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Proton Capture by ^{7}Be and the Solar Neutrino Problem*

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It is shown theoretically that the ^{7}Be$(p,\gamma)^8$B reaction cross section contains substantial contributions from p and d partial waves at laboratory energies, and extrapolations to stellar energies based on the assumption of pure s-wave capture are therefore erroneous. However, there is no change in the predicted solar neutrino flux, because the calculated low-energy cross-section factor, 31 eV b, is essentially the same as the empirical value in current use.

I. INTRODUCTION

The recent experiments of Davis[1] have set an upper limit of 1.0 SNU on the neutrino flux from the sun (1 SNU = 10^{-36} captures per target atom per sec), in sharp disagreement with the theoretical prediction of 9 SNU, calculated by Bahcall and Ulrich.[2] The rare termination of the p-p chain ^{7}Be$(p,\gamma)^8$B$(e^+\nu)2\alpha$ results in energetic neutrinos and is calculated to contribute 7.3 SNU. It is therefore important to have an accurate estimate of the rate of this reaction in the solar interior. Very detailed measurements of the cross section for ^{7}Be$(p,\gamma)^8$B have been carried out by Kavanagh *et al.*[3] (see Barnes[4]) at laboratory energies $E_p = 0.165$ to 10.0 MeV. A theoretical extrapolation to lower energies based on a calculation by Tombrello[5] yielded a zero-energy cross-section factor[6] $S(0)$ of 0.034 keV b, where, if σ is the cross section and E_p the lab proton energy in MeV,

$$S(E_p) = 0.87441\,\sigma E_p \exp(3.9734\,E_p^{-1/2})$$

for the ^{7}Be$(p,\gamma)^8$B reaction. A calculation by Aurdal[7] similar to that of Tombrello gave $S(0) = 0.044$ keV b, but the new data of Kavanagh *et al.* were not used in that extrapolation. The value $S(0) = 0.030$ keV b actually adopted by Bahcall and Ulrich[2] is lower than either of these, and is presumably the result of an empirical extrapolation.

Proton capture by ^{7}Be involves the radiative transition of a proton in a continuum state to the 2^+ ground state of ^{8}B, bound by 137.2 keV. Only dipole radiation is of importance at the energies considered here. Because the spin and parity of ^{7}Be are $\frac{3}{2}^-$, capture from the s and d partial waves leads to $E1$ radiation, and from the p wave, $M1$. Higher partial waves cannot contribute to dipole radiation. The calculations of Tombrello[5] and Aurdal[7] assumed that only s-wave capture was significant. The present work shows that while this is approximately true in the solar environment ($E_p \approx 20$ keV), it is not the case at laboratory energies, even as low as 150 keV. The small binding energy of ^{8}B results in a spatially extended wave function, enhancing capture from the p and d partial waves.

II. DERIVATION OF THE CROSS SECTION

The total cross section for dipole capture in the reaction $A(a,\gamma)B$ is

$$\sigma_1 = (16/9)\pi E_\gamma{}^3(\hbar c)^{-4}(M_a c^2/2E_a)^{1/2} \times \sum_{\mu m M_A M_B} (2s+1)^{-1}(2J_A+1)^{-1}|\,T_1^\mu\,|^2,$$

where E_γ is the photon energy, and M_a and E_a are the mass and lab energy of the incident particle. Particles A, a, and B have total angular momenta J_A, s, J_B, and projections M_A, m, M_B, respectively. The electromagnetic transition matrix element between initial and final states may be written

$$T^\mu_\lambda = \langle \psi(J_B M_B) | \sum_i t^\mu_\lambda(i) | \psi(J_A M_A, sm, \vec{k}, \vec{r}) \rangle ,$$

where $\vec{k}$ and $\vec{r}$ are the relative wave vector and separation of particles A and a. The single-particle operators $t^\mu_\lambda(i)$ describe electric or magnetic radiation of multipolarity λ, polarization μ by the ith nucleon. They have the standard forms, except that reduced mass corrections must be made to the electric dipole operator and the orbital part of the magnetic dipole operator.[8]

The initial state wave function includes an incoming distorted wave which is an eigenfunction of the Schrödinger equation for a proton moving in the Coulomb and nuclear potential of A. A real Woods-Saxon potential with a Thomas spin-orbit term is assumed for the nuclear potential:

$$V(r) = V_R f(r) - V_R \frac{\lambda}{45.2} \frac{1}{r} \frac{df}{dr} \vec{L} \cdot \vec{s} ,$$

where

$$f(r) = \left[1 + \exp \frac{(r-R)}{a} \right]^{-1} ,$$

while the Coulomb potential is assumed to be that of a uniform charged sphere of radius R.

Before considering the detailed form of the distorted waves, some discussion of the potentials that generate them is in order. It is customary to choose one potential for the bound state, so that the correct separation energy results, and another for the incident wave, to meet some other criterion, such as low-energy scattering properties. This procedure leads to continuum wave functions that are not necessarily orthogonal to the bound-state wave function. If differing spin-angular functions insure the orthogonality there is no serious problem, but for magnetic dipole transitions, nonorthogonality can lead to large values for transition probabilities which should be identically zero. Thus, if the ground state of ^{8}B were considered to be a pure $p_{3/2}$ proton single-particle state, there should be no capture from the $p_{3/2}$ partial wave. That prohibition must be relaxed, however, in the presence of a residual interaction, because it is only the 2^+ coupling of a $p_{3/2}$ proton to the ^{7}Be wave function which forms the ^{8}B ground state. Other couplings, 1^+ and 3^+, are unbound and appear as resonances in the (p,γ) excitation function. This effect may be taken into account in a phenomenological way by using a slightly different well depth for the incoming wave depending on the total J for the initial capturing state. The well depth for each J is adjusted to produce a resonance at the observed energy. Furthermore, one may take a more realistic view of the ^{7}Be and ^{8}B wave functions by using a reliable shell-model calculation, such as that of Cohen and Kurath.[9] However, in the spirit of DWBA, no attempt is made to diagonalize in the space of continuum wave functions. Thus one would not expect precise descriptions of resonance properties, but accurate results off resonance. It turns out that the resonances are actually quite well approximated by this first-order perturbation theory approach.

For the s wave, Tombrello[5] and Aurdal[7] have both made use of the fact that ^{7}Li exhibits a negative scattering length for low-energy neutrons. They have interpreted this as arising from a single-particle $l=0$ resonance in the optical potential, and have adjusted the well depth to reproduce the scattering length from either a $0s$ resonance[5] ($V_R \approx -3.5$ MeV) or a $1s$ resonance[7] ($V_R \approx -45.3$ MeV). However, low-energy neutron scattering is strongly influenced by properties of the compound nucleus, and it is not clear that the negative scattering length is due to a single-particle "shape" resonance as opposed to a compound-nucleus resonance. If the negative value is primarily a compound-nucleus phenomenon, adjusting the well depth to reproduce it is inappropriate in a direct-reaction analysis. Accordingly, in the present calculation the potential used for $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, and $p_{1/2}$ waves is that which correctly reproduces the center of gravity of $p_{3/2}$ states, as calculated by Cohen and Kurath.[9] The $p_{3/2}$ potential itself has been further refined to simulate the residual interaction in the manner described above.

To permit a J dependence in the optical potential

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the initial wave function is expanded as follows:

$$\psi(J_A M_A, sm, \vec{k}, \vec{r}) = \sum_{m' M'_A} (4\pi/kr) \sum_{JLMj} \langle Ls M m | jM+m \rangle \langle Ls M' m' | jM'+m' \rangle \langle j J_A M + m M_A | JM+m+M_A \rangle$$
$$\times \langle j J_A M' + m' M'_A | J M' + m' + M'_A \rangle \psi(J_A M'_A) \phi(sm') i^L Y_L^{M*}(\theta_k, \phi_k) Y_L^{M'}(\theta_r, \phi_r) \chi_{Ljj}(k,r) ,$$

where

$$\vec{L} + \vec{s} = \vec{j} , \qquad \vec{j} + \vec{J}_A = \vec{J} ,$$

and

$$M+m+M_A=M'+m'+M'_A\,.$$

In this expression, $\psi(J_AM_A)$ and $\phi(sm)$ are intrinsic wave functions for the target and projectile, respectively, and $\chi_{LjJ}(k,r)$ is the distorted-wave radial function, completely analogous in definition and normalization to the $\chi_{LJ}(k,r)$ defined by Satchler.[10] Making a fractional parentage expansion of states of B in terms of those of A coupled to a single-particle state b of angular momentum j_b, one finds by straightforward angular momentum reduction that

$$\begin{aligned}\sum_{\substack{\mu m\\ M_AM_B}}|T^\mu_\lambda|^2 &= (4\pi/k^2)\sum_{JL}(2J_B+1)(2J+1)\\ &\times\Bigg\{\sum_b\Bigg(\sum_\alpha\beta_{\alpha b}\begin{Bmatrix}\lambda & J_A & J_\alpha\\ j_b & J_B & J\end{Bmatrix}\langle J_\alpha\|\textstyle\sum t_\lambda\|J_A\rangle\Bigg)^2|I(LJj_bj_bk)|^2\\ &+\sum_{bb'\alpha}\beta_{\alpha b'}\beta_{0b}\begin{Bmatrix}\lambda & j_{b'} & j_b\\ J_A & J_B & J\end{Bmatrix}\begin{Bmatrix}\lambda & J_A & J_\alpha\\ j_{b'} & J_B & J\end{Bmatrix}\langle J_\alpha\|\textstyle\sum t_\lambda\|J_A\rangle\,(\langle j_b\|t_\lambda\|j=j_{b'}\rangle^*I(LJj_{b'}j_{b'}k)+\text{c.c.})\\ &+\sum_j\Bigg|\sum_b\beta_{0b}\begin{Bmatrix}\lambda & j & j_b\\ J_A & J_B & J\end{Bmatrix}\langle j_b\|t_\lambda\|j\rangle\Bigg|^2\Bigg\}\,.\end{aligned}\tag{1}$$

In this expression, $\alpha=0,1,\ldots$ indexes the complete set of states of A (^{7}Be), $b=1,2,\ldots$ indexes the single-particle states of the captured proton in B ($p_{1/2}$ or $p_{3/2}$), and $\beta_{\alpha b}$ is the spectroscopic amplitude connecting those states to the ground state of B (^{8}B). The isospin coupling coefficient $C\equiv\langle T_\alpha\tfrac{1}{2}M_{T\alpha}M_{TB}-M_{T\alpha}|T_BM_{TB}\rangle$ is treated as a factor in $\beta_{\alpha b}$. The reduced matrix elements are defined in the usual way.[11] Those of the type $\langle J_\alpha\|\sum t_\lambda\|J_A\rangle$ are evaluated between the (multiparticle) states of A, while the second kind $\langle j_b\|t_\lambda\|j\rangle$ are between single-particle bound states j_b and continuum states j. The quantity $I(LJj_bjk)$ is the overlap integral between the incoming distorted wave $\chi_{LjJ}(k,r)/r$ and the radial wave function of the bound j_b particle. Parity considerations dictate that only the last sum in Eq. (1) can contribute to $E1$ capture. If, furthermore, there is no J dependence in the s and d wave optical potentials, the summation over J can be carried out, giving:

$$\sum_{\substack{\mu m\\ M_AM_B}}|T^\mu_1|^2(E1) = (4\pi/k^2)\sum_{\substack{b,j\\ (L=0,2)}}(2J_B+1)(2j_b+1)^{-1}\beta_{0b}{}^2|\langle j_b\|t_1\|j\rangle|^2\,.$$

III. RESULTS AND DISCUSSION

Calculation of the bound and continuum radial wave functions was carried out with the program DWUCK72 of Kunz,[12] and reduced matrix elements of the second kind were evaluated with the program EMSPME of Larson.[13] Optical-model parameters used are listed in Table I. Well depths for $p_{3/2}$ ($J^\pi=1^+$, 2^+, and 3^+) were chosen to give the observed bound-state and resonance energies. The center of gravity of the $p_{3/2}$ strength as calculated from Cohen and Kurath's results[9] lies so close to the 1^+ resonance that the same potential has been used for $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, and $p_{1/2}$ partial waves, which are insensitive to small changes in the well depth.

Spectroscopic amplitudes β_{01} and β_{02} connecting the ground states of ^{7}Be and ^{8}B by $p_{1/2}$ and $p_{3/2}$ transfer were taken to be -0.237 and 0.9884, respectively.[9] The total $l=1$ strength has been verified experimentally in the ^{7}Li$(d,p)^8$Li reaction by Schiffer *et al.*[14] The reduced magnetic moment of ^{7}Be was estimated to be $-1.713\,\mu_N$ from a shell-model calculation by Hauge and Maripuu[15] and the known moment of the mirror nucleus ^{7}Li, using the relation

$$\mu(^7\text{Be})=\mu(^7\text{Be},\text{calc})+\mu(^7\text{Li},\text{calc})-\mu(^7\text{Li})\,.$$

The remaining values of $\beta_{\alpha b}$ and $\langle J_\alpha\|\sum t_\lambda\|J_A\rangle$ were evaluated directly from Hauge and Maripuu's wave functions; their net effect is small compared to the ground-state term.

Figure 1 shows the unnormalized calculation compared to the experimental data of Kavanagh *et al.*,[3] both expressed in terms of the approximately energy-independent $S(E)$ factor. To fit the low-energy data between 165 and 500 keV it is necessary to renormalize the calculation by a factor of 1.266. This yields a low-energy cross-section factor, evaluated at the Gamow peak ($E_p\approx 20$ keV), of

$$S(0.02)=0.031\ \text{keV b}\,.$$

TABLE I. Optical-model parameters used in the $^7Be(p,\gamma)^8B$ calculation.

R (fm)	a (fm)	λ	$-V_R$ (MeV) $p_{3/2}{}^{2+}$	$p_{3/2}{}^{1+}$	$p_{3/2}{}^{3+}$	$s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $p_{1/2}$
2.95[a]	0.52[a]	25	30.83	28.65	23.50	28.65

[a] References 5 and 7.

This value is almost identical with the 0.030 keV b used by Bahcall and Ulrich,[2] and there is, therefore, no appreciable change in the predicted solar neutrino flux from ^8B decay, despite the revision in the theoretical treatment. (Other effects, however, have recently been shown to reduce the neutrino capture rate. Lanford and Wildenthal[16] have found that the cross section for capture of ^8B neutrinos by ^{37}Cl is 15% lower than previously thought, and Gari and Huffman[17] have demonstrated that mesonic effects in the $p+p$ reaction depress the total neutrino flux by 21%.)

To have confidence in the result, one should examine the uncertainties in the model. The neglect of many-body degrees of freedom is justified because the capture at low energies (≈200 keV) takes place well outside the nuclear volume, with the transition density peaking around 25 fm. Secondly, any reasonable prescription for the s-wave potential, including that of Tombrello[5] or Aurdal,[7] gives essentially the same results at low energies, and that applies *a fortiori* to the d-wave potential. When the cross sections are normalized at 200 keV, the potentials of Tombrello and Aurdal both give $S(0.02) = 0.029$ keV b. The choice of potentials is not a significant uncertainty in the extrapolation. Third, the overestimated width of the 1^+ resonance is unimportant because p-wave capture is negligible at low energies in any case. Why, then, is a 27% renormalization required? One possibility is that the spectroscopic amplitudes β_{01} and β_{02} (calculated by Cohen and Kurath[9]) are too small, but they are in fact quite close to the maximum set by a pure-configuration limit, which requires that $\beta_{01}{}^2 + (\beta_{02}/1.15)^2 \leq 1$. The $^7Li(d,p)^8Li$ experiments of Schiffer *et al.*[14] also seem to preclude any increase in these quantities. The most likely explanation is the choice of optical-model parameters for the bound-state wave function. Increasing the radius to 3.33 fm and the diffuseness to 0.65 fm (and readjusting the well depths accordingly) produces the required renormalization, affecting capture from all the partial waves in the same proportion except in the region of a resonance. Such a change in the radius parameters seems reasonable for the weakly bound ^7Be+p system. For these

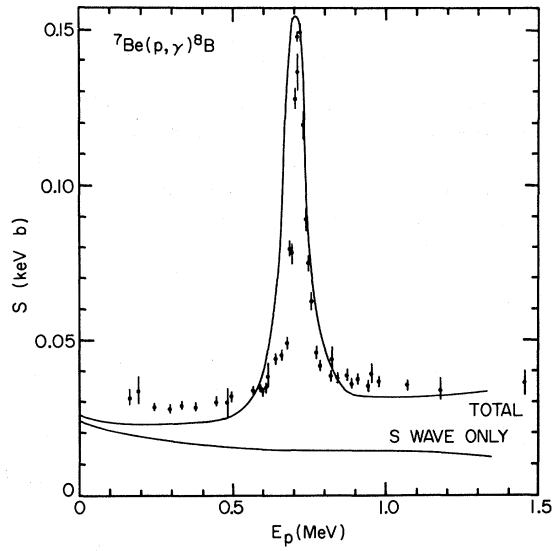

FIG. 1. Comparison between the experimental measurements of the $^7Be(p,\gamma)^8B$ cross section by Kavanagh *et al.* and the theoretical calculation (smooth curves). The calculation is shown here in absolute form, without normalization to the data. The abscissa is the lab proton energy and the ordinate is the cross-section factor as defined in the text.

reasons the renormalized extrapolation is thought to be accurate. Taking into account the experimental uncertainty, it is unlikely that the low-energy S factor is in error by more than 15%.

Also shown in Fig. 1 is the unnormalized contribution from s-wave capture alone. In support of the present choice of s-wave potential, it may be remarked that the potentials used by Tombrello and Aurdal give s-wave contributions similar at low energies but as much as 50% larger at $E_p \approx 1.2$ MeV.

Two final points are of interest. Exact agreement with the results of Tombrello was obtained (for s wave) when the calculations were repeated with his parameters, but it has not been possible to duplicate the results of Aurdal in the same way. It would thus appear that some numerical error is present in Aurdal's computations. The second point relates to the mass of ^8B: The extrapolation is almost independent of the binding energy because, by chance, the dependence on $E_\gamma{}^3$ cancels the dependence on the bound-state wave function to a high order. Although the mass is believed to be known to ±1.2 keV,[18] even a change 10 times that size would have no discernible effect on the cross sections.

Note added in proof: Private communications from J. N. Bahcall and W. A. Lanford indicate that Bahcall's estimate for the ^{37}Cl neutrino capture cross section[19] may be still the most reliable.

IV. ACKNOWLEDGMENTS

It is a pleasure to acknowledge informative discussions with Sam M. Austin and George F. Bertsch. Sven Maripuu kindly made available his shell-model wave functions for ^{7}Be and ^{8}B. I am very grateful to T. A. Tombrello for reading the manuscript and for his helpful comments.

*Work supported by the National Science Foundation.

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Fully Self-Consistent Brueckner-Hartree-Fock and Renormalized Brueckner-Hartree-Fock Calculation for ^{4}He and ^{16}O

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Generalized Brueckner-Hartree-Fock (BHF) and renormalized BHF (RBHF) calculations of ^{4}He and ^{16}O have been performed. They are exact, including fully self-consistent single-particle energies and wave functions, except for the truncation of the expansion into oscillator states and the assumption of spherical symmetry. The trends of earlier approximate calculations essentially are confirmed.

I. INTRODUCTION

All Brueckner-Hartree-Fock (BHF) or renormalized BHF (RBHF) calculations published so far[1] use certain approximations. They do not fully take into account the self-consistency of single-particle (s.p.) energies and wave functions, for instance in the Pauli projection operator of the Bethe-Goldstone (BG) equation. They also omit certain usually small (kinetic energy coupling and hole-hole ladder) terms which can be taken into account without solving Bethe-Faddeev equations. In this paper the e^S version of the many-body theory[2-5] will be used in the exact two-body correlation approximation and applied in the calculation of the binding energy (BE) and rms radius $\langle r^2 \rangle$ of ^{4}He and ^{16}O. This is done for several n-n potentials (always including the Coulomb interaction), with and without the small terms mentioned before, c.m. corrections (including the exact one) and renormalization. From this, final conclusions can be drawn whether or not the potentials and/or the restriction to two-body correlations suffice to explain data, since everything else is exact.