[10]F. G. Perey, unpublished.
[11]C. B. Fulmer, J. C. Hafele, and C. C. Foster, Bull. Am. Phys. Soc. 16, 1165 (1971); and to be published.
[12]M. E. Cage, D. L. Clough, A. J. Cole, J. B. A. England, G. J. Pyle, P. M. Rolph, L. H. Watson, and D. H. Worledge, Nucl. Phys. A183, 449 (1972).
[13]L. Rosen, in *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960,* edited by D. A. Bromley and E.W. Vogt (Univ. of Toronto Press, Toronto, Canada, 1960), p. 185.
[14]M. P. Fricke, E. E. Gross, B. J. Morton, and A. Zucker, Phys. Rev. 156, 1207 (1967).
[15]K. K. Seth, Nucl. Phys. 24, 169 (1961).
[16]R. B. Taylor, N. R. Fletcher, and R. H. Davis, Nucl. Phys. 65, 318 (1965).
[17]C. B. Fulmer and J. C. Hafele, Bull. Am. Phys. Soc. 16, 646 (1971); and to be published.
[18]H. R. Weller, Phys. Rev. Letters 28, 247 (1972).
[19]G. H. Rawitscher, Phys. Rev. C 6, 1212 (1972).
[20]P. E. Hodgson, *The Optical Model of Elastic Scattering* (Clarendon Press, Oxford, England, 1963), p. 58.

PHYSICAL REVIEW C VOLUME 7, NUMBER 2 FEBRUARY 1973

Study of ^{42}K, ^{42}Ca and ^{46}K, ^{46}Ca by Pickup Reactions

Y. Dupont,* P. Martin, and M. Chabre†
Institut des Sciences Nucléaires, BP n° 257, Centre de Tri, 38044 Grenoble, France
(Received 11 September 1972)

The ^{44,48}Ca$(p,t)^{42,46}$Ca and ^{43}Ca$(p,d)^{42}$Ca reactions have been investigated at 40 MeV, and the ^{44,48}Ca$(p,{}^3\mathrm{He})^{42,46}$K reactions at 40 and 56 MeV. Analysis of angular distributions leads to new spin assignments. Distorted-wave Born-approximation calculations are performed for (p,d) and (p,t) reactions. The nucleus ^{42}Ca being reached by both reactions, comparison is made between the results of calculations of the two processes. Potassium isotopes are studied through the comparison of analog states observed in the (p,t) reaction with the states reached in the $(p,{}^3\mathrm{He})$ reaction. In the $(p,{}^3\mathrm{He})$ spectra, the analogs of the ^{42}Ar and ^{46}Ar ground states have been identified.

I. INTRODUCTION

The calcium region has been the subject of numerous experimental investigations as well as extensive theoretical calculations.[1,2] However, only a few studies of the neutron states in the even isotopes through the two-neutron pickup reaction (p,t) have yet been published. In previous studies at comparatively low beam energies the analysis was restricted to the ground state and low-lying states due to lack of structure of the angular distributions.[3,4]

With a 40-MeV proton beam and good particle identification one can study states up to 15-MeV excitation energy in the residual nucleus. One then reaches the region of the analog states of the corresponding potassium isotopes. All angular distributions show enough structure to be characteristic of the L transfer; this allows unambiguous spin and parity assignments when starting from a doubly even target nucleus. The simultaneous study of the $(p,{}^3\mathrm{He})$ reaction and the comparison of the ^{3}He spectra with the triton spectra in the analog-state region makes it possible to assign spin and parity to states in potassium isotopes that otherwise can be reached only by complex reactions.[5]

In this paper we present studies on the ^{42}Ca and ^{46}Ca isotopes; since analog states are observed in both nuclei this study was extended to the ^{42}K and ^{46}K isotopes.

The nucleus ^{42}Ca was reached in the ^{44}Ca$(p,t)^{42}$Ca reaction and in the one-nucleon-transfer reaction ^{43}Ca$(p,d)^{42}$Ca. The neutron-hole amplitudes obtained in the (p,d) reaction may be used to check the normalization of the distorted-wave Born-approximation (DWBA) calculation of the (p,t) reaction and also to resolve ambiguities in the (p,t) analysis. For negative-parity states one can determine this way the relative phases of the two components of a configuration mixing whose percentage has been obtained in the analysis of the (p,d) reaction. The combined results of these two reactions for positive-parity states give a sensitive test of the wave functions calculated by Gerace and Green.[6]

The nuclei ^{46}Ca and ^{46}K have been studied in the ^{48}Ca$(p,t)^{46}$Ca and ^{48}Ca$(p,{}^3\mathrm{He})^{46}$K reactions. Due to the lack of wave-function calculations for ^{46}Ca, the conclusions drawn from the DWBA analysis of the (p,t) reaction are mostly restricted to spin and parity assignments; but a lot of new information on the low-lying states of ^{46}K is obtained from the comparison of the (p,t) and

TABLE I. Isotopic composition of the targets.

Target Isotope	^{43}Ca	^{44}Ca Composition (%)	^{48}Ca
^{40}Ca	12.7	1.33	2.71
^{42}Ca	9.4	0.05	0.04
^{43}Ca	61.6	0.03	<0.02
^{44}Ca	16.3	98.59	0.10
^{46}Ca	<0.1	<0.005	<0.02
^{48}Ca		0.006	97.16

$(p, {}^3\text{He})$ reactions.

In addition, in the high-excitation-energy part of the ^{44,48}Ca$(p, {}^3\text{He})^{42,46}$K spectra, analog states of ^{42}Ar and ^{46}Ar may be observed.

II. EXPERIMENTAL METHOD

The measurements have been performed using a beam of the Grenoble variable-energy cyclotron at a proton energy of 40 MeV for (p, d) and (p, t) reactions and 40 and 56 MeV for $(p, {}^3\text{He})$ reaction. Beam intensities were 100 to 600 nA. The calcium targets were self-supported metal foils rolled to a thickness of about 1 mg/cm^2. As they were appreciably contaminated by variable amounts of ^{12}C and ^{16}O, calibrated areas were cut out of each target after the experiments and chemical analysis of the calcium content of these samples were performed. The isotopic composition of each target is as shown in Table I. The reaction products were detected in a ΔE-E silicon counter telescope. A third counter operating in anticoincidence rejects the long-range particles, essentially elastically scattered protons; the use of this counter and of an antipileup circuit allows a high counting rate. The detectors were connected to a Goulding-Landis-type particle identifier and spectra for the (p, d), (p, t), and $(p, {}^3\text{He})$ reactions were simultaneously routed into a 4×1024 multichannel analyzer. Dead-time corrections were accurately measured by integrating the peak of a pulser which was sent through the whole electronic setup in one of the observed spectra, this pulser being triggered proportionally to the ΔE counting rate.

After data taking at each angle, the spectra were transferred into PDP-9 computer where a rapid preliminary analysis was made. A smoothing program based on Fourier transform of the spectra[7,8] eliminates most of the random fluctuations without affecting the spectra resolution. It makes easier energy location of the levels, background subtraction, and peak summation.

Typical energy spectra for the various reactions are shown in Figs. 1, 3, 5, and 6. The average energy resolution was 80 to 100 keV in (p, d) and (p, t) reaction, and 150 keV or more in $(p, {}^3\text{He})$ due to the target thickness. Angular distributions were taken by 4° steps from 10 to 60°. They are shown in Figs. 2, 4, 7, and 10.

III. METHODS OF ANALYSIS AND RESULTS

In the present study we have used (p, d), (p, t), and $(p, {}^3\text{He})$ reactions. Due to differences in selection rules or in theoretical analysis, these reactions give different or complementary information.

In a (p, t) reaction on a 0^+ target the selection rules only allow excitation of natural-parity states with an orbital momentum transfer L equal to the spin of the residual level J_f. Then in most cases this spin can be obtained by a simple comparison of the angular distributions with that for a known state.

The other results concern the study of state configurations. The uncertainties in the two-nucleon cross-section calculations are well known.[9] The approximations made in order to obtain a calculable transition amplitude and the interference

TABLE II. Optical-model parameters.

Potential[a]	V_C (MeV)	a_0 (fm)	r_0 (fm)	W_s (MeV)	W_d (MeV)	r'_0 (fm)	a'_0 (fm)	V_{so} (MeV)	r_{so} (fm)	a_{so} (fm)
Proton[b]	41.55	1.20	0.67	1.56	4.92	1.25	0.704	6.22	1.03	0.78
Deuteron[c]	104.8	1.063	0.72	0	7.81	1.41	0.86	6.51	1.06	0.72
Triton[d]	129.2	1.4	0.64	53.2	0	1.4	0.64	0	0	0

[a] $V(r) = -V_C(r)(e^x+1)^{-1} - i\left(W_s - 4W_d \frac{d}{dx'}\right)(e^{x'}+1)^{-1} + \frac{\hbar^2}{(m_\pi c)^2} V_{so}\, \sigma\cdot l\, \frac{1}{r}\frac{d}{dr}(e^{x_{so}}+1)^{-1}$,

with

$x = (r - r_0 A^{1/3})/a_0$.

[b] M. P. Fricke, E. E. Gross, B. J. Morton, and A. Zucker, Phys. Rev. 156, 1207 (1967).

[c] C. M. Perey and F. G. Perey, Phys. Rev. 152, 923 (1966).

[d] R. N. Glover and A. D. W. Jones, Nucl. Phys. 81, 286 (1966).

of the contributing configurations restrict the application of such calculations to precise cases:

(i) When the transferred angular momentum allows a single configuration, a spectroscopic factor can be obtained for that configuration.

(ii) To test wave functions obtained in nuclear-structure calculations, but due to the great sensitivity to weak components, the results are meaningless when too many configurations are to be taken into account.

(iii) In cases where the wave function is essentially built on two components and when one can obtain the absolute values of these components by a single-nucleon-transfer reaction, their relative sign can be deduced from the calculation of the two-nucleon-transfer reaction reaching the same state.

The normalization of the cross section

$$N=\left(\frac{d\sigma}{d\Omega}\right)_{\exp}\Big/\left(\frac{d\sigma}{d\Omega}\right)_{\rm DWBA}$$

being unknown, Flynn and Hansen[10] have tried to evaluate this normalization in analyzing a wide range of (t,p) reactions; the average value they have found is $N=310$. Since this factor is certainly dependent on the optical-model parameters it is necessary to find for each reaction a known transition [case (i)] in order to verify this empirical normalization.

All DWBA calculations were made with the DWUCK-2 code; optical-model parameters are listed in Table II. In the (p,d) reaction calculations, we used the constant well method, in choosing a unique average binding energy for each l

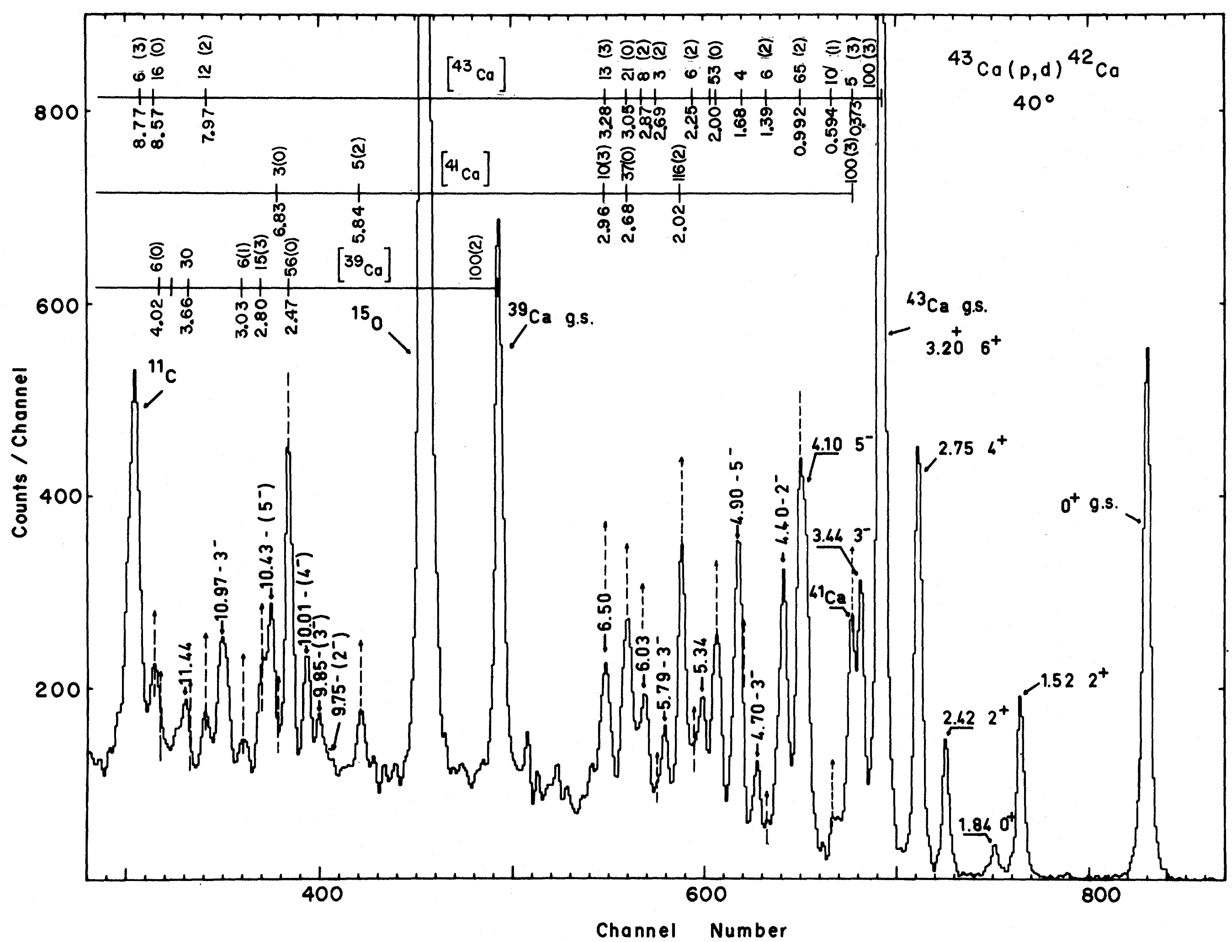

FIG. 1. Experimental spectrum for the ^{43}Ca$(p,d)^{42}$Ca reaction. The positions of peaks from calcium contaminants are marked by broken lines; energies, relative intensities, and orbital momentum transfer values are indicated at the top of the figure. Data are taken from Ref. 20.

value. The (p, t) DWBA cross sections were calculated by means of the Glendenning method.[9] The binding energies were $E_B = Q(\gamma, n) - E_X$.

A. ^{43}Ca$(p, d)^{42}$Ca Reaction

21 peaks are observed corresponding unambiguously to ^{42}Ca states. Several others are mixed with other calcium isotopes contaminants (Fig. 1). Angular distributions were obtained for all those peaks (Fig. 2). The cross section for the 3.2-MeV state has been corrected for the contamination by the ^{44}Ca$(p, d)^{43}$Ca ground-state transition. Between 0 and 3.2 MeV all angular distributions present the same pattern characteristic of their known $l = 3$ angular momentum transfer. This corresponds to positive-parity states excited by the $(f_{7/2}{}^2)_{J^\pi = 0^+, 2^+, 4^+, 6^+}$ configuration. At higher energies one observes angular distributions characteristic of either $l = 2$ or $l = 0 + 2$ transitions. The $T = 2$ states appear above 9.75 MeV.

B. ^{44}Ca$(p, t)^{42}$Ca Reaction

22 triton peaks are observed (Fig. 3) and 16 angular distributions are obtained (Fig. 4). By simply comparing with angular distributions for known-L transitions one can already assign new spins for the states at 4.90 (5^-), 5.79 (3^-), 6.08 (0^+), and 8.45 (0^+). Above 9-MeV excitation energy, five $T = 2$ analog states are seen which had never been observed before in a two-nucleon-transfer reaction. From the shape of the angular distribution new assignments are obtained for the 10.97-MeV (3^-) and 12.28-MeV (0^+) states.

C. ^{44}Ca$(p, {}^3$He$)^{42}$K Reaction

The energy resolution was too poor and the level density too high to observe excitation of individual states, except for the strong 0^+ state at 6.45 MeV which is the analog of the $T = 3$ ^{42}Ar ground state (Fig. 5).

D. ^{48}Ca$(p, t)^{46}$Ca Reaction

13 $T = 3$ states are seen in the ^{46}Ca spectra. Comparison of the angular distributions with those of known-L patterns leads to the following assignments: 4.43 MeV (3^-), 4.75 MeV (5^-), 5.38 MeV (3^-), and 7.83 MeV (0^+). The other states will be discussed with reference to the DWBA analysis. Two $T = 4$ states are seen in all spectra at 14.45 and 14.75 MeV.

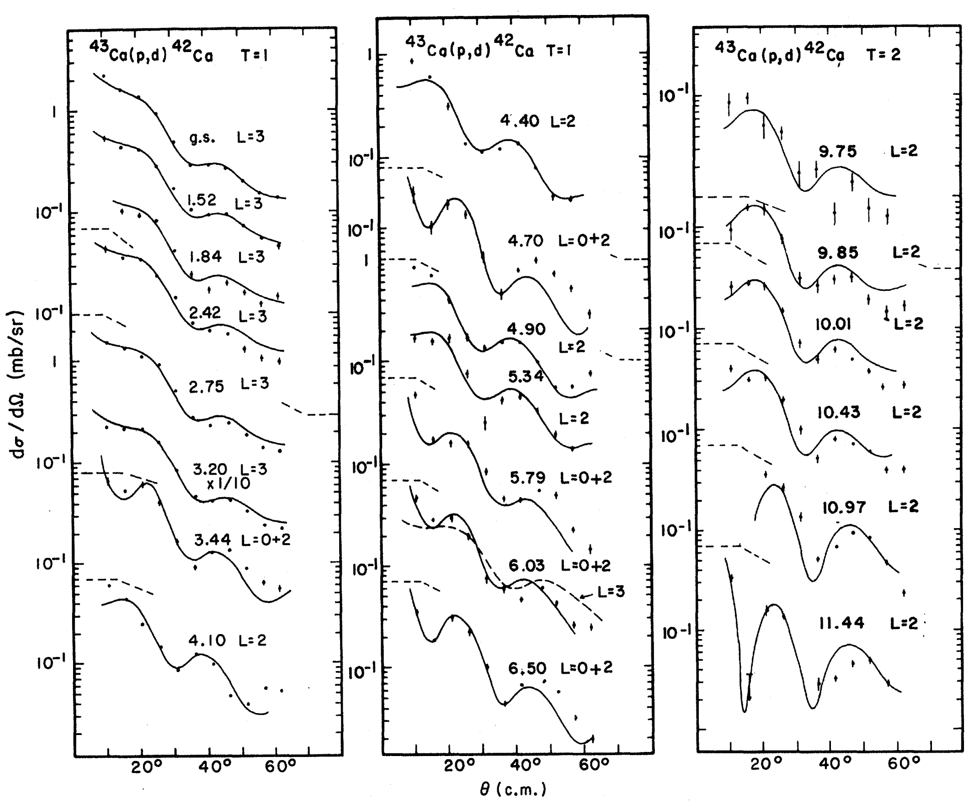

FIG. 2. Angular distributions and DWBA calculations for ^{43}Ca$(p, d)^{42}$Ca.

E. ^{48}Ca(p, ^{3}He)^{46}K Reaction

In the first experiment at 40 MeV seven $T=4$ states were observed. Two of them at, respectively, 1.35 and 2.26 MeV had never been observed before and were seen with only poor statistics. The experiment at 56 MeV confirms with better statistics the excitation of those two states (Fig. 6) and additional $T=4$ states were identified up to 3.5-MeV excitation energy. Higher up, a fairly well excited level is observed, its kinematic shift is only consistent with a state in ^{46}K at 11.47 MeV. This state is seen mostly at those angles where $L=0$ transitions have a maximum cross section, it might be the $T=5$ analog of the ^{46}Ar ground state.

IV. DISCUSSION OF ^{42}K AND ^{42}Ca

Since it can be studied by many types of reactions like one-or two-nucleon stripping or pickup, or by inelastic scattering, the nucleus ^{42}Ca occupies a favored place in the isotope chart.

The intent of this work is to present a detailed study of level configurations of the nucleus ^{42}Ca reached by (p, d) and (p, t) reactions. The relatively high energy of the beam allows a more precise analysis than in previous studies on this subject.

A. (p, d) Reaction

We only consider pickup in the $2s_{1/2}$, $1d_{3/2}$, and $1f_{7/2}$ shells. Only one state at 3.65 MeV was supposed to be excited via an $l=1$ transfer,[12,13] but due to the presence of contaminants it was impossible to analyze it in our experiment.

The spectroscopic factors are listed in Table III, in all cases we have supposed a pure $(f_{7/2}{}^3)_{7/2}$ configuration for the ^{43}Ca ground state.

For the positive-parity states we obtain the amplitude of the $(f_{7/2}{}^2)_J$ configuration in the different states. This amplitude is in general close to the value predicted by Gerace and Green. The only exception is the value found for the first 0^+ state which is a factor of 2 lower than the theoretical value given by Gerace and Green but closer to the value predicted in the more recent work of Flowers and Skouras.[14]

The negative-parity states are primarily excited via the $[2s_{1/2}{}^{-1}, (1f_{7/2}{}^3)_{7/2}]$ and $[1d_{3/2}{}^{-1}(1f_{7/2}{}^3)_{7/2}]$ configurations. These two configurations are found to be mixed in all 3^- $T=1$ states. The angular distributions for those states are composed of a sum of $l=0$ plus $l=2$ patterns; different percentages of these two transitions are to be checked in order to reproduce the experimental shapes. The $T=2$ states are much purer and no significant $l=0$ component was found in the 2^-, 3^-, 4^-, and 5^- $[1d_{3/2}{}^{-1}, (1f_{7/2}{}^3)_{7/2}]^{T=2}$ multiplet.

The sum rules are a test for the absolute values of the spectroscopic factors. The value found for the $l=3$ transfers $[\sum(C^2S)_{l=3}=1.88]$ is close to the theoretical value (2.07) obtained in taking into account the observed levels. Since nuclear-structure calculations are not available for negative-parity states, the comparison for $l=0$ and $l=2$ transfers is only possible with the maximum value. For these levels the ratio of spectroscopic factors for $T_<$ and $T_>$ states must be close to the theoretical value:

$$\sum_{T_<}(C^2S)_l / \sum_{T_>}(C^2S)_l = 3\,.$$

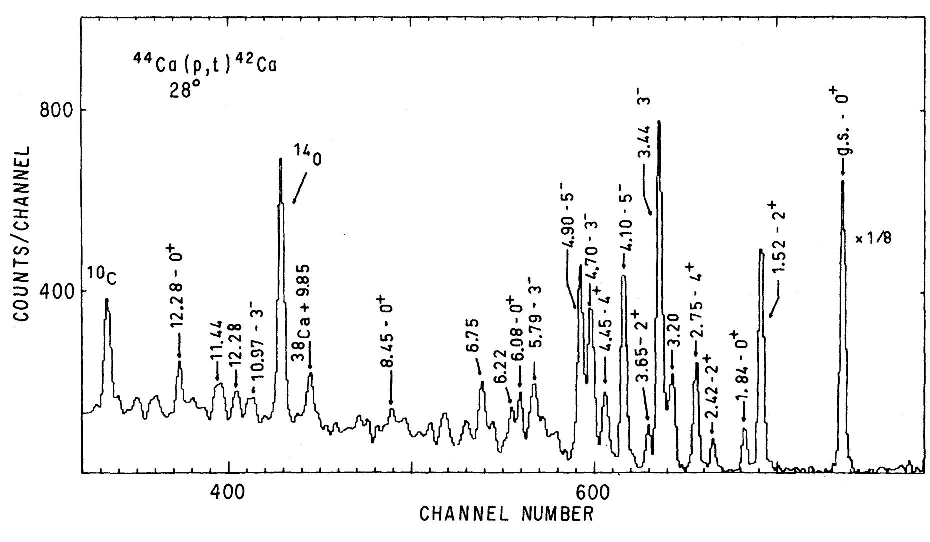

FIG. 3. Energy spectrum for the ^{44}Ca(p,t)^{42}Ca reaction.

TABLE III. Cumulative results of ^{43}Ca$(p,d)^{42}$Ca and ^{44}Ca$(p,t)^{42}$Ca reactions analysis. For (p,t) calculations and for positive-parity states, the numbers in parentheses indicate the amount of 6p-2h components taken for the ^{44}Ca(g.s.) configuration; for 3^- states the sign in parentheses indicates the phase between the two components whose amplitudes have been calculated in the (p,d) experiment. For calculations using pure configurations

$$a=\left[\left(\frac{d\sigma}{d\Omega}\right)_{\rm exp}\Big/310\left(\frac{d\sigma}{d\Omega}\right)_{\rm DWBA}\right]^{1/2}$$

represents the amplitude of that particular configuration in the final state. Phase and sign conventions are the same as in N. K. Glendenning, UCRL Report No. UCRL-19268 (unpublished). J^π values are from literature (Refs. 1, 4, and 12); those with an asterisk are from this work.

State E(MeV)	J^π	l_n	^{43}Ca$(p,d)^{42}$Ca C^2S	Amp.	Theory	L	^{44}Ca$(p,t)^{42}$Ca $a=\left[\left(\frac{d\sigma}{d\Omega}\right)_{\rm exp}/310\left(\frac{d\sigma}{d\Omega}\right)_{\rm DWBA}\right]^{1/2}$ With pure configurations		With Ref. 6 wave functions
g.s.	0^+	3	0.5	0.81	0.82 [a]	0	$(1f_{7/2}{}^2)_0$	1.55	1.33 (0), 1.16 (0.3)
1.52	2^+	3	0.15	0.60	0.66 [a]	2	$(1f_{7/2}{}^2)_2$	0.98	0.7 (0)
1.84	0^+	3	0.04	0.23	0.42 [a] 0.30 [b]	0	$(1f_{7/2}{}^2)_0$ $(1d_{3/2}{}^{-2})_0$	0.20 0.44	0.4 (0.3), 0.65 (0.5)
2.42	2^+	3	0.11	0.51	0.66 [a]	2	$(1f_{7/2}{}^2)_2$	0.39	0.96 (0)
2.75	4^+	3	0.42	0.74	0.75 [a]	4	$(1f_{7/2}{}^2)_4$	0.68	
3.20	6^+	3	0.66	0.78	0.97 [b]				Calculations using the (p,d) amplitudes
3.44	3^-	0 + 2	0.18 0.20	0.45 0.48		3	$[2s_{1/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$ $[1d_{3/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$	0.73 1.37	1.01 (−), 3.4 (+)
3.65	2^+	(1)				2	$(1f_{7/2},\ 2p_{3/2})_2$	0.19	
4.10	5^-	2	0.24	0.41		5	$[1d_{3/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_5$	0.37	
4.40	2^-	2	0.29	0.68					
4.45	4^+*					4	$(1f_{7/2}{}^2)_4$	0.55	
4.70	3^-	0 + 2	0.06 0.045	0.26 0.23		3	$[2s_{1/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$ $[1d_{3/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$	0.43 0.93	1.3 (−), >4 (+)
4.90	5^-*	2	0.33	0.49		5	$[1d_{3/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$	0.39	
5.34		2	0.10	0.31					
5.79	3^-*	0 + 2	0.057 0.07	0.25 0.28		(3)	$[2s_{1/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$ $[1d_{3/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$	0.34 0.64	0.8 (−), >4 (+)
6.03	$(3^-, 4^-)$*	(0 + 2)	0.09 0.11						
6.08	0^+*					0	$(2p_{3/2}{}^2)_0$	0.13	
6.50	$(3^-, 4^-)$*	0 + 2	0.12 0.07						
8.45	0^+*					0	Observed		
9.75	(2^-)	2	>0.03 <0.06	0.22 0.31	Maximum value =0.5 (for 9.75–10.43)				
9.85	(3^-)	2	0.10	0.33			Observed		
10.01	(4^-)	2	0.17	0.39					
10.43	(5^-)	2	0.23	0.41			Observed		
10.97	3^-*	0	0.10	0.34		(3)	$[2s_{1/2}{}^{-1},\ (1f_{7/2}{}^3)_{7/2}]_3$	0.20	
11.44	$(3^-, 4^-)$*	0	0.07				Observed		
12.28	0^+*					0	$[(1d_{3/2}{}^{-2})_0,\ (1f_{7/2}{}^4)_0]$	0.49	Max. value =0.57

[a] Reference 6.

[b] Reference 14.

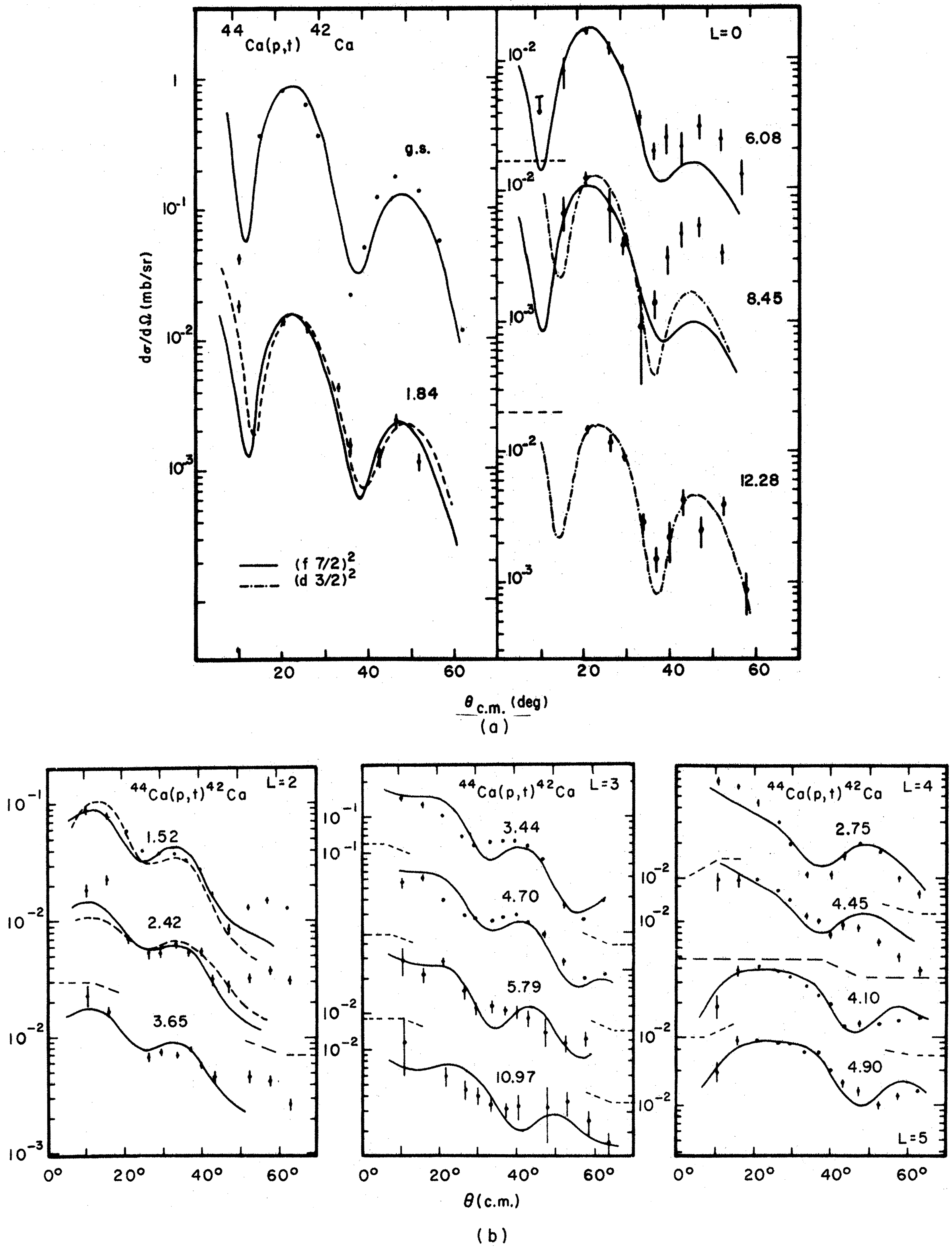

FIG. 4. (a), (b) Angular distributions and DWBA calculations for the ^{44}Ca$(p,t)^{42}$Ca reaction.

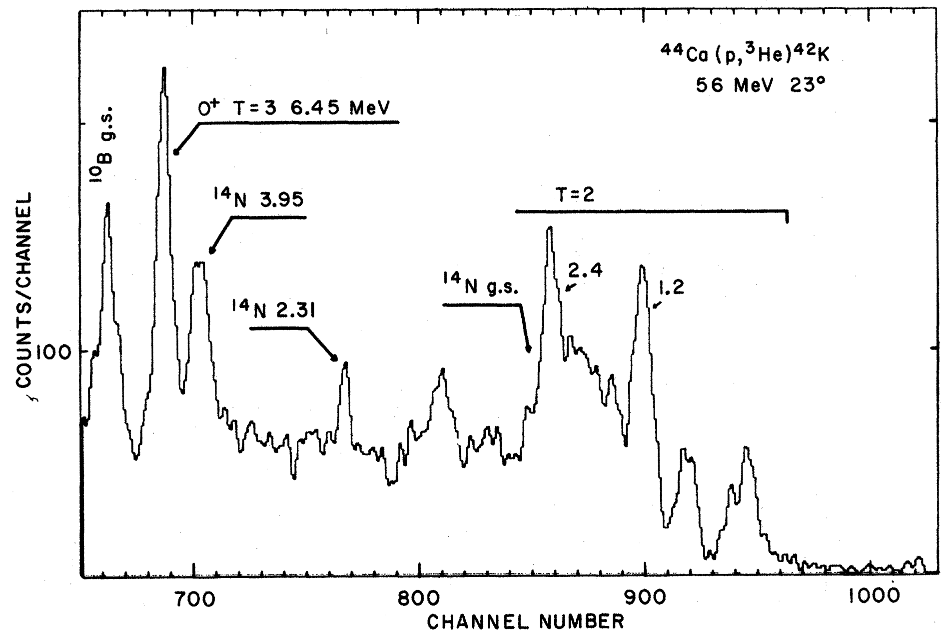

FIG. 5. Energy spectrum for the ^{44}Ca(p, ^{3}He)^{42}K reaction.

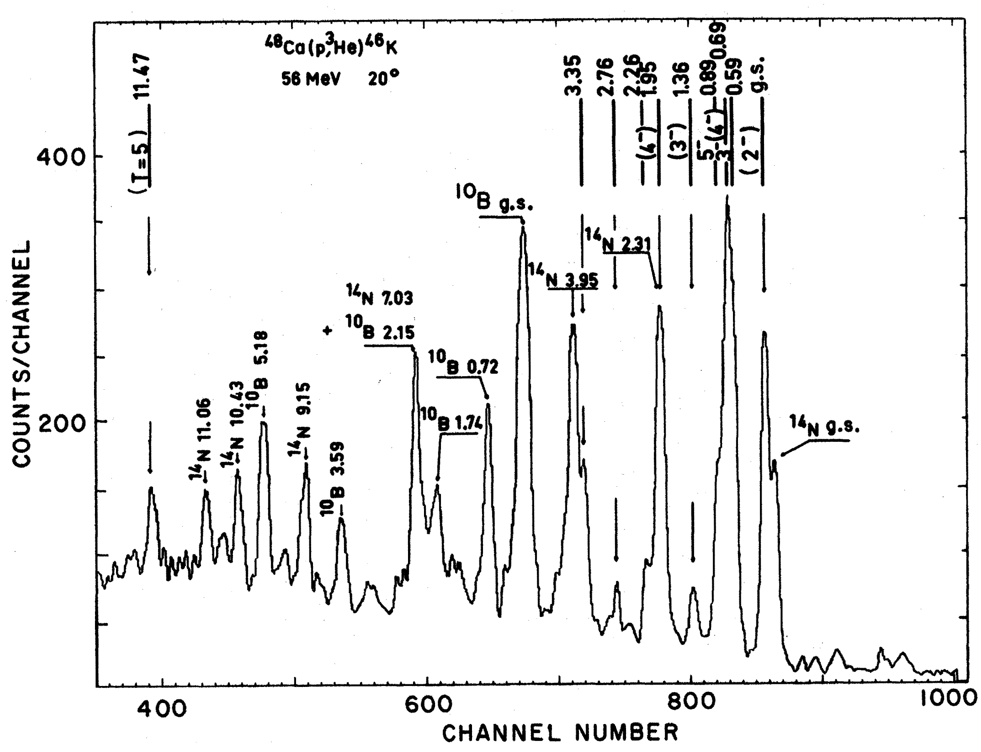

FIG. 6. Energy spectrum for ^{48}Ca(p, ^{3}He)^{46}K at $E_p = 56$ MeV.

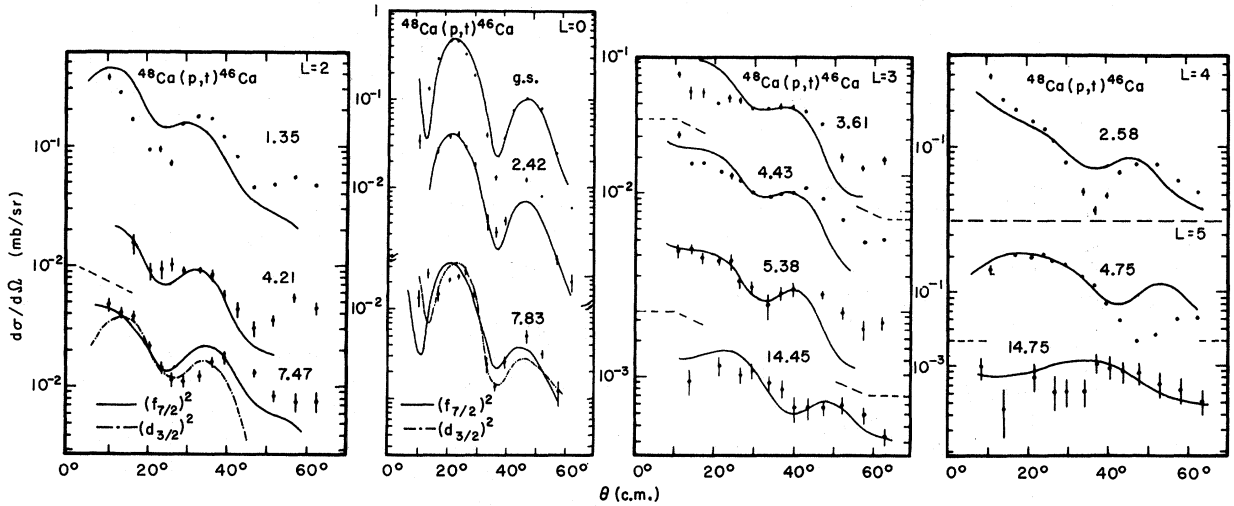

FIG. 7. Angular distributions and DWBA fits for the ^{48}Ca$(p,t)^{46}$Ca experiment.

The values found (3 for $l=0$ and 2.5 for $l=2$) confirm the accuracy of the relative values of the spectroscopic factors.

B. (p, t) Reaction

As many cross-checks are possible with the results of the ^{43}Ca$(p, d)^{42}$Ca experiment, this reaction will be a genuine test to determine what type of information one can reasonably expect from a two-nucleon-transfer reaction. Ground-state-to-ground-state transitions are ineffective to normalize DWBA calculations. As a matter of fact many configurations are to be taken into account and usually these transitions are greatly enhanced. A small change like 10% in the amplitude of the $(p_{3/2}{}^2)_0$ configuration induces a change of about 100% in the DWBA cross section. The $L=4$ and $L=5$ transitions are much more appropriate for that purpose; they essentially take place via the

TABLE IV. Comparison of amplitudes obtained in ^{43}Ca$(p, d)^{42}$Ca and ^{44}Ca$(p, t)^{42}$Ca experiments for states with single configuration. The (p, t) amplitudes are calculated using the empirical normalization of Flynn and Hansen (Ref. 10).

E (MeV)	Level J^{π}	Configuration	Amplitude (p, d)	(p, t)	Theory
2.75	4^+	$(f_{7/2}{}^2)_4$	0.74	0.68	0.75 [a]
4.10	5^-	$[(1d_{3/2}{}^{-1}), (1f_{7/2}{}^3)_{7/2}]_5$	0.41	0.37	
4.90	5^-	$[(1d_{3/2}{}^{-1}), (1f_{7/2}{}^3)_{7/2}]_5$	0.49	0.39	

[a] Reference 6.

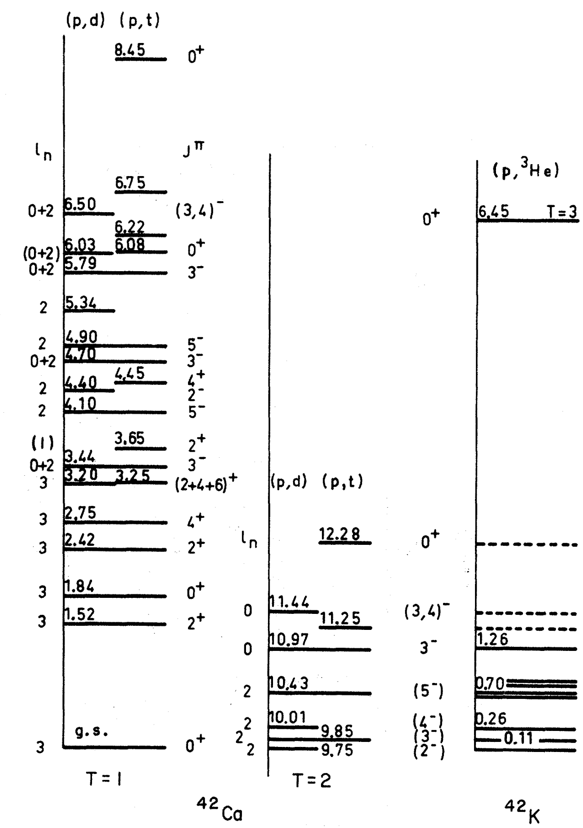

FIG. 8. Summary of states observed in ^{42}Ca and ^{42}K by the ^{44}Ca$(p,t)^{42}$Ca, ^{43}Ca$(p,d)^{42}$Ca, and ^{44}Ca$(p, {}^3$He$)^{42}$K reactions. Data on ^{42}K are taken from Ref. 12.

$(f_{7/2}{}^2)_{J=4}$ and $[1d_{3/2}{}^{-1}, (1f_{7/2}{}^3)_{7/2}]_{J=5}$ configurations. Table IV shows a comparison of the amplitudes obtained in the ^{43}Ca$(p, d)^{42}$Ca and ^{44}Ca$(p, t)^{42}$Ca reactions using the empirical normalization found by Flynn and Hansen.[10] The fact that the amplitudes obtained from the two reactions are very similar is a good test of the validity of the empirical normalization $N = 310$ for (p, t) reactions. We have therefore used this normalization to extract amplitudes of configurations when a single component is involved, or to test more complex wave functions.

C. Discussion of $T = 1$ States

In the ^{44}Ca$(p, t)^{42}$Ca reaction many configurations are involved in the excitation of positive-parity states. Calculations including 6p-2h configurations in the ^{44}Ca ground state are not available at present, it is still possible to use Federman and Pittel[2] wave functions and to add 6p-2h configurations in order to see their effect on the cross section. The results are summarized in Table III. Using a pure $(f_{7/2}{}^2)$ pickup, the DWBA calculation for the ground-state-to-ground-state transition indicates a strong enhancement. This enhancement is reduced in using a more elaborate wave function, a good agreement is obtained with a percentage of 30% of 6p-2h configuration in the ^{44}Ca(g.s.) wave function. The first excited 0^+ state is poorly reproduced in both (p, d) and (p, t) reactions and is probably more deformed than expected. Introduction of 30 to 50% of 6p-2h components in the ^{44}Ca(g.s.) improves appreciably the agreement with the experiment. Two additional 0^+ states are found at 6.08 and 8.45 MeV, one of them can be identified with a $(p_{3/2}{}^2)$ 0^+ state expected around 6 MeV.[2]

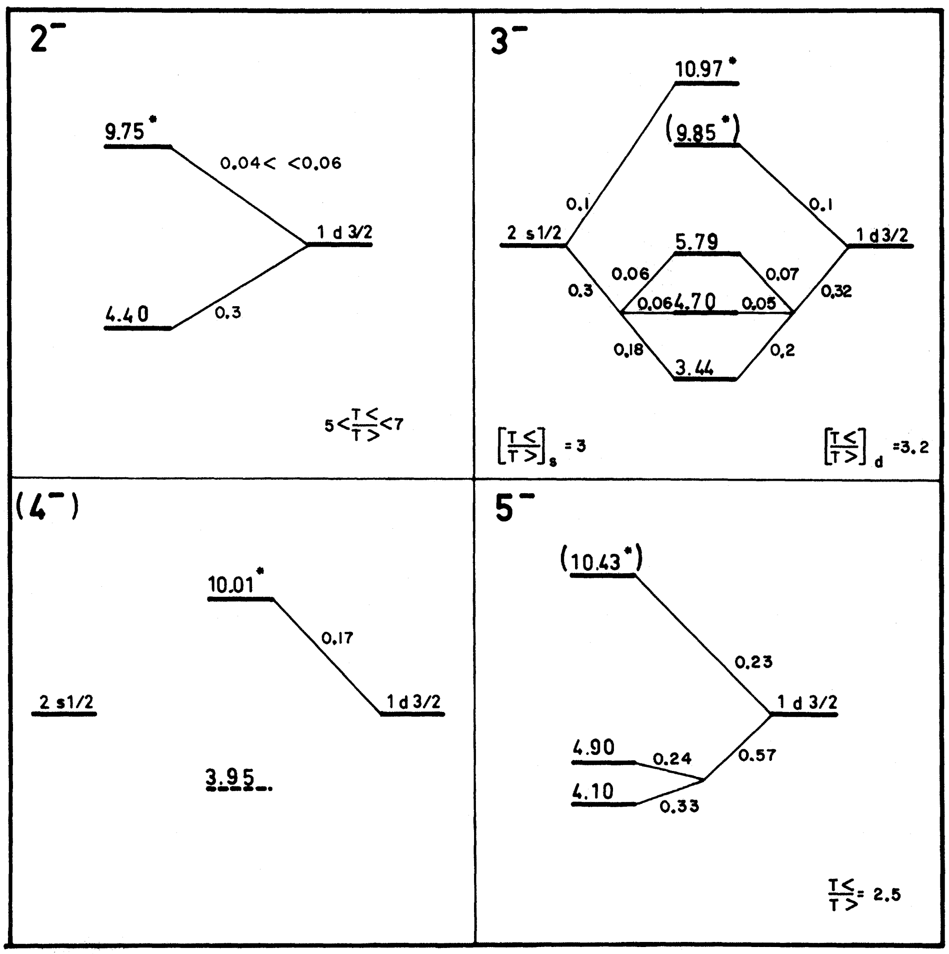

FIG. 9. Splitting of the $[1d_{3/2}{}^{-1}, (1f_{7/2}{}^3)_{7/2}]$ and $[2s_{1/2}{}^{-1}, (1f_{7/2}{}^3)_{7/2}]$ configurations in ^{42}Ca. For each state excitation energy and spectroscopic factors obtained in the ^{43}Ca$(p, d)^{42}$Ca reaction are indicated. ($T = 2$ states are noted with an asterisk.)

As indicated in Sec. III, we obtain for 3^- states complementary results from the analysis of (p, d) and (p, t) reactions. As seen in Table III, the difference between the positive and negative interference of the $[1d_{3/2}^{-1}, (1f_{7/2}^{3})_{7/2}]$ and $[2s_{1/2}^{-1}, (1f_{7/2}^{3})_{7/2}]$ configurations is larger than the uncertainty in the experiment or in the DWBA analysis. The fact that a negative sign is always found is at first sight surprising, but the ratio of the calculated cross sections when changing the sign indicates that 3^- states with a positive sign should be inhibited and certainly not seen in the (p, t) experiment.

D. Discussion of $T=2$ States and of ^{42}K

The $T=2$ states in ^{42}Ca were previously observed by Lynen *et al.*[12] in their ^{43}Ca(^{3}He, α)^{42}Ca experiment but due to the low incident energy the derived spectroscopic factors are not very reliable. We also observe $T=2$ states in our (p, d) and (p, t) experiments. The jj-coupling model for the $[1d_{3/2}^{-1}, (1f_{7/2}^{3})_{7/2}]$ configuration predicts a spin sequence 2^-, 3^-, 4^-, and 5^- with spectroscopic factors for a neutron pickup like 5/7/9/11. Table III shows that our results for spin assignments are in agreement with the results of Lynen *et al.*[12] In addition the (p, t) reaction confirms the natural parities for the states at 9.85 and 10.43 MeV.

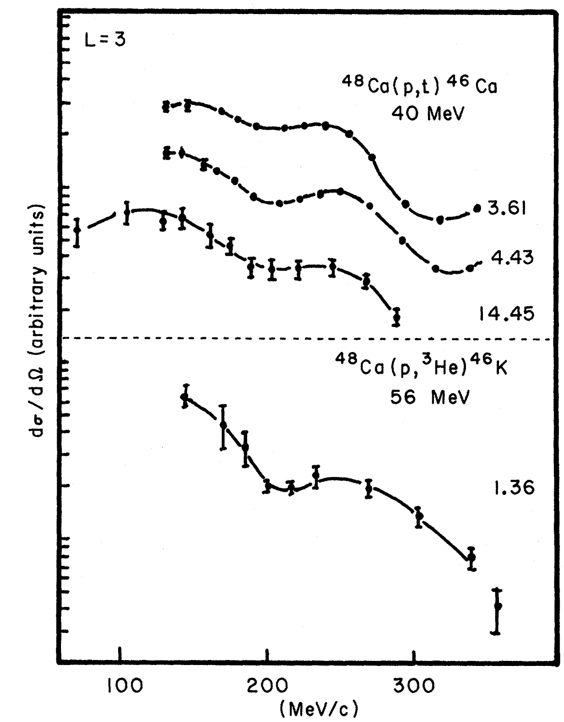

FIG. 10. Angular distribution for the state observed at 1.36 MeV in the ^{48}Ca$(p, ^3$He$)^{46}$K experiment at $E_p=56$ MeV compared with known $L=3$ transfers. The distributions are plotted versus momentum transfer.

TABLE V. Results of DWBA calculations for the ^{48}Ca$(p, t)^{46}$Ca reaction.

Level E (MeV)	J^π (this work)	Configuration	$\left[\left(\frac{d\sigma}{d\Omega}\right)_{exp}/310\left(\frac{d\sigma}{d\Omega}\right)_{DWBA}\right]$
0	0^+	$(1f_{7/2}^{-2})_0$	1.75
		Ref. 2 wave functions	1.27
1.35	2^+	$(1f_{7/2}^{-2})_2$	0.98
		Ref. 2 wave functions	1.10
2.42	0^+	$(1f_{7/2}^{-2})_0$	0.42
2.58	4^+	$(1f_{7/2}^{-2})_4$	0.62
3.61	3^-	$[(1d_{3/2}^{-1}), (1f_{7/2}^{-1})]_3$	0.74
4.21	(2^+)	$(1f_{7/2}^{-2})_2$	0.20
		Ref. 2 wave functions	0.5
4.43	3^-	$[(1d_{3/2}^{-1}), (1f_{7/2}^{-1})]_3$	1.29
4.75	5^-	$[(1d_{3/2}^{-1}), (1f_{7/2}^{-1})]_5$	0.57
7.47	(2^+)	$(1f_{7/2}^{-2})_2$	0.26
		$(1d_{3/2}^{-2})_2$	0.80
7.83	0^+	$(1f_{7/2}^{-2})_2$	0.25
		$(1d_{3/2}^{-2})_2$	0.63
14.45	3^-	$[(1d_{3/2}^{-1}), (1f_{7/2}^{-1})]_3$	0.44
14.75	5^-	$[(1d_{3/2}^{-1}), (1f_{7/2}^{-1})]_5$	0.18

Assuming a pure $(d_{3/2}^2)$ pickup for the 12.28-MeV 0^+ state, the calculations give an amplitude equal to 0.49 which is close to the maximum value $\langle 22\ 1-1|21\rangle=\sqrt{1/3}=0.57$ for the $T=2$ component of the $[(1d_{3/2}^{-2})_0,(1f_{7/2}^{4})_0]$ configuration.

Putting together all the data on the negative-parity $T=1$ and $T=2$ states one obtains a good description of the splitting of the (d,f) and (s,f) configurations. For each of them and for each J-coupling value the ratio

$$\sum_{T_<} C^2S_{lJT} / \sum_{T_>} C^2S_{lJT}$$

must be equal to 3 (C^2S_{lJT} being the spectroscopic factor found for each individual lJT state). Results are shown on Fig. 9. One sees that the situation is clear for 3^- and 5^- states, where the experimental ratios are close to 3. For the 2^- states the difference from this value is principally due to the uncertainty on the spectroscopic factor for the 9.75-MeV state.

In our experiment as in the results of Lynen *et al.*[12] no $T=1$, 4^- states have been observed (Figs. 8 and 9). It is quite surprising, since a 4^- state is to be expected at 3.95 MeV.[15] No peaks were seen in our data at this excitation energy. States at 6.03 and 6.50 MeV are possible 4^- states but due to their weak excitation the fact that they were not observed in the (p,t) experiment is not conclusive.

V. DISCUSSION OF ^{46}K AND ^{46}Ca

A. $T=3$ States

The level scheme of nucleus ^{46}Ca is expected to be very simple, low-lying states being built with two neutron holes in a doubly closed shell. At present the major part of the data are due to the study of the ^{44}Ca$(t,p)^{46}$Ca reaction[1]; ^{46}Ca has also been studied by inelastic scattering[16] or by ^{48}Ca$(p,t)^{46}$Ca.[4,17] Our results concern mainly spin-parity assignments. Some DWBA calcula-

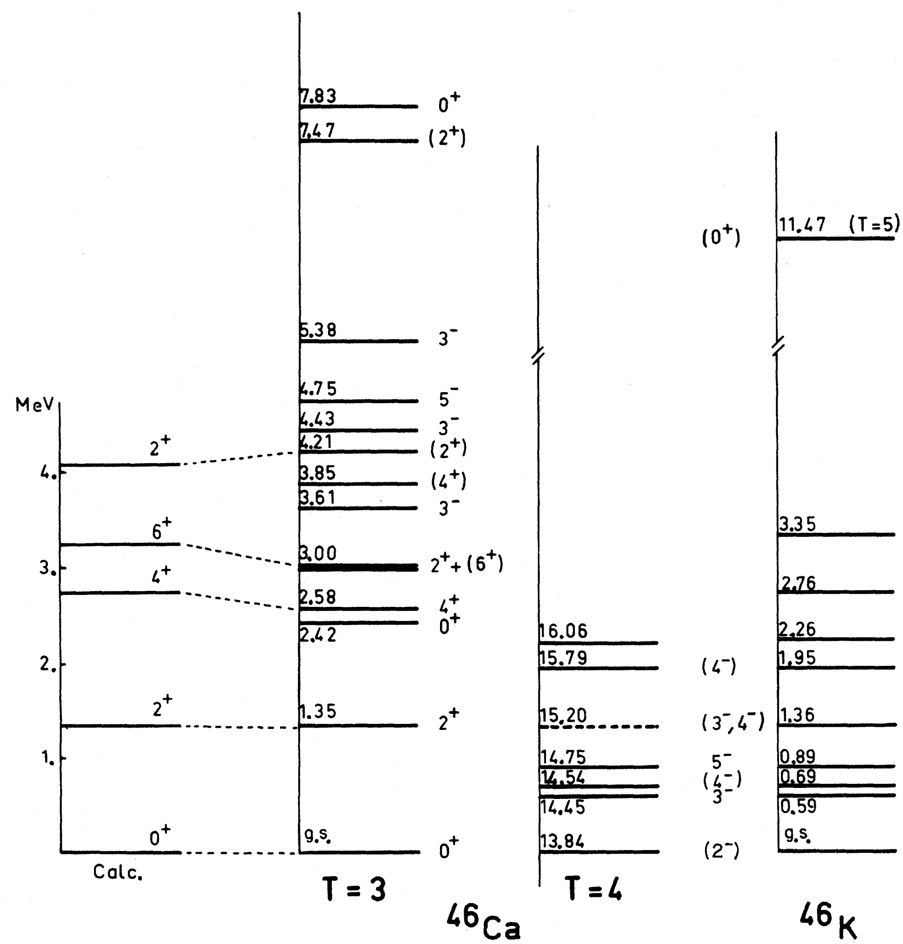

FIG. 11. Level scheme of ^{46}Ca and ^{46}K and comparison with the calculation of Federman and Pittel (Ref. 2).

tions were performed but they are restricted by the fact that nuclear-structure calculations including the (s, d) shell are not available at present. One sees in Table V that the cross section for the reaction to the 0^+ g.s. is enhanced even if one uses the Federman and Pittel wave functions.[2] Configurations like $[(d_{3/2}^{-2}), (f_{7/2}^{8})_0]$ are to be taken into account to explain this enhancement. Around 3.00 MeV a doublet is well excited. The spin of one of these states at 3.02 MeV is known as 2^+. The angular distribution obtained for this doublet is quite similar to an $L=2$ pattern at forward angles and displays a smooth maximum around 35°; this indicates that the second member is a high-spin state $(5^-, 6^+)$. This result is consistent with the results of Hefele *et al.*[4] and with the calculations of Federman and Pittel which give a 6^+ state at about 3 MeV (Fig. 11). At 4.21 MeV one sees a weak state for which the spin has been given as 5^- by Hefele *et al.* Figure 7 shows that its angular distribution is also compatible with a spin 2^+. If such is the case, it is well fitted by the Federman and Pittel calculations (Fig. 11).

At 4.43 MeV one finds a 3^- state whose cross section is greatly enhanced; this is due to the coherence of the $(1d_{3/2}^{-1}, 1f_{7/2}^{-1})_3$ and

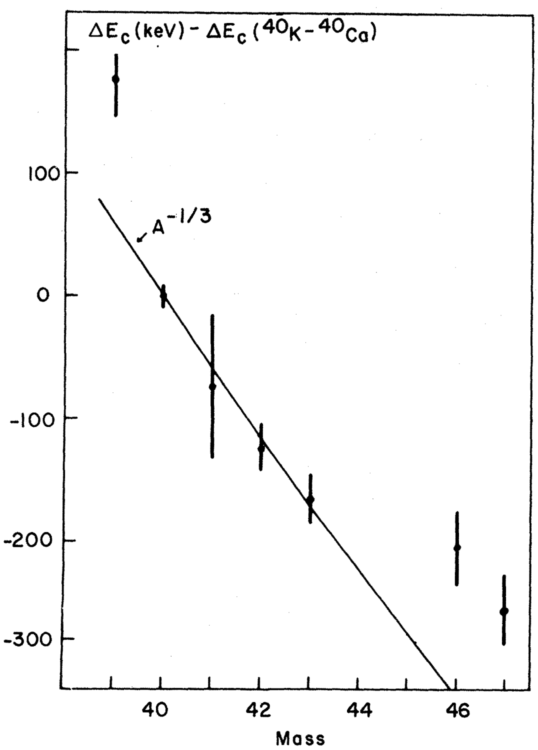

FIG. 12. The K-Ca Coulomb energy differences. $\Delta E_C(^{40}K-^{40}Ca)$ is equal to 7.114 ± 0.10 MeV.

$(2s_{1/2}^{-1}, 1f_{7/2}^{-1})_3$ configurations.

One should notice the disagreement between ^{44}Ca$(t, p)^{46}$Ca [1] and ^{48}Ca$(p, t)^{46}$Ca results concerning 4.43- and 4.75-MeV states. In the first experiment their spins are found to be 2^+ and 4^+, while in the second they are unambiguously 3^- and 5^-. In both experiments there is no evidence of presence of a doublet at these energies. One can observe that in ^{44}Ca$(t, p)^{46}$Ca experiment negative-parity states are poorly excited as they require the existence of strong (6p-2h) components in the ^{44}Ca(g.s.) wave function. On the other hand, positive-parity states above 3 MeV are very scarce when studying the ^{48}Ca$(p, t)^{46}$Ca experiment. They require the existence of pairs of nucleons in the (f, p) shell for the ^{48}Ca(g.s.). The disagreement between both results will only be removed by a γ-decay experiment.

The 0^+ and 2^+ states are observed at 7.83 and 7.47 MeV. They are probably excited via a $(1d_{3/2}^{2})$ pickup.

B. $T=4$ States and ^{46}K

Another interesting aspect of the study of ^{46}Ca is the observation of $T=4$ states, analogs to the ^{46}K low-lying states. Although ^{46}K is a doubly-odd nucleus the proximity of two closed shells should give a very simple level scheme. One should observe a hole-hole spectrum corresponding to the $(1d_{3/2}^{-1}, 1f_{7/2}^{-1})$ configuration. In an earlier paper[11] we have shown that it was necessary to include the $(2s_{1/2}^{-1}, 1f_{7/2}^{-1})$ configuration in order to explain the low-lying states of ^{46}K. The (s, d) shell being at about the same energy in this region, these configurations are expected to be strongly mixed in 3^- and 4^- states. Therefore, comparison with p-h spectrum in ^{40}K or p-p spectrum in ^{38}Cl will be meaningless.

By comparison of the (p, t) and $(p, {}^3\mathrm{He})$ experiments we have obtained some spins of the first excited states of ^{46}K.[11] In the 40-MeV ^{48}Ca$(p, {}^3\mathrm{He})$-^{46}K experiment additional weak states were observed at 1.36 and 2.26 MeV; in the 56-MeV experiment an angular distribution for the state at 1.36 MeV has been drawn which is characteristic of an $L=3$ transfer (Fig. 10). The weak excitation of this state does not allow a comparison with the (p, t) experiment and we are not able to distinguish between spin 3^- and 4^-.

Two papers were recently published on the study of ^{46}K, they essentially use ^{48}Ca$(d, \alpha)^{46}$K experiments. The results of one of them[18] are in complete disagreement with ours. The assignments are based on a comparison of the shapes of the angular distributions with those obtained for the ^{42}Ca$(d, \alpha)^{40}$K reaction, where the spins of residual states are well known. In the second paper Daehnick *et al.*[19] have confirmed our results in studying the ^{48}Ca(d, $\alpha\gamma$)^{46}K reaction, the only difference being an assignment of 3^- for the 1.95-MeV state; this is quite surprising since the strong inhibition of its analog in the ^{48}Ca(p, t)^{46}Ca experiment is in favor of an unnatural parity. In both papers the existence of an excited state at 1.36 ± 0.01-MeV excitation energy is confirmed.

VI. CONCLUDING REMARKS

Including 4p-2h deformed components Gerace and Green, and Flowers and Skouras were able to reproduce positive-parity states in the level scheme of ^{42}Ca. Our calculations on ^{43}Ca(p, d)^{42}Ca and ^{44}Ca(p, t)^{42}Ca reactions demonstrate the accuracy of this model. The introduction of about 30% of 6p-2h component in ^{44}Ca(g.s.) wave function is still necessary to explain properly the $0^+ \to 0^+$ transitions in the (p, t) experiment.

We now have a more precise idea on the configuration of the $T=1$ state of ^{42}Ca, by the same way we have obtained a clear spectrum of ^{46}Ca up to 8 MeV for $T=3$ states (Fig. 11). In these reactions $T=1$, 2, and 3 states have been identified in mass $A=42$ and $T=3$, 4, and 5 in mass $A=46$. These results and other results[20] on the calcium isotopes enable us to make a systematic study of Coulomb energy difference (ΔE_C) for the calcium and potassium isotopes. For ΔE_C (Ca-K), Fig. 12 shows a disagreement from the $A^{-1/3}$ law for masses 46 and 47. This phenomenon is related to a reduction of the nuclear radius r_0 in the neighborhood of the $f_{7/2}$ shell closure. The same fact has been pointed out before in electron scattering or muonic x-ray measurements.[21] Using the rough approximation of an uniformly charged sphere of radius $R=r_0A^{1/3}$, this corresponds to a reduction of about 2 to 3% in the nuclear radius r_0.

---

*Present address: Max-Planck-Institut für Medizinische Forschung – Heidelberg.

†Present address: Centre de Génétique Moléculaire, Gif-sur-Yvette.

[1]J. H. Bjerregaard, O. Hansen, O. Nathan, R. Chapman, S. Hinds, and R. Middleton, Nucl. Phys. A103, 33 (1967).

[2]P. Federman and S. Pittel, Nucl. Phys. A155, 161 (1970).

[3]S. M. Smith and A. M. Bernstein, Nucl. Phys. A125, 339 (1969).

[4]H. Hefele, U. Lynen, and R. Santo, Nucl. Phys. A157, 93 (1970).

[5]J. Cerny, R. H. Pehl, and G. T. Garvey, Phys. Rev. Letters 12, 619 (1964).

[6]W. J. Gerace and A. M. Green, Nucl. Phys. A93, 110 (1967).

[7]F. G. Perey, private communication.

[8]T. Inouye, T. Harper, and N. C. Rasmussen, Nucl. Instr. Methods 67, 125 (1969).

[9]N. K. Glendenning, Phys. Rev. 137, B102 (1965).

[10]E. R. Flynn and O. Hansen, Phys. Letters 31B, 135 (1970).

[11]Y. Dupont, P. Martin, and M. Chabre, Phys. Letters 31B, 68 (1970).

[12]U. Lynen, H. Oeschler, R. Santo, and R. Stock, Nucl. Phys. A127, 343 (1969).

[13]J. L. Yntema, Phys. Rev. 186, 1144 (1969).

[14]B. H. Flowers and L. D. Skouras, Nucl. Phys. A136, 353 (1969).

[15]A. Tellez, R. Ballini, J. Delaunay, and J. P. Fouan, Phys. Letters 29B, 655 (1969).

[16]J. A. Belote, J. H. Bjerregaard, O. Hansen, and G. R. Satchler, Phys. Rev. 138, 1067 (1965).

[17]R. J. Peterson, Phys. Rev. 170, 1003 (1968).

[18]M. Paul, A. Marinov, Ch. Drory, J. Burde, E. Navon, and Sh. Mardechai, Nucl. Phys. A168, 267 (1971).

[19]W. W. Daehnick, J. Orloff, T. Canada, and T. S. Bhatia, Bull. Am. Phys. Soc. 16, 555 (1971).

[20]P. Martin, M. Buenerd, Y. Dupont, and M. Chabre, Nucl. Phys. A185, 465 (1972).

[21]F. G. Perey and J. P. Schiffer, Phys. Rev. Letters 17, 324 (1966).