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Calculations of Allowed Beta Decay in the $(0d, 1s)$ Shell*

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Allowed β-decay transition rates and half-lives have been calculated for $(0d, 1s)$ shell nuclei with $A=17$–22, 23–24, 27–29, 30–34, and 35–39. For nuclei with $A=17$–22 and 34–39, the calculated $\log ft$ values have a rms deviation of 5% from experiment, with no discrepancies greater than 12%. For nuclei nearer the middle of the shell there are more significant discrepancies between experiment and theory. The calculated $\log ft$ values are used to predict the half-lives of some light elements. The predicted half-lives for which there are no experimental measurements are: ^{20}Mg (0.1 sec), ^{21}O (1.2 sec), and ^{22}O (0.15 sec). The $\log ft$ values relevant to the solar neutrino experiment are discussed.

I. INTRODUCTION

This paper presents calculations of strengths of allowed β-decay transitions for the $(0d, 1s)$ shell nuclei with $A=17$–22, 23–24, 27–29, 30–34, and 35–39. The shell-model wave functions of Wildenthal *et al.*[1-6] are used to describe the initial and final nuclear states. We present calculated $\log ft$ values for the approximately 100 transitions for which there are experimental measurements, and we give predictions for approximately an equal number of decays which may be measurable. We also use these calculated $\log ft$ values to predict β-decay half-lives. As will be seen, the agreement between the present calculations and experiments is consistently quite good for the nuclei for which the complete $(0d, 1s)$ shell-model basis space could be used. On the other hand, agreement with experiment is not as consistently good for calculations in the middle of the shell where significant truncations of the model space were necessary.

The calculation of β-decay transition rates is interesting for several reasons. (1) There are few uncertainties in the operators involved and in the connection between the experimentally measured quantities and those predicted by the theory. (2) Because the β-decay operators only connect single-particle states which have the same orbital angular momentum and because the initial and final states are in different nuclei, the matrix elements of the β-decay operators tend to be sensitive to aspects of wave functions not extensively tested in comparisons of theoretical results with nucleon transfer and γ-decay data. Hence, we have the opportunity to learn more about the detailed efficacy of the extant sets of wave functions in the $(0d, 1s)$ shell. (3) As indicated above, there are a large number of experimentally measured decays which can be compared with calculated values. (4) If the calculations turn out to be reasonably successful, the results can be used to predict the half-lives of some of the neutron- and proton-rich nuclei which have not yet been observed. Such predictions might aid in designing experiments to observe these nuclei. And (5) calculated β-decay transition rates are needed to evaluate the results of Davis's experiment[7] to measure the solar neutrino flux using the $^{37}\mathrm{Cl}+\nu \rightarrow {}^{37}\mathrm{Ar}+e$ reaction.

II. DESCRIPTION OF THE CALCULATION

A. Operators

β-decay transition rates are expressed in terms of a $\log ft$, where t is the partial half-life for the decay to a given final state and f is a "statistical rate function" which takes account of the energy released in the decay and the Coulomb field of the final nucleus.[8] For allowed decays, ft is given by[9,10]

$$ft = \frac{\pi^2\hbar^7 \ln 2/2m_e{}^5c^4}{(g_v{}^2/4\pi)\langle 1\rangle^2 + (g_A{}^2/4\pi)\langle\sigma\rangle^2}, \tag{1}$$

where $\langle 1\rangle$ and $\langle\sigma\rangle$ are the Fermi and Gamow-Teller matrix elements (described below), and g_v and g_A are the vector and axial vector β-decay coupling constants. These coupling constants can be evaluated by requiring Eq. (1) to describe both the $0^+ \to 0^+$ transitions between analog states (which depend only on $g_v{}^2$) and the lifetime of the neutron (which depends on $g_v{}^2 + 3g_A{}^2$). Using the most recent measurement for the lifetime of the neutron,[11] one gets $g_v = (1.36 \pm 0.02)\times 10^{-3}$ and $g_A/g_v = -1.23 \pm 0.01$. Or, rewriting Eq. (1),[10]

$$ft = \frac{6250}{\langle 1\rangle^2 + 1.51\langle\sigma\rangle^2}\ \mathrm{sec}. \tag{2}$$

The Fermi and Gamow-Teller matrix elements are given by

$$\begin{aligned}\langle 1\rangle &= \langle J_f T_f T_{zf} | \sum_a \tau_\pm^a | J_i T_i T_{zi}\rangle, \\ \langle\sigma\rangle &= \langle J_f T_f T_{zf} | \sum_a \sigma^a \tau_\pm^a | J_i T_i T_{zi}\rangle,\end{aligned} \tag{3}$$

where σ^a and τ^a are the components of the Pauli spin and isospin operators (using the convention $\vec{\sigma} = 2\vec{s}$). The Fermi matrix element can be immediately evaluated,

$$\langle 1\rangle^2 = [T_i(T_i+1) - T_{zi}T_{zf}]\delta_{if}, \tag{4}$$

where δ_{if} is zero unless all the quantum numbers (except T_z) of the initial and final states are the same. That is, Fermi decay contributes only to transitions connecting isobaric analog states (superallowed transitions).

From simple parity and angular momentum considerations, it follows that the Gamow-Teller matrix element is nonzero only when $\Delta\pi = 0$ and $J_f = J_i \pm 1, 0$ with no $0 \to 0$ transitions. We have evaluated $\langle\sigma\rangle$ using the shell-model wave functions described below.

B. Wave Functions

All of the wave functions used in the present investigation are the result of standard shell-model calculations. In these calculations, mass-independent Hamiltonians (over the regions listed) specified the one- and two-body interactions of $N = A - 16$ nucleons in $(0d, 1s)$ orbits outside an ^{16}O core. These calculations have been described in detail in Refs. 1–6. Results for some of the proton- or neutron-rich nuclei were not presented in these references, however, because of the lack of experimental data. Therefore, in order to obtain predicted half-lives, we have extended the previous calculations to some of these high-T nuclei (e.g., ^{21}O and ^{22}O).

Contained in Table I is a description of the vector spaces and the Hamiltonians used for the various calculations. (See Refs. 1–6 for details of the calculations; the abbreviations describing the Hamiltonians are the same as in these references.) In general, interactions based on those of Kuo (K) were used when complete $(0d, 1s)$ shell-model calculations were possible. For the nuclei with $27 \leq A \leq 34$, interactions based on the surface delta interaction (SDI) were used.

These wave functions have already been extensively tested[1–6] by comparing calculated and experimental energy spectra, spectroscopic factors for single-nucleon-transfer reactions, and magnetic dipole and electric quadrupole observables. The general results of these tests can be briefly summarized as follows: (1) The energy spectra and single-nucleon-transfer spectroscopic factors are in generally good agreement. (2) The electric quadrupole moments, magnetic dipole moments, and strong $BE(2)$ values are in reasonable agreement. (3) The $BM(1)$ values and moderate to weak $BE(2)$ values are in only qualitative agreement. (4) Generally, the agreement for all these observables is best for those nuclei which correspond to a few (3, 4, 5) particles (holes) outside the $^{16}O(^{40}Ca)$ core. That is, the calculations for the nuclei with $A = 27-34$, have less over-all success in accounting for experimental observations than those for lighter or heavier masses.

The present calculations of β-decay strengths are further tests of these wave functions which may provide more insight into their systematic virtues and defects.

III. RESULTS AND COMPARISON WITH EXPERIMENT

Included in Table II are the calculated $\log ft$ values for the allowed β-decay transitions for $(0d, 1s)$ shell nuclei with $A = 17-22$, 23–24, 27–29, 30–34, and 35–39. Also included are the experimental values, where available. The experimental values are from the compilation of Refs. 12–14, except for the more recent results which are individually referenced.[15–24] For nuclei corresponding to up to six particles (or holes) in the $(0d, 1s)$ shell, we calculated $\log ft$ values for all isotopes; for nuclei nearer the middle of the shell, we include calculations only for those nuclei for which there are some experimental data.

To show graphically the agreement between theory and experiment and to associate the assumptions made in calculating the wave functions with this agreement, we have plotted in Fig. 1 the fractional

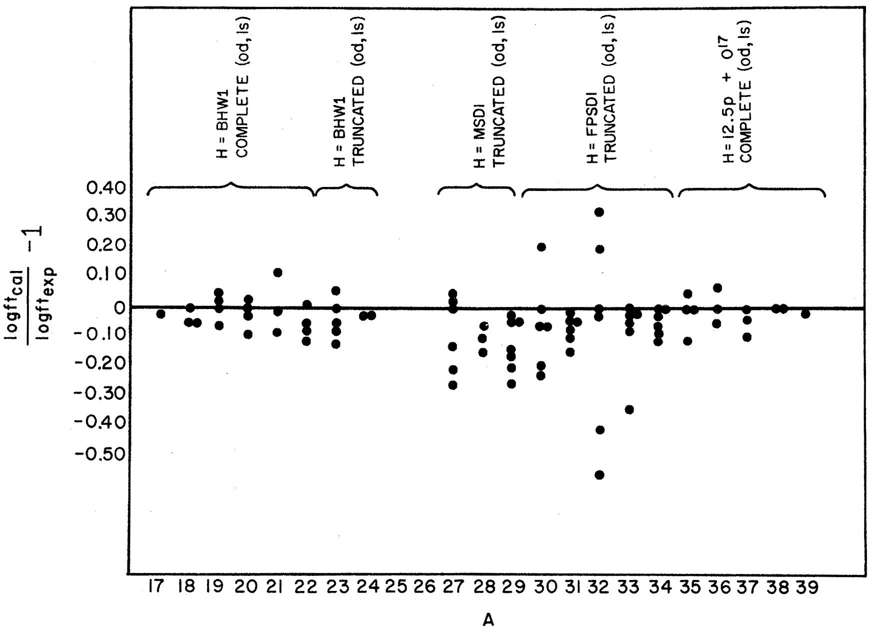

FIG. 1. The fractional deviation between the predicted and experimental logft values ($=\log ft_{\rm cal}/\log ft_{\rm exp} - 1$) versus the atomic mass number A. The interactions and the basis vector spaces used in the shell-model calculation of the nuclear wave functions are indicated at the top of the figure. See text.

deviation,

$$\left(\frac{\log ft_{\rm cal}}{\log ft_{\rm exp}} - 1\right) \text{ versus the mass number } A.$$

Given at the top of this figure are brief descriptions of the model spaces and Hamiltonians used to obtain the wave functions. (See Table I and Refs. 1–6 for more details.) For the nuclei with $A = 17$–22 and 35–39, for which complete $(0d, 1s)$ shell-model calculations were possible, the rms deviation between experimental and calculated logft values is about 5%, and there are no deviations greater than 12%. There are, however, several serious discrepancies for nuclei near the middle of the shell, for which only truncated shell-model calculations were possible.

We cannot say at present whether the poor agreement for some of the transitions in the $A = 30$, 32, and 33 nuclei results from the truncations of the allowed basis states or from the different two-body interactions used. It is known[2] that the same "realistic" interaction which works well for $A = 34$–39 does not yield satisfactory $A = 30$–33 nuclear wave functions in the truncated space. In addition, McGrory[25] has shown that severe truncations in the $(0d, 1s)$ shell can have large effects on the calculated β-decay transition rates. While

TABLE I. A brief description of the interactions and shell-model bases used in the calculations of the wave functions.

A	Hamiltonian	Configuration restrictions: $(0d_{5/2})^{n_1}(1s_{1/2})^{n_2}(0d_{3/2})^{n_3}$ $n_1+n_2+n_3=A-16$	
17–22	BHW1 [a]	No restrictions on n_i	See Ref. 5
23–24	BHW1 [a]	$n_2+n_3 \le 4$, $n_3 \le 2$	See Ref. 4
27	MSDI [b]	$n_1 \ge 7$	See Ref. 3
28	MSDI [b]	$n_1 \ge 8$	See Ref. 3
29	MSDI [b]	$n_1 \ge 9$ + the configuration with $n_1=8$, $n_2=4$, $n_3=1$	See Ref. 3
30–34	FPSDI [c]	$n_1 \ge 10$	See Ref. 2
35–39	12.5p + ^{17}O [d]	No restrictions on n_i	See Ref. 1

[a] Some two-body matrix elements were treated as free parameters, while the calculated spectra were required to have a minimum rms deviation from the experimental spectra for the nuclei with $A = 18$–22. This minimization was performed using the code SMIT using the two-body matrix elements of Kuo as the starting values in the search. The single-particle energies are from ^{17}O. Results are largely equivalent to $K + {}^{17}$O (Ref. 6) below $A = 22$.

[b] The usual modified surface δ interaction.

[c] A mix of free and modified-surface-δ-interaction matrix elements.

[d] The two-body matrix elements of Kuo plus the single-particle energies from ^{17}O. No free parameters.

TABLE II. Calculated and experimental $\log ft$ values. Also included are the predicted and observed excitation energies (E_x) for the relevant final states.

$^{A}Z_i$	J_i, T_i	$^{A}Z_f$	J_f, T_f	E_x calc	E_x [a] expt [a]	$\log ft$ calc	$\log ft$ expt [a]
^{17}F	$\frac{5}{2}, \frac{1}{2}$	^{17}O	$\frac{5}{2}, \frac{1}{2}$	0.0	0.0	3.30	3.37
^{18}F	1, 0	^{18}O	0, 1	0.0	0.0	3.37	3.56
^{18}Ne	0, 1	^{18}F	1, 0	0.0	0.0	2.89	3.04 [b]
			0, 1	0.89	0.937	3.49	3.49
^{19}O	$\frac{5}{2}, \frac{3}{2}$	^{19}F	$\frac{3}{2}, \frac{1}{2}$	1.56	1.56	4.61	4.61
			$\frac{5}{2}, \frac{1}{2}$	0.05	0.20	5.62	5.47
			$\frac{7}{2}, \frac{1}{2}$	4.41	4.39	3.83	3.64
^{19}Ne	$\frac{1}{2}, \frac{1}{2}$	^{19}F	$\frac{1}{2}, \frac{1}{2}$	0.0	0.0	3.08	3.24
			$\frac{3}{2}, \frac{1}{2}$	1.56	1.56	10.5	
^{20}O	0, 2	^{20}F	1, 1	1.31	1.06	3.86	3.74
^{20}F	2, 1	^{20}Ne	2, 0	1.65	1.63	4.98	4.98
^{20}Na	2, 1	^{20}Ne	2, 0	1.65	1.63	4.98	4.98 [c]
			3, 0	10.50		4.90	
			2, 1	9.96	10.28	3.45	3.47
			1, 1	11.27		3.73	
			3, 1	10.75		4.89	
			$(2, 1)_2$	12.28		5.34	
			$(1, 1)_2$	13.43		4.74	
			1, 0	12.36		3.85	
			$(2, 0)_2$	7.19		4.13	
			$(2, 0)_3$	9.92		4.96	
			$(2, 0)_4$	11.18		3.53	
^{20}Mg	0, 2	^{20}Na	1, 1	1.31		3.86	
			0, 2	6.83		3.19	
			$(1, 1)_2$	3.47		4.08	
			$(1, 1)_3$	4.43		4.15	
			$(1, 1)_4$	5.17		3.53	
^{21}O	$(\frac{5}{2}, \frac{5}{2})$	^{21}F	$\frac{3}{2}, \frac{3}{2}$	1.56		5.44	
			$\frac{5}{2}, \frac{3}{2}$	0.0	0.0	8.41	
			$\frac{7}{2}, \frac{3}{2}$	3.19		7.99	
			$(\frac{5}{2}, \frac{3}{2})_2$	3.23		4.89	
^{21}F	$\frac{5}{2}, \frac{3}{2}$	^{21}Ne	$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	5.55	
			$\frac{5}{2}, \frac{1}{2}$	0.31	0.35	4.70	5.0
			$\frac{7}{2}, \frac{1}{2}$	1.80	1.75	4.80	5.2
			$(\frac{3}{2}, \frac{1}{2})_2$	4.35		4.44	
			$(\frac{5}{2}, \frac{1}{2})_2$	3.59	3.74	8.34	
^{21}Na	$\frac{3}{2}, \frac{1}{2}$	^{21}Ne	$\frac{1}{2}, \frac{1}{2}$	2.83	2.80	4.34	
			$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	3.56	3.60
			$\frac{5}{2}, \frac{1}{2}$	0.31	0.35	4.54	4.94
^{21}Mg	$(\frac{5}{2}, \frac{3}{2})$	^{21}Na	$\frac{3}{2}, \frac{1}{2}$	0.0		5.55	
			$\frac{5}{2}, \frac{1}{2}$	0.31		4.70	
			$\frac{7}{2}, \frac{1}{2}$	1.80		4.80	
			$\frac{5}{2}, \frac{3}{2}$	8.99	(8.90)	3.26	(2.9)
			$(\frac{3}{2}, \frac{1}{2})_2$	4.35		4.44	
			$(\frac{5}{2}, \frac{1}{2})_2$	3.59		8.34	
^{21}Al	$(\frac{5}{2}, \frac{3}{2})$	^{21}Mg	$\frac{3}{2}, \frac{3}{2}$	1.56		5.44	
			$\frac{5}{2}, \frac{3}{2}$	0.0		8.41	
			$\frac{7}{2}, \frac{3}{2}$	3.19		7.99	
			$\frac{5}{2}, \frac{5}{2}$	11.14		3.08	
^{22}O	(0, 3)	^{22}F	1, 2	1.06		4.82	
			$(1, 2)_2$	2.09		3.72	
^{22}F	(3, 2)	^{22}Ne	2, 1	1.40	1.28	6.13	6.4
			3, 1	5.40		6.66	
			4, 1	3.49	3.34	6.18	5.9
			$(2, 1)_2$	4.14	4.47	4.98	
^{22}Na	3, 0	^{22}Ne	2, 1	1.40	1.28	6.61	7.40
^{22}Mg	0, 1	^{22}Na	1, 0	0.0	0.58	3.52	3.7
			0, 1	0.89	0.66	3.49	3.5
			$(1, 0)_2$	1.63	1.93	3.30	3.5
			$(1, 0)_3$	3.57		3.61	
^{23}Ne	$\frac{5}{2}, \frac{3}{2}$	^{23}Na	$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	5.02	5.3
			$\frac{5}{2}, \frac{1}{2}$	0.38	0.44	4.76	5.39
			$\frac{7}{2}, \frac{1}{2}$	1.95	2.08	5.44	5.86
			$(\frac{3}{2}, \frac{1}{2})_2$	3.31		5.57	
			$(\frac{5}{2}, \frac{1}{2})_2$	3.36		6.78	
			$(\frac{7}{2}, \frac{1}{2})_2$	4.18		5.26	
^{23}Mg	$\frac{3}{2}, \frac{1}{2}$	^{23}Na	$\frac{1}{2}, \frac{1}{2}$	2.28	2.39	4.69	
			$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	3.68	3.70
			$\frac{5}{2}, \frac{1}{2}$	0.38	0.44	4.68	4.41
^{24}Ne	0, 2	^{24}Na	1, 1		0.47	4.29	4.4
			$(1, 1)_2$		1.35	4.30	4.4
^{27}Mg	$(\frac{1}{2}, \frac{3}{2})$	^{27}Al	$\frac{1}{2}, \frac{1}{2}$	0.52	0.84	4.03	4.74
			$\frac{3}{2}, \frac{1}{2}$	1.03	1.01	4.18	4.91
^{27}Si	$\frac{5}{2}, \frac{1}{2}$	^{27}Al	$\frac{3}{2}, \frac{1}{2}$	1.03	1.01	5.01	6.80 [d]
			$\frac{5}{2}, \frac{1}{2}$	0.0	0.0	3.51	3.53
			$\frac{7}{2}, \frac{1}{2}$	1.84	2.21	5.04	4.94
			$(\frac{3}{2}, \frac{1}{2})_2$	2.75	2.98	4.76	4.51
			$(\frac{5}{2}, \frac{1}{2})_2$	3.07	3.72	5.08	5.4
			$(\frac{7}{2}, \frac{1}{2})_2$	4.52		5.24	
^{28}Mg	0, 2	^{28}Al	1, 1	0.91	1.37	3.78	4.45
^{28}Al	3, 1	^{28}Si	2, 0	2.53	1.78	4.42	4.86
^{28}P	(3, 1)	^{28}Si	2, 0	2.53	1.78	4.42	4.7
			3, 0	5.11	(6.27)	4.06	
			4, 0	5.58	(4.61)	5.51	
^{29}Al	$\frac{5}{2}, \frac{3}{2}$	^{29}Si	$\frac{3}{2}, \frac{1}{2}$	1.96	1.28	4.34	5.03
			$\frac{5}{2}, \frac{1}{2}$	1.80	2.03	4.62	>6.0
			$\frac{7}{2}, \frac{1}{2}$	4.73		5.79	
			$(\frac{3}{2}, \frac{1}{2})_2$	2.83	2.43	4.21	4.99
			$(\frac{5}{2}, \frac{1}{2})_2$	3.61		4.58	

TABLE II *(Continued)*

$^{A}Z_i$	J_i, T_i	$^{A}Z_f$	J_f, T_f	E_x calc	E_x expt[a]	$\log ft$ calc	$\log ft$ expt[a]
^{29}P	$\frac{1}{2}, \frac{1}{2}$	^{29}Si	$\frac{1}{2}, \frac{1}{2}$	0.00	0.00	3.62	3.72
			$\frac{3}{2}, \frac{1}{2}$	1.96	1.28	4.64	4.89
			$(\frac{3}{2}, \frac{1}{2})_2$	2.83	2.43	4.05	4.27
^{30}Al	(2, 2)?	^{30}Si	1, 1	3.81	3.77	4.85	
			2, 1	2.48	2.23	4.91	5.15
			3, 1	4.64	(4.83)	5.51	
			$(2, 1)_2$	3.84	3.51	4.80	3.86
			$(2, 1)_3$	4.65	4.81	4.69	
			$(3, 1)_2$	5.01	(5.23)	7.38	
^{30}Al	(3, 2)?	^{30}Si	2, 1	2.48	2.23	6.10	5.15
			3, 1	4.64	(4.83)	4.46	
			4, 1	5.43	5.28	4.86	
			$(2, 1)_2$	3.84	3.51	4.38	3.89
			$(2, 1)_3$	4.65	4.81	5.27	
			$(3, 1)_2$	5.01	(5.23)	7.56	
^{30}P	1, 0	^{30}Si	0, 1	0.0	0.0	3.84	4.85
			1, 1	3.81	3.77	5.11	
			2, 1	2.48	2.23	4.61	4.95
			$(2, 1)_2$	3.48	3.51	4.36	
			$(0, 1)_2$	3.52	3.79	4.53	
^{30}S	0, 1	^{30}P	1, 0	0.0	0.0	3.36	4.37[e]
			0, 1	1.00	0.69	3.49	3.48
			1, 1	4.81		4.64	
			$(1, 0)_2$	1.02	0.71	5.34	5.7
			$(1, 0)_3$	3.22	3.02	4.25	3.49
^{31}Si	$\frac{3}{2}, \frac{3}{2}$	^{31}P	$\frac{1}{2}, \frac{1}{2}$	0.0	0.0	4.94	5.51
			$\frac{3}{2}, \frac{1}{2}$	1.24	1.27	5.28	5.54
^{31}S	$\frac{1}{2}, \frac{1}{2}$	^{31}P	$\frac{1}{2}, \frac{1}{2}$	0.0	0.0	3.45	3.71[d]
			$\frac{3}{2}, \frac{1}{2}$	1.24	1.27	4.79	4.99
			$(\frac{1}{2}, \frac{1}{2})_2$	3.40	3.13	4.12	4.86
			$(\frac{1}{2}, \frac{3}{2})_2$	3.81	3.51	4.57	4.68
^{32}Si	0, 2	^{32}P	1, 1	0.0	0.0	3.89	8.7
^{32}P	1, 1	^{32}S	0, 0	0.0	0.0	4.65	7.9
^{32}Cl	(2, 1)	^{32}S	1, 0	4.81	4.71	4.31	
			2, 0	2.20	2.24	6.47	4.48
			3, 0	5.20	5.41	5.71	
			$(2, 0)_2$	4.55	4.29	6.52	4.8
			$(2, 0)_3$	5.44	5.55	6.05	4.5
			1, 1	6.88		4.65	
			2, 1	6.70		3.38	3.44[f]
			3, 1	8.02		5.48	
^{33}Si	$\frac{5}{2}, \frac{5}{2}$	^{33}P	$\frac{1}{2}, \frac{3}{2}$	0.00	0.00	5.40	
			$\frac{3}{2}, \frac{3}{2}$	1.18	1.43	5.36	
			$\frac{5}{2}, \frac{3}{2}$	1.72	1.84	3.91	
^{33}P	$\frac{1}{2}, \frac{3}{2}$	^{33}S	$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	4.64	5.00
^{33}Cl	$\frac{3}{2}, \frac{1}{2}$	^{33}S	$\frac{1}{2}, \frac{1}{2}$	1.00	0.81	4.94	5.59[g]
			$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	3.65	3.74
			$\frac{5}{2}, \frac{1}{2}$	2.02	1.97	4.29	4.87
			$(\frac{3}{2}, \frac{1}{2})_2$	2.31	2.31	4.90	5.6
			$(\frac{5}{2}, \frac{1}{2})_2$	2.74	2.87	4.35	4.05
^{33}Ar	$\frac{1}{2}, \frac{3}{2}$	^{33}Cl	$\frac{1}{2}, \frac{1}{2}$	1.00	0.81	4.31	4.44[h]
			$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	4.64	5.03
			$(\frac{1}{2}, \frac{1}{2})_2$	3.78		4.38	
			$(\frac{3}{2}, \frac{1}{2})_2$	2.31		4.56	
			$\frac{1}{2}, \frac{3}{2}$	5.50	5.51	3.28	3.34
^{34}Si	(0, 3)	^{34}P	1, 2	0.0		7.21	
			$(1, 2)_2$	0.27		3.38	
^{34}P	1, 2	^{34}S	0, 1	0.0	0.0	5.51	5.2
			1, 1	3.83	4.08	4.86	
			2, 1	1.99	2.13	4.25	4.5
			$(2, 1)_2$	2.96	3.30	5.33	
			$(0, 1)_2$	4.04	3.92	6.42	
^{34}Cl	0, 1	^{34}S	0, 1	0.0	0.0	3.49	3.49
			1, 1	3.83	4.08	4.46	
^{34m}Cl	3, 0	^{34}S	2, 1	1.99	2.13	5.88	6.0
			3, 1	4.28		4.93	
			4, 1	4.36		6.48	
			$(2, 1)_2$	2.96	3.30	4.26	4.8
			$(2, 1)_3$	4.19	4.12	8.14	
^{34}Ar	0, 1	^{34}Cl	0, 1	0.0	0.0	3.49	3.45[e]
			1, 0	0.21	0.46	3.64	5.6
			$(1, 0)_2$	0.49	0.66	4.29	5.0
^{35}P	$(\frac{1}{2}, \frac{5}{2})$	^{35}S	$\frac{1}{2}, \frac{3}{2}$	1.88	1.57	4.19	4.1[i]
			$\frac{3}{2}, \frac{3}{2}$	0.0	0.0	6.34	
^{35}S	$\frac{3}{2}, \frac{3}{2}$	^{35}Cl	$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	5.03	5.01
^{35}Ar	$\frac{3}{2}, \frac{1}{2}$	^{35}Cl	$\frac{1}{2}, \frac{1}{2}$	1.11	1.22	4.49	5.09[d]
			$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	3.75	3.78
			$\frac{5}{2}, \frac{1}{2}$	1.70	1.76	5.84	5.52
			$(\frac{1}{2}, \frac{1}{2})_2$	4.01	3.96	4.55	
			$(\frac{3}{2}, \frac{1}{2})_2$	2.17	2.69	4.29	
			$(\frac{5}{2}, \frac{1}{2})_2$	2.43	3.00	4.16	4.96
^{36}K	2, 1	^{36}Ar	2, 0	1.87	1.97	4.79	4.48
			$(2, 0)_2$	5.31	4.40	4.76	(5.0)
			1, 1	8.68		4.85	
			2, 1	7.53	6.61	3.47	3.48[f]
			3, 1	8.36		4.23	
^{36}Ca	(0, 2)	^{36}K	1, 1	1.15		5.06	
			0, 2	3.40		3.19	
			1, 2	7.75		4.87	
^{37}K	$\frac{3}{2}, \frac{1}{2}$	^{37}Ar	$\frac{1}{2}, \frac{1}{2}$	1.43	1.41	6.32	
			$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	3.53	3.66
			$\frac{5}{2}, \frac{1}{2}$	2.03	2.80	3.41	3.80
			$(\frac{1}{2}, \frac{1}{2})_2$	5.31		4.77	
			$(\frac{3}{2}, \frac{1}{2})_2$	4.63		4.64	
			$(\frac{5}{2}, \frac{1}{2})_2$	3.04		5.29	
^{37}Ca	$\frac{3}{2}, \frac{3}{2}$	^{37}K	$\frac{1}{2}, \frac{1}{2}$	1.43		6.37	
			$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	5.55	5.06[j]
			$\frac{5}{2}, \frac{1}{2}$	2.03		4.54	
			$\frac{1}{2}, \frac{3}{2}$	7.58		5.52	

TABLE II *(Continued)*

A_{Z_i}	J_i, T_i	A_{Z_f}	J_f, T_f	E_x calc	E_x expt [a]	$\log ft$ calc	$\log ft$ expt [a]
^{37}Ca	$\frac{3}{2}, \frac{3}{2}$	^{37}K	$\frac{3}{2}, \frac{3}{2}$	4.84	5.02	3.30	
			$\frac{5}{2}, \frac{3}{2}$	8.80		4.81	
			$(\frac{1}{2}, \frac{1}{2})_2$	5.31		3.69	
			$(\frac{3}{2}, \frac{1}{2})_2$	4.63		3.96	
			$(\frac{5}{2}, \frac{1}{2})_2$	3.04		4.84	

A_{Z_i}	J_i, T_i	A_{Z_f}	J_f, T_f	E_x calc	E_x expt [a]	$\log ft$ calc	$\log ft$ expt [a]
^{38}K	3, 0	^{38}Ar	2, 1	1.56	2.17	4.97	4.99
			$(2, 1)_2$	5.09	(3.95)	5.76	(5.3)
^{38m}K	0, 1	^{38}Ar	0, 1	0.0	0.0	3.49	3.50
^{38}Ca	0, 1	^{38}K	0, 1	0.80	0.13	3.49	3.49 [k]
			1, 0	1.53	0.45	6.18	>4.8
			(1, 0)	1.93	1.70	3.03	3.41
^{39}Ca	$\frac{3}{2}, \frac{1}{2}$	^{39}K	$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	3.52	3.64

[a] Unless otherwise noted the experimental results are from Refs. 12–14.
[b] From Ref. 15.
[c] From Ref. 16.
[d] From Ref. 17.
[e] From Ref. 18.
[f] From Ref. 19.
[g] From Ref. 20.
[h] From Ref. 21.
[i] From Ref. 22.
[j] From Ref. 23.
[k] From Ref. 24.

both these observations indicate that the problem is not being able to include a large enough basis in the shell-model calculation, the possibility of successful calculations in the truncated space with a modified Hamiltonian is certainly not eliminated. In fact, it should be noted that the over-all agreement for these nuclei in the middle of the shell is respectable, even though there are some marked failures. The percentage rms deviation between experiment and theory of the nuclei with $A = 27-33$ is 18%, but as can be seen from Fig. 1, a large part of the rms deviation comes from the four very bad disagreements for $A = 32$ and 33.

In order to see if there are particular problems in predicting either very strong or very weak allowed transitions, we have plotted in Fig. 2 the fractional deviation

$$\left(\frac{\log ft_{\text{cal}}}{\log ft_{\text{exp}}} - 1\right) \text{ versus } \log ft_{\text{exp}} .$$

These are plotted separately for $A = 17-23$, 24–33, and 34–39. Inspection of Fig. 2 shows that, at least for the nuclei in the upper and lower ends of the shell, the percentage deviation between calculated and experimental $\log ft$ values is independent of the strength of the transition.

For the nuclei in the middle of the shell, the very fast transitions are generally reproduced better than the ones with $\log ft > 4.0$. Most of these fast transitions are superallowed decays which depend strongly on T but are almost independent of the details of the nuclear wave functions. It is noteworthy that the two worst discrepancies are for the two weakest allowed β decays in the whole $(0d, 1s)$ shell ($\log ft = 8.7$ and 7.9), and both of these decays involve the ^{32}P ground state. One is tempted to conclude that the ^{32}P ground-state wave function is badly in error, but there is reasonably good agreement between experimental results and model predictions for other observables which involve this state.[2] At present we do not understand even qualitatively why the $^{32}\text{Si} \xrightarrow{\beta^-} {}^{32}\text{P}$ and $^{32}\text{P} \xrightarrow{\beta^-} {}^{32}\text{S}$ decays are so inhibited.

There have been two other systematic studies of shell-model calculations of $\log ft$ values in the $(0d, 1s)$ shell, one by McGrory,[25] who calculated some of the transitions for $A = 18-21$ nuclei, and one by Engelbertink and Brussaard,[26] who studied some of the transitions for $A = 29-39$ nuclei. For the transitions reported by McGrory, our results are very similar to those of his calculated with complete $(0d, 1s)^n$-shell-model wave functions. This is not surprising, since the wave functions used in both calculations are very similar. Engelbertink and Brussaard used wave functions based on an inert core of ^{28}Si. A comparison of the present results with theirs and with experiment shows that the present results are generally in better agreement with experiment, particularly for the heavier nuclei ($A = 34-39$). For the cases in which the present calculations show serious discrepancies with measured values, Engelbertink and Brussaard experience similar problems.

IV. COMMENTS ON PARTICULAR TRANSITIONS

The calculations indicate that in several cases there are pure Gamow-Teller transitions which are stronger than the superallowed decays to isobaric analog states. In fact, with the possible exception of the superallowed decay of ^{21}Mg, the strongest known transition in the $(0d, 1s)$ shell in the Fermi-forbidden $^{18}\text{N} \xrightarrow{\beta^-} {}^{18}\text{F}$ decay to the ground state ($\log ft_{\text{exp}} = 3.04 \pm 0.03$, $\log ft_{\text{cal}} = 2.89$). We point this out to show that one must be careful in assigning strong β-decay branches to analog transitions.

$^{37}Ca \xrightarrow{\beta^-} {}^{37}K$ Decay and the Solar Neutrino Experiment

Davis, Jr., Harmer, and Hoffman are presently using the $\nu + {}^{37}Cl \to {}^{37}Ar + e$ reaction to measure the neutrino flux from the sun.[7] To calculate the absorption cross section for this process, it is necessary to know the $\log ft$ values connecting the ground state of ^{37}Cl to the various states in ^{37}Ar. Assuming isospin is good, these $\log ft$ values are exactly the same as those in the $^{37}Ca \xrightarrow{\beta^-} {}^{37}K$ decay. At present there are delayed proton measurements[27] of the $\log ft$ values to the states between about 3 and 7 MeV of excitation in ^{37}K relative to the superallowed decay to the analog state at 5.05 MeV. The ground-state-to-ground-state $\log ft$[23] can be inferred from the electron-capture rate of ^{37}Ar. It is then of interest to have theoretical calculations of the unobserved $\log ft$ values.

The relevant transitions are indicated in Table III. There are three columns of calculated $\log ft$ values. Column 2 gives the values calculated by Bahcall[23] in his original estimate of the neutrino absorption cross section. The $\log ft$ values in column 3 are from Table II. Because of the interest in these particular transitions we also calculated $\log ft$'s for the decay of ^{37}Ca using wave functions derived using the Hamiltonian 11.0h + ASPE of Ref. 1 (column 4). This Hamiltonian differs from $12.5p + {}^{17}O$ in that the matrix elements were renormalized relative to a ^{40}Ca core with $\hbar\omega = 11.0$ MeV and the three single-particle energies were treated as free parameters. More details are available in Ref. 1.

By comparing columns in Table III, one sees that both columns 3 and 4 give reasonable agreement with the ground-state $(J_f, T_f = \frac{3}{2}, \frac{1}{2})$ $\log ft$. There is also agreement between columns 3 and 4 and Bahcall for transitions to the $J = \frac{3}{2}$, $T = \frac{3}{2}$ state and to the $J = \frac{5}{2}$ state. However, the present results indicate that the transition to the $J = \frac{1}{2}$ state is much weaker (larger $\log ft$) than Bahcall estimated.

The important source of high-energy solar neutrinos is believed to be the $^{8}B \to {}^{8}Be + e + \nu$. Using the results of Ref. 23 and Bahcall,[28] and the calculated $\log ft$ values to the $\frac{1}{2}^+$ state, we find the cross section for absorption of a ^{8}B neutrino, averaged over the neutrino spectrum as in Ref. 23, is 1.2×10^{-42} cm^2 using calculation from column 3 and 1.25×10^{-42} cm^2 using calculation from column 4. That is, the larger $\log ft$ predicted for the $J = \frac{1}{2}$ state decreases the predicted average cross section for absorption of a ^{8}B neutrino by 8–12% from Bahcall's calculation.[28]

TABLE III. A comparison of the present calculations with the results of Bahcall for the $\log ft$ values needed to predict the absorption cross section for solar neutrinos on ^{37}Cl.

J_f, T_f	Calculated $\log ft$ Bahcall	Ref. a	Ref. b
$\frac{3}{2}, \frac{3}{2}$	3.28	3.30	3.30
$\frac{3}{2}, \frac{1}{2}$	(5.06) [c]	5.55	5.14
$\frac{1}{2}, \frac{1}{2}$	4.48	6.37	5.44
$\frac{5}{2}, \frac{1}{2}$	4.34	4.54	4.36

[a] Calculated using $H = 12.5p + {}^{17}O$.
[b] Calculated using $H = 11.0h$ + ASPE of Ref. 1.
[c] From electron capture of ^{37}Ar (Ref. 23).

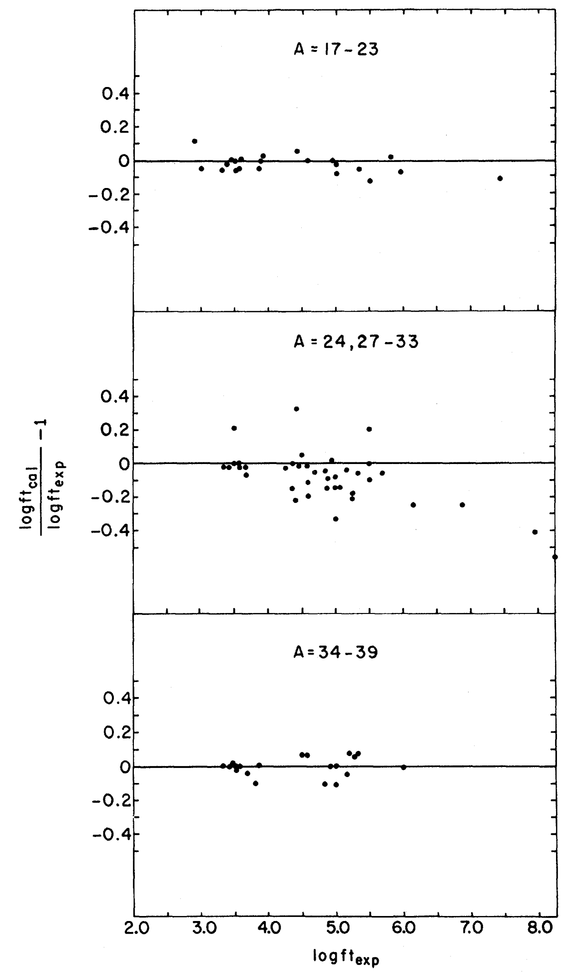

FIG. 2. The fractional deviation between the predicted and experimental $\log ft$ values $(= \log ft_{cal}/\log ft_{exp} - 1)$ versus $\log ft_{exp}$. This is plotted separately for the three mass regions $A = 17$–23, 24–33, and 34–39.

V. β-DECAY HALF-LIVES

An alternate way to present these calculated $\log ft$ values is to use them to predict β-decay half-lives. Given the predicted $\log ft$ values, one needs only the β-decay end-point energies and the "statistical rate function" f, to deduce t_i, the partial half-life.[8] The partial half-lives t_i can then be combined by $(1/t) = (1/t_1) + (1/t_2) + \cdots$ to give the total half-life t.

These predicted half-lives are given in Table IV along with the experimentally observed half-lives. Inspection of Table IV shows that one gets particularly good agreement for the lighter and heavier nuclei, and poorer agreement for some of the nuclei in the middle of the shell. Where possible, the known masses were used to deduce the β-decay end-point energies. For unknown masses we used the estimate, based on systematics, of Wapstra and Gove[29] or, in a few cases, the Garvey-Kelson prediction.[30]

Of particular interest may be the rather long half-life predicted for ^{21}O (1.2 sec). We also note that soon after these calculations were completed, the half-life of ^{35}P was reported to be 47 sec,[31,32] in excellent agreement with our prediction of 54 sec.

VI. CONCLUSIONS

Experiments generally measure β-decay $\log ft$ values only to low-lying states in the final nucleus. Consequently, the present comparison between calculated and experimental β-decay rates tests only the wave functions for these low-lying states. The remarkably good agreement between experiment and theory for the $\log ft$ values for the nuclei in the regions $A = 18-22$ and 34–39 is further confirmation that the wave functions for the low-lying states in these nuclei are well understood in terms of the conventional shell model. Conversely, the several disagreements between experiment and theory around $A = 30$ show that some of the states of nuclei in the middle of the $(0d, 1s)$ shell are not as well understood. The inhibition of β decays to and from the ^{32}P ground state is especially puzzling.

TABLE IV. The calculated and experimental β-decay half-lives.

	$t_{\rm calc}$	$t_{\rm exp}$ [a]		$t_{\rm calc}$	$t_{\rm exp}$ [a]
^{17}F	56 sec	66 sec	^{29}P	3.5 sec	4.4 sec
^{18}F	70 min	110 min	^{30}P	14 sec	2.5 min
^{18}Ne	1.0 sec	1.67 sec [b]	^{30}S	0.4 sec	1.4 sec
^{19}O	32 sec	29 sec	^{31}Si	0.7 h	2.62 h
^{19}Ne	12 sec	17 sec	^{31}S	1.4 sec	2.7 sec
^{20}O	19 sec	14 sec	^{32}Si	3.5 day	650 yr
^{20}F	9.1 sec	11 sec	^{32}P	12 min	14.3 day
^{20}Na	0.3 sec	0.44 sec [c]	^{32}Cl	0.4 sec	0.31 sec
^{20}Mg	0.1 sec		^{33}P	10.3 day	25 day
^{21}O	1.2 sec		^{33}Cl	2 sec	2.5 sec
^{21}F	4.5 sec	4.4 sec	^{33}Ar	0.1 sec	0.18 sec
^{21}Na	20 sec	23 sec	^{34}P	16 sec	12.4 sec
^{21}Mg	0.1 sec	0.12 sec	^{34}Cl	1.5 sec	1.56 sec
^{22}O	0.15 sec [d]		^{34m}Cl	25 min	32 min
^{22}F	3.2 sec [e]	4.0 sec	^{34}Ar	0.62 sec	1.2 min
^{22}Na	0.44 yr	2.6 yr	^{35}P	54 sec	47 sec [f]
^{22}Mg	3.2 sec	4.0 sec	^{35}S	94 day	88 day
^{23}Ne	15 sec	37.6 sec	^{35}Ar	1.6 sec	1.83 sec
^{23}Mg	11 sec	12.1 sec	^{36}K	0.3 sec	0.34 sec [c]
^{24}Ne	2.8 min	3.38 min	^{37}Ca	0.18 sec	0.173 sec
^{27}Mg	1.8 min	9.5 min	^{37}K	0.9 sec	1.23 sec
^{27}Si	3.4 sec	4.2 sec	^{38}K	7.5 min	7.7 min
^{28}Mg	7.2 h	21 h	^{38m}K	0.9 sec	0.95 sec
^{28}Al	0.8 min	2.31 min	^{38}Ca	0.39 sec	0.44 sec [g]
^{29}Al	1.2 min	6.6 min	^{39}Ca	0.62 sec	0.87 sec

[a] From compilations of Refs. 12 and 13 unless otherwise referenced.
[b] From Ref. 15.
[c] From Ref. 19.
[d] Mass taken from Garvey-Kelson Mass Table.
[e] Assuming g.s. has $J = 3$.
[f] This half-life was reported after we had finished our calculations. See Refs. 31 and 32.
[g] From Ref. 24.

It is now clear that modern, large configuration space shell models can describe not only the spectra and transitions within individual nuclei but also the decay schemes which connect isobars. It is hoped that the predicted β-decay rates for some exotic nuclei (such as ^{20}Mg, ^{21}O, and ^{22}O) will help in designing experiments to study their decays. It may be of interest that ^{20}Mg, ^{21}O, and ^{22}O are all predicted to undergo β decay predominately to excited states. Hence, there should be a γ ray associated with the β decay of each of these isotopes.

Finally, our results for the β decay of ^{37}Ca indicate that while Bahcall's estimates for these $\log ft$ values were based on a considerably simpler model of nuclear structure, his results are substantially correct when applied to calculating the absorption cross section of ^{8}B neutrinos on ^{37}Cl. There do not seem to be any strong transitions omitted in his calculation and, except for one transition, his predictions for the β-decay rates agree quite well with our more complete calculation.

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