

Seemingly Table VII represents a haphazard collection of parameters. Still the potentials can give equally good fits to the observed properties because they enter into the calculations in particular combinations. Of course, one expects that the interaction in nuclei will be different from the free two-body force. However, we hope that a comparison of the different effective potentials may still prove useful in giving information about the gross nature of the effective force, which a "reasonable" potential should possess.

In all cases the triplet-even force is attractive and is generally stronger than that in the singlet-even state. In these states the equivalent Sussex and Yale potentials are similar. However, in singlet-odd and in triplet-odd states the potentials are very diverse. Thus, the "CAL" and "COP" interactions of Gillet, Green, and Sanderson[22] have opposite signs in singlet-odd state and this is also the case with Schmittroth's interactions[19] in *p*- and *sd*-shell nuclei. In analyzing the structure of the two-body matrix elements, Schmittroth finds that the $(2^+,0)$ level is very sensitive to this interaction e.g., if the $(2^+,0)$ matrix element is to be changed from -3.7 to -1.25, the potential strength, V_{so} has got to be varied from -91 to $+10$ MeV. Inoue *et al.*[16] use, for V_{so}, values ranging from 0 to 17 MeV. However, it is repulsive in both the equivalent interactions for Yale and Sussex.[24]

In the triplet-odd state the various forces have much more variation. However, one observes that in general the force in this state is weak. For Sussex and Yale equivalents it is seen to be attractive, though much weaker in the former case.

In terms of exchange mixtures, the Wigner force component is, in all cases, positive except for the Rosenfeld mixture and the equivalent Sussex potential. However, they are weak. The Rosenfeld mixture was derived to explain the singlet-triplet splittings for the deuteron without the use of the tensor force and also gives saturation for the nuclear binding energy; it is near to the free two-body force.[25] However, the Rosenfeld force has been found unsuitable by Abulaffio[26] to explain the level order in ^{22}Ne, ^{24}Mg, and ^{26}Mg. They assume a force of the form

$$V=(W+M)f(r) \tag{5}$$

and find that $M \geq 0.8$ (with normalization, $W+M=1$). It has been pointed out by Parikh and Bhatt[27] that Kuo and Brown's renormalized force[1] including core excitation, has essentially a Majorana character. Schmittroth's force for the *sd* shell, on the contrary, has very small Majorana-force content. This is surprising as it has been derived from the Talmi-fit matrix elements and as such should include the core-renormalization effects.

From the inelastic scattering experiments in the *sd*-shell nuclei, Anderson *et al.*[28] have found that M and H components are both attractive and that M is about four times stronger than H.[29] From Table VII it is observed that for the various forces generally $M>H$, though not by as much as observed by Anderson *et al.*[28] The other components, B and H have very diverse values.

As pointed out earlier, particular combinations, e.g. $(B-M)/(B+M)$, rather than the individual components are important for most of the nuclear properties. We also tabulate this parameter for various interactions in Table VII.

V. CONCLUSION

We have derived a potential of simple form that is equivalent to the Sussex interaction. This interaction is compared with the quivalent potential derived from the Yale interaction by a similar method and with other central effective interactions in common use. The interaction in even states is attractive and strong; in odd states it is weaker. In the singlet-odd state the interactions have varying signs but are repulsive for both Sussex and Yale equivalents. In the triplet-odd state both Yale and Sussex have a still weaker attractive force. In terms of the exchange components, the Majorana force is predominant consistent with the findings from the scattering experiments[19,28] and with Kuo and Brown's force.[1]

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