Obtainability of Two-Body Off-Shell Information from Elastic Nucleon-Nucleus Scattering at Intermediate Energy

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In this paper, we include off-shell two-body T matrices in both the unary and binary terms of the optical potential derived from Watson's multiple-scattering theory. We take a wholly momentum-space-oriented approach: All calculations are performed in terms of momentum variables and spatial coordinates do not appear. We conclude that it is not possible at present to obtain definitive information on the off-shell nucleon-nucleon T matrix from intermediate-energy elastic nucleon-nucleus scattering. On the other hand, the size of the off-shell effect is sufficiently large that it is not possible to unambiguously select a model for the pair-correlation function without a definite model for the off-shell nucleon-nucleon scattering amplitude.

I. INTRODUCTION

Watson[1] has derived an optical-potential operator U for elastic nucleon-nucleus scattering as an expansion in terms of the scattering operator $\tau_i(E^0)$ which is defined by

$$\tau_i(E^0) = v_i + v_i Q_0 G(E^0)\tau_i(E^0) \tag{1}$$

with

$$G(E^0) = (E^0 + \epsilon_0 + i\eta - \hat{E} - H_N)^{-1}. \tag{2}$$

Both the potential, U, and the scattering operators are defined in the nucleon-nucleus barycentric subspace at the kinetic-energy eigenvalue E^0 of the nucleon-nucleus center-of-mass energy operator $\hat{E}$, where a caret is used to denote the operator of a kinematic variable. v_i is the free two-body interaction between the incident and ith target nucleon. Q_0 projects off the ground state $|0\rangle$ with energy ϵ_0 of the nuclear Hamiltonian H_N, while $P_0 = |0\rangle\langle 0|$ projects onto the ground state.

To second order in the $\tau_i(E^0)$, the Watson expansion for U is

$$U(E^0) = \sum \tau_i(E^0) + \sum{}' \tau_i(E^0) Q_0 G(E^0) \tau_j(E^0). \tag{3}$$

Unprimed summations extend over all target-nucleon indices, $1, \ldots, A$, while primed summations do not allow any consecutive indices to be equal.

In this paper we examine the possibility of obtaining off-shell two-body information from nucleon-nucleus scattering. We consider the intermediate-energy elastic case in detail and we conclude that off-shell information cannot be extracted from this process. The possibility of using nonelastic collisions to obtain off-shell information is also discussed. In Sec. II, a review is made of the extended-impulse-approximation (EIA) approach introduced by Adelberg and Saperstein[2] (henceforth referred to as AS1). In Sec. III, the optical potential is expressed up to second order in the EIA scattering operators, and the forms of the potential terms are given. In Sec. IV, an outline of the computational procedure is given with the goal of describing the numerical aspects of the calculation and hence showing what approximations should be made for intermediate-energy elastic nucleon-nucleus scattering. These approximations are made in Sec. V to the forms given in Sec. III. Finally, in Secs. VI and VII, respectively, the results of several calculations and our conclusions are discussed.

II. REVIEW OF THE EIA APPROACH

In AS1 we introduced the modified scattering operator

$$\bar{\tau}_i(E^0) = v_i + v_i \tilde{G}_i(E^0)\bar{\tau}_i(E^0) \tag{4}$$

with

$$\tilde{G}_i(E^0) = (E^0 + \epsilon_0 + i\eta - \hat{E} - \tilde{H}_i)^{-1}; \tag{5}$$

$\bar{\tau}_i$ is a many-body operator defined in the nucleon-nucleus barycentric subspace. $\tilde{G}_i$ differs from G in that H_N has been replaced by the model-dependent operator $\tilde{H}_i$ which must be diagonal in all momenta so that $\bar{\tau}_i$ has the same momentum-conserving properties as the free two-body scattering operator t_i given by

$$t_i(w_i) = v_i + v_i g_i(w_i) t_i(w_i) \tag{6}$$

with

$$g_i(w_i) = (w_i + i\eta - \hat{e}_i)^{-1}, \tag{7}$$

where $\hat{e}_i$ is the operator of the two-body center-of-mass energy. Both $\bar{\tau}_i$ and t_i are defined only

on the momentum shell of the incident and ith target nucleons and do not change the momenta of the other target nucleons. The energy parameter w_i in contrast to E^0 which is fixed, varies according to the values of the kinematic variables of the optical-potential matrix elements, when t_i is used in the approximate construction of U.

The essence of the EIA approach is to establish an unambiguous relationship between the matrix elements of $\bar{\tau}_i$ and those of t_i. This was accomplished using nonrelativistic kinematics which imply that the nucleon-nucleon final and initial momenta, $\vec{k}_i$ and $\vec{k}_i'$, and the nucleon-nucleus final and initial relative momenta, $\vec{K}$ and $\vec{K}'$, are related by

$$\vec{k}_i = c\vec{K} - \tfrac{1}{2}\vec{l}_i \quad \text{and} \quad \vec{k}_i' = c\vec{K}' - \tfrac{1}{2}\vec{l}_i' , \tag{8}$$

where

$$c = (A+1)/2A \tag{9}$$

and where $\vec{l}_i$ and $\vec{l}_i'$ are the final and initial momenta of the ith target nucleon relative to the nuclear center of mass. By definition, these internal momenta are not independent since they must obey

$$\sum \vec{l}_i = 0 = \sum \vec{l}_i' . \tag{10}$$

Because the only absolute momenta changed by $\bar{\tau}_i$ are those of the incident and ith target nucleons, the momentum transfer, $\vec{q}$, of the incident nucleon to the nucleus is entirely taken up by the struck nucleon; this implies that

$$\vec{k}_i - \vec{k}_i' = \vec{q} = \vec{K} - \vec{K}' . \tag{11}$$

The kinematic relations are wholly contained in (8) and (11); no approximation besides that of nonrelativistic kinematics need be made.[3] However, in order to arrive at an expression which is practical for use in an optical potential, the dependence of the nucleon-nucleon kinematic variables on internal momenta is neglected.

Because the optical potential for elastic scattering,

$$U_0(E^0) = \langle 0|U(E^0)|0\rangle , \tag{12}$$

must be reciprocally invariant,[4] the neglect of internal momenta should maintain reciprocal symmetry; this can be accomplished by interpreting the ground-state wave function as

$$\langle \bar{\vec{l}}_1, \ldots, \bar{\vec{l}}_A|0\rangle ,$$

where

$$\bar{\vec{l}}_i = \tfrac{1}{2}(\vec{l}_i + \vec{l}_i'), \quad i = 1, \ldots, A. \tag{13}$$

Thus, internal momenta are neglected by taking $\bar{\vec{l}}_i \to 0$ in the kinematic relations, rather than taking both $\vec{l}_i \to 0$ and $\vec{l}_i' \to 0$. In this way, we derived in AS1 the relationship

$$\langle \vec{K}|\bar{\tau}_i(E^0)|\vec{K}'\rangle = \langle \vec{k}_i|t_i(w_i)|\vec{k}_i'\rangle \tag{14}$$

between matrix elements, where the scalars of the kinematic variables are related by (dropping the particle index)

$$k^2 = \tfrac{1}{2}(c^2+c)K^2 + \tfrac{1}{2}(c^2-c)K'^2 + \tfrac{1}{4}(1-c^2)q^2 ,$$

$$k'^2 = \tfrac{1}{2}(c^2-c)K^2 + \tfrac{1}{2}(c^2+c)K'^2 + \tfrac{1}{4}(1-c^2)q^2 , \tag{15}$$

and

$$kk' \cos\theta = \tfrac{1}{2}c^2(K^2+K'^2) - \tfrac{1}{4}(1+c^2)q^2 .$$

Also, neglecting a very small q-dependent term, we obtained

$$w = (\tfrac{1}{2}c - \tfrac{1}{2})(K^2+K'^2)/2\mu + K_0^2/2\mu , \tag{16}$$

where μ is the reduced mass equal to $(1+1/A)^{-1}$ times the nucleon mass m and where K_0 is the momentum corresponding to E^0. We obtained (16) by interpreting

$$\overline{\sum l_i^2}/2m - \sum \bar{l}_i^2/2m = \tfrac{1}{4}(1-1/A)q^2/2m \tag{17}$$

as the excitation of the kinetic-energy part of the nucleus and by assuming that the total excitation should be approximately double the right-hand side of (17). Thus, $\vec{H}_i$ was chosen to have the same kinematic part as H_N and a dynamic part approximately equal to the right-hand side of (17).

The kinematic relations (15) and (16) imply that the matrix elements of t_i given by (14) are generally off their energy shell. In other words, one or both of the equalities, $k^2/m = w$ and $k'^2/m = w$, fail. The matrix elements are said to be "on-shell" only when both equalities hold; otherwise, they are "off-shell." If only one fails, they are "half-shell," while if both fail they are "fully off-shell" in general and "symmetrically off-shell" only when $k = k'$. These situations will be denoted by the lower case abbreviations: oes, fes, hes, ffs, and sfs, respectively.

Also, (16) shows that the two-body energy parameter w is not fixed at the energy $e^0 = cE^0$, which corresponds to the free scattering of two nucleons. In fact, w is negative when $K^2+K'^2$ is greater than $4A/(A-1)$ times K_0^2, and w has the maximum value of E^0 when both K and K' are zero.

III. EIA POTENTIAL TO SECOND ORDER

We expand the potential operator U in terms of $\bar{\tau}_i$ by using the identity

$$\tau_i(E^0) = \bar{\tau}_i(E^0) + \bar{\tau}_i(E^0)[Q_0 G(E^0) - \tilde{G}_i(E^0)]\tau_i(E^0) \tag{18}$$

to replace the τ_i by the EIA operators. To second

order in the $\bar{\tau}_i$,

$$\tau_i = \bar{\tau}_i - \bar{\tau}_i P_0 G \bar{\tau}_i + \bar{\tau}_i (H_N - \tilde{H}_i)\bar{\tau}_i \ . \tag{19}$$

The second term has previously been considered [5] as a correction to the impulse approximation of the form $\tau_i \to \bar{\tau}_i$. This interpretation depends on the initial choice of modified scattering operator in terms of which U is expanded. In this paper, only the last term of (19) is considered as an impulse correction. This term represents [6] a higher-order dynamical correction, which hopefully could be made small by the appropriate choice of $\tilde{H}_i$. The impulse approximation used here is "extended" in the sense that the kinematics of a nucleon scattering from a nucleus are treated consistent with momentum conservation and the nonrelativistic definitions of relative momenta.

Thus, neglecting the last term of (19) the EIA potential operator, to second order in the $\bar{\tau}_i$, is

$$U(E^0) = \sum \bar{\tau}_i(E^0) + \sum{}' \bar{\tau}_i(E^0) G(E^0) \bar{\tau}_j(E^0) - \sum \bar{\tau}_i(E^0) P_0 G(E^0) \bar{\tau}_i(E^0) \ . \tag{20}$$

The optical potential for elastic scattering is obtained by taking the ground-state expectation value of (20). Matrix elements of the potential with respect to the nucleon-nucleus relative momenta are then given by

$$\langle \vec{k}|U_0(E^0)|\vec{k}'\rangle = U_1(\vec{K}, \vec{K}'; E^0) + U_2(\vec{K}, \vec{K}'; E^0). \tag{21}$$

The first-order term is

$$U_1(\vec{K}, \vec{K}'; E^0) = \langle \vec{k}0|\sum \bar{\tau}_i|0\vec{k}'\rangle \ , \tag{22}$$

while the second-order potential has the terms

$$U_2(\vec{K}, \vec{K}'; E^0) = U_C(\vec{K}, \vec{K}'; E^0) - U_P(\vec{K}, \vec{K}'; E^0), \tag{23}$$

which are given by

$$U_C(\vec{K}, \vec{K}'; E^0) = \langle \vec{k}0|\sum{}' \bar{\tau}_i G(E^0) \bar{\tau}_j|0\vec{K}'\rangle \ , \tag{24}$$

$$U_P(\vec{K}, \vec{K}'; E^0) - \langle \vec{k}0|\sum \bar{\tau}_i|0\rangle G_0(E^0)\langle 0|\sum \bar{\tau}_i|0K'\rangle, \tag{25}$$

where

$$G_0(E^0) = \langle 0|G(E^0)|0\rangle = (E^0 + i\eta - \hat{E})^{-1} \ . \tag{26}$$

For the second-order terms, the label "C" in (24) refers to the fact that this part of the optical potential depends upon correlations between two target nucleons. The potential in (25) with the label "P" is independent of correlations because of the projection operator, P_0, onto the nuclear ground state.

The first-order potential was evaluated in AS1. Complete sets of final and initial internal momentum states were inserted to the left and right, respectively, of the EIA operators in (22). The momentum conserving properties of the $\bar{\tau}_i$ and Gallilean invariance were used. Finally, the evaluation was made in what we shall refer to as "the form-factor approximation," that is, the vector-average internal momenta are taken as zero in the kinematic relations Eqs. (15) and (16) between the nucleon-nucleon and nucleon-nucleus momentum and energy variables. The resulting first-order potential is

$$U_1(\vec{K}, \vec{K}'; E^0) = \overline{M}_S(k, k', \theta; w) F_1(q) \ . \tag{27}$$

F_1 is the "one-body" form factor, which is the Fourier transform of the one-body probability density in coordinate space. $\overline{M}_S$ is the spin-isospin average of $\sum \bar{\tau}_i$ times factors of proportionality determined by normalization conventions; it is constructed from the generally fes two-body partial-wave amplitudes as a linear combination whose coefficients depend only on symmetries of the nuclear state and the two-body interaction, so that the coefficients are the same fes as well as oes.[7]

The second-order terms, like the first-order potential, are evaluated by inserting complete sets of final and initial internal momentum states, but it is also necessary to insert a complete set of intermediate nucleon-nucleus momentum states. Denoting the intermediate momentum as $\vec{K}''$, the momentum transfers of the second and first scatterings are given, respectively, by

$$\vec{q}_a = \vec{K} - \vec{K}'', \quad \vec{q}_b = \vec{K}'' - \vec{K}' \ . \tag{28}$$

An additional difficulty is incurred in the U_C term, because the Green's function $G(E^0)$ depends upon the intermediate state of the nucleus. Thus, we make the model-dependent approximation,

$$G(E^0) = (E^0 + i\eta - \Delta - \hat{E})^{-1}, \tag{29}$$

which is a modification of the closure approximation in which Δ is taken as zero. Our model for Δ is discussed in Sec. V. The resulting second-order terms are

$$U_C(K, K'; E^0) = \int d^3K'' (E^0 + i\eta - \Delta - E'')^{-1} \times [\overline{M}_D F_{2D}(q_a, q_b) + \overline{M}_E F_{2E}(q_a, q_b)] \tag{30}$$

and

$$U_P(K, K'; E^0) = \int d^3K'' (E^0 + i\eta - E'')^{-1} \overline{M}_P F_1(q_a) F_1(q_b) \ . \tag{31}$$

$\overline{M}_D, \overline{M}_E$, and $\overline{M}_P$ are spin-isospin averages that are quadratic in the nucleon-nucleon amplitude; they

depend upon nucleon-nucleon kinematic variables k_a, k'_a, θ_a, and w_a for the second scattering and k_b, k'_b, θ_b, and w_b for the first scattering, that are given by relations analogous to (15) and (16), using K, K'', q_a and K'', K', q_b, respectively, in place of K, K', q. F_{2D} and F_{2E} are two-body form factors ("D" for direct and "E" for exchange) that are the Fourier transforms of corresponding two-body coordinate space densities, which in turn are related to the one-body density and so-called correlation functions. Our models for the correlation function and final approximations are given in Sec. V.

IV. OVERVIEW OF THE MOMENTUM-SPACE-IMPULSE APPROACH

In AS1, we found that the fes effects on the elastic observables calculated from the unary potential were of insufficient size to allow fes information to be extracted from the calculation, especially since double scattering, depending on unknown nuclear correlation functions, was not considered. In this paper, we seek the answer to the converse: can nuclear-structure information be extracted from the intermediate-energy elastic calculation in view of the fes uncertainty?

In this section we discuss the calculational procedure of the momentum-space approach. Our objectives are to ascertain what information we can reasonably hope to obtain about the off-energy-shell parts of the nucleon-nucleon interaction from nucleon-nucleus collisions, in particular from intermediate-energy elastic scattering of nucleons from completely spinless nuclei and to see what this combination of data and fes information can tell us about nuclear structure, specifically, nuclear correlation functions. By completely spinless we mean that total neutron and proton spins are separately zero and that total isospin is also zero. Moreover, we calculate observables only for ^{4}He, ^{12}C, and ^{40}Ca, which are not only completely spinless but also have equal numbers of neutrons and protons. Thus, outside of the Coulomb interference region, observables should be the same for either incident neutrons or incident protons. We make these specializations in order to simplify the calculation, in particular, the evaluation of the spin averages, and to facilitate comparison of cross sections and polarizations, as calculated from potentials which keep the two-body matrices oes, with those that extrapolate the t_i fes.

Because of rotational symmetry, the optical potential is a function of three scalar kinematic variables, K, K', and Θ. To solve scattering equations in momentum space, finite sets of points are selected to represent the continuous variables. For example, we choose a set of momentum values in $(0, K_0)$ as well as their reciprocals in (K_0, ∞). We call K_0, which corresponds to E^0, the on-shell value of the nucleon-nucleus momentum; thus, we are referring to the nucleon-nucleus energy shell, which must be clearly distinguished from the nucleon-nucleon energy shell. We shall use the uppercase abbreviations, OES, FES, HES, SFS, and FFS, to refer to the many-body shell analogously to the lowercase abbreviations used for the two-body shell.

In the simplest coordinate-space approach,[8] K and K' are fixed at K_0, and the OES matrix elements of the potential, illustrated by the dotted line in Fig. 1, are Fourier transformed with respect to q to give a local potential. A Schrödinger equation is then solved. In momentum space, nonlocality of the potential plays no special role. Without extra work, we can retain the information contained in the FES matrix elements of the optical potential. This is important because (as discussed in AS1) the fes effects on the elastic nucleon-nucleus observables are essentially nonlocal, i.e., they stem from the FES potential matrix elements. The coordinate-space approach "works" because the form factors are rapidly decreasing functions of momentum transfer, thereby making the potential a very peaked function in q, hence only weakly dependent on K and K' when q is small and the matrix elements significantly large. In Fig. 1, the regions where q is small are inside

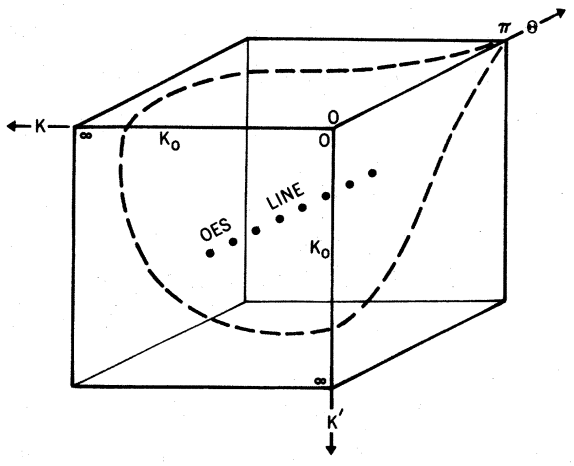

FIG. 1. Three-dimensional domain of the optical potential. Here K is the initial momentum, K' the final momentum, and θ is the scattering angle in the nucleon-nucleus center-of-mass frame. K_0 is the on-shell momentum, corresponding to the incident energy E^0. The OES line represents the range in scattering angle from 0 to π for $K = K_0 = K'$. The dashed surface encloses the volume outside of which the matrix elements are insignificant.

the dashed lines; outside, the matrix elements are nearly zero.

The momentum-space method continues by reducing the three-dimensional potential to partial-wave potentials U_{LJ} indexed by nucleon-nucleus orbital angular momentum L and total angular momentum, $J=L\pm\frac{1}{2}$ (for spinless nuclei). The next step is to obtain the solution (for which the entire FFS reduced potential matrix is required) to the integral equation with the standing wave Green's function G_S,

$$R_{LJ}(K,K')=U_{LJ}(K,K') + U_{LJ}(K,K'')G_S(K'')R_{LJ}(K'',K) \tag{32}$$

for the HES vector $R_{LJ}(K)=R_{LJ}(K,K_0)$, of the reduced reaction matrix. Principal value numbers, Π_{LJ}, are obtained through the principal-value quadrature,

$$\Pi_{LJ}=\int_0^\infty K^2dK(K_0^2-K^2)^{-1}U_{LJ}(K)R_{LJ}(K), \tag{33}$$

where $U_{LJ}(K)$ is the HES vector of the reduced potential. From the π_{LJ} and the OES U_{LJ}, we obtain the OES, reduced reaction and T matrix elements, which are given, respectively, by

$$R_{LJ}=U_{LJ}+\Pi_{LJ}, \quad T_{LJ}=R_{LJ}/(1+iR_{LJ}). \tag{34}$$

Finally, from the numbers T_{LJ}, the observables are calculated.

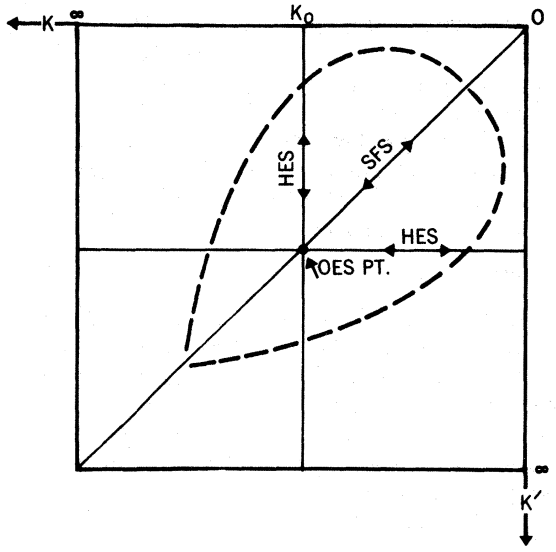

FIG. 2. Two-dimensional domain of the reduced optical potential. K and K' are the initial and final momenta, K_0 is the on-shell value, the half-shell line represents $K=K_0$ or $K'=K_0$, and the symmetric-off-shell line is the locus of $K=K'$.

For a given choice of points used to represent the momentum variable on $(0,K_0)$, a matrix equation approximates the integral equation (32) and a sum approximates the quadrature (33). The choice of points is discussed[9] with the goal of improving the performance of the program for a given desired accuracy in the calculated observables. In particular, a method is given for exploiting the reciprocal symmetry of the optical potential: Each matrix element of the potential $U_{LJ}(K,K')$ is multiplied by two weighting factors which are $+(K_0^2-K^2)^{-1/2}$ when $K<K_0$, $-(K_0^2-K^2)^{-1/2}$ when $K>K_0$, unity when $K=K_0$, and with the identical convention used for K'. Thus, the reduced potential matrix as well as the calculated reaction matrix are both symmetric with respect to K and K'.

In Fig. 2, the domain of a reduced (potential or reaction) matrix is illustrated. The matrix is symmetrical with respect to the main diagonal. Also, corresponding to the dashed curves of Fig. 1, there is a dashed curve in Fig. 2; outside of this oval, the potential matrix elements are nearly zero, and they do not significantly effect the HES reaction matrix elements. Thus, the significantly large matrix elements (potential and reaction) are near the main diagonal. If the information we seek is far from the main diagonal (i.e., outside the oval) elastic scattering cannot provide us with additional knowledge.

Moreover, from (34) we see that all of the physical information obtainable from the elastic process is contained in the two sets of numbers, U_{LJ} and Π_{LJ}. For the middle values of L (about 5–10) where most of the intermediate energy scattering occurs, our calculations show that the Π_{LJ} are about an order of magnitude smaller than the U_{LJ}. Therefore, for any input to have a large effect on the elastic observables, it must significantly change the U_{LJ} and/or drastically alter the Π_{LJ}.

Our calculations have clearly shown that the U_{LJ} are quite insensitive (always less than 5%) to fes (vs oes) input. Thus, the differences found in the observables due to going fes are due to changes in the Π_{LJ}, which can change as much as 50% for the small L, but only about 5–20% for the middle L. Since most of the intermediate-energy elastic scattering occurs in the middle values of L, this process is not sensitive to the fes extrapolation of the t_i.

It should be noted that, in addition to the rapid diminuition of U_0 with increasing q, there is another reason that matrix elements near and on the main diagonal are the most significant. The two-body t matrix, like the optical potential, must be reciprocally symmetric. In a partial-wave representation, this implies that the diagonal matrix elements (which are generally somewhat

larger than the nondiagonal that occur only in the coupled triplet channels) of the reduced t are symmetric with respect to the two-nucleon momentum variables k and k'. Therefore, the most rapid variation of the fes extrapolation occurs sfs. It follows from (15) that the $k=k'$ if and only if $K=K'$. Thus, the largest changes in the optical potential are found in SFS matrix elements. The $U_{LJ}(K,K)$ stay significantly large to greater FES distances than do the HES matrix elements; moreover, the $U_{LJ}(K,K)$ are constructed from only sfs t_i, while the HES matrix elements of U_{LJ} are constructed from t_i whose k and k' are on the opposite sides of the oes value κ given by $w=\kappa^2/m$. This discussion explains why we found changes due to fes extrapolation in the HES $R_{LJ}(K)$ were 2–4 times larger than changes in the HES $U_{LJ}(K)$. Our conclusions are that fes effects and the nonlocality of the optical potential are made manifest by the SFS matrix elements $U_{LJ}(K,K)$.

From the above analysis, we conclude that elastic scattering is unlikely to provide information on the fes nature of the nucleon-nucleon interaction. This has been suspected for some time now.[8] What is of current and possibly future interest is that the fes input manifests itself primarily in a nonlocal way, as described above, at least for the elastic process. Thus, in attempts to obtain fes information from nonelastic nucleon-nucleus processes, it is our opinion that the nonlocality of the potential must be maintained unless it can be shown that, for these processes, the Born term (which is HES not OES) manifests fes effects that dominate those due to the distortion term, which can be calculated via an optical potential.

However, an examination of the HES vectors (with *fixed* two-nucleon input) for the elastic case has shown their shape to be a plateau about the OES point. Thus, for the elastic process, the $U_{LJ}(K)$ are at or near a maximum for $K=K_0$, while $\Pi_{LJ}(K')$ defined by (33) with $U_{LJ}(K',K)$, is at or near a minimum when $K'=K_0$. Thus, unless the inelastic transition matrix elements radically alter the situation, the distortion term should be relatively more important for a nonelastic process than for the elastic case.

V. MODELS AND APPROXIMATIONS

In this section, we discuss our models and approximations made in the construction of the optical potential. We do not attempt to obtain a potential whose calculated observables very closely (within a few percent) fit the experimental data; To accomplish this fit, we would have to include in the potential many corrections whose evaluations are very tedious, but whose sizes have been estimated to be several percent of the unary term. Our primary goal is to describe reasonable models and to demonstrate that, *vis-a-vis* the double-scattering corrections, the fes effects cannot be ignored. We also would like to demonstrate the practicality of performing the entire potential construction in terms of momentum-space variables. Thus, we evaluate (30) and (31) by momentum-space principal-value quadratures rather than the oscillatory quadratures that appear in the coordinate-space approach.

The two-body information consists of the recent phase parameter set determined at Livermore[10] and one- and two-term separable potentials used by Mongan[11] to fit these phases. From these separable models, so-called half-shell factors can be calculated for positive-energy partial waves with orbital angular momentum less than six. In AS1, we discussed and justified the use of the approximation[12]

$$t_{\rm fes}(k,k';w)=h(k;w)t_{\rm oes}(w)h(k';w) \tag{35}$$

for all diagonal nucleon-nucleon partial-wave t matrix elements, while the nondiagonal matrix elements in the coupled triplet cases are left oes. In this paper, the half-shell factor is approximated by

$$h(k;w)\approx 1+h'_+(e^0)(1-\kappa/k)+h''_+(e^0)(1-\kappa/k)^2 \text{ for } k>\kappa,$$

and (36)

$$h(k;w)\approx 1+h'_-(^0)(k/\kappa-1)+h''_-(e^0)(k/\kappa-1)^2 \text{ for } k<\kappa,$$

where e^0 is half the lab energy and κ is the two-body momentum corresponding to w. The parameters, $h'_\pm$ and $h''_\pm$, were chosen to fit the half-shell factor curves at values of k/k_0 (k_0 corresponding to e^0) roughly equal to 0.1, 0.5, 1.1, and 2.

The nuclear-structure models consist of form factors and correlation function. For the one-body form factor, we use a modified Gaussian form suggested by Ravenhall[13]

$$F_1(q)=(1-yq^2)\exp(-R_0^2q^2/4). \tag{37}$$

The parameter y was chosen to fit the position of the first diffraction minimum determined from elastic e^--nucleus scattering,[13,14] and then R_0 was taken to give the rms radius, also obtained from electron experiments.[15] Our models of the two-body form factors are the Fourier transforms of two-body densities of the special form (expected to be valid in the interior of the nucleus)

$$D_{ab}(\vec{r}_a,\vec{r}_b)=[1+C_{ab}(s)]\,\rho(r_a)\rho(r_b), \tag{38}$$

where $s=|\vec{r}_a-\vec{r}_b|$. Since (38) is valid only for a large nucleus, $1/A$ corrections are not considered. C_{ab} is called the correlation function between particles "a" and "b." Because of the identity of all

nucleons, C_{ab} can depend only on s and the relative spatial, spin, and isospin symmetries. Following Lax and Feshbach,[16] we will consider only spatial symmetry to be significant, so that the correlation function is of the form

$$C(s) = C_D(s) + C_E(s)\tilde{P}, \tag{39}$$

where $\tilde{P}$ is the operator for spatial exchange.

The special form (38) leads to a correspondingly special form in momentum space, when ρ in (38) is taken to be the Gaussian

$$\rho(r) = (\pi^{1/2}R)^{-3}\exp(-r^2/R^2). \tag{40}$$

We choose R so that (40) gives the same rms radius as given by (37). Defining $\vec{Q}$ by

$$\vec{Q} = \tfrac{1}{2}(\vec{q}_a - \vec{q}_b), \tag{41}$$

we obtain from (38), (39), and (40)

$$F_2(\vec{q}_a, \vec{q}_b) = B(Q)\exp(-R^2q^2/8), \tag{42}$$

where

$$B(Q) = (2\pi R^2)^{-3/2}\int d^3s\, e^{i\vec{Q}\cdot\vec{s}}\exp(-\tfrac{1}{2}R^2s^2)[1 + C(s)]. \tag{43}$$

We call B the momentum-space correlation function and consider two models.

Our first model is of the Brueckner-Gammel type. It arises from short-range forces due to nucleonic cores. We consider the soft-core finite (SCF) model used by Chalmers[17]

$$C_B(s) = -(1 - s^2/b^2)e^{-s^2/b^2}, \tag{44}$$

which via (43) leads to

$$B_B(Q) = \exp(-\tfrac{1}{2}R^2Q^2) - (R'^5Q^2/3R^3)\exp(-\tfrac{1}{2}R'^2Q^2), \tag{45}$$

where

$$1/R'^2 = 1/R^2 + 2/b^2. \tag{46}$$

C_B depends only on s, so that C_B as well as B_B is the same for either space symmetric or antisymmetric states.

Our second model is of the Fermi type. It arises from the Pauli principle which, when applied to a Fermi gas of identical nucleons, leads to space symmetric and antisymmetric correlation functions. These functions are, respectively, given by[18]

$$C_S(s) = (\tfrac{1}{4}A - 1)(\tfrac{1}{4}A + 1)^{-1}C_A(s) \tag{47}$$

and

$$C_A(s) = [\tfrac{1}{4}A(1 - \tfrac{1}{4}A)]^{-1}\sum_{k_F, k_F'}[e^{i(\vec{k}_F - \vec{k}_F')\cdot\vec{s}}], \tag{48}$$

where the summation in (48) is over all nonequal values of $\vec{k}_F$ and $\vec{k}_F'$ whose magnitudes are less than K_F, the so-called Fermi momentum. Johnston and Watson evaluate this summation by replacing it by integration over a zero-temperature Fermi distribution. We perform our calculation with a Gaussian momentum distribution corresponding to the density (40), so that we get a Gaussian result for $C_A(s)$. Applying (43) to this result, we obtain

$$B_A(Q) = \exp(-\tfrac{1}{2}R^2Q^2) - 5^{-3/2}\exp(-R^2Q^2/10). \tag{49}$$

Our momentum-space Fermi-Gauss model (FG) can be written in a direct-plus-exchange form; it follows from (47)–(49) that this form is

$$B_F(Q) = \exp(-\tfrac{1}{2}R^2Q^2) - 5^{-3/2}(1 + \tfrac{1}{4}A)^{-1}\exp(-R^2Q^2/10) + 5^{-3/2}(1 + 4/A)^{-1}\exp(-R^2Q^2/10)\tilde{P}. \tag{50}$$

The special form (42) is very convenient in the integrations of (30) and (31). First, these integrations are written in terms of $\vec{Q}$ by noting from the definition (41) that

$$\vec{Q} = \tfrac{1}{2}(\vec{K} + \vec{K}') - \vec{K}'' = \vec{K}_1 - \vec{K}'', \tag{51}$$

so that

$$\int d^3K'' = \int_0^\infty K''dK''\int_{K_1 - K''}^{K_1 + K''} Q\,dQ/K_1\int_0^{2\pi} d\phi'', \tag{52}$$

where ϕ'' is the azimuthal angle measured from the nucleon-nucleus plane of scattering determined by $\vec{K}$ and $\vec{K}'$. The calculation of the integrals is intractable when the entire angular variation of the spin averages is considered. As an improvement over the forward-angle approximation, where $\vec{q}_a = 0 = \vec{q}_b$, in the evaluation of the binary terms, Chalmers and Saperstein[5] introduced the "half-angle" approximation

$$\vec{q}_a = \tfrac{1}{2}\vec{q} = \vec{q}_b \tag{53}$$

so that the spin averages are evaluated with $\vec{Q} = 0$ and taken outside of the integrations of (30) and (31). However, taking $\vec{Q} = 0$ is identical to taking $\vec{K}'' = \vec{K}_1$; this suggests that (53) may be generalized by taking

$$\vec{K}'' = K''\vec{K}_1/K_1 \tag{54}$$

in the spin averages, so that they are evaluated at the value

$$\vec{Q}_1 = \vec{K}_1(1 - K''/K_1). \tag{55}$$

Note that (54) maintains both first and second scatterings in the plane of $\vec{K}$ and $\vec{K}'$, so that the azimuthal dependence of the two-body matrices need not be explicitly considered. Note also that the magnitude of $\vec{Q}_1$ is the minimum value that Q can have for given $\vec{K}$ and $\vec{K}'$. The evaluation of the spin averages can thus be made for each value of

K'' used to perform the quadratures in (30) and (31), use being made of kinematic relations analogous to (15) and (16) with q_a and q_b given by

$$q_a^2 = \tfrac{1}{4}q^2 + (K_1 - K'')^2 + \tfrac{1}{4}(K^2 - K'^2)(1 - K''/K_1) \tag{56}$$

and

$$q_b^2 = \tfrac{1}{4}q^2 + (K_1 - K'')^2 - \tfrac{1}{4}(K^2 - K'^2)(1 - K''/K_1).$$

Our model for the excitation energy is derived in the same way that the local part of the energy parameter was derived in AS1. We interpret the ground state as being a distribution in the average internal momenta, so that $\sum \bar{l}_i^2/2m - \sum l_i'^2/2m$ represents excitation of the kinetic-energy part of the nuclear Hamiltonian in the intermediate state. An evaluation of this difference, Δ_K, gives

$$\Delta_K = [q^2/8 + (A-2)Q^2/2A + \vec{Q}\cdot(\vec{\bar{l}}_a + \vec{\bar{l}}_b) + \tfrac{1}{2}\vec{q}\cdot(\vec{\bar{l}}_a - \vec{\bar{l}}_b)]/2m. \tag{57}$$

The last two terms involve integration over the average internal momenta, and in consistency with (15) and (16), where internal momenta are neglected in a reciprocally invariant manner, these terms are dropped. Our final model is obtained by assuming that the potential excitation equals the kinetic part and by evaluating the result at $\vec{Q}_1$. This leads to a model excitation given by

$$\Delta_1 = q^2/8m + (A-2)Q_1^2/2Am. \tag{58}$$

The model Δ_1 significantly alters the singularity structure of the integrand in (30), whereas (31) still has singularities at $K'' = \pm K_0$. Thus, $(E^0 - \Delta_1 - K''^2/2\mu)^{-1}$ has poles at positions K_P and K_P' given by

$$K_P = \tfrac{1}{2}A_2 + \tfrac{1}{2}D_0 \quad \text{and} \quad K_P' = \tfrac{1}{2}A_2 - \tfrac{1}{2}D_0, \tag{59}$$

where

$$A_2 = (A-2)/(A-\tfrac{1}{2})$$

and

$$D_0 = \{(A_2^2 - 2A_2)K_1^2 + [(A+1)K_0^2 - Aq^2]/(\tfrac{1}{2}A - \tfrac{1}{4})\}^{1/2}. \tag{60}$$

In terms of the quantities K_R and K_R' given by

$$K_R = \{[(A+\tfrac{1}{2})K_0^2 - Aq^2]/(A-2)\}^{1/2}$$

and (61)

$$K_R' = K_0/(1 - \tfrac{1}{2}A_2)^{1/2}$$

the model exhibits one positive and one negative pole for K_1 in $(0, K_R)$; two positive poles for K_1 in (K_R, K_R'); and no real poles for $K_1 > K_R'$. When $K_1 = K_R'$, the residues from the two poles cancel exactly.

Now we shall show how the models so far described in this section are used to construct the binary terms of the optical potential. Each of the three ("D", "E," and "P") binary terms is of the form

$$U_{DEP}(\vec{K}, \vec{K}'; E^0) = 2\pi K_1^{-1}\int_0^\infty K''dK''(E + i\eta - \Delta - E'')^{-1} \times \bar{M}(Q_1)\bar{B}(Q_1, Q_1'). \tag{62}$$

The factor 2π comes from the azimuthal integration, $\int d\phi''$; Δ is given by Δ_1 for the U_C terms and by zero for the U_P term; and $\bar{M}$ is one of the binary-spin averages, which depends upon K, K', K'', and q. Through relations analogous to (15) and (16) two-body first and second scattering-momentum variables are calculated; these variables are then used to construct the $\bar{M}$. The $\bar{B}$ are integrals of the B; they are defined by

$$\bar{B}(Q_1, Q_1') = \int_{Q_1}^{Q_1'} Qd\,Q\,B(Q), \tag{63}$$

where $Q_1' = K_1 + K''$ is the maximum value that Q attains.

The approach based on (62) has several advantages over the approach in which the intermediate-angle quadrature is handled in terms of coordinate-space variables. In the latter method, the integrands of (30) and (31) are subjected to Fourier transformation, but the $\bar{M}$ must be fixed and Δ must be taken as zero for all terms. The reason for this is that the Fourier transform is used to avoid the singularity (here, at $K_P = K_0$ since $\Delta = 0$); inclusion of Δ or the K'' angular variation of the $\bar{M}$ would result in extremely tedious computations in obtaining the transform. The Fourier transformation is unnecessary; the singularities in (62) are readily handled by principal-value quadrature. In fact, we have used Gauss-Legendre quadrature in segments for the $\Delta = 0$ and $\bar{M}$ fixed case, for both the coordinate-space oscillatory integrals and the momentum-space principal-value integrals; the former take about twice as many grid points to achieve four place accuracy as does the latter. Finally, we point out that for the $\bar{M}$ fixed case, there are two coordinate quadratures (one real and one imaginary), while (62) leads to a single real principal-value quadrature plus imaginary contributions from the residues of poles.

Although the computation involved in (62) is presently possible, it would be expensive and difficult. Also, the most significant matrix elements of the potential are OES for which K_1 is nearly K_0 and

q/K_0 is nearly zero. Thus, inclusion of the Q_1 variation of the $\overline{M}$ in the quadrature is a second-order correction to the distortion term, which, for intermediate-energy elastic scattering, is dominated by the Born term, which is OES. In view of other approximations made in constructing the potential, the effort to evaluate (62) with $\overline{M}$ varied is probably not justified – at least for the process under consideration.

We evaluate $\overline{M}$ with (53), but we still use Δ_1 given by (58). Our expression for U_2 now takes the form

$$U_2 = g(K, K', q; \Delta)\overline{M}(0), \tag{64}$$

where

$$g(K, K', q; \Delta) = 2\pi K_1^{-1} \int_0^\infty K''dK''(E + i\eta - \Delta - E'')^{-1} \times \overline{B}(Q_1, Q_1'), \tag{65}$$

where Δ is given by Δ_1 or 0, and where $Q = 0$ in the $\overline{M}$. The integrations are performed using 6 four-point Gauss-Legendre segments when K_1 is in (K_R, K_R') and 3 eight-point segments otherwise; this yields the real part of g. The imaginary part is given by the sum of residues,

$$-i\pi[(A+1)/(2A-1)][K_P\overline{B}|_{K_P} - K_P'\overline{B}|_{K_P'}]D_0^{-1}, \tag{66}$$

where either residue is included only if the pole occurs at a positive position.

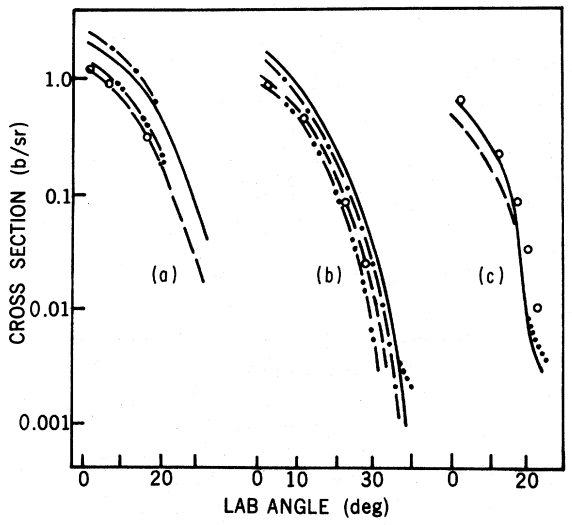

FIG. 3. Elastic differential cross section for n-^{12}C. (a) Incident lab energy is 95 MeV, data are from Ref. 19. (b) Incident lab energy is 137 MeV, data are from Ref. 20. (c) Incident lab energy is 350 MeV, data are from Ref. 22. The solid and dashed lines, respectively, use the SCF and FG correlation models with oes t matrices and no energy-parameter variation. SCF and FG models with fes t matrices and energy-parameter variation are shown with single dot-dash and double dot-dash curves, respectively. The dotted extensions to other curves represent the inclusion of excitation energy in the intermeditate state. The data is indicated by open circles.

VI. RESULTS AND COMPARISON WITH EXPERIMENT

We have calculated the elastic cross section and polarization for ^{4}He, ^{12}C, and ^{40}Ca using the matrix methods described in Sec. IV for various energies between 95 and 350 MeV. These calculations used both the oes and fes two-body matrix elements to construct the unary-plus-binary potential, and both the effect of excitation and the effect of energy-parameter variation were considered. In these calculations, only the case I models of Mongan were used.

Our results using oes matrix elements and neglecting excitation and energy-parameter variation are quite similar to those of Chalmers.[5,17] This was expected because these calculations differ essentially only in the kinematics and the use of the EIA kinematics instead of the kinematics of two free nucleons was seen in AS1 to produce a very small effect in observables calculated from the unary potential. In Figs. 3 and 4, respectively, cross sections and polarizations are calculated for n-^{12}C from the unary-plus-binary potential; in these and subsequent figures, data are indicated by open circles. The 95-, 137-, 210-, and 350-MeV data are taken from Refs. 19, 20, 21, and 22, respectively. The oes curves without excitation or energy parameter variation for SCF

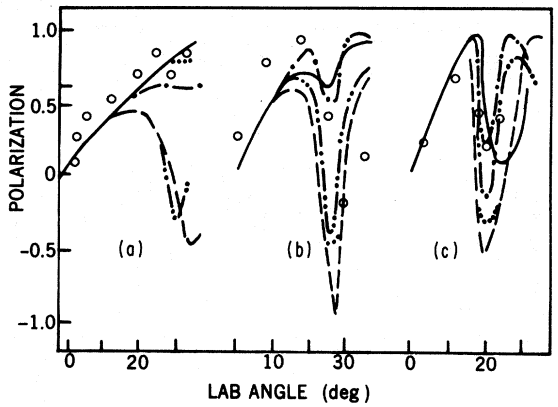

FIG. 4. Elastic nucleon-carbon polarization: (a) Incident lab energy is 137 MeV, data are from Ref. 20. (b) Incident lab energy is 210 MeV, data are from Ref. 21. (c) Incident lab energy is 350 MeV, data are from Ref. 22. Notation is the same as in Fig. 3.

and FG correlations, respectively, are illustrated with solid and dashed lines. The most important thing to note is that the data are generally between or near these two lines. Hence, the fes, energy parameter, and excitation effects are not required to explain the experimental results, especially since these effects are small relative to that caused by the addition of the binary potential to the unary. Therefore, the choice of correlation function is the dominant consideration for the elastic intermediate-energy calculations of nucleon-nucleus scattering.

The inclusion of the fes and energy-parameter variations are illustrated with single-dot-dash and double-dot-dash lines for the SCF and FG correlations, respectively. At 137 MeV, there is a slight dip in the SCF polarization at the position of the former maximum; also, there is a reduction of 13% in the small-angle cross sections. Both of these changes are improvements in the fit of the SCF curve to the data. At 95 MeV, the fes cross section lays about 20% higher than the oes; again the FG curve gives a better fit than the SCF. At 350 MeV, the fes and energy-parameter effects on cross section are too small to be plotted in the figure, while there is an appreciable effect on polarization. Here, for both observables, the SCF curve gives a better fit to the data than the FG.

The effects of including the excitation Δ_1 in U_C are disappointingly small. They are illustrated, for several curves, with dotted-line extensions to the curves without excitation. It is of interest to note that inclusion of the excitation leads to a smoothing of the polarization curves and a small tail in the cross sections. Although both effects are in the right direction, they are far too small to appreciably reduce the differences between calculated and experimental observables.

In Fig. 5, calculated observables for neutrons incident at 142 MeV on ^{4}He are compared with the p-^{4}He data of Cormack *et al.*[23] The fes FG gives the best over-all fit, although the cross section is still too high and the polarization is still too low. Even discounting the Coulomb interference region,

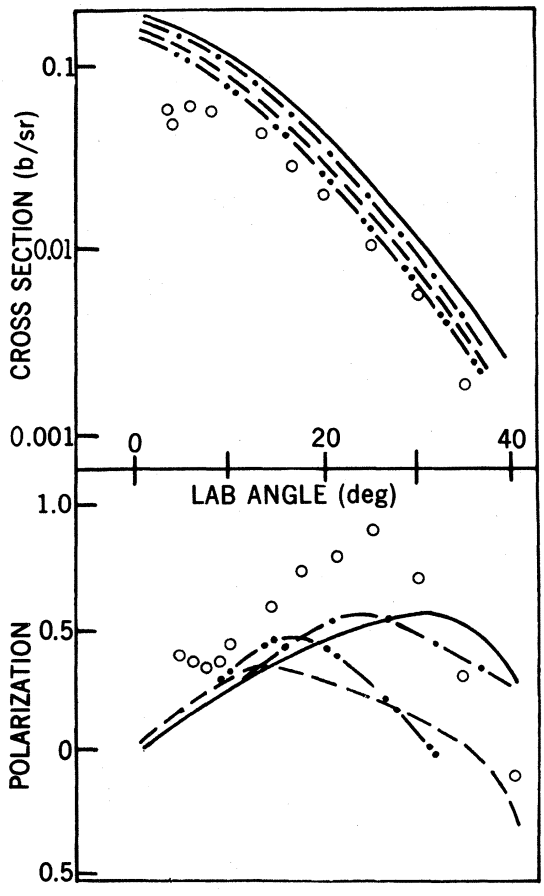

FIG. 5. Nucleon-^{4}He elastic cross section and polarization at 142 MeV. Data are from Ref. 23. Notation is the same as in Fig. 3.

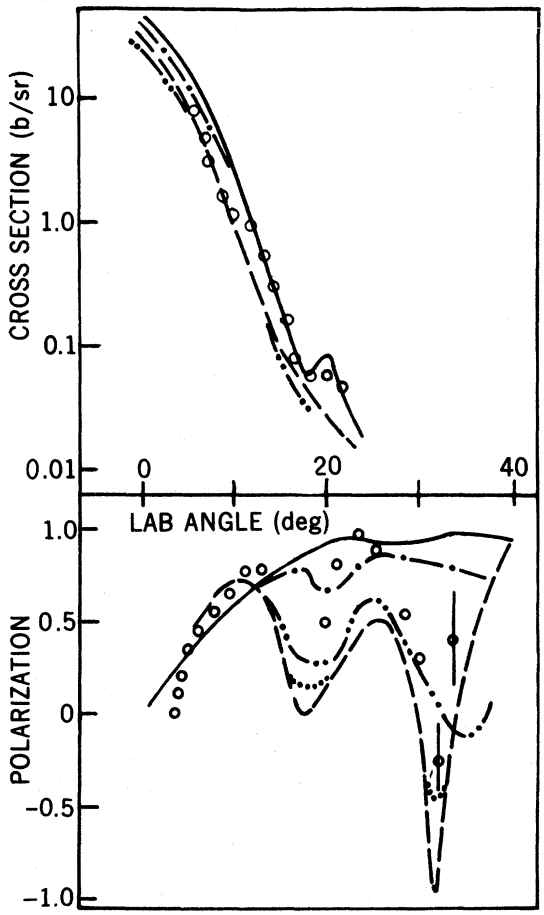

FIG. 6. Nucleon-^{40}Ca elastic cross section and polarization at 220 MeV. Data are from Ref. 21. Notation is the same as in Fig. 3.

the low-energy ^{4}He fits are the worst we have seen, which is not surprising considering the $1/A$ approximations we have made.

In Fig. 6, calculated observables for neutrons incident at 220 MeV on ^{40}Ca are compared with the p-^{40}Ca data of Ref. 21. This calculation is of particular interest because data are available at large angles for a heavier nucleus than we have previously considered. The SCF (both oes and fes) produce a definite diffraction minimum, while the FG curves do not. Considering the whole angular range, the fes SCF curve is somewhat favored by the cross section. On the other hand, the polarization shape of the experimental data is most closely reproduced by the fes FG curve. In particular, both FG curves show two pronounced dips in the polarization, while the oes SCF curve is rather flat and the fes SCF curve has a single shallow dip. The dotted lines on the oes FG curve indicate the smoothing effect of Δ_1 on the sharp spikes. However, this effect is smaller than that due to including the fes and energy-parameter variations.

VII. CONCLUSIONS AND REMARKS

This paper was based on the hope that use of an optical potential, derived from multiple-scattering theory and applied to intermediate-energy elastic nucleon-nucleus scattering, would lead to information on the fes matrix elements of the nucleon-nucleon t matrix. A second hope was to obtain very close fits between calculated and experimental observables. Neither goal can be achieved at this time.

Little, if any, information on fes matrix elements can at this time be obtained from the elastic or any other process. This is true even if the fes effect should prove very large, because there is far too much variability allowed in the Fredholm reduction (35) or in the approach of Baranger *et al.*[24] What is needed is an off-shell extrapolation that depends on a few adjustable parameters – instead of many unknown functions. Thus we "adjusted" our aim to be the investigation of the probable size of fes effects in the elastic process.

Some of the curves in Figs. 3 to 6 do give good fits to the data. Yet, any very good fit should be considered fortuitous, because numerous corrections,[25] each of which has been estimated to be within a few to ten percent, have to be made. Among the approximations made are the multiple-scattering theory itself,[26] exchange between the incident and target nucleons,[27] relativistic corrections to the kinematics,[8] inclusion of average internal momenta in the kinematics (and, hence, in the spin averages), and "$1/A$" corrections to the formalism.[28] A computation involving all of these corrections is too formidable at this time.

However, in our investigation of the elastic problem, we feel that some valuable things have been learned. The fes effects are certainly too small to be disentangled from other considerations, particularly, the uncertainty in nuclear structure brought in by the correlation functions. On the other hand, the fes effects are probably too large to allow a definitive choice to be made for the correlation function, without also considering the possible effect of extrapolation of the two-body t matrix fes. Most importantly and most definitely, we found that fes effects are manifest in the nonlocal manner described in Sec. IV.

Our investigation has achieved two goals that we did not anticipate. First, in order to take the two-body matrix fes, it was found necessary to handle the kinematics more carefully than previous works have, and to do this we had to interpret the essence of the impulse-approximation concept more carefully. Our solution was the EIA formalism discussed in Sec. II and AS1. This formalism has the advantage of being a minimal change from previous treatments, but allowing the unambiguous and straight-forward fes definition of the two-body kinematic variables. Finally, the formalism can be extended to consider, simultaneously with the fes effects, corrections to a number of approximations, such as including the internal momenta in the evaluation of the two-body matrix elements. The second goal we achieved was the momentum-space-oriented approach to the evaluation of the binary terms. This approach allows more generality in the computation (and is much faster) than the coordinate-space treatment of binary terms and correlation functions. In addition, the approach introduced in this paper has led to the development of a *vector*-momentum principal-value quadrature, which could possibly be modified for computing the distortion term in the scattering equation for a nonelastic process.

Finally, we repeat that the elastic process is the least likely process to manifest fes effects, because the Born term is OES and dominates the distortion term. We feel, however, that the careful treatment of nonlocality by means of the momentum-space approach will prove to be of value in calculations of observables for nonelastic processes.

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Λ-d Bound States ($J=\frac{1}{2},\frac{3}{2}$) and Faddeev's Approach to the Three-Body Problem

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The binding energy problem of $^3_\Lambda$H in the ground state and in a possible excited state has been investigated in the light of the recent experimental and theoretical results available for the low-energy parameters of Λ-nucleon interaction. The calculations are performed in Faddeev's formalism with two-body nonlocal separable potentials between pairs of particles, two of which are of unequal masses. For each of the sets of Λ-N potentials, the binding energy of the Λ particle within the hypertriton has been evaluated. One of our results for the binding energy of the hypertriton in an excited state ($J=\frac{3}{2}$) is in reasonably good agreement with that of Toepfer and Schick. We have also discussed the possible qualitative effects of charge-symmetry breaking in the Λ-N interaction in relation to the quantitative comparisons of the Λ binding energies.

1. INTRODUCTION

Making use of Faddeev's[1] elegant approach, a series of papers[2] have appeared on the three-body bound-state problem. This technique has been applied to the Λ-d scattering problems by Hetherington and Schick.[3,4] They make use of a multiple-scattering formalism.[5] The two-body interactions are taken to be spin-dependent nonlocal separable (NLS) s-wave potentials.[6]

The low-energy Λ-N parameters are now better known from the recent experimental studies of Λ-N elastic scattering. In the light of the presently available Λ-nucleon scattering lengths and effective ranges,[7-14] we evaluate binding energies[15] of the hypertriton in the ground state ($J=\frac{1}{2}$) and in the possible excited state ($J=\frac{3}{2}$), taking into consideration only the attractive potential between any two of the three particles.

The ground-state[16] spin of the hypertriton is J