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## $\Lambda$ - $d$ Bound States ( $J = \frac{1}{2}, \frac{3}{2}$ ) and Faddeev's Approach to the Three-Body Problem

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The binding energy problem of  $^3\Lambda\text{H}$  in the ground state and in a possible excited state has been investigated in the light of the recent experimental and theoretical results available for the low-energy parameters of  $\Lambda$ -nucleon interaction. The calculations are performed in Faddeev's formalism with two-body nonlocal separable potentials between pairs of particles, two of which are of unequal masses. For each of the sets of  $\Lambda$ - $N$  potentials, the binding energy of the  $\Lambda$  particle within the hypertriton has been evaluated. One of our results for the binding energy of the hypertriton in an excited state ( $J = \frac{3}{2}$ ) is in reasonably good agreement with that of Toepfer and Schick. We have also discussed the possible qualitative effects of charge-symmetry breaking in the  $\Lambda$ - $N$  interaction in relation to the quantitative comparisons of the  $\Lambda$  binding energies.

### 1. INTRODUCTION

Making use of Faddeev's<sup>1</sup> elegant approach, a series of papers<sup>2</sup> have appeared on the three-body bound-state problem. This technique has been applied to the  $\Lambda$ - $d$  scattering problems by Hetherington and Schick.<sup>3,4</sup> They make use of a multiple-scattering formalism.<sup>5</sup> The two-body interactions are taken to be spin-dependent nonlocal separable (NLS)  $s$ -wave potentials.<sup>6</sup>

The low-energy  $\Lambda$ - $N$  parameters are now better known from the recent experimental studies of  $\Lambda$ - $N$  elastic scattering. In the light of the presently available  $\Lambda$ -nucleon scattering lengths and effective ranges,<sup>7-14</sup> we evaluate binding energies<sup>15</sup> of the hypertriton in the ground state ( $J = \frac{1}{2}$ ) and in the possible excited state ( $J = \frac{3}{2}$ ), taking into consideration only the attractive potential between any two of the three particles.

The ground-state<sup>16</sup> spin of the hypertriton is  $J$

$= \frac{1}{2}$ . There is some evidence for an excited state of  ${}^3_\Lambda\text{H}$  in  $J = \frac{3}{2}$  state. Herndon and Tang<sup>17</sup> have made a theoretical analysis of the binding-energy data of the  $s$ -shell hypernuclei using the hard-core  $\Lambda$ -nucleon potentials with an attractive part of Yukawa shape. They remark on the existence of a possible excited state of  ${}^3_\Lambda\text{H}$  with  $J = \frac{3}{2}$  and  $I = 0$ . In the experimental investigations of the  ${}^3_\Lambda\text{H}$  lifetime, Keyes *et al.*<sup>18</sup> have found evidence of what they call a speculative possibility of the spin- $\frac{3}{2}$   ${}^3_\Lambda\text{H}$  bound state.

The hypertriton consists of a  $\Lambda$  particle and two nucleons. The nucleons are labeled 1 and 3; the  $\Lambda$  particle is numbered 2. The nucleons form the two-dimensional representation of SU(2) and  $\Lambda$  belongs to the singlet representation.

Each of the three particles is a spin- $\frac{1}{2}$  fermion, so the spin of the three-body  $\Lambda np$  system is either  $\frac{1}{2}$  or  $\frac{3}{2}$ . The two-body potentials between pairs of particles are taken to be  $s$ -wave, spin-dependent. The neutron-proton mass difference has been neglected; the  $\Lambda$ - $N$  potentials are assumed to be charge symmetric. For the calculations of the  $\Lambda np$  system in  $J = \frac{1}{2}$  state we have taken into account the following two-body potentials: the  $N$ - $N$  potential in the triplet spin state and the  $\Lambda$ - $N$  potential in singlet and triplet spin states. It is obvious that the  $\Lambda$ - $N$  potential in the singlet state will not appear while a study of the  $\Lambda np$  system in  $J = \frac{3}{2}$  state is carried out.

For the two-body energy to be negative we get

$$\tau_\nu(q') = \lambda_\nu \left( 1 + \frac{\mu_\nu \lambda_\nu}{4\pi \beta_\nu} \right) \beta_\nu + \left[ 2\mu_\nu \left( \frac{M}{m_\nu M_\nu} \frac{q'^2}{2} - E \right) \right]^{1/2} \left\{^{-2} \right\}^{-1}, \quad (5)$$

where  $m_\nu$  is the mass of the spectator particle in the  $\nu$ th two-particle channel,  $M$  is the total mass of all three particles, and  $M_\nu = M - m_\nu$ . Greek indices in this paper run 1-3 for  $\Lambda$ - $N$  singlet,  $N$ - $N$  triplet, and  $\Lambda$ - $N$  triplet interactions, respectively. Details of the kinematics one can find in Ref. 5.

The  $N$ - $N$   ${}^3S_1$  potential parameters used in this paper are  $\beta = 1.4494 \text{ fm}^{-1}$  and  $\lambda = -21.6173 \text{ fm}^{-2}$ . They are obtained by fitting the  $np$  triplet scattering length 5.37 fm and the effective range 1.716 fm.

For the  $\Lambda$ - $N$  potential parameters we do not have any unique choice. In such circumstances we have taken into account a number of sets for  $\Lambda$ - $N$  parameters<sup>7-14</sup> which are given in Table I. Their merits and demerits<sup>15</sup> are investigated on the basis of a comparison made with the recent experimental results obtained for the  $\Lambda$ - $N$  elastic scattering cross sections and  ${}^3_\Lambda\text{H}$  binding energies in the ground and in the excited states corresponding to these sets.

These parameters appearing in Table I are found out from the low-energy experimental data for  $\Lambda$ - $p$  elastic scattering, and the results available from the analysis of the binding energies of the light hypernuclei. Moreover, if there is a charge-symmetry breaking  $\Lambda$ - $N$  force,<sup>8</sup> the  $\Lambda$ - $p$  scattering data as such can not be used for the charge-symmetric parameters required for our three-body calculations.

### 3. THREE-BODY EQUATIONS

To obtain the binding energy of the hypertriton  ${}^3_\Lambda\text{H}$  we have used the formalism of Schick and Hetherington.<sup>3-5</sup> A Faddeev-type multiple-scattering analysis provides the following set of coupled integral equa-

### 2. TWO-BODY INPUT FORCES

The two-body  $s$ -wave potentials that we have taken into account in our calculation of  ${}^3_\Lambda\text{H}$  binding energy are  $\Lambda$ - $N$  singlet and triplet potentials and  $N$ - $N$  triplet potential only. Each of these is taken to be an NLS potential of the Yamaguchi<sup>6</sup> form; i.e., in a given spin channel the kernel of the two-body potential in a relative-momentum space representation has the form

$$\langle \vec{p} | V | \vec{q} \rangle = \lambda g(p)g(q), \quad (1)$$

where

$$g(p) = 1/(p^2 + \beta^2). \quad (2)$$

The potential parameters  $\lambda$  and  $\beta$  are related to the scattering length  $a$  and effective range  $r$  by the well-known relations

$$a = \frac{2}{\beta} \left( 1 + \frac{4\pi\beta^3}{\mu\lambda} \right)^{-1}, \quad (3a)$$

$$r = \frac{1}{\beta} \left( 1 - \frac{8\pi\beta^3}{\mu\lambda} \right), \quad (3b)$$

where  $\mu$  is the reduced mass of two interacting particles.

The two-body momentum space  $s$ -wave  $t$  matrix element at three-body energy  $E$  for a single NLS potential, such as given in Eq. (1), may be written

$$\langle \vec{p} | t_\nu | \vec{q} \rangle = t_\nu(p, q) = g_\nu(p)\tau_\nu g_\nu(q). \quad (4)$$

tions for the  $\Lambda$ - $d$  scattering amplitude<sup>4</sup>:

$$R_{\alpha\beta}(q, q') = K_{\alpha\beta}(q, q') + \sum_{\gamma\lambda} \int_0^\infty K_{\alpha\gamma}(q, k) \tau^{\gamma\lambda}(k) R_{\lambda\beta}(k, q') (2\pi)^{-2} k^2 dk \quad (6)$$

with

$$K_{\alpha\beta}(q, q') = W_{\alpha\beta} \int_{-1}^1 \frac{D^{-1} d \cos \theta}{\left[ q'^2 + \left( \frac{m_\beta}{M_\alpha} \right)^2 q^2 + 2 \frac{m_\beta}{M_\alpha} q q' \cos \theta + \beta_\alpha^2 \right] \left[ q^2 + \left( \frac{m_\alpha}{M_\beta} \right)^2 q'^2 + 2 \frac{m_\alpha}{M_\beta} q q' \cos \theta + \beta_\beta^2 \right]}, \quad (7)$$

where

$$D = E - \frac{M_\beta}{2m_{\alpha\beta}m_\alpha} q^2 - \frac{M_\alpha}{2m_{\alpha\beta}m_\beta} q'^2 - \frac{1}{m_{\alpha\beta}} q q' \cos \theta.$$

In Eq. (6) the matrix  $\tau^{\alpha\beta}$  is given by

$$\tau^{\alpha\beta} = \begin{bmatrix} \tau_1 & 0 & 0 \\ 0 & \tau_2 & 0 \\ 0 & 0 & \tau_3 \end{bmatrix}, \quad (8)$$

where  $\tau_i$ 's are as defined in Eq. (5). In Eq. (7)  $\theta$  is the angle between  $q$  and  $q'$  and  $m_{\alpha\beta} = M - m_\alpha - m_\beta$ . One can find the matrix  $[W_{\alpha\beta}]$  for the doublet and for the quartet spin states in Ref. 3.

To obtain the  ${}^3_\Lambda\text{H}$  binding energies in the ground and in the excited states, it merely needs to be noted that, at the bound-state energy,  $R$  is singu-

lar. The bound state occurs at the energy  $E_0$  for which the Fredholm determinant for the set of integral equations vanishes. Then  $B_\Lambda$  is given by  $B_\Lambda = E_0 - \epsilon$  where  $\epsilon$  is the deuteron binding energy. Numerical calculations<sup>19</sup> have been performed using the Gauss-Legendre quadrature method. From now on we will use  ${}_e B_\Lambda$  for the binding energy of the  $\Lambda$  particle in the ground state of  ${}^3_\Lambda\text{H}$  and  ${}_e B_\Lambda$  for that in the excited state of  ${}^3_\Lambda\text{H}$ .

#### 4. RESULTS AND DISCUSSION

The sets of the two-body  $\Lambda$ - $N$  parameters considered in this paper can be divided into two classes.

TABLE I. Values of  $B_\Lambda$  for several sets of  $\Lambda$ - $N$  parameters. The  $N$ - $N$   ${}^3S_1$  potential parameters used here are  $\beta = 1.4494 \text{ fm}^{-1}$ ,  $\lambda = -21.6173 \text{ fm}^{-2}$ .

| Set            | Reference | $\Lambda$ - $N$ ( ${}^1S_0$ ) |          |                              |                                | $\Lambda$ - $N$ ( ${}^3S_1$ ) |          |                              |                                | $B_\Lambda$ (MeV)  |                    | Ground-state spin of $\Lambda n p$ system |
|----------------|-----------|-------------------------------|----------|------------------------------|--------------------------------|-------------------------------|----------|------------------------------|--------------------------------|--------------------|--------------------|---|
|                |           | $a$ (fm)                      | $r$ (fm) | $\beta$ ( $\text{fm}^{-1}$ ) | $\lambda$ ( $\text{fm}^{-2}$ ) | $a$ (fm)                      | $r$ (fm) | $\beta$ ( $\text{fm}^{-1}$ ) | $\lambda$ ( $\text{fm}^{-2}$ ) | $J = \frac{1}{2}$  | $J = \frac{3}{2}$  |   |
| 1              | 7         | -2.46                         | 3.87     | 1.1429                       | -4.2393                        | -2.07                         | 4.50     | 1.0687                       | -3.1153                        | 0.203              | 0.060 <sup>a</sup> | $\frac{1}{2}$                             |
| 2              | 8         | -2.76                         | 3.05     | 1.3385                       | -7.5605                        | -1.96                         | 3.50     | 1.3043                       | -6.0499                        | 0.625              | 0.160 <sup>b</sup> | $\frac{1}{2}$                             |
| 3              | 9         | -3.30                         | 1.83     | 1.9746                       | -28.6284                       | -0.64                         | 3.70     | 1.7669                       | -9.6825                        | 1.195              | <0                 | $\frac{1}{2}$                             |
| 4              | 10        | -1.80                         | 2.80     | 1.5753                       | -11.1406                       | -1.60                         | 3.30     | 1.4366                       | -7.7047                        | 0.188              | 0.052              | $\frac{1}{2}$                             |
| 5 <sup>c</sup> | 11        | -1.70                         | 2.50     | 1.7406                       | -15.2919                       | -1.50                         | 2.00     | 2.1268                       | -28.7365                       | 0.302              | 0.343              | $\frac{3}{2}$                             |
| 6              | 12        | -4.60                         | 1.70     | 2.0177                       | -32.8415                       | -0.53                         | 3.88     | 1.8339                       | -9.8030                        | 1.941              | <0                 | $\frac{1}{2}$                             |
| 7 <sup>c</sup> | 13        | -2.00                         | 5.00     | 1.0000                       | -2.4299                        | -2.20                         | 3.50     | 1.2670                       | -5.7550                        | 0.042              | 0.284              | $\frac{3}{2}$                             |
| 8 <sup>c</sup> | 14        | -1.36                         | 3.06     | 1.5863                       | -10.0668                       | -1.62                         | 2.93     | 1.5635                       | -10.3789                       | 0.016              | 0.120              | $\frac{3}{2}$                             |
| 9              | d         | -3.60                         | 2.00     | 1.8073                       | -21.9426                       | -0.53                         | 5.00     | 1.5647                       | -5.4567                        | 1.010 <sup>e</sup> | <0                 | $\frac{1}{2}$                             |
| 10             | f         | -2.89                         | 1.94     | 1.9187                       | -25.2275                       | -0.71                         | 3.75     | 1.6893                       | -8.7826                        | 0.900 <sup>e</sup> | <0                 | $\frac{1}{2}$                             |
| 11             | 3         | -1.80                         | 2.06     | 1.9965                       | -24.8468                       | -0.40                         | 4.00     | 2.0000                       | -11.1079                       | 0.050              | <0                 | $\frac{1}{2}$                             |

<sup>a</sup> See Refs. 25 and 26.

<sup>b</sup> See Refs. 26 and 27.

<sup>c</sup> It may be remarked that these sets as such do not qualify to be considered for the calculations of the  ${}^3_\Lambda\text{H}$  binding energy. See Ref. 15, and the discussions on pages 7 and 11.

<sup>d</sup> J. J. de Swart and C. Dullemond, Ann. Phys. (N.Y.) **19**, 458 (1962).

<sup>e</sup> See Ref. 3.

<sup>f</sup> R. C. Herndon, Y. C. Tang, and E. W. Schmid, Phys. Rev. **137**, B294 (1965).

(i) One having the potential in the triplet spin state weaker than the potential in the singlet state. Sets 1-4, 6, and 9-11 fall in this category and fulfill the requirement of Dalitz,<sup>16, 20</sup> who concluded the dominance of the singlet  $\Lambda$ - $N$  spin-dependent interaction. From the decay branching ratios, the spin of the  ${}^3_\Lambda\text{H}$  ground state has been established to be  $J = \frac{1}{2}$ . Therefore, the singlet  $\Lambda$ - $N$  interaction is more attractive than the triplet interaction.

(ii) Another class having the potential in the singlet spin state weaker than the potential in the triplet state. Sets 5, 7, and 8 of Table I belong to this class. These sets should have been rejected outright because they contradict the analysis of Rayet and Dalitz.<sup>16</sup> For Sets 5, 7, and 8 the ground-state spin of  ${}^3_\Lambda\text{H}$  would be  $\frac{3}{2}$  and the excited state's spin would be  $\frac{1}{2}$ . The Sets 5, 7, and 8, along with their  ${}_g B_\Lambda$  and  ${}_e B_\Lambda$  values could have been omitted from this paper, but we have kept them for the sake of completeness. To avoid confusion the ground-state spin of the  ${}^3_\Lambda\text{H}$  for each set has been given in the last column of Table I. The rejection of these three sets is similar to the dropping of potential  $G$  in Ref. 8 because it led to  $J=1$  for the ground-state spin of  ${}^4_\Lambda\text{H}$  in contradiction to the value  $J=0$  deduced from the decay branching ratios.

The values of the binding energies  ${}_g B_\Lambda$  and  ${}_e B_\Lambda$  of the  $\Lambda$  particle in the hypertriton obtained by us are given in Table I. In Fig. 1 we have plotted the low-energy  $\Lambda$ - $p$  elastic scattering data and the cross sections obtained from the NLS  $s$ -wave  $\Lambda$ - $N$  potentials fitted in turn to each of the Sets 1, 2, ..., 11. The figure is broken into two parts just to show the curves distinctly.

Hetherington and Schick did the  $\Lambda$ - $d$  bound-state problem analysis<sup>3, 4</sup> and compared their results with the experimental data<sup>21</sup> existing at that time. Since then a considerable change has been observed in the experimental data. Thus this problem has been investigated to see whether one can fit both the low-energy  $\Lambda$ - $p$  elastic scattering cross section and binding energy of the  $\Lambda$  in the hypertriton with only attractive two-body potentials using Faddeev's formalism for the three-body bound-state problem.

We have included for comparison the results obtained by Hetherington and Schick in Table I and in Fig. 1. Their curves in Fig. 1 corresponding to Sets 9 and 10 do not match the recent experimental data, but they were good fits to the data<sup>21</sup> of the year 1964. Moreover, these sets yield very high values<sup>3</sup> for  ${}_g B_\Lambda$ . In order to get a good fit for the value of  ${}_g B_\Lambda$  with the then existing experimental value of 0.20 MeV they varied the scattering lengths keeping the  $\Lambda$ - $N$  range parameter  $\beta^{-1}$

fixed at 0.5 fm and by trial and error procedure they got, corresponding to Set 11 in Table I, a value of 0.05 MeV for  ${}_g B_\Lambda$  which is quite consistent with the recent experimental value. But this set yields  $\Lambda$ - $p$  elastic scattering cross section (curve 11) too low when compared to the experimental data, as is evident from Fig. 1.

The experimental situation is changing for  ${}_g B_\Lambda$ . From  $(0.20 \pm 0.12)$  MeV<sup>22</sup> it has come to  $(0.01$

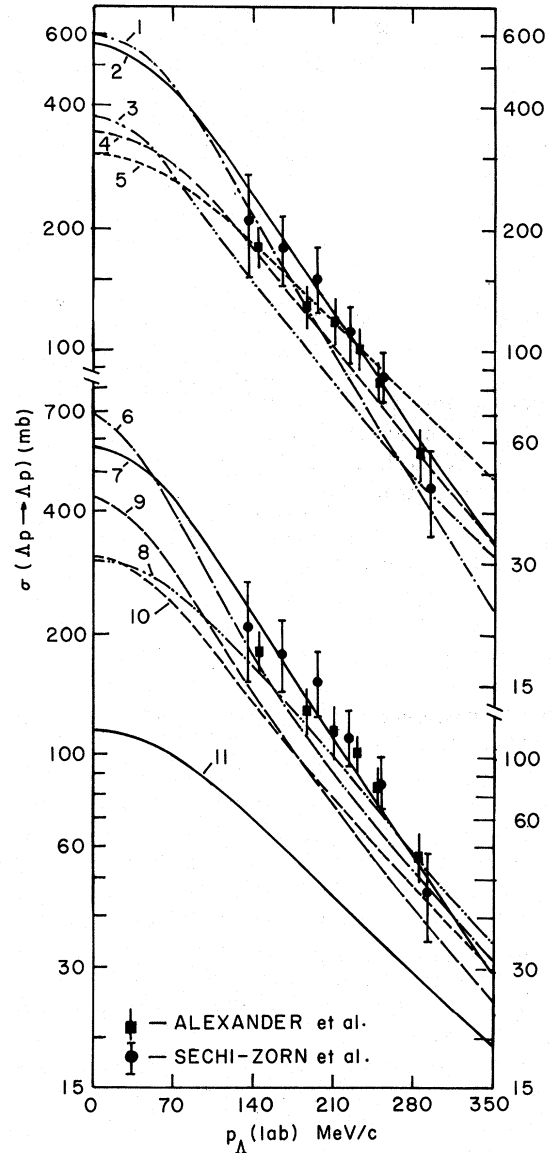


FIG. 1. Low-energy  $\Lambda$ - $p$  elastic scattering cross sections derived from the nonlocal separable potentials corresponding to the Sets 1-8 of  $\Lambda$ - $N$  parameters listed in Table I. The experimental results are from Refs. 10 and 13. Curves 9, 10, and 11 are from Ref. 3. The figure is broken into two parts just to show the curves distinctly.

$\pm 0.07$ ) MeV.<sup>23</sup> Presently it is  $(0.06 \pm 0.06)$  MeV.<sup>24</sup> Among the sets which fit the low-energy  $\Lambda$ - $p$  cross-section data satisfactorily (Fig. 1), Sets 1 and 4 yield the  ${}_g B_\Lambda$  values 0.203 and 0.188 MeV, respectively, which are nearest to the experimental result. For each of the Sets 1, 2, and 4 there is an excited bound state with  $J = \frac{3}{2}$ . According to the Sets 3, 6, and 9-11, the  $\Lambda$ - $N$  triplet interaction is sufficiently weaker than the singlet one, thus these sets do not predict the existence of an excited bound state with  $J = \frac{3}{2}$ .

The potential parameters fitted to the  $\Lambda$ - $N$  scattering data of Alexander *et al.*<sup>10</sup> given as Set 4 in the table yield the value of  ${}_e B_\Lambda$  ( $J = \frac{3}{2}$ ) as 0.052 MeV which is in good agreement with the value of 0.06 MeV obtained by Toepfer and Schick<sup>25, 26</sup> (Set 1 in Table I). Here it may be remarked that Set 4 yields  $\Lambda$ - $p$  cross sections in better agreement with the experimental data than that obtained corresponding to Set 1.

Toepfer and Schick<sup>25</sup> have estimated the effect of  $\Lambda$ - $\Sigma$  conversion both with and without a  $\Lambda NN$  force on  ${}_g B_\Lambda$  and  ${}_e B_\Lambda$  values in  $J = \frac{1}{2}$  and  $\frac{3}{2}$  states, respectively. The  $\Lambda$ - $N$  parameters used in their calculations are those of Alexander *et al.*<sup>7</sup> The change in  ${}_g B_\Lambda$  ( $J = \frac{1}{2}$ ) because of a  $\Lambda NN$  force is not of a simple nature. It increases  ${}_g B_\Lambda$  when used in the  $S = 1$   $YN$  channel only, and decreases  ${}_g B_\Lambda$  if used in the  $S = 0$  channel only. But when used in both channels the  ${}_g B_\Lambda$  gets reduced even more than when used in the  $S = 0$  channel alone.  $S = 1$   $\Lambda$ - $\Sigma$  conversion in the  $\Lambda np$   $J = \frac{3}{2}$  state decreases  ${}_e B_\Lambda$ , while in the  $\Lambda np$   $J = \frac{1}{2}$  state it increases  ${}_g B_\Lambda$ . Toepfer<sup>27</sup> has extended this work further and has reported that for  $\Lambda np$  ( $J = \frac{3}{2}$ ) system (i.e., the excited bound state of the  ${}^3_\Lambda\text{H}$ ) the inclusion of the  $\Lambda$ - $\Sigma$  virtual transitions with a potential having a short-range repulsion makes a change in the binding energy  ${}_e B_\Lambda$  of the same order of magnitude as when there was no repulsion at all.  $\Lambda$ - $N$  charge-symmetric parameters used by Toepfer<sup>27</sup> were those of Herndon and Tang.<sup>8</sup> Thus the binding energy  ${}_e B_\Lambda$  obtained from the consideration of the attractive potential only can be further reduced in magnitude by the techniques discussed above.

Further we discuss the possible qualitative effects of charge-symmetry breaking in the  $\Lambda$ - $N$  interaction in relation to the quantitative comparisons we have made.

Assuming the two body  $\Lambda$ - $N$  forces to be charge symmetric the  $\Lambda$ - $p$  scattering data give the low-energy parameters for the  $\Lambda$ -nucleon scattering. If charge-symmetry breaking is present in the  $\Lambda$ - $N$  interaction, the  $\Lambda$ - $p$  parameters cannot be used in place of charge-symmetric  $\Lambda$ - $N$  parameters. The charge-symmetry breaking effects for

$\Lambda$ - $N$  interaction have been discussed in detail by Herndon and Tang.<sup>8</sup> They have used an exponential well outside a hard core to fit the binding energies of the  $s$ -shell hypernuclei and the  $\Lambda$ - $p$  scattering data. Set 2 given in Table I for  $\Lambda$ - $N$  charge-symmetric effective range parameters can be calculated from the charge-symmetric potential parameters given for potential "H" in Eq. (16) of Ref. 8. Corresponding to Set 2,  $\Lambda$ - $p$  parameters are obtained by Herndon and Tang<sup>8</sup> where the charge-symmetry breaking effects are taken into account; these parameters are  $a_s^p = -2.25$  fm,  $r_s^p = 3.29$  fm,  $a_t^p = -2.08$  fm, and  $r_t^p = 3.40$  fm. Under the assumption of charge symmetry, if these  $\Lambda$ - $p$  parameters are used to find out the binding energy  ${}_g B_\Lambda$ , the value comes out to be 0.338 MeV, which is 0.287 MeV smaller than the  ${}_g B_\Lambda$  value 0.625 MeV for the corresponding  $\Lambda$ - $N$  charge-symmetric parameters (Set 2).  $\Lambda$ - $N$  charge-symmetric parameters given in Table I as Set 2, yield the binding energy  ${}_e B_\Lambda$  to be 0.16 MeV, but the  $\Lambda$ - $N$  potential with charge-symmetry-breaking part yields the  ${}_e B_\Lambda$  value to be 0.252 MeV. The value of  ${}_e B_\Lambda$  for  $\Lambda$ - $p$  charge-symmetry-breaking parameters is larger than its counterpart for  $\Lambda$ - $N$  charge-symmetric parameters unlike what we have observed for the  $\Lambda np$  system in  $J = \frac{1}{2}$  state. Hence the  $\Lambda$ - $p$  potential with charge-symmetry-breaking effect is more attractive in the triplet spin state and the corresponding  $\Lambda$ - $N$  charge-symmetric potential is more attractive in the singlet spin state.

If the charge-symmetry breaking is important in the  $\Lambda$ - $N$  interactions, some adjustments of the sort given in Ref. 8 in the scattering parameters of the sets obtained from the  $\Lambda$ - $p$  scattering data are necessary in order to get the corresponding charge-symmetric parameters required for the evaluation of  ${}_g B_\Lambda$  and  ${}_e B_\Lambda$ . One interesting feature of this adjustment is that the singlet to triplet interaction strength ratio for  $\Lambda$ - $N$  charge-symmetric parameter is larger than the same ratio for the  $\Lambda$ - $p$  charge-symmetry-breaking parameter. Qualitatively speaking, by this sort of adjustment in the  $\Lambda$ - $p$  scattering parameters of Sets 5, 7, and 8 one may obtain the respective  $\Lambda$ - $N$  charge-symmetric parameters with  $\Lambda$ - $N$  singlet spin interaction dominance and thus making the ground-state spin of  ${}^3_\Lambda\text{H}$  to be  $\frac{1}{2}$ , consistent with experimental findings.<sup>16</sup>

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calculations of the hypertriton binding energy should have the singlet part of the interaction more attractive than the triplet one. But for the sake of completeness we have included three sets numbered 5, 7, and 8 in Table I which do not satisfy this condition. More about them appears later. Also see Ref. 16.

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