PHYSICAL REVIEW C VOLUME 7, NUMBER 2 FEBRUARY 1973

Shell-Model Calculation for Masses 27, 28, and 29: General Methods and Specific Applications to ^{27}Al, ^{28}Si, and ^{29}Si

B. H. Wildenthal*
Cyclotron Laboratory and Physics Department, Michigan State University, East Lansing, Michigan 48823

and

J. B. McGrory†
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 26 July 1972)

A shell-model calculation for $A=27$, 28, and 29 nuclei has been carried out in a truncated $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ basis space with a modified-surface-δ-interaction Hamiltonian. A comparison of the calculated results for level energies, single-nucleon spectroscopic factors and $E2$ and $M1$ transition strengths in ^{27}Al, ^{28}Si, and ^{29}Si with the corresponding experimental values indicates that a unified and quantitive explanation of nuclear structure around $A=28$ can be obtained via shell-model techniques.

I. INTRODUCTION

We present here a description of a shell-model calculation for the mass region $A=27-29$ and some specific results for ^{27}Al, ^{28}Si, and ^{29}Si. In associated papers are presented: (1) a detailed study of $E2$ and $M1$ decays for all nuclei of this region using the present wave functions,[1] and (2) theoretical predictions based on the present work for the level energies, wave functions, and spectroscopic factors in ^{27}Mg, ^{28}Al, ^{28}Mg, and ^{29}Al.[2]

Despite extensive experimental investigation[3] (and in another sense, because of it) and a multitude of strong-coupling and weak-coupling collective-model calculations, it seems fair to say that a fully satisfying and internally consistent understanding of the nuclear structure around ^{28}Si has not yet been achieved. The structure of ^{27}Al seems better described in a weak-coupling picture[4,5] built on the 2^+ first excited state of ^{28}Si than it does in a simple Nilsson picture[6,7] with the usual assumption of a prolate ground-state rotational band. However, either of these simple approaches leaves many significant features unexplained, and even more complicated collective-model calculations involving mixing[8] of Nilsson bands or rotation-vibration mixing[9] still fail to account for important aspects of the experimental situation. A recent suggestion[10] has been that an oblate-shape assumption for the low-lying states of ^{27}Al can produce an accounting for $B(E2)$ and spectroscopic-factor observations.

The energy levels of ^{28}Si are clearly not typical of either the simple rotational or simple vibrational model. Calculations have been carried out with Hartree-Fock[11] and SU_3[12] techniques with some success. Projected Hartree-Fock techniques together with vibration-interaction corrections[13,14] give a better accounting for energies and $B(E2)$ properties in ^{28}Si, but the single-particle characteristics of these wave functions have not been thoroughly explored yet.

The structure of ^{29}Si is generally well accounted for with either a straightforward band-mixed Nilsson calculation[15,16] or with an intermediate core-coupling approach[17,18] and seems to offer the least resistance to theoretical interpretations of any set of phenomena in this region.

A particularly interesting feature of this region is that ^{29}Si has long been known to require an oblate deformation for a Nilsson-type interpretation[19] and ^{27}Al, to the extent that the same model applied, seemed to require a prolate shape. Hartree-Fock calculations for ^{28}Si tended to yield ambiguous results concerning the shape of this nucleus until measurements of the quadrupole moment of the $J^\pi=2^+$ first excited state indicated oblateness. Thus one outstanding challenge to a theory for the region is to predict, rather than incorporate as parameters, the phenomena corresponding to this nominal change of shape occurring in the progression from ^{27}Al to ^{29}Si.

Our present study presents a simultaneous, internally consistent treatment of this mass region, in which the only essential variation between ^{27}Al, ^{28}Si, and ^{29}Si is the change in active particles from 11 to 12 to 13. We examine level energies, single-nucleon-transfer properties, and electromagnetic decays. The results are in consistent reasonable accord with experimental data and

seem to indicate that a comprehensive quantitative theory for all these nuclei can emerge naturally from a single microscopic shell model.

II. GENERAL COMMENTS

In order to keep to a minimum ambiguities devolving from parameters in the theory which are not known *a priori*, we work with a "modified"-surface-δ-interaction (MSDI) model Hamiltonian.[20,21] The specification of this Hamiltonian requires setting values to seven parameters: The three single-particle energies of the *sd* shell, $E(d_{5/2})$, $E(s_{1/2})$, and $E(d_{3/2})$, two strengths V_T, for the $T=1$ and $T=0$ surface-δ-interaction (SDI) interactions, and two values C_T, for the $T=1$ and $T=0$ monopole "modifications" to the SDI. Of these seven parameters only four (the two splittings of the single-particle energies and the two strengths V_T) affect the aspects of nuclear structure which are of interest to us here, namely wave functions and the energy spacings within a system of given A and T. The other parameters of the Hamiltonian serve to produce correct ground-state binding energies with respect to changes of A and T.

In a manner analogous to previous studies[22-24] of nuclei in the $A=18-22$ and $A=30-39$ regions we have determined values for the parameters of the MSDI Hamiltonian by requiring a simultaneous least-squares fit between energies of low-lying states in ^{27}Al, ^{28}Si, and ^{29}Si and the corresponding shell-model eigenvalues. The shell-model matrices were constructed with the computer codes described by French, Halbert, McGrory, and Wong[25] and the parameter search was carried out with the code of Glaudemans and Wildenthal.[26] The parameter values obtained for this region (see Table I) are reasonably consistent with the corresponding values obtained for the other mass regions of the *sd* shell. Obviously, with such a tightly constrained assumption for the shell-model Hamiltonian we cannot hope for extremely high accuracy in reproducing experimental energies. However, previous successes with the MSDI lead us to hope that the general features of experimental level sequences, spacing, and densities can be accounted for with this Hamiltonian and that many essential features of the wave functions will be in accord with measured observable values, regardless of the mass region.

One key to a successful shell-model calculation is an adequate model space. There is ample experimental evidence that all three *sd*-shell orbits are important in low-lying levels of nuclei around ^{28}Si and are vital to a realistic theoretical treatment of their structure. At the same time, it is practically impossible to work in an unrestricted $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ space. We have met this problem by the simple expedient of restricting the numbers of nucleons which are allowed to be excited out of the $1d_{5/2}$ orbit in the model space.

The configurations $(d_{5/2})^{n_1}(s_{1/2})^{n_2}(d_{3/2})^{n_3}$ which are included in these calculations are: for ^{27}Al, all 11-particle combinations for which $n_1 \geq 8$; for ^{28}Si, all 12-particle combinations for which $n_1 \geq 8$; and for ^{29}Si, $n_1=8$, $n_2=4$, $n_3=1$, and all 13-particle combinations for which $n_1 \geq 9$. We think that the numbers of $d_{5/2}$ holes included in these spaces is very nearly the minimum necessary for successful results. Even these spaces constitute very severe truncations, and the poorly understood effects arising from this aspect of the present calculations are an additional reason, aside from the simple Hamiltonian, why highly accurate agreement between calculated observable values and experiment is not expected. To some extent, however, we would hope that the parametrized shifts of the single-particle energy splittings compensate for some truncation effects. (Indeed, the

TABLE I. Parameters of MSDI Hamiltonians as empirically optimized for shell-model calculations in $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ spaces.

Mass range	Space	$V(T=1)$	$V(T=0)$	$C(T=1)$	$C(T=0)$	$E(d_{5/2})$	$E(s_{1/2})$	$E(d_{3/2})$
18–22 [a]	Full $\frac{5}{2}$-$\frac{1}{2}$-$\frac{3}{2}$ model space	0.954	0.774	0.370	−2.503	−4.49	−3.16	+1.04
27–29 [b]	$\frac{5}{2}$-$\frac{1}{2}$-$\frac{3}{2}$ space with truncation on number of $d_{5/2}$ holes	0.948	1.01	0.767	−1.694	−6.22	−3.65	−0.19
30–34 [c]	$\frac{5}{2}$-$\frac{1}{2}$-$\frac{3}{2}$ space with ≤ 2 $d_{5/2}$ holes	0.906	0.646	0.770	−1.470	−7.56	−6.03	−3.96
35–39 [d]	Full $\frac{5}{2}$-$\frac{1}{2}$-$\frac{3}{2}$ model space	0.960	0.379	0.461	−1.818	−6.82	−2.00	+0.65

[a] Reference 22.
[b] Present work.
[c] Reference 24.
[d] Reference 23.

gross truncation effect of reduced binding energies is evidently compensated for in part by more tightly bound single-particle orbits.)

III. RESULTS AND DISCUSSION

The three types of experimental data which we use to test our model wave functions are level excitation energies, single-nucleon spectroscopic factors, and $E2$ and $M1$ moments and transition rates. For each type of observable we discuss the results for each mass in turn, to emphasize that the single formulation of the nuclear structure problem used here has the capability of explaining what have usually been considered as quite disparate sets of phenomena.

TABLE II. Energies and single-nucleon spectroscopic factors for states of $A=27$, $T=\frac{1}{2}$ (^{27}Al-^{27}Si.) The calculated and observed binding energies relative to ^{16}O (with Coulomb effects removed) are listed for ground state, and the excitations of the other states, relative to these ground-state values, are listed for excited states. Asterisks denote experimental level energies used in the determination of the optimum MSDI Hamiltonian parameters. Experimental energies enclosed in parentheses indicate the lack of a firm experimental spin-parity assignment. A reference footnote to an experimental S factor refers to the entire column of data under that entry. The experimental energies are taken from ^{27}Al. The target state for the stripping transitions is the $J^\pi=0^+$, $T=1$ ground state of ^{26}Mg. The target state for the pickup transfers is the $J^\pi=0^+$, $T=0$ ground state of ^{28}Si.

	Energy		$100\times(2J_f+1)S_j$ (stripping)			$100\times S_j$ (pickup)		
J	Calc.	Expt.	Calc.	Expt.		Calc.	Expt.	
$\frac{1}{2}$	0.50	0.84*	160	126 [a]	130 [b]	109	98 [c]	158 [d]
$\frac{1}{2}$	2.85	3.68*	6	8	10	3	2	
$\frac{1}{2}$	5.35	5.75	0			9		
$\frac{1}{2}$	5.99	6.59 [b]	0			1		
$\frac{3}{2}$	1.02	1.01*	4	40	40	5	112	150
$\frac{3}{2}$	2.78	2.98*	192	312	264	34	70	48
$\frac{3}{2}$	3.81	(3.96) [a]	28	48	28	6		
$\frac{3}{2}$	4.62		96			6		
$\frac{5}{2}$	119.19	119.27*	252	288	252	695	750	624
$\frac{5}{2}$	3.07	2.73*	12	18	18	71	122	150
$\frac{5}{2}$	3.98	4.41 [e]	12	54	30	12	70	
$\frac{5}{2}$	4.59	4.81 [e]	0			11		
$\frac{7}{2}$	1.84	2.21						
$\frac{7}{2}$	4.52	4.58 [e]						
$\frac{7}{2}$	4.82							
$\frac{7}{2}$	5.22							
$\frac{9}{2}$	2.76	3.00						
$\frac{9}{2}$	4.91	5.43 [e]						
$\frac{9}{2}$	5.94	5.67 [e]						
$\frac{9}{2}$	6.41	7.17 [e]						
$\frac{11}{2}$	3.91	(5.50) [e]						
$\frac{11}{2}$	5.25	4.51 [e]						
$\frac{11}{2}$	7.11	6.51 [e]						
$\frac{11}{2}$	7.41							
$\frac{13}{2}$	6.87							
$\frac{13}{2}$	7.44							

[a] Reference 28.
[b] Reference 27.
[c] Reference 30.
[d] Reference 31.
[e] Reference 29.

A. Level Energies

The highly overdetermined nature of the empirical MSDI Hamiltonian implies that a successful calculation of the general aspects of energy level densities and spin sequences of these nuclei would be a significant accomplishment in itself. Since there are only four adjustable parameters relevant to the excitation spectra, there is little likelihood that any combination of values could account for the large amount of known energy level data if the model did not encompass some essential degrees of freedom which underlie the totality of observed phenomena of this region.

The level energies for ^{27}Al-^{27}Si, ^{28}Si, and ^{29}Si-^{29}P which are obtained in the present calculation are presented and compared to experimental data in Tables II–IV and in Figs. 1–3. In all cases, only four states of each J have been calculated. While there are exceptions, we do not expect a meaningful correspondence between model and observed levels to persist past this point. The experimental and calculated ground-state energies have been set equal in the figures, while in the tables, experimental and calculated energies of excited states are given relative to the respective ground-state energies. The calculated and observed (Coulomb effects subtracted) binding energies of the ground states are listed, relative to ^{16}O. Experimental energies are entered in the tables only when there seems adequate experimental reason to expect a correspondence between an observed level and a calculated state. Experimental energies used in the search for the optimum MSDI Hamiltonian parameters are marked with asterisks. In the figures, all known experimental levels are included up to the maximum excitation energy plotted. All experimental information on energies and spectroscopic factors used in the present paper which is not explicitly referenced has been obtained from the compilation of Endt and van der Leun.[3] Information on electromagnetic

TABLE III. Energies and single-nucleon spectroscopic factors for states of $A=28$, $T=0$ (^{28}Si). The target state for the stripping transfers is the $J^\pi=\frac{5}{2}^+$, $T=\frac{1}{2}$ ground state of ^{27}Al. The target state for the pickup transfers is the $J^\pi=\frac{1}{2}^+$, $T=\frac{1}{2}$ ground state of ^{29}Si. Other conventions of the presentation are noted in the caption of Table II.

			$100\times\frac{(2J_f+1)}{(2J_i+1)}S_j$ (stripping)					$100\times S_j$ (pickup)				
	Energy		Calc.			Expt.		Calc.			Expt.	
J	Calc.	Expt.	$j=\frac{1}{2}$	$j=\frac{3}{2}$	$j=\frac{5}{2}$	$j=\frac{1}{2}$	$j=\frac{3}{2}$ and/or $\frac{5}{2}$	$j=\frac{1}{2}$	$j=\frac{3}{2}$	$j=\frac{5}{2}$	$j=\frac{1}{2}$	$j=\frac{3}{2}$ and/or $\frac{5}{2}$
0	136.67	136.48*	···	···	106	···[a,b]	88,[a] 50[b]	50	···	···	45,[c] 45[d]	···,[c] ···[d]
0	5.75	4.98*	···	···	10	···	10, 11	9	···	···	12, 22	···, ···
0	7.28	6.69*	···	···	1			1	···	···	, 1	···, ···
0	9.04		···	···	9			4	···	···		
1	7.36	8.33*	···	1	0			4	0	···		
1	9.05		···	17	0			4	0	···	, 6	
1	9.17		···	4	1			0	1	···		
1	10.05		···	4	0			2	0	···		
2	2.54	1.78*	76	2	9	76, 31	14, 14	···	13	98		86, 87
2	6.78	7.38*	2	11	8	···	24, 22	···	1	0		, (25)
2	6.87	7.42*	2	13	3	20, 10	14, 13	···	2	7		, 26
2	7.12	7.93	0	27	0			···	1	1		
3	5.11	6.28	68	23	5	28, 28	14, 15	···	···	107		83, 74
3	7.47	7.80	10	20	5	12, 10	8, 9	···	···	10		, 8
3	7.90	8.59	7	42	0	40, 41	56, 47	···	···	22		, 3
3	9.20		2	1	0			···	···	1		
4	5.59	4.62	···	59	6	···	64, 65					
4	6.45	6.89	···	52	2	···	100, 48					
4	8.63		···	1	1							
4	9.07		···	3	0							
5	8.38		···	···	0							
5	8.86		···	···	3							
6	8.83	8.54										
6	10.08											
7	14.59											
8	14.59											
8												

[a] Reference 32.
[b] Reference 27.
[c] Reference 33.
[d] Reference 34.

properties is taken from the detailed compilation given in Ref. 1.

1. *Excitation Energies in ^{27}Al-^{27}Si*

The excitation energies of the calculated and observed spectra of $A=27$, $T=\frac{1}{2}$ are presented in Table II and Fig. 1. Since information about ^{27}Al is more complete than is the case for ^{27}Si, the ^{27}Al energies are used in the figure and in the associated Table II. Of course, as far as energies and spectroscopic factors are concerned, our isospin-formalism results pertain equally to ^{27}Al and ^{27}Si.

At present[3,27-31] there are experimentally observed levels in ^{27}Al which appear to be counterparts to each of the four calculated $J=\frac{1}{2}$ states, to the first three calculated $J=\frac{3}{2}$ states and to the four calculated $\frac{5}{2}^+$ states. Some of these correspondences (mainly to the higher-lying states) are based solely on energy proximities, but most are buttressed by correlations in other observables. The largest deviations between theory and experiment in this group occur for the second $J=\frac{1}{2}$ and second $J=\frac{5}{2}$ states. The model energies predicted for $J=\frac{7}{2}$ and $\frac{9}{2}$ states also match up well with six experimentally determined energies for levels with these spins. In particular, only one $J=\frac{7}{2}$ state and one $J=\frac{9}{2}$ state are predicted to occur in the low-energy part of the spectrum, thus hopefully adding a final chapter to a long story of the-

TABLE IV. Energies and single-nucleon spectroscopic factors for the states of $A=29$, $T=\frac{1}{2}$ (^{29}Si-^{29}P). Experimental energies are taken from ^{29}Si. The target state for the stripping transfers is the $J=0^+$, $T=0$ ground state of ^{28}Si. The target state for the pickup transfer is the $J^\pi=0$, $T=1$ ground state of ^{30}Si. Other conventions of the presentation are noted in the caption to Table II.

	Energy		$100\times(2J_f+1)S_j$ (stripping)		$100\times S_j$ (pickup)	
J	Calc.	Expt.	Calc.	Expt.	Calc.	Expt.
$\frac{1}{2}$	144.82	144.93*	100	105 [a]	123	80,[b] 70,[b] 20,[c] 68 [d]
$\frac{1}{2}$	4.83	4.84* [c]	2		13	, , 10,
$\frac{1}{2}$	5.51	6.44 [c]	0		3	, , 2,
$\frac{1}{2}$	6.99		14		8	
$\frac{3}{2}$	1.96	1.27*	236	295	27	70, 120, 80, 70
$\frac{3}{2}$	2.82	2.43*	12	<5	1	17, 21, 8, 13
$\frac{3}{2}$	5.55	(5.93) [c,e]	0		1	
$\frac{3}{2}$	6.14	(6.11) [e]	0		0	
$\frac{5}{2}$	1.80	2.03*	66	73	275	170, 170, 170, 177
$\frac{5}{2}$	3.61	3.07*	12	35	0	10, 18, 8, 8
$\frac{5}{2}$	4.43	4.89* [b,e]	6		56	100, 100, 100, 119
$\frac{5}{2}$	6.10	6.72 [b]	0		4	, , 40, 35
$\frac{7}{2}$	4.37	4.08 [f]				
$\frac{7}{2}$	4.56	5.28 [e,g]				
$\frac{7}{2}$	5.97	5.81 [e]				
$\frac{7}{2}$	6.58					
$\frac{9}{2}$	5.11	4.74 [f,e]				
$\frac{9}{2}$	6.33	5.65 [e,g]				
$\frac{9}{2}$	6.79					
$\frac{9}{2}$	7.54					
$\frac{11}{2}$	6.66	7.14 [g]				
$\frac{11}{2}$	8.50					
$\frac{13}{2}$	9.12					

[a] Reference 38.
[b] Reference 39.
[c] Reference 37.
[d] Reference 33.
[e] Reference 40.
[f] Reference 16.
[g] Reference 41.

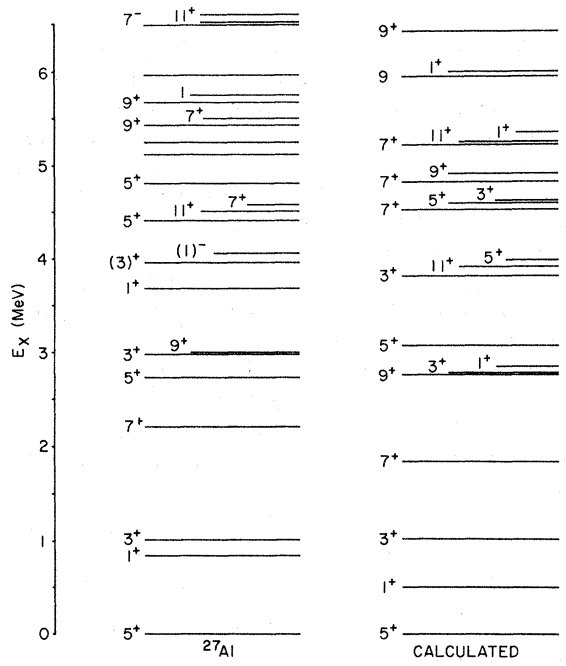

FIG. 1. Observed (as in ^{27}Al) and calculated spectra of $A=27$, $T=\frac{1}{2}$. All observed levels up to 6 MeV of excitation, and calculated states up to the fourth of each J value which fall within this same range, are entered. The numbers listed are two times the J values of the states.

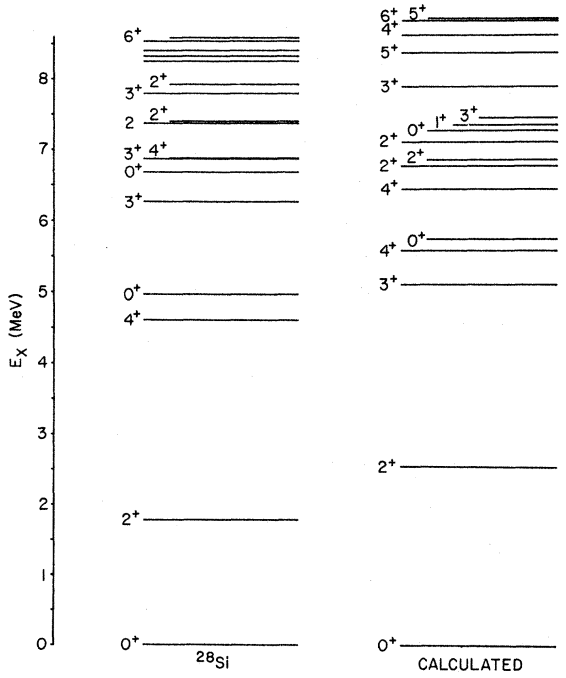

FIG. 2. Observed and calculated spectra of $A=28$, $T=0$. All observed levels up to 8.5 MeV of excitation and calculated states up to the fourth of each J value which fall within this same range, are entered.

oretical controversy and experimental confusion over whether the 3.00-MeV experimental level had $J=\frac{7}{2}$ or $\frac{9}{2}$. Two and possibly three $J=\frac{11}{2}$ states have been located experimentally in ^{27}Al, but the model spectrum is not simply correlated with these data if the measured and calculated $B(E2)$ rates are also considered. This matter will be discussed in more detail later.

The energy level predictions from the present calculation for $A=27$, $T=\frac{1}{2}$ can be summarized as quite successful in accounting for the density of levels of various spins in the first 6 MeV of excitation and as moderately successful in ordering the states of the spectrum according to J. The typical deviations between calculated and observed excitation energies are large enough to cause several inversions in order in the model spectrum.

2. *Excitation Energies of ^{28}Si*

The excitation energies of the calculated and observed spectra of $A=28$, $T=0$ are presented in Fig. 2 and in Table III. The level sequence of ^{28}Si is marked by a scarcity of levels below 6 MeV excitation. Above 6 MeV, numerous positive-parity states are found. The model spectrum for ^{28}Si re-

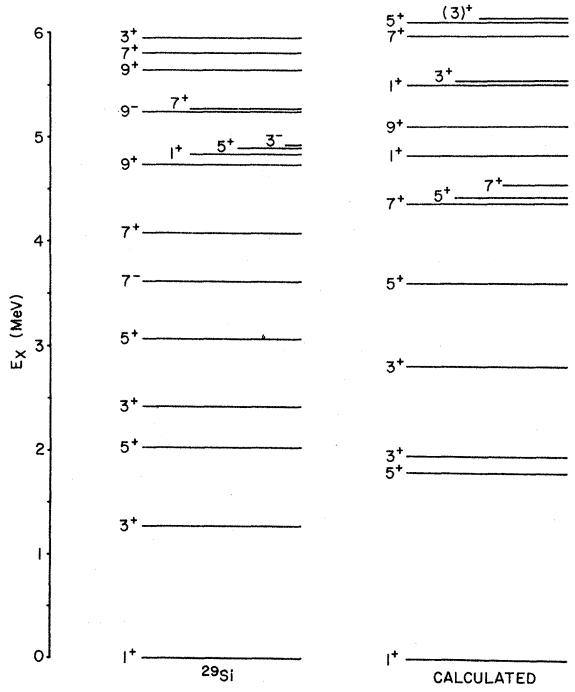

FIG. 3. Observed (as in ^{29}Si) and calculated spectra of $A=29$, $T=\frac{1}{2}$. All observed levels up to 6 MeV of excitation, and calculated states up to the fourth of each J value, are entered. The numbers listed are 2 times the J values of the states.

produces the density distribution of levels quite well. Deviations between individual corresponding pairs of calculated and observed energies are typically 0.5 to 1.0 MeV. These discrepancies are quite a bit larger than those observed in the shell-model studies of other portions of the *sd* shell previously referred to. On the other hand, the excitations here are larger than those typically studied in higher- and lower-mass nuclei of the shell. In summary, the model spectrum comes quite close in predicting the total number of levels observed in the first 8–9 MeV of excitation. In addition, all experimentally observed levels of known J have possible counterparts at qualitatively the right places in the model spectrum. Nonetheless, the deviations between some model and observed excitation energies are large enough to cause concern about the detailed value of the associated model wave functions. Hence recourse would have to be made to spectroscopic factors[27,32-34] and electromagnetic data[35,36] before more than qualitative success could be claimed for this system, if it were considered independent of ^{27}Al and ^{29}Si.

3. *Excitation Energies of ^{29}Si-^{29}P*

The excitation energies calculated and observed for $A=29$, $T=\frac{1}{2}$ are presented in Fig. 3 and Table IV. The experimental values are taken from the spectrum of ^{29}Si. The 16 experimentally observed levels which have probable J^π assignments[3,18,37-42] all have possible counterparts in the model spectrum. The typical deviations between model and experimental pairs is comparable to the situation for ^{27}Al. Again, the over-all density of levels is predicted quite well, although some steps in the experimental sequence of observed J values are inverted in the model spectrum.

B. Single-Nucleon-Transfer Characteristics

The single-particle characteristics of nuclear levels, as implied by spectroscopic factors extracted from measurements of the angular distributions of single-nucleon-transfer direct reactions, comprise a critically important set of experimental features that a microscopic shell-model calculation should reproduce. That is to say, if such model wave functions do not account for the essential features of the observed single-particle characteristics of the states in question, there seems little point in pursuing questions involving electromagnetic decays (for which less clear relations between observed or calculated values and a specific component or sets of components in the model wave function exist) and questionable meaning to agreements obtained between calculated and observed level energies. There are presently available data on both stripping and pickup to each of the systems we consider in this paper. Hence a thorough critique of this aspect of the calculations is possible.

In conjunction with our discussion of calculated and observed spectroscopic factors, we will refer to tables listing and identifying major components of the various calculated wave functions. These wave function listings are of interest from a variety of viewpoints, of course, but seem most relevant to ideas involving single-nucleon transfer.

In Tables II–IV, spectroscopic factors are listed in the isospin representation; i.e., S is intended, not C^2S. Hence, for example, the sum rule limit on pickup from ^{28}Si is $\sum_i S_i = 12$. The fact that $\sum_{i=1}^{12} S_i = 9.5$, as obtained from the 12 states listed in Table II, merely reflects the amount of strength fragmented into the several hundred higher-lying states not listed in the tables.

We are not primarily concerned with absolute agreement between experimentally obtained and calculated spectroscopic factors, but rather attach primary importance to relative values from state to state. We regard 25% agreement between relative trends as good agreement for strong states and 50% as adequate for weak states. As it happens, absolute agreements typically fall within this range also.

1. *Single-Nucleon Transfer to $A=27$, $T=\frac{1}{2}$*

The relevant experiments populating ^{27}Al-^{27}Si are proton stripping on ^{26}Mg and proton or neutron pickup on ^{28}Si. Experimentally determined spectroscopic factors and predictions are given in Table II. Stripping experiments show very definitely that the "single-particle" $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ states of $A=27$, $T=\frac{1}{2}$, relative to the $A=26$, $J=0$, $T=1$ ground state, are, respectively, the $\frac{5}{2}^+$ ground state, the $\frac{1}{2}^+$ first excited state, and the *second* $\frac{3}{2}^+$ state in the spectrum. The calculations predict exactly this behavior. The quantitative values are in good agreement for $\frac{1}{2}^+$ and $\frac{5}{2}^+$, while the predicted $\frac{3}{2}^+$ strength may be somewhat low. The predictions for weakly excited states are in general accord also.

An inspection of observed pickup strengths to the $\frac{5}{2}^+$ and $\frac{1}{2}^+$ states shows agreement with model predictions and consistency with the pictures of these states drawn from the stripping results. The $\frac{5}{2}^+$ ground state is well represented both as a $d_{5/2}$ hole in ^{28}Si and as a $d_{5/2}$ particle on ^{26}Mg. The first $\frac{1}{2}^+$ state can likewise be considered in terms of a $s_{1/2}$ hole or particle.

TABLE V. Major components of $A=27$, $T=\frac{1}{2}$ wave functions. A model state is labeled by its mass number (A), twice its total angular momentum ($2J$), twice its total isospin ($2J$), its calculated binding energy (E), and an ordinal number which denotes whether it is the first, second, third, etc. lowest state for the particular A, J, T combination in question. Also noted are the number of core particles in the model (16 here) and the dimensionality (number of basis states) for this A, J, T. Following this first line of information the eight largest (absolute magnitude) amplitudes of the wave function are listed and identified. Under each "amplitude" value is a triplet of columns, one for each of the three sd-shell orbits. The information in these columns serves to completely specify the basis vector associated with the particular amplitude in question as follows. The "configuration" triplet gives the occupation number for each of the orbits. The 2(S-shell J's) triplet gives the angular momentum to which the particles in the single orbits (shells) are separately coupled. The 2(coupled J's) doublet then gives first the angular momentum J_{12} which results from coupling the angular momentum of shell 1 ($0d_{5/2}$) to that of shell 2 ($1s_{1/2}$) and then gives the total angular momentum J which results from coupling J_{12} to the angular momentum of shell 3 ($0d_{3/2}$). The 2(S-shell T's) triplet and the 2(coupled T's) doublet give analogous information about the isospin couplings. The S-S seniorities triplet lists the seniorities of the particle coupling within each orbit. The concluding information given about the state comes on the last two lines after the components have been listed. The percentage of the total wave function structure that is identified in the preceding listing is noted on the first of these lines, and on the second (and last line for the particular state in question) we list the average occupation number for each orbit as calculated from the complete wave function.

$A=27$, $2J^{\pi}=1^{+}$, $2T=1$, $E=118.688$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 165.

Amplitude	0.725			−0.341			−0.262			0.230			−0.192			0.179			−0.171			−0.136		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	1	0	8	3	0	9	2	0	10	1	0	8	1	2	9	2	0	8	1	2	9	1	1
2(S-shell J's)	0	1	0	0	1	0	1	2	0	2	1	0	0	1	0	1	0	0	0	1	0	5	1	3
2(coupled J's)		1	1		1	1		1	1		1	1		1	1		1	1		1	1		4	1
2(S-shell T's)	2	1	0	0	1	0	1	0	0	0	1	0	0	1	2	1	2	0	4	1	2	1	1	1
2(coupled T's)		1	1		1	1		1	1		1	1		1	1		1	1		3	1		2	1
S-S seniorities	0	1	0	0	1	0	3	2	0	2	1	0	0	1	0	3	0	0	0	1	0	1	1	1

Listed components account for 88.0% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle = 9.32$, $\langle s_{1/2}\rangle = 1.37$, $\langle d_{3/2}\rangle = 0.30$.

$A=27$, $2J^{\pi}=3^{+}$, $2T=1$, $E=118.163$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 297.

Amplitude	0.640			0.288			0.271			−0.258			−0.218			0.180			−0.156			−0.156		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	1	0	10	1	0	9	2	0	8	3	0	8	3	0	9	2	0	9	2	0	8	3	0
2(S-shell J's)	4	1	0	2	1	0	3	0	0	4	1	0	4	1	0	5	2	0	5	2	0	2	1	0
2(coupled J's)		3	3		3	3		3	3		3	3		3	3		3	3		3	3		3	3
2(S-shell T's)	2	1	0	0	1	0	3	2	0	2	1	0	0	1	0	1	0	0	1	0	0	2	1	0
2(coupled T's)		1	1		1	1		1	1		1	1		1	1		1	1		1	1		1	1
S-S seniorities	2	1	0	2	1	0	3	0	0	2	1	0	2	1	0	3	2	0	1	2	0	2	1	0

Listed components account for 76.1% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle = 9.23$, $\langle s_{1/2}\rangle = 1.49$, $\langle d_{3/2}\rangle = 0.28$.

$A=27$, $2J^{\pi}=5^{+}$, $2T=1$, $E=119.187$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 376.

Amplitude	−0.664			−0.340			0.236			−0.207			−0.204			−0.201			0.170			−0.139		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	11	0	0	9	2	0	9	2	0	10	1	0	9	0	2	10	1	0	9	0	2	9	2	0
2(S-shell J's)	5	0	0	5	0	0	5	0	0	6	1	0	5	0	0	4	1	0	5	0	0	3	2	0
2(coupled J's)		5	5		5	5		5	5		5	5		5	5		5	5		5	5		5	5
2(S-shell T's)	1	0	0	1	2	0	3	2	0	0	1	0	1	0	2	2	1	0	3	0	2	1	0	0
2(coupled T's)		1	1		1	1		1	1		1	1		1	1		1	1		3	1		1	1
S-S seniorities	1	0	0	1	0	0	1	0	0	2	1	0	1	0	0	2	1	0	1	0	0	3	2	0

Listed components account for 78.5% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle = 9.90$, $\langle s_{1/2}\rangle = 0.76$, $\langle d_{3/2}\rangle = 0.34$.

TABLE V *(Continued)*

$A=27$, $2J^{\pi}=3^{+}$, $2T=1$, $E=116.411$, eigenvector No. 2 of this A, J, T. Model core = 16, dimension = 297.

Amplitude	0.523			0.420			0.306			0.248			−0.194			−0.168			−0.165			−0.164		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	0	1	10	1	0	9	2	0	8	2	1	9	1	1	8	2	1	8	2	1	10	0	1
2(S-shell J's)	0	0	3	2	1	0	5	2	0	0	0	3	1	1	3	2	2	3	0	0	3	2	0	3
2(coupled J's)		0	3		3	3		3	3		0	3		0	3		0	3		0	3		2	3
2(S-shell T's)	2	0	1	0	1	0	1	0	0	0	2	1	1	1	1	2	0	1	4	2	1	0	0	1
2(coupled T's)		2	1		1	1		1	1		2	1		2	1		2	1		2	1		0	1
S-S seniorities	0	0	1	2	1	0	1	2	0	0	0	1	3	1	1	2	2	1	0	0	1	2	0	1

Listed components account for 72.6% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=9.22$, $\langle s_{1/2}\rangle=0.95$, $\langle d_{3/2}\rangle=0.83$.

$A=27$, $2J^{\pi}=7^{+}$, $2T=1$, $E=-117.347$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 389.

Amplitude	−0.694			−0.264			0.244			0.215			−0.201			0.139			0.137			−0.135		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	1	0	8	3	0	8	3	0	9	2	0	10	1	0	8	1	2	8	1	2	10	0	1
2(S-shell J's)	8	1	0	8	1	0	8	1	0	9	2	0	6	1	0	8	1	0	8	1	0	6	0	3
2(coupled J's)		7	7		7	7		7	7		7	7		7	7		7	7		7	7		6	7
2(S-shell T's)	2	1	0	2	1	0	0	1	0	1	0	0	0	1	0	0	1	2	2	1	2	0	0	1
2(coupled T's)		1	1		1	1		1	1		1	1		1	1		1	1		1	1		0	1
S-S seniorities	2	1	0	2	1	0	2	1	0	3	2	0	2	1	0	2	1	0	2	1	0	2	0	1

Listed components account for 75.4% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=9.26$, $\langle s_{1/2}\rangle=1.37$, $\langle d_{3/2}\rangle=0.36$.

$A=27$, $2J^{\pi}=9^{+}$, $2T=1$, $E=-116.427$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 351.

Amplitude	0.657			0.290			0.254			0.217			0.192			0.170			−0.153			−0.142		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	1	0	10	1	0	9	2	0	8	3	0	10	0	1	8	1	2	9	2	0	8	3	0
2(S-shell J's)	10	1	0	8	1	0	9	0	0	10	1	0	8	0	3	10	1	0	9	2	0	8	1	0
2(coupled J's)		9	9		9	9		9	9		9	9		8	9		9	9		9	9		9	9
2(S-shell T's)	0	1	0	2	1	0	1	2	0	2	1	0	2	0	1	2	1	2	1	0	0	0	1	0
2(coupled T's)		1	1		1	1		1	1		1	1		2	1		3	1		1	1		1	1
S-S seniorities	2	1	0	2	1	0	3	0	0	2	1	0	2	0	1	2	1	0	3	2	0	2	1	0

Listed components account for 73.6% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=9.34$, $\langle s_{1/2}\rangle=1.28$, $\langle d_{3/2}\rangle=0.38$.

The experimental results for pickup to the $\frac{3}{2}^{+}$ states are in clear disagreement with the model predictions and also, at first glance, seem to be inconsistent with the stripping data. The first $\frac{3}{2}^{+}$ state is more strongly populated in pickup than the second by a factor of 2. While the experimental results for the second observed $\frac{3}{2}^{+}$ state are roughly in agreement with the calculated spectroscopic factor for the second model $\frac{3}{2}^{+}$, and consistent with the stripping strength to this same level, the observed strong pickup cross section for the first $\frac{3}{2}^{+}$ state appears to be inconsistent both with the small predicted value and with the small experimental stripping strength for this state. The inconsistencies between the stripping and pickup data for the lowest $\frac{3}{2}^{+}$ level have stimulated the suggestion[28] that the observed pickup strength is to be attributed to some non-single-step process, rather than to the direct, single-step transition assumed in the extraction of spectroscopic factors. This sort of phenomenon is always difficult to rule out. Effects of this magnitude, should they manifest themselves with any frequency, could severely abridge the usefulness of single-nucleon-transfer data, to say the least. In this present instance, the possibility is exceptionally troubling in that a long string of experiments with different projectiles and at different energies have all noted basically the same strength of excitation for this level.

From the viewpoint of the present calculations, the discounting of the observed pickup strength of the first $\frac{3}{2}^{+}$ level, on the grounds of reaction-mechanism effects, is, of course, appealing.

TABLE VI. Major components of $A=28$, $T=0$ wave functions. The conventions of the presentation are explained in Table V.

$A=28$, $2J^{\pi}=0^{+}$, $2T=0$, $E=-136.664$, eigenvector No. 1 of this A, J, T. Model core=16, dimension=132.

Amplitude	0.519			−0.510			−0.312			−0.266			0.196			0.184			0.159			−0.157		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	12	0	0	10	2	0	10	0	2	10	2	0	8	4	0	9	3	0	8	2	2	10	1	1
2(S-shell J's)	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	1	1	0	0	0	0	4	1	3
2(coupled J's)		0	0		0	0		0	0		0	0		0	0		0	0		0	0		3	0
2(S-shell T's)	0	0	0	2	2	0	2	0	2	0	0	0	0	0	0	1	1	0	4	2	2	2	1	1
2(coupled T's)		0	0		0	0		2	0		0	0		0	0		0	0		2	0		1	0
S-S seniorities	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	3	1	0	0	0	0	2	1	1

Listed components account for 81.9% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=10.14$, $\langle s_{1/2}\rangle=1.27$, $\langle d_{3/2}\rangle=0.59$.

$A=28$, $2J^{\pi}=0^{+}$, $2T=0$, $E=-130.911$, eigenvector No. 2 of this A, J, T. Model core=16, dimension=132.

Amplitude	−0.410			0.407			−0.397			0.321			−0.272			0.239			0.233			0.187		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	1	1	12	0	0	8	4	0	10	2	0	9	3	0	8	2	2	8	3	1	9	2	1
2(S-shell J's)	4	1	3	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	4	1	3	5	2	3
2(coupled J's)		3	0		0	0		0	0		0	0		0	0		0	0		3	0		3	0
2(S-shell T's)	2	1	1	0	0	0	0	0	0	2	2	0	1	1	0	0	2	2	0	1	1	1	0	1
2(coupled T's)		1	0		0	0		0	0		0	0		0	0		2	0		1	0		1	0
S-S seniorities	2	1	1	0	0	0	0	0	0	0	0	0	3	1	0	0	0	0	2	1	1	1	2	1

Listed components account for 81.5% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=9.46$, $\langle s_{1/2}\rangle=1.80$, $\langle d_{3/2}\rangle=0.75$.

$A=28$, $2J^{\pi}=2^{+}$, $2T=0$, $E=129.301$, eigenvector No. 1 of this A, J, T. Model core=16, dimension=299.

Amplitude	0.566			0.294			−0.225			0.222			0.204			0.182			0.180			−0.158		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	2	0	10	1	1	9	3	0	10	1	1	9	2	1	10	1	1	10	1	1	8	3	1
2(S-shell J's)	2	2	0	2	1	3	1	1	0	4	1	3	3	0	3	0	1	3	2	1	3	4	1	3
2(coupled J's)		2	2		3	2		2	2		3	2		3	2		1	2		1	2		3	2
2(S-shell T's)	0	0	0	0	1	1	1	1	0	2	1	1	1	2	1	2	1	1	0	1	1	0	1	1
2(coupled T's)		0	0		1	0		0	0		1	0		1	0		1	0		1	0		1	0
S-S seniorities	2	2	0	2	1	1	3	1	0	2	1	1	3	0	1	0	1	1	2	1	1	2	1	1

Listed components account for 63.8% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=9.41$, $\langle s_{1/2}\rangle=1.75$, $\langle d_{3/2}\rangle=0.83$.

$A=28$, $2J^{\pi}=4^{+}$, $2T=0$, $E=134.127$, eigenvector No. 1 of this A, J, T. Model core=16, dimension=474.

Amplitude	0.591			−0.399			0.276			0.216			−0.204			−0.159			−0.157			−0.143		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	11	1	0	9	3	0	10	2	0	10	2	0	9	1	2	8	4	0	10	1	1	9	1	2
2(S-shell J's)	5	1	0	5	1	0	4	0	0	6	2	0	5	1	0	4	0	0	0	1	3	5	1	0
2(coupled J's)		4	4		4	4		4	4		4	4		4	4		4	4		1	4		4	4
2(S-shell T's)	1	1	0	1	1	0	2	2	0	0	0	0	1	1	2	0	0	0	2	1	1	3	1	2
2(coupled T's)		0	0		0	0		0	0		0	0		2	0		0	0		1	0		2	0
S-S seniorities	1	1	0	1	1	0	2	0	0	2	2	0	1	1	0	2	0	0	0	1	1	1	1	0

Listed components account for 74.3% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=9.80$, $\langle s_{1/2}\rangle=1.71$, $\langle d_{3/2}\rangle=0.48$.

TABLE VI *(Continued)*

$A=28$, $2J^{\pi}=4^+$, $2T=0$, $E=129.889$, eigenvector No. 2 of this A, J, T. Model core = 16, dimension = 474.

Amplitude	0.441			−0.292			−0.239			−0.230			0.218			−0.210			0.200			0.179		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	1	1	10	2	0	8	3	1	11	0	1	10	2	0	9	2	1	9	2	1	9	2	1
2(S-shell J's)	0	1	3	2	2	0	0	1	3	5	0	3	4	0	0	5	0	3	5	2	3	5	2	3
2(coupled J's)		1	4		4	4		1	4		5	4		4	4		5	4		3	4		5	4
2(S-shell T's)	2	1	1	0	0	0	0	1	1	1	0	1	2	2	0	1	2	1	1	0	1	1	0	1
2(coupled T's)		1	0		0	0		1	0		1	0		0	0		1	0		1	0		1	0
S-S seniorities	0	1	1	2	2	0	0	1	1	1	0	1	2	0	0	1	0	1	1	2	1	1	2	1

Listed components account for 55.4% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle = 9.31$, $\langle s_{1/2}\rangle = 1.58$, $\langle d_{3/2}\rangle = 1.11$.

$A=28$, $2J^{\pi}=6^+$, $2T=0$, $E=131.553$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 511.

Amplitude	−0.532			0.327			−0.250			0.243			−0.218			−0.188			−0.170			−0.169		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	11	1	0	9	3	0	11	0	1	9	1	2	9	2	1	10	1	1	10	1	1	10	2	0
2(S-shell J's)	5	1	0	5	1	0	5	0	3	5	1	0	5	0	3	8	1	3	6	1	3	6	2	0
2(coupled J's)		6	6		6	6		5	6		6	6		5	6		7	6		5	6		6	6
2(S-shell T's)	1	1	0	1	1	0	1	0	1	1	1	2	1	2	1	2	1	1	0	1	1	0	0	0
2(coupled T's)		0	0		0	0		1	0		2	0		1	0		1	0		1	0		0	0
S-S seniorities	1	1	0	1	1	0	1	0	1	1	1	0	1	0	1	2	1	1	2	1	1	2	2	0

Listed components account for 65.2% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle = 9.77$, $\langle s_{1/2}\rangle = 1.38$, $\langle d_{3/2}\rangle = 0.84$.

$A=28$, $2J^{\pi}=8^+$, $2T=0$, $E=131.080$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 523.

Amplitude	0.459			−0.405			0.293			−0.266			−0.227			−0.179			−0.167			−0.151		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	2	0	11	0	1	10	2	0	9	3	0	9	2	1	10	1	1	8	4	0	10	1	1
2(S-shell J's)	10	2	0	5	0	3	8	0	0	9	1	0	5	0	3	8	1	3	8	0	0	4	1	3
2(coupled J's)		8	8		5	8		8	8		8	8		5	8		7	8		8	8		5	8
2(S-shell T's)	0	0	0	1	0	1	2	2	0	1	1	0	1	2	1	2	1	1	0	0	0	2	1	1
2(coupled T's)		0	0		1	0		0	0		0	0		1	0		1	0		0	0		1	0
S-S seniorities	2	2	0	1	0	1	2	0	0	3	1	0	1	0	1	2	1	1	2	0	0	2	1	1

Listed components account for 66.6% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle = 9.55$, $\langle s_{1/2}\rangle = 1.66$, $\langle d_{3/2}\rangle = 0.80$.

Indeed, calculations of spectroscopic factors between the 2^+ first excited state of ^{28}Si and the $A=27$, $T=\frac{1}{2}$ states show that this $\frac{3}{2}^+$ level should be populated quite strongly, relative to its neighbors via a process involving an initial inelastic scattering followed by pickup. At the same time, an examination of the model wave functions suggests an alternative explanation of the problem. From inspection of Table V it can be seen that the lowest $\frac{5}{2}^+$ and $\frac{1}{2}^+$ states in $A=27$ are most simply characterized as $(d_{5/2})^{11}{}_{J=5/2}$ and $(d_{5/2})^{10}{}_{J=0,T=1}(s_{1/2})^1{}_{J=1/2}$ states, respectively. The state best characterized as $(d_{5/2})^{10}{}_{J=0,T=1}(d_{3/2})^1{}_{J=3/2}$ is, as expected from the calculated stripping spectroscopic factors, the second $\frac{3}{2}^+$ state. Now the expectation that the *same* states of odd-mass nuclei should be excited strongly in stripping *and* pickup arises both from general empirical experience and the theoretical idea that particles mix into 0^+ ground states in pairs. Thus the $(d_{5/2})^{10}(s_{1/2})^1$ and $(d_{5/2})^{10}(d_{3/2})^1$ states are expected to be populated most strongly in pickup because the $(d_{5/2})^{10}{}_{J=0}(s_{1/2})^2{}_{J=0}$ and $(d_{5/2})^{10}{}_{J=0}(d_{3/2})^2{}_{J=0}$ components are expected to be the largest admixtures to the predominantly $(d_{5/2})^{12}{}_{J=0}$ ground state of ^{28}Si. Indeed, this conventional picture is just what our calculation predicts (see also Table VI) and accounts for our spectroscopic factors being "consistent" between stripping and pickup.

The dominant component of the *first* $A=27$, $J=\frac{3}{2}^+$ state is $(d_{5/2})^{10}{}_{J=2,T=1}(s_{1/2})^1{}_{J=1/2}$. This component is unreachable via stripping from ^{26}Mg. On the other hand, it can be reached from a

$$(d_{5/2})^{10}{}_{J=2,T=1}(s_{1/2})^1{}_{J=1/2}(d_{3/2})^1{}_{J=3/2}$$

TABLE VII. Major components of $A=29$, $T=\frac{1}{2}$ wave functions. The conventions of the presentation are explained in Table V.

$A=29$, $2J^{\pi}=1^{+}$, $2T=1$, $E=144.820$, eigenvector No. 1 of this A, J, T. Model core=16, dimension=202.

Amplitude	0.673			−0.394			−0.279			−0.262			0.208			−0.153			0.127			−0.116		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	12	1	0	10	3	0	11	1	1	10	1	2	10	3	0	10	2	1	9	3	1	11	0	2
$2(S$-shell J's)	0	1	0	0	1	0	5	1	3	0	1	0	2	1	0	4	0	3	5	1	3	5	0	4
2(coupled J's)		1	1		1	1		4	1		1	1		1	1		4	1		4	1		5	1
$2(S$-shell T's)	0	1	0	2	1	0	1	1	1	2	1	2	0	1	0	2	2	1	1	1	1	1	0	2
2(coupled T's)		1	1		1	1		0	1		3	1		1	1		0	1		0	1		1	1
S-S seniorities	0	1	0	0	1	0	1	1	1	0	1	0	2	1	0	2	0	1	1	1	1	1	0	2

Listed components account for 85.1% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=10.89$, $\langle s_{1/2}\rangle=1.59$, $\langle d_{3/2}\rangle=0.52$.

$A=29$, $2J^{\pi}=1^{+}$, $2T=1$, $E=139.988$, eigenvector No. 2 of this A, J, T. Model core=16, dimension=202.

Amplitude	0.486			0.409			−0.261			−0.248			−0.230			−0.211			−0.192			−0.177		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	10	3	0	12	1	0	9	4	0	10	1	2	10	2	1	9	3	1	10	2	1	10	2	1
$2(S$-shell J's)	0	1	0	0	1	0	1	0	0	0	1	0	4	0	3	5	1	3	4	2	3	4	2	3
2(coupled J's)		1	1		1	1		1	1		1	1		4	1		4	1		2	1		4	1
$2(S$-shell T's)	2	1	0	0	1	0	1	0	0	2	1	2	2	2	1	1	1	1	2	0	1	2	0	1
2(coupled T's)		1	1		1	1		1	1		1	1		2	1		0	1		2	1		2	1
S-S seniorities	0	1	0	0	1	0	3	0	0	0	1	0	2	0	1	1	1	1	2	2	1	2	2	1

Listed components account for 69.9% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=10.06$, $\langle s_{1/2}\rangle=2.19$, $\langle d_{3/2}\rangle=0.75$.

$A=29$, $2J^{\pi}=3^{+}$, $2T=1$, $E=142.863$, eigenvector No. 1 of this A, J, T. Model core=16, dimension=359.

Amplitude	−0.529			0.449			0.357			0.275			−0.202			−0.175			0.166			−0.141		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	12	0	1	10	2	1	11	2	0	10	2	1	9	3	1	8	4	1	10	0	3	11	1	1
$2(S$-shell J's)	0	0	3	0	0	3	5	2	0	2	2	3	1	1	3	0	0	3	0	0	3	5	1	3
2(coupled J's)		0	3		0	3		3	3		0	3		0	3		0	3		0	3		4	3
$2(S$-shell T's)	0	0	1	2	2	1	1	0	0	0	0	1	1	1	1	0	0	1	2	0	1	1	1	1
2(coupled T's)		0	1		0	1		1	1		0	1		0	1		0	1		2	1		0	1
S-S seniorities	0	0	1	0	0	1	1	2	0	2	2	1	3	1	1	0	0	1	0	0	1	1	1	1

Listed components account for 80.4% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=10.52$, $\langle s_{1/2}\rangle=1.43$, $\langle d_{3/2}\rangle=1.05$.

$A=29$, $2J^{\pi}=3^{+}$, $2T=1$, $E=141.997$, eigenvector No. 2 of this A, J, T. Model core=16, dimension=359.

Amplitude	0.532			0.487			−0.287			−0.212			0.195			0.160			0.136			−0.132		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	11	2	0	11	1	1	9	3	1	10	3	0	10	2	1	10	2	1	9	4	0	10	2	1
$2(S$-shell J's)	5	2	0	5	1	3	5	1	3	4	1	0	6	2	3	4	0	3	3	0	0	0	0	3
2(coupled J's)		3	3		4	3		4	3		3	3		4	3		4	3		3	3		0	3
$2(S$-shell T's)	1	0	0	1	1	1	1	1	1	2	1	0	0	0	1	2	2	1	1	0	0	2	2	1
2(coupled T's)		1	1		0	1		0	1		1	1		0	1		0	1		1	1		0	1
S-S seniorities	1	2	0	1	1	1	1	1	1	2	1	0	2	2	1	2	0	1	3	0	0	0	0	1

Listed components account for 74.8% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=10.27$, $\langle s_{1/2}\rangle=1.90$, $\langle d_{3/2}\rangle=0.83$.

TABLE VII *(Continued)*

$A=29$, $2J^{\pi}=5^{+}$, $2T=1$, $E=143.024$, eigenvector No. 1 of this A, J, T. Model core = 16, dimension = 442.

Amplitude	−0.623			−0.326			−0.307			−0.211			0.195			0.176			−0.164			0.154		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	11	2	0	9	4	0	11	2	0	10	3	0	9	2	2	10	3	0	11	0	2	11	1	1
2(S-shell J's)	5	0	0	5	0	0	5	2	0	6	1	0	5	0	0	4	1	0	5	0	0	5	1	3
2(coupled J's)		5	5		5	5		5	5		5	5		5	5		5	5		5	5		4	5
2(S-shell T's)	1	2	0	1	0	0	1	0	0	0	1	0	1	2	2	2	1	0	1	0	2	1	1	1
2(coupled T's)		1	1		1	1		1	1		1	1		3	1		1	1		1	1		0	1
S-S seniorities	1	0	0	1	0	0	1	2	0	2	1	0	1	0	0	2	1	0	1	0	0	1	1	1

Listed components account for 75.2% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=10.25$, $\langle s_{1/2}\rangle=2.23$, $\langle d_{3/2}\rangle=0.52$.

$A=29$, $2J^{\pi}=5^{+}$, $2T=1$, $E=141.214$, eigenvector No. 2 of this A, J, T. Model core = 16, dimension = 442.

Amplitude	−0.566			0.394			−0.305			−0.246			−0.225			0.152			−0.141			0.129		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	11	1	1	9	3	1	11	2	0	10	2	1	10	2	1	8	4	1	9	3	1	10	2	1
2(S-shell J's)	5	1	3	5	1	3	5	2	0	4	0	3	6	2	3	4	0	3	3	1	3	8	0	3
2(coupled J's)		4	5		4	5		5	5		4	5		4	5		4	5		4	5		8	5
2(S-shell T's)	1	1	1	1	1	1	1	0	0	2	2	1	0	0	1	0	0	1	1	1	1	2	2	1
2(coupled T's)		0	1		0	1		1	1		0	1		0	1		0	1		0	1		0	1
S-S seniorities	1	1	1	1	1	1	1	2	0	2	0	1	2	2	1	2	0	1	3	1	1	2	0	1

Listed components account for 73.9% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=10.07$, $\langle s_{1/2}\rangle=1.82$, $\langle d_{3/2}\rangle=1.11$.

$A=29$, $2J^{\pi}=5^{+}$, $2T=1$, $E=140.394$, eigenvector No. 3 of this A, J, T. Model core = 16, dimension = 442.

Amplitude	0.535			−0.248			−0.232			0.232			0.229			−0.226			−0.183			0.136		
S-shell labels	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
Configuration	11	2	0	11	1	1	11	2	0	11	1	1	11	1	1	11	0	2	11	1	1	10	2	1
2(S-shell J's)	5	2	0	5	1	3	5	0	0	5	1	3	5	1	3	5	0	0	5	1	3	6	2	3
2(coupled J's)		5	5		4	5		5	5		4	5		6	5		5	5		6	5		6	5
2(S-shell T's)	1	0	0	1	1	1	1	2	0	1	1	1	1	1	1	1	0	2	1	1	1	0	0	1
2(coupled T's)		1	1		0	1		1	1		2	1		0	1		1	1		2	1		0	1
S-S seniorities	1	2	0	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	1	1	1	2	2	1

Listed components account for 61.1% of the wave function.
The occupation numbers are: $\langle d_{5/2}\rangle=1041$, $\langle s_{1/2}\rangle=1.64$, $\langle d_{3/2}\rangle=0.95$.

component in ^{28}Si. Thus *if* such a component had a moderate amplitude in ^{28}Si, then the absence of stripping strength to the first $\frac{3}{2}^{+}$ together with the presence of pickup strength to the same state would be easily accounted for. As we will see later, there seems no other grounds to suspect qualitative error in the wave function of this lowest $\frac{3}{2}^{+}$ state of ^{27}Al-^{27}Si. Thus we advance the possibility that this combination of component amplitudes might also explain this puzzle. Questions which then arise are "can the needed amount of the $(d_{5/2})^{10}(s_{1/2})^{1}(d_{3/2})^{1}$ component be introduced into the ground state of ^{28}Si without ruining other features?" and "what aspect of the two-body interaction would need to be modified to best accomplish such an admixture?"

In summary, with the exception of the pickup strength to the lowest $\frac{3}{2}^{+}$ state, the present calculation very nicely predicts the single-particle characteristics of ^{27}Al-^{27}Si. There is some indication that the $d_{3/2}$ orbit is not quite as strongly mixed into the ground-state and low-lying-excited-state wave functions of the model as it should be. It would seem desirable to attempt to definitively settle the question of pickup to the lowest $\frac{3}{2}^{+}$ state from an experimental-reaction-theory standpoint and to search experimentally for the fourth $\frac{3}{2}^{+}$ state, which is predicted to have a significant spectroscopic factor for stripping.

2. *Single-Nucleon Transfer to $A=28$, $T=0$*

The relevant experiments populating ^{28}Si are proton stripping on ^{27}Al and neutron pickup on ^{29}Si.

Calculated spectroscopic factors are compared to experimentally determined values in Table III. Stripping to the ground state of ^{28}Si reveals the same structure information as that obtained via pickup from ^{28}Si to the ground state of ^{27}Al-^{27}Si. The stripping data are consistent with the results of the inverse reaction and with our model results. Stripping to the second 0^+ in the spectrum proceeds with an accurately predicted smaller intensity.

While it seems futile to hope that experiment can ever distinguish between a mixture of $d_{3/2}$ and $d_{5/2}$ transfer in a single transition, relative $l=2$ $(d_{3/2}+d_{5/2})$ versus $l=0$ $(s_{1/2})$ spectroscopic factors can, with difficulty, be extracted for 2^+ and 3^+ states, with $l=0$ values presumably being more reliable. The model predictions for the 2^+ first excited state look good in comparison to experiment, in particular as regards the $s_{1/2}$ strength. We note that the wave function for this state (see Table VI) is most simply characterized as $(d_{5/2})^{11}{}_{J=5/2}(s_{1/2})^1{}_{J=1/2}$. There is qualitative or better agreement for other 2^+ states and for the 1^+ and 4^+ states. The $s_{1/2}$ contribution to the 3^+ states is predicted to be concentrated into the lowest such level, while experiment shows more fragmentation.

The information available from pickup experiments to these same ^{28}Si states is more limited because of the $J=\frac{1}{2}^+$ spin of the ground state of ^{29}Si. The available data are in generally good agreement with the model predictions.

In summary, the observed single-particle aspects of the lowest energy levels of ^{28}Si are well accounted for in the present calculation.

3. *Single-Nucleon Transfer to* $A=29$, $T=\frac{1}{2}$

The $A=29$, $T=\frac{1}{2}$ system is reached by proton or neutron stripping on ^{28}Si and by neutron pickup on ^{30}Si. Calculated and experimental[37-39] results are presented in Table IV. The results of stripping experiments[38] identify the first $\frac{1}{2}^+$, the first $\frac{3}{2}^+$, and the first $\frac{5}{2}^+$ levels as "single-particle" $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ states relative to the ^{28}Si ground state. The model predictions are in complete and accurate agreement with the data. We note that the "unclosed" nature of ^{28}Si revealed by these stripping data, and by the pickup experiments on the same nucleus as well, is uniformly well accounted for.

The quantitative agreement between model predictions and results of pickup experiments is not too good, although the qualitative allocation of strength between various states is acceptable. The model correctly places the $d_{3/2}$ hole strength in the lowest $\frac{3}{2}^+$ level and predicts a large strength for the third $\frac{5}{2}^+$ level. The ground state $\frac{1}{2}^+$ and the lowest $\frac{5}{2}^+$ are predicted to be the strongest levels excited. The model quite definitely fails to put enough $d_{3/2}$ strength into the low-energy part of the $A=30-29$ spectra and does not mix the $d_{5/2}$-hole strengths of the first and third $\frac{5}{2}^+$ states enough. The nature of the various model wave functions can be examined in Table VII.

C. Electric Quadrupole and Magnetic Dipole Observables

The $B(E2)$ and $B(M1)$ values for transitions between states in the $A=27-29$ region have been calculated from the present wave functions under various assumptions. The results presented in this paper are restricted to ^{27}Al, ^{28}Si, and ^{29}Si. Bare-nucleon values are assumed for the $M1$ operator. Effective charges of $e_p=1.5e$ and $e_n=0.5e$ and harmonic-oscillator single-particle wave functions $(\hbar\omega=41A^{-1/3})$ are assumed for the $E2$ operator. All components of the wave function are included. In an associated paper,[1] more extensive calculated results are presented and the sensitivities of the predicted values of these observables to different aspects of the calculation are investigated.

The electric quadrupole moments for the ground state $(\frac{5}{2}^+)$ of ^{27}Al and the first excited state (2^+) of ^{28}Si have been measured.[3,43,44] The predicted values for these observables, compared to the observed values, are $+13.8\ e\,\mathrm{fm}^2$ versus $+15.2\ e\,\mathrm{fm}^2$ for ^{27}Al and $+14.3\ e\,\mathrm{fm}^2$ versus $18\pm6\ e\,\mathrm{fm}^2$ for ^{28}Si. We note that the signs of these moments imply a change from a prolate to an oblate intrinsic quadrupole shape in going from ^{27}Al to ^{28}Si if simple unmixed Nilsson orbitals are assumed for the intrinsic wave functions.

The observed magnetic dipole moments of the ^{27}Al and ^{29}Si ground states ($\frac{5}{2}^+$ and $\frac{1}{2}^+$, respectively) are $+3.64\mu_N$ and $-0.56\mu_N$, respectively. The Schmidt estimates are $+4.74$ and $-1.91\mu_N$. The values predicted from the present wave functions are $+3.75\mu_N$ and $=-0.64\mu_N$, respectively.

Thus, it appears that the few static properties which are experimentally known in this region can be well accounted for with the present wave functions. In the following subsections we briefly note comparisons between predicted $B(E2)$'s and $B(M1)$'s and experimental values extracted from lifetime and mixing ratio and branching ratio data. Unannotated experimental results are taken from the compendium presented in Table I of Ref. 1. Agreements between theory and experiment of 30% will generally be taken to indicate meaningful correspondence between model and observed states.

1. *Electromagnetic Transition Strengths in ^{27}Al*

Calculated $B(E2)$ and $B(M1)$ values for ^{27}Al are presented in Table VIII and compared to available data. Our results show a chain of strong $E2$ transitions connecting the $\frac{5}{2}^+$ ground state with the first $\frac{7}{2}^+$, the first $\frac{9}{2}^+$, and the *second* $\frac{11}{2}^+$ states. The first $\frac{1}{2}^+$ state, first $\frac{3}{2}^+$ state, and second $\frac{5}{2}^+$ state also all have sizable $E2$ strengths connecting them to the ground state, and in addition significant strengths for their own interconnecting transitions. The available data for transitions between low-lying states are in general agreement with the shell-model results with the exception that the lowest $\frac{11}{2}^+$ level known experimentally (at $E_x = 4.51$ MeV) exhibits a strong $E2$ transition to the first $\frac{7}{2}^+$ and $\frac{9}{2}^+$ levels. However, it is the *second* model $\frac{11}{2}^+$ state, at 5.25 MeV excitation, which has this property, not the first $\frac{11}{2}^+$, predicted at 3.91 MeV. The possible existance of another $\frac{11}{2}^+$ level in the 4–5-MeV region of excitation might be an interesting question to pursue experimentally.

The patterns of predicted and observed $B(M1)$ values are not so easy to characterize as are the $B(E2)$'s. About half of the transitions seem well accounted for, while the remainder are missed by factors of 2 to 5. In Ref. 1 it is shown that relatively small changes in the $M1$ operator can rectify most of these discrepancies, but of course it can by no means be ruled out that better wave functions would suffice to yield better agreement without an effective $M1$ operator. At any rate, the $M1$ results, taken as a whole, do not indicate any basic misidentification between model and observed states in the low-lying spectrum, and we conclude that the essentials of the low-energy region ($\leq$4.5 MeV excitation) of the $A = 27$, $T = \frac{1}{2}$ system are explained with the present model.

TABLE VIII. $B(E2)$ and $B(M1)$ values for transitions connecting states J_ν and $J'_{\nu'}$ of ^{27}Al. Listed energies are experimental values.

Initial state		Final state		$B(E2)$ (e^2 fm^4)		$B(M1)$ (μ_N^2)	
J_ν	E	J_ν	E	Th.	Expt.[a]	Th.	Expt.[a]
$(\frac{1}{2})_1$	0.84	$(\frac{5}{2})_1$	0.00	29	42	0	0
$(\frac{3}{2})_1$	1.01	$(\frac{5}{2})_1$	0.00	34	44	0.16	0.02
$(\frac{3}{2})_1$	1.01	$(\frac{1}{2})_1$	0.84	24	···	0.15	0.15
$(\frac{7}{2})_1$	2.21	$(\frac{5}{2})_1$	0.00	61	53	0.06	0.11
$(\frac{7}{2})_1$	2.21	$(\frac{3}{2})_1$	1.01		···	0	0
$(\frac{5}{2})_2$	2.73	$(\frac{5}{2})_1$	0.00	6	0.7	0.02	0.5
$(\frac{5}{2})_2$	2.73	$(\frac{1}{2})_1$	0.84	18	···	0	0
$(\frac{5}{2})_2$	2.73	$(\frac{3}{2})_1$	1.01	19	34	1.09	0.59
$(\frac{3}{2})_2$	2.98	$(\frac{5}{2})_1$	0.00	1.9	0.2	0.08	0.38
$(\frac{3}{2})_2$	2.98	$(\frac{1}{2})_1$	0.84	16	···	0.17	···
$(\frac{3}{2})_2$	2.98	$(\frac{3}{2})_1$	1.01	0.1	···	0.00	···
$(\frac{9}{2})_1$	3.00	$(\frac{5}{2})_1$	0.00	20	35	0	0
$(\frac{9}{2})_1$	3.00	$(\frac{7}{2})_1$	2.21	0.4	1.5	0.55	0.12
$(\frac{1}{2})_2$	3.68	$(\frac{1}{2})_1$	0.84	0	0	0.89	···
$(\frac{1}{2})_2$	3.68	$(\frac{5}{2})_1$	0.00	0.4	···	0	0
$(\frac{1}{2})_2$	3.68	$(\frac{3}{2})_1$	1.01	2.5	···	0.86	···
$(\frac{11}{2})_1$	4.51	$(\frac{7}{2})_1$	2.21	2.9	31	0	0
$(\frac{11}{2})_2$		$(\frac{7}{2})_1$	2.21	30	···	0	0
$(\frac{11}{2})_1$	4.51	$(\frac{9}{2})_1$	3.00	1.9	16	0.08	0.17
$(\frac{11}{2})_2$		$(\frac{9}{2})_1$	3.00	29	···	0.09	···
$(\frac{7}{2})_2$	4.58	$(\frac{5}{2})_1$	0.00	0.5	0.9	0.06	0.02
$(\frac{9}{2})_2$		$(\frac{5}{2})_1$	0.00	5.2	···	0	0
$(\frac{9}{2})_2$		$(\frac{7}{2})_1$	2.21	0.6	···	0.05	···
$(\frac{5}{2})_3$	4.41	$(\frac{5}{2})_1$	0.00	0.4	···	···	···

[a] Reference 1.

2. *Electromagnetic Transitions in ^{28}Si*

Comparison of the calculated $E2$ strengths from the ^{28}Si wave functions with present data (see Table IX) raises several interesting questions. It

TABLE IX. $B(E2)$ values for transitions connecting states J_ν and $J'_{\nu'}$ of ^{28}Si. Listed energies are experimental values.

Initial state		Final state		$B(E2)$ (e^2 fm^4)	
J_ν	$E_{\text{expt.}}$ (MeV)	J_ν	$E_{\text{expt.}}$ (MeV)	Th.	Expt.[a]
2_1	1.78	0_1	0.00	55	66
4_1	4.62	2_1	1.78	74	74
0_2	4.98	2_1	1.78	3.2	60
0_3	6.69	2_1	1.78	1.6	
4_2	6.89	2_1	1.78	2.2	3.5
1_1	8.33	0_1	0.00	0	
1_1	8.33	2_1	1.78	0.2	0.2
4_2	6.89	4_1	4.62	4.5	<22
2_2	7.38	0_1	0.00	4.6	1.6
2_2	7.38	2_1	1.78	11	<12
2_2	7.38	0_2	4.98	0.0	
6_1	8.54	4_1	4.62	11	14
6_2		4_1	4.62	56	
8_1		6_1	8.54	0.0	
8_1		6_2		16	
8_2		6_1	8.54	8.4	
8_2		6_2		20	
8_3		6_2		16	
2_3		0_1	0.00	1.6	
2_3		2_1	1.78	2.4	
2_3		0_2	4.98	8	
2_4		0_2	4.98	37	

[a] Reference 1.

might first be noted that, among the lowest three states, the shell-model results give a slightly larger $B(E2)$ for the 4^+ to 2^+ transition than for the 2^+ to 0^+ transition. The measured values are tentatively in accord with this relationship. The projected Hartree-Fock techniques of Castel and Parikh[14] produce a larger 2^+ to 0^+ than 4^+ to 2^+ strength. Second, the latest reported lifetimes for the first excited 0^+ level yield a much greater $B(E2)$ for the transition to the 2^+ first excited state than we predict. This strength does not show up in the third and fourth model 0^+ states either. This represents a significant failure for the model. Other straightforward calculations of ^{28}Si structure experience the same difficulty and it is only with specially concocted wave functions that the strength of this transition can be understood even qualitatively.

The third question of interest concerns the higher-spin states. There are at present two conflicting reports[35,36] on the lifetime of the 6^+ state observed at 8.54 MeV. One reported value is in good agreement with our predicted strength for the first model 6^+ state ($E_{\text{calc}} = 8.84$ MeV) and the other with our prediction for the *second* model 6^+ state ($E_{\text{calc}} = 10.08$ MeV). Basically, the model results predict that the lowest 6^+ state is only weakly related to the 0^+ ground state, first 2^+, and first 4^+, while the second 6^+ is related much more strongly to these same states, and hence, might be called a "member of the ground-state band."

If the experimental result which indicates a weak $B(E2)$ from 6^+ (8.54 MeV) to 4^+ (4.62 MeV) is correct, then the present results are in excellent agreement for this level and, in addition, predict the "ground-state band" 6^+ to lie in the vicinity of 10 MeV. If, on the other hand, the experimental result which indicates a strong $B(E2)$ for 6^+ (8.54-MeV) to 4^+ (4.62-MeV) transition is correct, then the first two 6^+ model states are inverted in ener-

TABLE X. $B(E2)$ and $B(M1)$ values for transitions connecting states J_ν and $J'_{\nu'}$ of ^{29}Si. Listed energies are experimental values.

Initial state J_ν	$E_{\text{expt.}}$ (MeV)	Final state J_ν	$E_{\text{expt.}}$ (MeV)	$B(E2)$ (e^2 fm^4) Th.	Expt.[a]	$B(M1)$ (μ_N^2) Th.	Expt.[a]
$(\frac{3}{2})_1$	1.27	$(\frac{1}{2})_1$	0.00	26	27	0.03	0.07
$(\frac{5}{2})_1$	2.03	$(\frac{1}{2})_1$	0.00	40	52	0	0
$(\frac{5}{2})_1$	2.03	$(\frac{3}{2})_1$	1.27	5	···	0.01	0.02
$(\frac{3}{2})_2$	2.42	$(\frac{1}{2})_1$	0.00	23	47	0.15	0.14
$(\frac{3}{2})_2$	2.42	$(\frac{3}{2})_1$	1.27	40	47	0.30	0.23
$(\frac{3}{2})_2$	2.42	$(\frac{5}{2})_1$	2.03	7	···	0.47	···
$(\frac{5}{2})_2$	3.07	$(\frac{1}{2})_1$	0.00	7	···	0	0
$(\frac{5}{2})_2$	3.07	$(\frac{3}{2})_1$	1.27	33	68	0.03	0.34
$(\frac{5}{2})_2$	3.07	$(\frac{5}{2})_1$	2.03	1	···	0.06	···
$(\frac{7}{2})_1$	4.08	$(\frac{3}{2})_1$	1.27	31	55	0	0
$(\frac{7}{2})_1$	4.08	$(\frac{5}{2})_1$	2.03	14	2.1		0.05
$(\frac{7}{2})_1$	4.08	$(\frac{3}{2})_2$	2.42	1	···		
$(\frac{7}{2})_1$	4.08	$(\frac{5}{2})_2$	3.07	40	···		
$(\frac{9}{2})_1$	4.74	$(\frac{5}{2})_1$	2.03	50	105	0	0
$(\frac{9}{2})_1$	4.74	$(\frac{5}{2})_2$	3.07	0.1		0	0
$(\frac{9}{2})_1$	4.74	$(\frac{7}{2})_1$	4.08	0.2			
$(\frac{5}{2})_3$	4.90	$(\frac{1}{2})_1$	0.00	2		0	0
$(\frac{5}{2})_3$	4.90	$(\frac{3}{2})_1$	1.27	10		0.06	
$(\frac{5}{2})_3$	4.90	$(\frac{5}{2})_1$	2.03	0.3			
$(\frac{1}{2})_2$	4.83	$(\frac{1}{2})_1$	0.00	0	0	0.25	
$(\frac{1}{2})_2$	4.83	$(\frac{3}{2})_1$	1.27	6.0			
$(\frac{1}{2})_2$	4.83	$(\frac{5}{2})_1$	2.03	8.0			
$(\frac{7}{2})_2$		$(\frac{3}{2})_1$	1.27	0.1		0	0
$(\frac{7}{2})_2$		$(\frac{5}{2})_1$	2.03	1.8			
$(\frac{7}{2})_2$		$(\frac{5}{2})_2$	3.07	1.6			
$(\frac{7}{2})_2$		$(\frac{9}{2})_1$	4.74	8.7			
$(\frac{9}{2})_2$		$(\frac{5}{2})_1$	2.03	1.2		0	0
$(\frac{9}{2})_2$		$(\frac{5}{2})_2$	3.07	2.5		0	0
$(\frac{9}{2})_2$		$(\frac{7}{2})_1$	4.08	5.3			
$(\frac{9}{2})_2$		$(\frac{7}{2})_2$		30			
$(\frac{9}{2})_3$		$(\frac{7}{2})_1$	4.08	18			
$(\frac{9}{2})_3$		$(\frac{7}{2})_2$		17			
$(\frac{11}{2})_1$		$(\frac{7}{2})_1$	4.08	0.9		0	0
$(\frac{11}{2})_1$		$(\frac{7}{2})_2$		2.2		0	0
$(\frac{11}{2})_1$		$(\frac{7}{2})_2$		0.3			
$(\frac{11}{2})_1$		$(\frac{9}{2})_2$		0.4			
$(\frac{11}{2})_2$		$(\frac{7}{2})_1$	4.08	43		0	0
$(\frac{11}{2})_2$		$(\frac{7}{2})_2$		1.0		0	0
$(\frac{11}{2})_2$		$(\frac{9}{2})_1$	4.74	16			
$(\frac{11}{2})_2$		$(\frac{9}{2})_2$		4.0			
$(\frac{11}{2})_2$		$(\frac{9}{2})_3$		13			
$(\frac{11}{2})_3$		$(\frac{9}{2})_2$		57			

[a] Reference 1.

gy by about ±1 MeV. (We recall a somewhat analogous situation with the $\frac{11}{2}^+$ model states in ^{27}Al.)

The predictions of the model indicate a breaking up of any meaningful rotational band structure at all after $J=6$, in that the $8^+ \rightarrow 6^+$ strength is shared by three 8^+ states. Several other moderate $B(E2)$ values are predicted for transitions involving other excited states (with generally good agreement with experiment) but no clear pattern of an excited state band emerges from these calculations.

3. *Electromagnetic Transitions in ^{29}Si*

The early expectations[19] that a well-defined rotational band should be formed from the $\frac{1}{2}^+$ ground state of ^{29}Si has been qualitatively confirmed by ensuing experimental work. Something like this picture emerges from the shell-model wave functions (see Table X). Strong $E2$ transitions connect the first $\frac{1}{2}^+$, second $\frac{3}{2}^+$, first $\frac{5}{2}^+$, and first $\frac{9}{2}^+$ model states, although none of the first four $\frac{7}{2}^+$ states seem related to this series in a clear way. An excited state rotational band built on the first $\frac{3}{2}^+$ level, which includes the second $\frac{5}{2}^+$, the first $\frac{7}{2}^+$, the second $\frac{9}{2}^+$, and an $\frac{11}{2}^+$ level at 7.14 MeV, has also been postulated, and evidence supporting the idea has been accumulated. Again some aspects of this structure emerge in our shell model, as strong $B(E2)$'s are predicted to connect the chain of levels starting with the lowest $\frac{3}{2}^+$ state and proceeding through the second $\frac{5}{2}^+$, the first $\frac{7}{2}^+$, the *third* model $\frac{9}{2}^+$, and the *second* model $\frac{11}{2}^+$ states.

Recent experiments have added other complications to the simple band picture by revealing strong transitions between the ground-state band levels and other levels.[15, 16, 40-42] These complexities have stimulated "band-mixed" Nilsson calculations[15, 16] which are able to successfully account for the lack of clean separation between different groups of levels. It can be seen that this "band-mixed" aspect of ^{29}Si emerges naturally from the present calculation. Indeed, the shell model predicts somewhat too much mixing above the lowest few levels on the basis of present experimental evidence. The model results for the higher $\frac{7}{2}^+$ and $\frac{9}{2}^+$ states do not seem to be simply correlated with the two main band sequences, but branching ratios of the experimental states indicate that a large amount of band mixing continues up through the higher-spin members of the sequences and it is not clear at present whether the model results are in really serious disagreement or not.

IV. SUMMARY AND CONCLUSIONS

The present calculation produces level densities and spin sequences for the low-excitation regions of the energy level diagrams of $A=27$, $T=\frac{1}{2}$; $A=28$, $T=0$; and $A=29$, $T=\frac{1}{2}$ nuclei which are in moderately good agreement with experimental observation. This agreement is obtained with an empirical zero-range-type interaction which is parametrized (for those aspects which pertain to level excitation data) in terms of four variables. The values of these variables were determined in a fit to about half (20) of the presently assigned energy levels in the A-T systems of interest. The values obtained are consistent with values obtained in similar studies of lighter and heavier nuclei in the *sd* shell. The agreement between corresponding model and experimental energies for states not included in the fit is approximately as good as for those states used in the fit. There is essentially no evidence from the present comparison of calculated and experimental level energies that essential degrees of freedom have been omitted from the calculation. This refers to the specification of the form of the Hamiltonian, the restriction of the model space to the *sd* shell, and the specific truncation of the *sd* shell used. The only clear cut exception to this evaluation is the anomalous strength observed experimentally for the $0_2^+ \rightarrow 2_1^+$ transition in ^{28}Si.

The characteristics of these nuclei which are revealed by single-nucleon-transfer experiments are almost universally well accounted for by the model predictions. The predicted occupation numbers (see Tables V–VII) of the three *sd* orbits in the ground states of $A=27$–29 (and by implication, those of ^{26}Mg and ^{30}Si) agree with the average of the experimental estimates to well within the experimental uncertainties for $d_{5/2}$ and $s_{1/2}$. The predicted $d_{3/2}$ occupation may be $\approx 30\%$ too low; predicted stripping and pickup S factors for this orbit both tend to be somewhat lower than those observed. There is no suggestion in our results (or, of course, in nature) of massive emptying of the $d_{5/2}$ orbit into $s_{1/2}$ and $d_{3/2}$ orbits. Of course, our space truncation would not allow a total population inversion, but the occupation values actually obtained for the three orbits are not close to the truncated-configuration-space limits.

The static electric quadrupole and magnetic dipole moments predicted with the present model are in good agreement with measured values. Predicted magnetic dipole transition strengths do not consistently agree with experimental numbers, but the discrepancies are not unexpected in view of the strong cancellations which occur in evaluating this operator with shell-model wave functions. The electric quadrupole transition strengths calculated between low-lying states with the assumption of $0.5e$ added charge for both the proton and neutrons agree well with experimental values. This is the same effective charge for the $E2$ oper-

ator which gives satisfactory results for $A = 18-22$ and $A = 35-38$ shell-model calculations in a full sd-shell basis, and for $A = 30-34$ calculations in a truncated basis something like that used here. Close inspection of the present results, however, indicates that a uniform increase in the charge added to the neutron and proton in these calculations, say from $0.5e$ to $0.6e$, would produce even better agreement between theory and experiment.

The model results for $B(E2)$ values give little indication of well-defined "rotational bands" in these nuclei. This is in contrast to similar calculations for the lighter sd-shell nuclei in which the well defined rotational-like properties of the experimental spectra were equally apparent in the shell-model results. A general prediction of the present calculation which can and should be tested with further experimental study is that the "ground-state band" $B(E2)$ strength is fragmented over several levels for the higher J values. Although there has been a long history of rotational model interpretations of $A = 27-29$, the status of experimental knowledge and, indeed, the present stage of band-mixed Nilsson-model calculations both seem to indicate that the mixing of rotational bands in this region is so severe as to greatly abridge the usual benefits of a Nilsson-model approach. Over-all, the degree to which "band-structure" can be discerned in the experimental data for this region, and, more importantly, the many aspects in which breakdowns of pure band structure are evident, both emerge naturally from our very straightforward shell-model approach.

In conclusion, energy level spectra, occupation numbers for the sd-shell orbits, specifics of the distribution of single-nucleon-transfer strength, general trends of $M1$ transition strengths and considerable detailed $B(E2)$ data are all accounted for here with a tightly unified approach that is consistent with successful theories for the lighter and heavier nuclei in the sd shell. It would seem that with appropriate refinements of model Hamiltonian and basis space which are now feasible, the present techniques offer promise of giving an even more accurate and complete accounting for the nuclear structure phenomena of this region.

ACKNOWLEDGMENTS

We thank Edith Halbert for many helpful discussions and considerable practical assistance in performing the calculations.

---

*Research supported in part by the National Science Foundation.

†Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

[1]M. J. A. de Voigt, P. W. M. Glaudemans, J. de Boer, and B. H. Wildenthal, Nucl. Phys. A186, 365 (1972).

[2]M. J. A. de Voigt and B. H. Wildenthal, to be published.

[3]P. M. Endt and C. van der Leun, Nucl. Phys. A105, 1 (1967).

[4]V. K. Thankappan, Phys. Rev. 141, 957 (1966).

[5]D. Evers, J. Hertel, T. W. Retz-Schmidt, and S. J. Skorka, Nucl. Phys. A91, 472 (1967).

[6]K. H. Bhatt, Nucl. Phys. 39, 375 (1962).

[7]T. R. Ophel and B. T. Lawergren, Nucl. Phys. 52, 417 (1964).

[8]M. B. Lewis, N. R. Roberson, and D. R. Tilley, Phys. Rev. 163, 1238 (1967).

[9]H. Ropke, V. Glattes, and G. Hammel, Nucl. Phys. A156, 477 (1970).

[10]D. Dehnhard, Phys. Letters 38B, 389 (1972).

[11]J. P. Bernier and M. Harvey, Nucl. Phys. A94, 593 (1967).

[12]S. Das Gupta and M. Harvey, Nucl. Phys. A94, 602 (1967).

[13]B. Castel and J. P. Svenne, Nucl. Phys. A127, 141 (1969).

[14]B. Castel and J. C. Parikh, Phys. Rev. C 1, 990 (1970).

[15]A. De-Shalit and I. Talmi, Yale University (unpublished).

[16]A. A. Pilt, R. H. Spear, R. V. Elliott, and J. A. Kuehner, Can. J. Phys. 49, 1263 (1971).

[17]F. G. Bailey and D. C. Choudhury, Nucl. Phys. A144, 628 (1970).

[18]B. Castel and K. W. C. Stewart, Phys. Rev. C 3, 964 (1971); B. Castel, K. W. C. Stewart, and M. Harvey, Can. J. Phys. 48, 1490 (1970).

[19]D. A. Bromley, H. E. Gove, and A. E. L. Litherland, Can. J. Phys. 35, 1057 (1957).

[20]R. Arvieu and S. A. Moszkowski, Phys. Rev. 145, 830 (1966).

[21]P. W. M. Glaudemans, B. H. Wildenthal, and J. B. McGrory, Phys. Letters 21, 427 (1966).

[22]E. C. Halbert, J. B. McGrory, B. H. Wildenthal, and S. P. Pandya, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1971), Vol. IV.

[23]B. H. Wildenthal, E. C. Halbert, J. B. McGrory, and T. T. S. Kuo, Phys. Rev. C 4, 1266 (1971).

[24]B. H. Wildenthal, J. B. McGrory, E. C. Halbert, and H. D. Graber, Phys. Rev. C 4, 1708 (1971).

[25]J. B. French, E. C. Halbert, J. B. McGrory, and S. S. M. Wong, in *Advances in Nuclear Physics* (see Ref. 22), Vol. III.

[26]P. W. M. Glaudemans and B. H. Wildenthal, unpublished. A brief description of this computer program is given in the Appendix of Ref. 22.

[27]W. Bohne *et al.*, Nucl. Phys. A131, 273 (1969).

[28]W. P. Alford *et al.*, Nucl. Phys. A130, 119 (1969).
[29]M. J. A. de Voigt, J. W. Maas, De. Veenhof, and C. van der Leun, Nucl. Phys. A170, 449 (1971).
[30]B. H. Wildenthal and E. Newman, Phys. Rev. 167, 1027 (1968).
[31]H. E. Gove *et al.*, Nucl. Phys. A116, 369 (1968).
[32]R. W. Barnard and G. D. Jones, Nucl. Phys. A108, 641 (1968).
[33]F. Pellegrini *et al.*, Phys. Rev. C 2, 1440 (1970).
[34]G. J. Wagner, Nucl. Phys. A176, 47 (1971).
[35]F. C. P. Huang and D. K. McDaniels, Phys. Rev. C 2, 1342 (1970); F. C. P. Huang, E. F. Gibson, and D. K. McDaniels, *ibid.* 3, 1222 (1971).
[36]S. T. Lam, A. E. Litherland, and T. K. Alexander, Can. J. Phys. 47, 1342 (1970).
[37]D. Dehnhard and J. L. Yntema, Phys. Rev. C 2, 1390 (1970).
[38]M. C. Mermaz, C. A. Whitten, Jr., J. W. Champlin, A. J. Howard, and D. A. Bromley, Phys. Rev. C 4, 1778 (1971).
[39]D. Dehnhard and J. L. Yntema, Phys. Rev. 163, 1198 (1967).
[40]T. T. Bardin, J. A. Becker, T. R. Fisher, and A. D. W. Jones, Phys. Rev. C 4, 1625 (1971).
[41]D. C. Bailey *et al.*, J. Phys. A5, 596 (1972).
[42]I. G. Main *et al.*, Nucl. Phys. A158, 364 (1970).
[43]O. Häusser *et al.*, Phys. Rev. Letters 23, 320 (1969).
[44]D. Schwalm *et al.*, Nucl. Phys. A192, 449 (1972).