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Elastic Scattering of Nucleons from Correlated Nuclei*

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A theory of the optical potential for scattering of nucleons from nuclei is presented. A development appropriate for intermediate- and high-energy scattering is given in this paper. For low energies the theory provides the leading term of the optical potential which must then be supplemented by the effects of compound-nucleus formation and inelastic scattering to low-lying states of the target. Particular attention is paid to the Pauli principle, and it is shown how standard multiple-scattering theories must be modified if the projectile is identical to the target particles. The development proceeds in the spirit of the many-body theory of finite nuclei, but deals with the problem of the addition of a (continuum) nucleon to an already known strongly correlated nuclear state. The manner in which the correlations between target and projectile nucleons develop is explicitly displayed. It is the consideration of these additional two-body correlations which allows us to introduce the Bethe-Goldstone type of reaction matrices for the scattering problem.

I. INTRODUCTION

In a previous publication[1] we constructed an orthonormal set of $(A+1)$-body states $|X^{(+)}_{\vec{k},A}\rangle$, and indicated that these states could be used in a theory of elastic scattering of a nucleon (fermion) from a nucleus (a collection of fermions in a correlated bound state). In this paper, we describe in some detail the way these states are used to calculate an optical potential from two-body forces. In the interest of clarity, some of the results of Ref. 1 will be recalled, although few of the details to be found in that treatment will be reproduced here.

The basic premise of our argument is that although the ground-state wave function of a heavy nucleus may be considered, to a high degree of approximation, to be a single Slater determinant the correlations cannot be ignored. (If we were dealing with a situation in which all the particles moved in a Hartree-Fock field, the problem at hand would be trivial. We seek guidance from this trivial problem in order to find a good beginning point for the real problem we are attacking.) We note that there exists a set of single-particle bound states $|\phi_b\rangle$ which may be obtained from a Brueckner-Hartree-Fock calculation, for example. However, single Slater determinants of these states are not taken as the ground-state wave functions of the target, but rather as inputs (model wave functions) for a more elaborate procedure for the calculation of the correlated nuclear states. We apply the same general philosophy to the calculation of the scattering states.

Thus we postulate single-particle continuum states $|\chi^{(+)}_{\vec{k}}\rangle$ which together with the discrete single-particle states $|\phi_b\rangle$ form a complete orthonormal single-particle basis. The prescription used to construct the continuum states $|\chi^{(+)}_{\vec{k}}\rangle$ is not crucial to the argument. We have made the explicit suggestion[1] that the states $|\chi^{(+)}_{\vec{k}}\rangle$ and $|\phi_b\rangle$ be eigenstates of the model single-particle Hamiltonian, h_M,

$$h_M=(1-\sum_b|\phi_b\rangle\langle\phi_b|)h_0(1-\sum_b|\phi_b\rangle\langle\phi_b|)+\sum_b|\phi_b\rangle\epsilon_b\langle\phi_b|, \tag{1.1}$$

so that the outgoing-wave–continuum-state vectors $|\chi^{(+)}_{\vec{k}}\rangle$ satisfy the convenient "separable" integral equation,

$$|\chi^{(+)}_{\vec{k}}\rangle=|\vec{k}\rangle-\sum_b\frac{1}{\epsilon_{\vec{k}}-h_0+i\epsilon}|\phi_b\rangle\langle\phi_b|h_0|\chi^{(+)}_{\vec{k}}\rangle. \tag{1.2}$$

We take h_0 to be the kinetic energy operator in which case $|\vec{k}\rangle$ is a plane-wave state. This is not a necessary choice, however.

We note that the η's defined through the relations

$$|\chi^{(+)}_{\vec{k}}\rangle\equiv\eta^{\dagger}_{\vec{k}}|0\rangle, \tag{1.3}$$

and

$$|\phi_b\rangle\equiv\eta^{\dagger}_b|0\rangle, \tag{1.4}$$

where $|0\rangle$ is the vacuum state, are fermion operators. Then we define antisymmetric $(A+1)$-body state vectors $|\chi^{(+)}_{\vec{k},A}\rangle$ and $|\Phi_{b,A}\rangle$ as

$$|\chi^{(+)}_{\vec{k},A}\rangle\equiv\eta^{\dagger}_{\vec{k}}|\Phi_A\rangle \tag{1.5}$$

and

$$|\Phi_{b,A}\rangle\equiv\eta^{\dagger}_b|\Phi_A\rangle, \tag{1.6}$$

where we take $|\Phi_A\rangle$ to be the exact ground-state eigenstate of the complete A-body Hamiltonian. If

$|\Phi_A\rangle$ were a Slater determinant of the states $|\phi_b\rangle$, then we would obviously have in the $(A+1)$-body states of Eqs. (1.5) and (1.6) an orthonormal set. However, because we insist that $|\Phi_A\rangle$ be an eigenstate of H, $|\Phi_A\rangle$ must represent a correlated state, and so the $(A+1)$-body states of Eqs. (1.5) and (1.6) cannot be orthonormal.

The states given in Eqs. (1.5) and (1.6) define a subspace of the full $(A+1)$-body Hilbert space. It is shown in Ref. 1 that an orthonormal set of states which span the same subspace can be constructed from these states by writing

$$|X_{b,A}\rangle = |\Phi_{b,A}\rangle + \sum_{b''} |\Phi_{b'',A}\rangle\langle\phi_{b''}|u|\phi_b\rangle \tag{1.7}$$

and

$$|X^{(+)}_{\vec{k},A}\rangle = |\chi^{(+)}_{\vec{k},A}\rangle + \int |\chi^{(+)}_{\vec{k}',A}\rangle\, d\vec{k}'\langle\chi^{(+)}_{\vec{k}'}|v|\chi^{(+)}_{\vec{k}}\rangle + \sum_{b'} |\Phi_{b',A}\rangle\langle\phi_{b'}|w|\chi^{(+)}_{\vec{k}}\rangle . \tag{1.8}$$

Straightforward prescriptions for obtaining the matrix elements of the one-body operators u, v, and w, which appear in Eqs. (1.7) and (1.8), are given in Ref. 1. It is also shown in Ref. 1 that, when the states $|X^{(+)}_{\vec{k},A}\rangle$ are calculated according to the prescriptions therein, then

$$\langle\Phi_A|\, a(\vec{r})|X^{(+)}_{\vec{k},A}\rangle \xrightarrow[r\to\infty]{} \langle\vec{r}|\chi^{(+)}_{\vec{k}}\rangle , \tag{1.9}$$

where $a(\vec{r})$ is the destruction operator for a particle at $\vec{r}$. Thus the states $|X^{(+)}_{\vec{k},A}\rangle$ may serve as channel states for elastic scattering.

The states $|X^{(+)}_{\vec{k},A}\rangle$ have been used to define a projection operator for elastic scattering, P,

$$P = \int |X^{(+)}_{\vec{k},A}\rangle\, d\vec{k}\langle X^{(+)}_{\vec{k},A}| . \tag{1.10}$$

We construct a Hermitian Hamiltonian $\mathcal{H}_0$ which does not connect states in P with states in the remainder of the Hilbert space, i.e., we write a Hamiltonian of the form

$$\mathcal{H}_0 = P\mathcal{H}_0 P + Q\mathcal{H}_0 Q , \tag{1.11}$$

where $P+Q=1$. We then define $\bar{H}_0$ to be,

$$\bar{H}_0 \equiv P\mathcal{H}_0 P \equiv \int |X^{(+)}_{\vec{k},A}\rangle\, d\vec{k}(\epsilon_{\vec{k}} + E_A)\langle X^{(+)}_{\vec{k},A}| . \tag{1.12}$$

Thus, we may write a formal integral equation for $|\Psi^{(+)}_{\vec{k},A}\rangle$ as

$$|\Psi^{(+)}_{\vec{k},A}\rangle = |X^{(+)}_{\vec{k},A}\rangle + \frac{1}{E-\mathcal{H}_0+i\epsilon}(H-\mathcal{H}_0)|\Psi^{(+)}_{\vec{k},A}\rangle , \tag{1.13}$$

where $|\Psi^{(+)}_{\vec{k},A}\rangle$ is the complete eigenstate of the full Hamiltonian H, satisfying the usual many-body Schrödinger equation

$$(E-H)|\Psi^{(+)}_{\vec{k},A}\rangle = 0 . \tag{1.14}$$

We may also find a formal integral equation for $P|\Psi^{(+)}_{\vec{k},A}\rangle$, which suffices for a description of elastic scattering. The formal integral equation for $P|\Psi^{(+)}_{\vec{k},A}\rangle$ is readily obtained by operating on Eq. (1.14) with P and Q separately to obtain

$$(E-PHP)P|\Psi^{(+)}_{\vec{k},A}\rangle = PHQ|\Psi^{(+)}_{\vec{k},A}\rangle \tag{1.15}$$

and

$$(E-QHQ)Q|\Psi^{(+)}_{\vec{k},A}\rangle = QHP|\Psi^{(+)}_{\vec{k},A}\rangle . \tag{1.16}$$

From Eq. (1.16) we observe that

$$Q|\Psi^{(+)}_{\vec{k},A}\rangle = \frac{1}{E-QHQ+i\epsilon}QHP|\Psi^{(+)}_{\vec{k},A}\rangle , \tag{1.17}$$

so that by substituting Eq. (1.17) into Eq. (1.15) we obtain an equation containing $P|\Psi\rangle$ alone. This equation is

$$(E-PHP)P|\Psi^{(+)}_{\vec{k},A}\rangle = PH\frac{Q}{E-QHQ+i\epsilon}HP|\Psi^{(+)}_{\vec{k},A}\rangle , \tag{1.18}$$

or

$$P|\Psi^{(+)}_{\vec{k},A}\rangle = |X^{(+)}_{\vec{k},A}\rangle + \frac{P}{E-\mathcal{H}_0+i\epsilon}\left[H-\mathcal{H}_0+H\frac{Q}{E-QHQ+i\epsilon}H\right]P|\Psi^{(+)}_{\vec{k},A}\rangle . \tag{1.19}$$

Now we also obtain an expression for the elastic scattering T matrix elements by noting that

$$\begin{aligned}\langle\vec{k}'|T_{\rm el}|\vec{k}\rangle &= \langle\vec{k}'|T_{\rm ORTH}|\vec{k}\rangle + \langle X^{(-)}_{\vec{k}',A}|H-\mathcal{H}_0|\Psi^{(+)}_{\vec{k},A}\rangle \\ &= \langle\vec{k}'|T_{\rm ORTH}|\vec{k}\rangle + \langle X^{(-)}_{\vec{k}',A}|(H-\mathcal{H}_0)P|\Psi^{(+)}_{\vec{k},A}\rangle + \langle X^{(-)}_{\vec{k},A}|(H-\mathcal{H}_0)Q|\Psi^{(+)}_{\vec{k},A}\rangle .\end{aligned} \tag{1.20}$$

Here T_{ORTH} refers to the so-called orthogonality scattering of Ref. 1. This contribution to the elastic scattering amplitude is a consequence of the use of the distorted waves $|X^{(+)}_{\vec{k},A}\rangle$. Substitution of Eq. (1.17) into Eq. (1.20) then gives

$$\langle\vec{k}'|T_{\text{el}}||\vec{k}\rangle=\langle\vec{k}'|T_{\text{ORTH}}|\vec{k}\rangle+\left\langle X^{(-)}_{\vec{k}',A}\right|P\left[H-\mathcal{H}_0+H\frac{Q}{E-QHQ+i\epsilon}H\right]P\left|\Psi^{(+)}_{\vec{k},A}\right\rangle . \tag{1.21}$$

Thus we see that if our interest lies only in elastic scattering we may use

$$V_{\text{eff}}=P\left[H-\bar{H}_0+H\frac{Q}{E-QHQ+i\epsilon}H\right]P \tag{1.22}$$

as an effective interaction in the P space.

In this representation the wave function $P|\Psi^{(+)}_{\vec{k},A}\rangle$ is given by

$$|\bar{\Psi}^{(+)}_{\vec{k},A}\rangle\equiv P|\Psi^{(+)}_{\vec{k},A}\rangle=|X^{(+)}_{\vec{k},A}\rangle+\frac{P}{E-\bar{H}_0+i\epsilon}V_{\text{eff}}|\bar{\Psi}^{(+)}_{\vec{k},A}\rangle , \tag{1.23}$$

and the elastic T matrix elements are

$$\langle\vec{k}'|T_{\text{el}}|\vec{k}\rangle=\langle\vec{k}'|T_{\text{ORTH}}|\vec{k}\rangle+\langle X^{(-)}_{\vec{k}',A}|V_{\text{eff}}|\bar{\Psi}^{(+)}_{\vec{k},A}\rangle . \tag{1.24}$$

II. OPTICAL POTENTIAL

The effective single-particle potential (optical-model potential) v_{opt} is defined to be a one-body potential which can be used in the one-body Schrödinger equation

$$(E-h_{\text{opt}})|\hat{\psi}\rangle=(E-h_0-v_{\text{opt}})|\hat{\psi}\rangle=0 , \tag{2.1}$$

where h_{opt} is a one-body Hamiltonian and h_0 is the kinetic energy operator for a single particle. The only properties that the one-body potential v_{opt} must have is that it be short-ranged and that the elastic scattering calculated from Eq. (2.1) be identical to the elastic scattering calculated from "first principles." We may also require that Eq. (2.1) have *discrete* solutions $|\phi_b\rangle$ as well as *continuum* solutions $|\hat{\psi}_{\vec{k}}\rangle$ and that

$$\langle\phi_b|\hat{\psi}_{\vec{k}}\rangle=0 . \tag{2.2}$$

In that case we may rewrite Eq. (2.1) as

$$\left(E-\sum_b|\phi_b\rangle\epsilon_b\langle\phi_b|-ph_{\text{opt}}p\right)|\hat{\psi}\rangle=0 , \tag{2.3}$$

where

$$p\equiv 1-\sum_b|\phi_b\rangle\langle\phi_b| . \tag{2.4}$$

The continuum solutions of Eq. (2.3), $|\hat{\psi}_{\vec{k}}\rangle$, have the property

$$|\hat{\psi}_{\vec{k}}\rangle=p|\hat{\psi}_{\vec{k}}\rangle , \tag{2.5}$$

so that we may reexpress Eq. (2.3) as

$$|\hat{\psi}^{(+)}_{\vec{k}}\rangle=|\chi^{(+)}_{\vec{k}}\rangle+\frac{1}{E-ph_0p+i\epsilon}(pv_{\text{opt}}p)|\hat{\psi}^{(+)}_{\vec{k}}\rangle , \tag{2.6}$$

where the states $|\chi^{(+)}_{\vec{k}}\rangle$ are the eigenstates of the operator ph_0p, and are given by Eq. (1.2). The T matrix for elastic scattering is then immediately seen, by inspection of Eq. (2.6), to be

$$\langle\vec{k}'|T_{\text{el}}|\vec{k}\rangle=\langle\vec{k}'|T_{\text{ORTH}}|\vec{k}\rangle+\langle\chi^{(-)}_{\vec{k}'}|pv_{\text{opt}}p|\hat{\psi}^{(+)}_{\vec{k}}\rangle . \tag{2.7}$$

The Born expansion of Eq. (2.7) is readily seen to be

$$\langle\vec{k}'|T_{\text{el}}|\vec{k}\rangle=\langle\vec{k}'|T_{\text{ORTH}}|\vec{k}\rangle+\langle\chi^{(-)}_{\vec{k}'}|pv_{\text{opt}}p|\chi^{(+)}_{\vec{k}}\rangle+\left\langle\chi^{(-)}_{\vec{k}'}\right|pv_{\text{opt}}p\frac{1}{E-ph_0p+i\epsilon}pv_{\text{opt}}p\left|\chi^{(+)}_{\vec{k}}\right\rangle+\cdots . \tag{2.8}$$

This expansion may be compared, term by term, with the analogous expansion for the expression for the elastic scattering matrix element given in Eq. (1.24). That expansion is

$$\langle\vec{k}'|T_{\rm el}|\vec{k}\rangle=\langle\vec{k}'|T_{\rm ORTH}|\vec{k}\rangle+\langle X^{(-)}_{\vec{k}',A}|PV_{\rm eff}P|X^{(+)}_{\vec{k},A}\rangle+\left\langle X^{(-)}_{\vec{k}',A}\left|PV_{\rm eff}P\frac{1}{E-\bar{H}_0+i\epsilon}PV_{\rm eff}P\right|X^{(+)}_{\vec{k},A}\right\rangle+\cdots. \tag{2.9}$$

Now we note that we have arranged matters so that

$$\left\langle X^{(+)}_{\vec{k}'',A}\left|\frac{1}{E-\bar{H}_0+i\epsilon}\right|X^{(+)}_{\vec{k}''',A}\right\rangle=\left\langle \chi^{(+)}_{\vec{k}''}\left|\frac{1}{E-ph_0p+i\epsilon}\right|\chi^{(+)}_{\vec{k}'''}\right\rangle, \tag{2.10}$$

where for simplicity we have taken $E_A=0$, above. Thus the two different expressions for $\langle\vec{k}'|T_{\rm el}|\vec{k}\rangle$ are identical if

$$\langle\chi^{(+)}_{\vec{k}'}|pv_{\rm opt}p|\chi^{(+)}_{\vec{k}}\rangle=\langle X^{(+)}_{\vec{k}',A}|PV_{\rm eff}P|X^{(+)}_{\vec{k},A}\rangle \tag{2.11}$$

and

$$\langle\chi^{(-)}_{\vec{k}'}|\chi^{(+)}_{\vec{k}}\rangle=\langle X^{(-)}_{\vec{k}',A}|X^{(+)}_{\vec{k},A}\rangle. \tag{2.12}$$

Now we know that Eq. (2.12) is an identity because we recognize the right- and left-hand sides of this equation as two different expressions for the same matrix element of the orthogonality scattering S matrix. Therefore it suffices for us to *define* $pv_{\rm opt}p$ by means of Eq. (2.11). With this definition of the one-body operator, $pv_{\rm opt}p$, we have achieved our goal, and we recognize that Eq. (2.11) is the relation we seek. We shall discuss methods for calculating the right-hand side of Eq. (2.11) in the next section.

At this point, it is perhaps in order to belabor the obvious and point out that the optical potential, as we have constructed it, is given as

$$v_{\rm opt}=pv_{\rm opt}p+\left[\sum_b|\phi_b\rangle\epsilon_b\langle\phi_b|+(ph_0p-h_0)\right]. \tag{2.13}$$

With this definition of $v_{\rm opt}$, one automatically satisfies Eq. (2.2). This is not a necessary condition, however, and various "optical-model wave functions" may be defined which have the *same asymptotic behavior* but differ at small distances.

This point can be clarified further if we note that a one-body "wave function" may be defined in the χ representation as

$$\langle\chi^{(+)}_{\vec{k}'}|\psi^{(+)}_{\vec{k}}\rangle\equiv\langle X^{(+)}_{\vec{k}',A}|\Psi^{(+)}_{\vec{k},A}\rangle=\langle X^{(+)}_{\vec{k}',A}|P|\Psi^{(+)}_{\vec{k},A}\rangle=\langle X^{(+)}_{\vec{k}',A}|\bar{\Psi}^{(+)}_{\vec{k},A}\rangle. \tag{2.14}$$

With this definition, we may reexpress Eq. (1.23) in the X representation as

$$\langle X^{(+)}_{\vec{k}',A}|\bar{\Psi}^{(+)}_{\vec{k},A}\rangle=\delta(\vec{k}-\vec{k}')+\frac{1}{\epsilon_{\vec{k}}-\epsilon_{\vec{k}'}+i\epsilon}\int\langle X^{(+)}_{\vec{k}',A}|PV_{\rm eff}P|X^{(+)}_{\vec{k}'',A}\rangle\,d\vec{k}''\langle X^{(+)}_{\vec{k}'',A}|\bar{\Psi}^{(+)}_{\vec{k},A}\rangle, \tag{2.15}$$

or

$$\langle\chi^{(+)}_{\vec{k}'}|\psi^{(+)}_{\vec{k}}\rangle=\langle\chi^{(+)}_{\vec{k}'}|\chi^{(+)}_{\vec{k}}\rangle+\frac{1}{\epsilon_{\vec{k}}-\epsilon_{\vec{k}'}+i\epsilon}\int\langle X^{(+)}_{\vec{k}',A}|PV_{\rm eff}P|X^{(+)}_{\vec{k}'',A}\rangle\langle\chi^{(+)}_{\vec{k}''}|\psi^{(+)}_{\vec{k}}\rangle\,d\vec{k}''. \tag{2.16}$$

If we then *define* the optical potential as in Eq. (2.11), we obtain the result that

$$p|\psi^{(+)}_{\vec{k}}\rangle\equiv|\hat{\psi}^{(+)}_{\vec{k}}\rangle=|\chi^{(+)}_{\vec{k}}\rangle+\frac{1}{\epsilon_{\vec{k}}-ph_0p+i\epsilon}pv_{\rm opt}p|\hat{\psi}^{(+)}_{\vec{k}}\rangle. \tag{2.17}$$

This, of course, is just Eq. (2.6).

This analysis only defines the p-space representation of $|\psi^{(+)}_{\vec{k}}\rangle$, $p|\psi^{(+)}_{\vec{k}}\rangle=|\hat{\psi}^{(+)}_{\vec{k}}\rangle$; $|\hat{\psi}^{(+)}_{\vec{k}}\rangle$ is an eigenstate (with eigenvalue ϵ_k) of the one-body Hamiltonian

$$h=p(h_0+v_{\rm opt})p. \tag{2.18}$$

In the following we discuss various possible specifications of $q|\psi^{(+)}_{\vec{k}}\rangle$.

We note that just as Eq. (1.18) or (1.22) is an equation for $P|\Psi^{(+)}_{\vec{k},A}\rangle$, so Eq. (2.6) or (2.17) is an equation for $|\hat{\psi}^{(+)}_{\vec{k}}\rangle\equiv p|\psi^{(+)}_{\vec{k}}\rangle$. Thus we could, so far as this discussion is concerned, add to $|\hat{\psi}^{(+)}_{\vec{k}}\rangle$ an arbitrary amount of the single-particle bound-state wave functions $|\phi_b\rangle$ in order to obtain an equally acceptable optical wave function $|\psi^{(+)}_{\vec{k}}\rangle$. We recognize, of course, that the coordinate space representatives of $|\psi^{(+)}_{\vec{k}}\rangle$ and $|\hat{\psi}^{(+)}_{\vec{k}}\rangle$ would have the same asymptotic behavior and would hence correspond to the same elastic scattering.

Another insight into the inherent ambiguity in the "optical-model wave function," even if that "optical-model wave function" is calculated from first principles, becomes apparent from the following considerations.

Let us define an *orthonormal* set of $(A+1)$-body states $|X^{(+)}_{\vec{r},A}\rangle$, which span the same space as the states $\{|X^{(+)}_{\vec{k},A}\rangle, |X_{b,A}\rangle\}$. We require that these states have the property that

$$\langle X^{(+)}_{\vec{r},A}|X^{(+)}_{\vec{k},A}\rangle \equiv \langle \vec{r}|\chi^{(+)}_{\vec{k}}\rangle \tag{2.19}$$

and

$$\langle X^{(+)}_{\vec{r},A}|X_{b,A}\rangle \equiv \langle \vec{r}|\phi_b\rangle . \tag{2.20}$$

Thus,

$$|X^{(+)}_{\vec{k},A}\rangle = \int |X^{(+)}_{\vec{r},A}\rangle\, d\vec{r}\,\langle \vec{r}|\chi^{(+)}_{\vec{k}}\rangle , \tag{2.21}$$

and

$$|X_{b,A}\rangle = \quad |X^{(+)}_{\vec{r},A}\rangle\, d\vec{r}\,\langle \vec{r}|\phi_b\rangle , \tag{2.22}$$

and conversely,

$$|X^{(+)}_{\vec{r},A}\rangle = \int |X^{(+)}_{\vec{k},A}\rangle\, d\vec{k}\,\langle \chi^{(+)}_{\vec{k}}|\vec{r}\rangle + \sum_b |X_{b,A}\rangle\langle \phi_b|\vec{r}\rangle . \tag{2.23}$$

It follows immediately from the above properties that these states are normalized such that[2]

$$\langle X^{(+)}_{\vec{r}',A}|X^{(+)}_{\vec{r},A}\rangle = \delta(\vec{r}-\vec{r}') . \tag{2.24}$$

If we now *define* the spatial representative of the one-body amplitude to be

$$\langle \vec{r}|\psi^{(+)}_{\vec{k}}\rangle \equiv \langle X^{(+)}_{\vec{r},A}|\Psi^{(+)}_{\vec{k},A}\rangle , \tag{2.25}$$

we see immediately that

$$|\psi^{(+)}_{\vec{k}}\rangle = \int |\chi^{(+)}_{\vec{k}'}\rangle\, d\vec{k}'\,\langle X^{(+)}_{\vec{k}',A}|\Psi^{(+)}_{\vec{k},A}\rangle + \sum_b |\phi_b\rangle\langle X_{b,A}|\Psi^{(+)}_{\vec{k},A}\rangle \tag{2.26}$$

or

$$|\psi^{(+)}_{\vec{k}}\rangle = |\hat{\psi}^{(+)}_{\vec{k}}\rangle + \sum_b |\phi_b\rangle\langle X_{b,A}|\Psi^{(+)}_{\vec{k},A}\rangle , \tag{2.27}$$

where $|\hat{\psi}^{(+)}_{\vec{k}}\rangle$ has been defined above as the solution of Eq. (2.17). This ansatz then yields a perfectly definite prescription for obtaining the one-body amplitude $|\psi^{(+)}_{\vec{k}}\rangle$ from the complete $(A+1)$-body wave function $|\Psi^{(+)}_{\vec{k},A}\rangle$.

Such a prescription may be perfectly definite, but it is far from unique. Another definition that suggests itself as an equally valid candidate for this one-body amplitude is

$$\langle \vec{r}|\psi'^{(+)}_{\vec{k}}\rangle \equiv \langle \Phi_A|a(\vec{r})|\Psi^{(+)}_{\vec{k},A}\rangle . \tag{2.28}$$

With this definition we obtain after a straightforward calculation that

$$|\psi'^{(+)}_{\vec{k}}\rangle = (1-\rho)[(1+q\beta q\beta q\rho)p\gamma p + q\beta q]|\psi^{(+)}_{\vec{k}}\rangle , \tag{2.29}$$

where p, q, β, γ, ρ are as defined in Ref. 1, and $|\psi^{(+)}_{\vec{k}}\rangle$ is given by Eqs. (2.26) and (2.27).

Another one-body amplitude has been given in Ref. 1. This amplitude is defined through the Feshbach-Friedman states $|\xi_{\vec{r},A}\rangle$,

$$\langle \vec{r}|\psi''^{(+)}_{\vec{k}}\rangle = \langle \xi_{\vec{r},A}|P|\Psi^{(+)}_{\vec{k},A}\rangle . \tag{2.30}$$

This amplitude is easily seen to be related to the amplitude we have defined in Eq. (2.26), viz.,

$$|\psi''^{(+)}_{\vec{k}}\rangle = F^{-1}(1+q\beta q\beta q\rho)p\gamma p|\psi^{(+)}_{\vec{k}}\rangle , \tag{2.31}$$

where F is defined such that

$$F(1-\rho)F = 1 ,$$

and (2.32)

$$F^{-1} = (1-\rho)^{1/2} .$$

Likewise the one-body amplitude defined as

$$\langle \vec{r}|\psi'''^{(+)}_{\vec{k}}\rangle = \langle \xi_{\vec{r},A}|\Psi^{(+)}_{\vec{k},A}\rangle \tag{2.33}$$

is given by

$$|\psi'''^{(+)}_{\vec{k}}\rangle = F^{-1}[(1+q\beta q\beta q\rho)p\gamma p + q\beta q]|\psi^{(+)}_{\vec{k}}\rangle . \tag{2.34}$$

This amplitude may serve equally well as a one-body amplitude.

All these amplitudes are identical at large distances but differ inside the nuclear interaction radius. These remarks may perhaps serve to make more definite the very well-known argument that the "optical-model wave function" may not be treated as a true one-body wave function. We have shown that several single-particle amplitudes can be obtained from the exact many-body wave function. Different "spectroscopic" amplitudes may be obtained for different purposes. It is also worthy of brief note that for a Slater determinant $p=q$, $\gamma=\beta=1$, and therefore in that limit we have

$$|\psi^{(+)}_{\vec{k}}\rangle = p|\psi^{(+)}_{\vec{k}}\rangle = |\psi'^{(+)}_{\vec{k}}\rangle = |\psi''^{(+)}_{\vec{k}}\rangle = |\psi'''^{(+)}_{\vec{k}}\rangle .$$

In the following sections we will study the quantity $\langle X^{(+)}_{\vec{k}',A}|PV_{\text{eff}}P|X^{(+)}_{\vec{k},A}\rangle$ from which we may obtain the operator $pv_{\text{opt}}p$, which was defined in Eq. (2.11).

III. CALCULATION OF THE OPTICAL POTENTIAL

We consider the expression

$$\langle X^{(+)}_{\vec{k}',A} | V_{\text{eff}} | X^{(+)}_{\vec{k},A}\rangle = \left\langle X^{(+)}_{\vec{k}',A}\right| \left[P(H-\bar{H}_0)P + PHQ\frac{1}{E-QHQ+i\epsilon}QHP\right]\left| X^{(+)}_{\vec{k},A}\right\rangle , \tag{3.1}$$

with the aim of providing a diagrammatic analysis appropriate to a systematic hole-line expansion. Such a treatment is suited to the study of the optical model at intermediate and high energies.[3] To this end, it is useful to divide the Q space into orthogonal subspaces containing various numbers of holes created on the *correlated* ground state, $|\Phi_A\rangle$. Thus we may let

$$Q \equiv 1 - P = Q_b + Q_1 + Q_2 + \cdots , \tag{3.2}$$

where Q_b is the projection operator onto the space spanned by the states $|X_{b,A}\rangle$, i.e.,

$$Q_b = \sum_b |X_{b,A}\rangle\langle X_{b,A}| . \tag{3.3}$$

Note that states of a particle created on the correlated ground state are in the space spanned by $P+Q_b$. Insofar as the virtual occupation of the states $|X_{b,A}\rangle$ is unimportant in the description of high-energy scattering, we may neglect this space in our discussion.

It is useful at this stage to make a distinction among the bound states, b. If we neglect the diffuseness of the Fermi surface due to long-range correlations (e.g., pairing), we may divide the *bound* states into those that are largely occupied and those that are largely unoccupied. The former states will be denoted by B and the latter by $\bar{B}$. (The states $\eta_B|\Phi_A\rangle$ and $\eta^\dagger_{\bar{B}}|\Phi_A\rangle$ will have norms close to unity while the states $\eta_{\bar{B}}|\Phi_A\rangle$ and $\eta^\dagger_B|\Phi_A\rangle$ will have small norms.) This distinction is a meaningful one for nuclei, in which the depletion of an orbit due to short-range correlations is expected to be about 10–15%.

The states spanning Q_1 are constructed as follows. We define the state

$$|\hat{Y}^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle = \eta^\dagger_{\vec{k}_1}\eta^\dagger_{\vec{k}_2}\eta_B|\Phi_A\rangle\rho_B^{-1/2} , \tag{3.4}$$

where

$$\rho_B \equiv \langle\Phi_A|\eta^\dagger_B\eta_B|\Phi_A\rangle , \tag{3.5}$$

and proceed to construct an orthonormal set of states, $\{|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle\}$, based on these states. We also require that the states $|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle$ be orthogonal to the states $|X^{(+)}_{\vec{k},A}\rangle$ and $|X_{b,A}\rangle$.

At this point it is useful to introduce a simplifying assumption as to the structure of the various density matrices for the correlated state $|\Phi_A\rangle$. It should be understood, however, that this assumption is not necessary to the treatment proposed here. Rather, the assumption we suggest is largely for the purpose of keeping our treatment from becoming unnecessarily complicated, where the complications would be concerned mainly with very small effects. It should be kept in mind that there is no difficulty, in principle, involved in including such effects in the present treatment.

We shall *assume* for simplicity that both

$$\langle\chi^{(+)}_{\vec{k}}|\rho|\phi_b\rangle \equiv \langle\Phi_A|\eta^\dagger_b\eta_{\vec{k}}|\Phi_A\rangle = 0 \tag{3.6}$$

and

$$\langle\Phi_A|\eta_{\vec{k}_1}\eta^\dagger_{\vec{k}_2}\eta^\dagger_{\vec{k}_3}\eta_b|\Phi_A\rangle = 0 . \tag{3.7}$$

The quantities appearing in Eqs. (3.6) and (3.7) are expected to be much smaller than quantities such as $\langle\Phi_A|\eta^\dagger_{B_1}\eta_{B_2}|\Phi_A\rangle$, $\langle\Phi_A|\eta^\dagger_{\vec{k}}\eta_{\vec{k}'}|\Phi_A\rangle$, $\langle\Phi_A|\eta^\dagger_{\vec{k}_1}\eta^\dagger_{\vec{k}_2}\eta_{\vec{k}_3}\eta_{\vec{k}_4}|\Phi_A\rangle$, $\langle\Phi_A|\eta^\dagger_{B_1}\eta^\dagger_{B_2}\eta_{B_3}\eta_{B_4}|\Phi_A\rangle$, etc., so that the loss of generality in making these assumptions is of little practical importance. With this ansatz, we are now able to neglect the distinction between the states $|\chi^{(+)}_{\vec{k},A}\rangle$ and $|\bar{\chi}^{(+)}_{\vec{k},A}\rangle$ of Ref. 1.

In analogy to the construction of the states $|X^{(+)}_{\vec{k},A}\rangle$ from the states $|\chi^{(+)}_{\vec{k},A}\rangle \equiv \eta^\dagger_k|\Phi_A\rangle$, i.e. in the notation of Ref. 1,

$$|X^{(+)}_{\vec{k},A}\rangle = \int |\chi^{(+)}_{\vec{k}',A}\rangle\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}}\rangle d\vec{k}' , \tag{3.8}$$

we introduce a matrix σ such that the states

$$|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle \equiv \sum_{B'}\iint |\hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',B'}\rangle \times d\vec{k}_1' d\vec{k}_2' \langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\sigma_{B'B}|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle \tag{3.9}$$

form an orthonormal set.[4] Note that these are automatically orthogonal to the $|X^{(+)}_{\vec{k},A}\rangle$ and the $|X_{b',A}\rangle$ in the approximation of Eqs. (3.6) and (3.7).

Thus we define

$$Q_1 = \frac{1}{2}\sum_B \int |Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle d\vec{k}_1 d\vec{k}_2 \langle Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}| . \tag{3.10}$$

In a similar fashion, starting from states with two destruction operators, such as $\eta^\dagger_{\vec{k}_1}\eta^\dagger_{\vec{k}_2}\eta^\dagger_{\vec{k}_3}\eta_{B_1}\eta_{B_2}|\Phi_A\rangle$, we proceed to construct orthonormal states orthogonal to the $|X^{(+)}_{\vec{k},A}\rangle$ and $|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle$. From these states we form the projection operator Q_2. This procedure may obviously be continued until we have spanned the entire $(A+1)$-body space.

From Eq. (3.1) we see that we can define an effective Hamiltonian H_{eff} by

$$H_{\text{eff}} \equiv H + HQ\frac{1}{E - QHQ + i\epsilon}QH\,, \tag{3.11}$$

so that

$$V_{\text{eff}} = H_{\text{eff}} - \mathcal{H}_0\,. \tag{3.12}$$

Here

$$Q = 1 - P\,, \tag{3.13}$$

and $\mathcal{H}_0$ is the zeroth-order Hamiltonian defined in Eqs. (1.10) through (1.12). We now have written the projection operator Q as the sum of projection operators

$$Q = \sum_{\alpha=1}^{\nu} Q_\alpha\,, \tag{3.14}$$

and Eq. (3.11) becomes

$$H_{\text{eff}} = H + H(\sum_\alpha Q_\alpha)\frac{1}{E - (\sum_\beta Q_\beta)H(\sum_\gamma Q_\gamma) + i\epsilon}(\sum_\delta Q_\delta)H\,. \tag{3.15}$$

Since Eq. (3.15) is most awkward, it may be rearranged according to the procedure indicated below.

We write the equation

$$(E - H)|\psi\rangle = 0\,, \tag{3.16}$$

as the set of coupled equations,

$$(E - PHP)P|\psi\rangle = PH\sum_\alpha Q_\alpha|\psi\rangle\,,$$

$$(E - Q_\beta H Q_\beta)Q_\beta|\psi\rangle = Q_\beta HP|\psi\rangle + Q_\beta H\sum_{\alpha\neq\beta} Q_\alpha|\psi\rangle\,. \tag{3.17}$$

These coupled equations may be solved for $P|\psi\rangle$, and we observe that $P|\psi\rangle$ is the solution of the equation

$$[E - PH_{\text{eff}}P]P|\psi\rangle = 0\,, \tag{3.18}$$

where H_{eff} is obtained from the iteration scheme given below, viz.,

$$H_{\text{eff}}^{(0)} = H\,, \tag{3.19}$$

$$H_{\text{eff}}^{(n)} = H_{\text{eff}}^{(n-1)} + H_{\text{eff}}^{(n-1)}Q_n\frac{1}{E - Q_n H_{\text{eff}}^{(n-1)}Q_n + i\epsilon}Q_n H_{\text{eff}}^{(n-1)}\,, \tag{3.20}$$

$$H_{\text{eff}} = H_{\text{eff}}^{(\nu)}\,, \tag{3.21}$$

and ν is defined in Eq. (3.14). Truncation of this expression for H_{eff} by approximation of H_{eff} as $H_{\text{eff}}^{(n)}$ is alternative to the direct truncation of Eq. (3.15).

For the present, we shall content ourselves with the truncation[5] $H_{\text{eff}} = H_{\text{eff}}^{(1)}$, in which case we may write

$$PV_{\text{eff}}P = P(H - \overline{H}_0)P + PHQ_1\frac{1}{E - Q_1HQ_1 + i\epsilon}Q_1HP + \cdots\,. \tag{3.22}$$

We will now study the various terms in this series.

The use of Eq. (3.8) and the definition of $\overline{H}_0$ leads to

$$\begin{aligned}\langle X^{(+)}_{\vec{k}',A}|P(H - \overline{H}_0)P|X^{(+)}_{\vec{k},A}\rangle &= \langle X^{(+)}_{\vec{k}',A}|H|X^{(+)}_{\vec{k},A}\rangle - (\epsilon_{\vec{k}} + E_A)\delta(\vec{k} - \vec{k}')\\ &= \int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle d\vec{k}''\,\langle\chi^{(+)}_{\vec{k}'',A}|H|\chi^{(+)}_{\vec{k}''',A}\rangle d\vec{k}'''\,\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle - (\bar{\epsilon}_{\vec{k}} + E_A)\delta(\vec{k} - \vec{k}')\,.\end{aligned} \tag{3.23}$$

By making use of the definition of the states $|\chi^{(+)}_{\vec{k},A}\rangle$ and the fact that $H|\Phi_A\rangle = E_A|\Phi_A\rangle$, we then have

$$\langle X^{(+)}_{\vec{k}',A}|P(H - \overline{H}_0)P|X^{(+)}_{\vec{k},A}\rangle = \int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle\langle\Phi_A|\eta_{\vec{k}''}[H, \eta^\dagger_{\vec{k}'''}]|\Phi_A\rangle\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle d\vec{k}'''\,d\vec{k}'' - \epsilon_{\vec{k}}\delta(\vec{k} - \vec{k}')\,. \tag{3.24}$$

Now if we separate H as $H = \mathcal{K} + V$, where $\mathcal{K}$ is the kinetic energy operator, and use Eq. (3.6) we may easily show that

$$\begin{aligned}\langle X^{(+)}_{\vec{k}',A}|P(H - \overline{H}_0)P|X^{(+)}_{\vec{k},A}\rangle &= \int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle\langle\chi^{(+)}_{\vec{k}''}|(1-\rho)|\chi^{(+)}_{\vec{k}'''}\rangle\epsilon_{\vec{k}'''}\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle d\vec{k}'''\,d\vec{k}'' - \epsilon_{\vec{k}}\delta(\vec{k} - \vec{k}')\\ &\quad + \int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle\langle\Phi_A|\eta_{\vec{k}''}[V, \eta^\dagger_{\vec{k}'''}]|\Phi_A\rangle\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle d\vec{k}''\,d\vec{k}'''\,.\end{aligned} \tag{3.25}$$

In obtaining Eq. (3.25) we have used the fact that the one-body kinetic energy operator is diagonal in the states $|\chi^{(+)}_{\vec{k}}\rangle$, i.e. $\langle\chi^{(+)}_{\vec{k}'}|h_0|\chi^{(+)}_{\vec{k}}\rangle = \epsilon_{\vec{k}}\delta(\vec{k}' - \vec{k})$.

The part of Eq. (3.25) independent of V may be written, with $\gamma_p \equiv p\gamma p$, $\rho_p \equiv p\rho p$, etc., as

$$\langle\chi_{\vec{k}'}^{(+)}|\gamma_p(1-\rho_p)(h_0-\epsilon_{\vec{k}})\gamma_p|\chi_{\vec{k}}^{(+)}\rangle = \langle\chi_{\vec{k}'}^{(+)}|\gamma_p^{-1}(h_0-\epsilon_{\vec{k}})\gamma_p|\chi_{\vec{k}}^{(+)}\rangle$$

$$= \langle\chi_{\vec{k}'}^{(+)}|\gamma_p^{-1}(h_0-\epsilon_{\vec{k}})(\gamma_p-1)|\chi_{\vec{k}}^{(+)}\rangle\,, \tag{3.26}$$

where we have recalled the relation,

$$\gamma_p(p-\rho_p)\gamma_p = p\,, \quad \text{or} \quad \gamma_p(p-\rho_p) = \gamma_p^{-1}\,, \tag{3.27}$$

and

$$\langle\chi_{\vec{k}''}^{(+)}|h_0|\chi_{\vec{k}'''}^{(+)}\rangle = \epsilon_{\vec{k}'''}\delta(\vec{k}''-\vec{k}''')\,. \tag{3.28}$$

We have also seen earlier[1] that $\gamma_p - 1 \approx \frac{1}{2}\rho_p$ is a small quantity, so that to lowest order

$$\langle\chi_{\vec{k}'}^{(+)}|\gamma_p^{-1}(h_0-\epsilon_{\vec{k}})(\gamma_p-1)|\chi_{\vec{k}}^{(+)}\rangle \approx (\epsilon_{\vec{k}'}-\epsilon_{\vec{k}})\langle\chi_{\vec{k}'}^{(+)}|(\gamma_p-1)|\chi_{\vec{k}}^{(+)}\rangle$$

$$\approx \tfrac{1}{2}(\epsilon_{\vec{k}'}-\epsilon_{\vec{k}})\langle\chi_{\vec{k}'}^{(+)}|\rho|\chi_{\vec{k}}^{(+)}\rangle\,. \tag{3.29}$$

The foregoing approximations thus yield the result that

$$\langle X_{\vec{k}',A}^{(+)}|P(H-\bar{H}_0)P|X_{\vec{k},A}^{(+)}\rangle \cong \tfrac{1}{2}(\epsilon_{\vec{k}'}-\epsilon_{\vec{k}})\langle\chi_{\vec{k}'}^{(+)}|\rho|\chi_{\vec{k}}^{(+)}\rangle + \int\langle\chi_{\vec{k}'}^{(+)}|\gamma|\chi_{\vec{k}''}^{(+)}\rangle\langle\Phi_A|\eta_{\vec{k}''}[V,\eta_{\vec{k}'''}^{\dagger}]|\Phi_A\rangle\langle\chi_{\vec{k}'''}^{(+)}|\gamma|\chi_{\vec{k}}^{(+)}\rangle d\vec{k}''\,d\vec{k}'''\,. \tag{3.30}$$

The second term of Eq. (3.30) may be developed further by writing

$$\langle\Phi_A|\eta_{\vec{k}''}[V,\eta_{\vec{k}'''}^{\dagger}]|\Phi_A\rangle = \langle\Phi_A|\{\eta_{\vec{k}''},[V,\eta_{\vec{k}'''}^{\dagger}]\}_+|\Phi_A\rangle - \langle\Phi_A|[V,\eta_{\vec{k}'''}^{\dagger}]\eta_{\vec{k}''}|\Phi_A\rangle\,. \tag{3.31}$$

The treatment of Eq. (3.31) may be facilitated by means of the well-known relation,

$$\{\eta_{\vec{k}''},[V,\eta_{\vec{k}'''}^{\dagger}]\}_+ = \sum_{\alpha\beta}\langle\chi_{\vec{k}''}^{(+)}\alpha|v|\chi_{\vec{k}'''}^{(+)}\beta\rangle_A\eta_\alpha^{\dagger}\eta_\beta\,, \tag{3.32}$$

where α and β run over the complete set of single-particle states $|\chi_{\vec{k}}^{(+)}\rangle$ and $|\phi_b\rangle$ and the subscript A indicates the antisymmetrized state vector, $|\chi_{\vec{k}'''}^{(+)}\beta\rangle_A = |\chi_{\vec{k}'''}^{(+)}\beta\rangle - |\beta\chi_{\vec{k}'''}^{(+)}\rangle$. The matrix element of interest is thus

$$\langle\Phi_A\,|\{\eta_{\vec{k}''},[V,\eta_{\vec{k}'''}^{\dagger}]\}_+\,\Phi_A\rangle = \sum_{\alpha\beta}\langle\chi_{\vec{k}}^{(+)}\alpha|v|\chi_{\vec{k}'''}^{(+)}\beta\rangle_A\langle\beta|\rho|\alpha\rangle\,, \tag{3.33}$$

where, as usual, $\langle\beta|\rho|\alpha\rangle$ represents an element of the density matrix.[6]

At this point we have, then, that

$$\langle X_{\vec{k}',A}^{(+)}|P(H-\bar{H}_0)P|X_{\vec{k},A}^{(+)}\rangle \approx (\epsilon_{\vec{k}'}-\epsilon_{\vec{k}})\langle\chi_{\vec{k}'}^{(+)}|(\gamma_p-1)|\chi_{\vec{k}}^{(+)}\rangle$$

$$+\sum_{\alpha\beta}\int\langle\chi_{\vec{k}'}^{(+)}|\gamma|\chi_{\vec{k}''}^{(+)}\rangle d\vec{k}''\,\langle\chi_{\vec{k}''}^{(+)}\alpha|v|\chi_{\vec{k}'''}^{(+)}\beta\rangle\langle\beta|\rho|\alpha\rangle d\vec{k}'''\,\langle\chi_{\vec{k}'''}^{(+)}|\gamma|\chi_{\vec{k}}^{(+)}\rangle$$

$$-\int\langle\chi_{\vec{k}}^{(+)}|\gamma|\chi_{\vec{k}''}^{(+)}\rangle d\vec{k}''\,\langle\Phi_A|[V,\eta_{\vec{k}'''}^{\dagger}]\eta_{\vec{k}''}|\Phi_A\rangle d\vec{k}'''\,\langle\chi_{\vec{k}'''}^{(+)}|\gamma|\chi_{\vec{k}}^{(+)}\rangle\,. \tag{3.34}$$

To complete this part of the discussion we now need to look at the last term of Eq. (3.34). We again use Greek letters to denote the complete set of single-particle states and, in this notation, we may write

$$[V,\eta_{\vec{k}'''}^{\dagger}]\eta_{\vec{k}''} = \tfrac{1}{2}\sum_{\alpha\beta\gamma}\eta_\alpha^{\dagger}\eta_\beta^{\dagger}\langle\alpha\beta|v|\chi_{\vec{k}'''}^{(+)}\gamma\rangle_A\eta_\gamma\eta_{\vec{k}''}\,. \tag{3.35}$$

From Eq. (3.35) we see that in order to evaluate the right-hand term in Eq. (3.34) we need to know the quantity $\langle\Phi_A|\eta_\alpha^{\dagger}\eta_\beta^{\dagger}\eta_\gamma\eta_\delta|\Phi_A\rangle$. This quantity is of course a two-body correlation function. If we define this correlation function as

$$\langle\gamma\delta|\rho^{(2)}|\beta\alpha\rangle \equiv \langle\Phi_A|\eta_\alpha^{\dagger}\eta_\beta^{\dagger}\eta_\gamma\eta_\delta|\Phi_A\rangle\,, \tag{3.36}$$

then we may rewrite Eq. (3.34) in the form

$$\langle X_{\vec{k}',A}^{(+)}|P(H-H_0)P|X_{\vec{k},A}^{(+)}\rangle = (\epsilon_{\vec{k}'}-\epsilon_{\vec{k}})\langle\chi_{\vec{k}'}^{(+)}|(\gamma_p-1)|\chi_{\vec{k}}^{(+)}\rangle$$

$$+\int\langle\chi_{\vec{k}'}^{(+)}|\gamma|\chi_{\vec{k}''}^{(+)}\rangle d\vec{k}''\Bigg\{\sum_{\alpha\beta}\langle\chi_{\vec{k}''}^{(+)}\alpha|v|\chi_{\vec{k}'''}^{(+)}\beta\rangle_A\langle\beta|\rho|\alpha\rangle$$

$$-\frac{1}{2}\sum_{\alpha,\beta,\gamma}\langle\alpha\beta|v|\chi_{\vec{k}'''}^{(+)}\gamma\rangle_A\langle\chi_{\vec{k}''}^{(+)}\gamma|\rho^{(2)}|\alpha\beta\rangle\Bigg\}d\vec{k}'''\,\langle\chi_{\vec{k}'''}^{(+)}|\gamma|\chi_{\vec{k}}^{(+)}\rangle\,. \tag{3.37}$$

The correlation function of Eq. (3.36) has been investigated in connection with nuclear structure studies. This correlation function is given through two-body cluster terms, by[7]

$$\langle\gamma\delta|\rho^{(2)}|\beta\alpha\rangle=\tfrac{1}{2}\sum_{\substack{B_{1'}B_{2'}\\B_1,B_2}}{}_A\langle\phi_{B_1}\phi_{B_2}|(1+f_{12}^{\dagger})|\alpha\beta\rangle\langle\delta\gamma|(1+f_{12})|\phi_{B_{1'}}\phi_{B_{2'}}\rangle_A\langle\phi_{B_{1'}}|\rho|\phi_{B_1}\rangle\langle\phi_{B_{2'}}|\rho|\phi_{B_2}\rangle+\cdots\,, \tag{3.38}$$

where $(1+f_{12})$ is the wave matrix for the Bethe-Goldstone equation, viz.

$$(1+f_{12})=1-\frac{Q_{12}}{e}K_{12}\,. \tag{3.39}$$

In Eq. (3.39), K_{12} is the Bethe-Goldstone reaction matrix as defined for finite systems. With this reduction we have

$$\begin{aligned}\langle\Phi_A|[V,\eta_{\vec{k}'''}^{\dagger}]\eta_{\vec{k}''}|\Phi_A\rangle&=\frac{1}{2}\sum_{\alpha\beta\gamma}\langle\alpha\beta|v|\chi_{\vec{k}'''}^{(+)}\gamma\rangle_A\langle\gamma\chi_{\vec{k}''}^{(+)}|\rho^{(2)}|\beta\alpha\rangle\\&=\frac{1}{4}\sum_{\substack{B_{1'}B_{2'}\\B_1B_2}}\int{}_A\langle\phi_{B_1}\phi_{B_2}|(1+f_{12}^{\dagger})v_{12}|\chi_{\vec{k}'''}^{(+)}\chi_{\vec{k}'}^{(+)}\rangle_A d\vec{k}'\langle\chi_{\vec{k}''}^{(+)}\chi_{\vec{k}'}^{(+)}|f_{12}|\phi_{B_{1'}}\phi_{B_{2'}}\rangle_A\\&\qquad\times\langle\phi_{B_{1'}}|\rho|\phi_{B_1}\rangle\langle\phi_{B_{2'}}|\rho|\phi_{B_2}\rangle\\&=-\frac{1}{2}\sum_{B_1B_2}\int{}_A\langle\phi_{B_1}\phi_{B_2}|K_{12}|\chi_{\vec{k}'''}^{(+)}\chi_{\vec{k}'}^{(+)}\rangle\frac{d\vec{k}''}{e}\langle\chi_{\vec{k}''}^{(+)}\chi_{\vec{k}'}^{(+)}|K_{12}|\phi_{B_1}\phi_{B_2}\rangle_A\,\rho_{B_1}\rho_{B_2}\,,\end{aligned} \tag{3.40}$$

where we have used the fact that $K_{12}=v_{12}-v_{12}(Q/e)K_{12}$, and have also assumed that ρ is diagonal[8] in the space spanned by the $|\phi_b\rangle$. The energy denominator in Eqs. (3.38) and (3.39) is $e=\epsilon_{\vec{k}'''}+\epsilon_{\vec{k}'}-(\epsilon_{B_1}+\epsilon_{B_2})$.

The expression given in Eq. (3.40) may be inserted in Eq. (3.34) to complete the specification of $\langle X_{\vec{k}',A}^{(+)}|P(H-\bar{H}_0)P|X_{\vec{k},A}^{(+)}\rangle$. We may develop this expression further by writing the continuum matrix elements of γ as

$$\begin{aligned}\langle\chi_{\vec{k}'}^{(+)}|\gamma|\chi_{\vec{k}}^{(+)}\rangle&=\delta(\vec{k}-\vec{k}')+\langle\chi_{\vec{k}'}^{(+)}|(\gamma_p-1)|\chi_{\vec{k}}^{(+)}\rangle\\&=\delta(\vec{k}-\vec{k}')+\tfrac{1}{2}\langle\chi_{\vec{k}'}^{(+)}|\rho|\chi_{\vec{k}}^{(+)}\rangle+\cdots\,,\end{aligned} \tag{3.41}$$

where we have used the fact that the (γ_p-1) term is small. The various terms of Eq. (3.34) may be expressed diagrammatically. This development is presented in Figs. 1 and 2. It should be noted that the presence of the $(\gamma-1)$ terms, which appear in the external lines, is particular to the theory developed in this work.

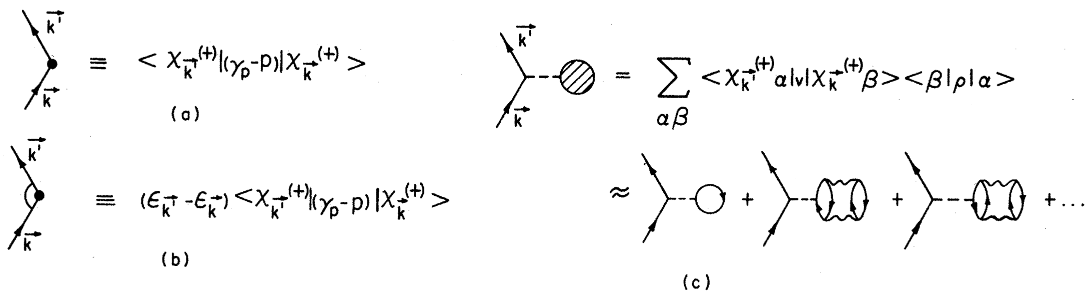

FIG. 1. A schematic representation of some of the elements appearing in Eq. (3.34). Upward going lines represent particles in the orbits $|\chi_{\vec{k}}^{(+)}\rangle$ and downward lines represent holes in the orbits $|\phi_b\rangle$. (a) The black dot represents the one-body operator (γ_p-p). (b) In this figure we represent the first term of Eq. (3.34). (c) Here the cross-hatched circle represents the ground-state density matrix and the horizontal dashed line represents the interaction, v. We also indicate an expansion of the density matrix in a conventional representation. (Only direct terms are drawn.) The wavy line represents the reaction matrix, K. The first and third terms in (c) may be written as a single term if we associate the occupation factors with the hole lines. These occupation factors should also be associated with the down going lines in the second diagram of (c).

IV. RESUMMATION OF THE PERTURBATION SERIES

It is evident that Eq. (3.34) may not be considered alone, since it contains the matrix elements of the potential which may be singular (due to a hard core) or, at least, very large. We must consider the effects of the spaces, Q_1, Q_2, etc., to provide the possibility of developing an expansion in reaction matrices such as is conventional for high-energy scattering of strongly interacting particles. The division of the Q space into various orthogonal Hilbert spaces was made with just this purpose in mind.

We now wish to consider the second term of Eq. (3.22), $PHQ_1(E-Q_1HQ_1+i\epsilon)^{-1}Q_1HP$. There are two types of matrix elements which occur in the analysis of that term. These are matrix elements of the type $\langle Y^{(+)}_{\vec k_1,\vec k_2,B}|H|X^{(+)}_{\vec k,A}\rangle$ and $\langle Y^{(+)}_{\vec k_1,\vec k_2,B}|H|Y^{(+)}_{\vec k_3,\vec k_4,B'}\rangle$. For example, we have

$$\langle Y^{(+)}_{\vec k_1,\vec k_2,B}|H|X^{(+)}_{\vec k,A}\rangle=\sum_{B'}\int\langle\chi^{(+)}_{\vec k_1}\chi^{(+)}_{\vec k_2}|\sigma_{BB'}|\chi^{(+)}_{\vec k_1'}\chi^{(+)}_{\vec k_2'}\rangle d\vec k_1'd\vec k_2'\langle\hat Y_{\vec k_1',\vec k_2',B'}|H|\chi^{(+)}_{\vec k',A}\rangle d\vec k'\langle\chi^{(+)}_{\vec k'}|\gamma|\chi^{(+)}_{\vec k}\rangle, \tag{4.1}$$

and

$$\begin{aligned}\langle\hat Y^{(+)}_{\vec k_1,\vec k_2,B}|H|\chi^{(+)}_{\vec k,A}\rangle&=\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}H\eta^\dagger_{\vec k}|\Phi_A\rangle/\rho_B^{1/2}\\&=\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}[H,\eta^\dagger_{\vec k}]|\Phi_A\rangle/\rho_B^{1/2}+E_A\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}\eta^\dagger_{\vec k}|\Phi_A\rangle/\rho_B^{1/2}.\end{aligned} \tag{4.2}$$

The last term in Eq. (4.2) may be discarded at this stage since it does not contribute to Eq. (4.1), since $|Y^{(+)}_{\vec k_1',\vec k_2',B'}\rangle$ and $|X^{(+)}_{\vec k_1',A}\rangle$ are orthogonal.

Further, the kinetic energy contribution to Eq. (4.1) may be obtained from

$$\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}[\mathcal{K},\eta^\dagger_{\vec k}]|\Phi_A\rangle=\sum_{b'}\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}\eta^\dagger_{b'}|\Phi_A\rangle\langle\phi_{b'}|h_0|\chi^{(+)}_{\vec k}\rangle+\epsilon_{\vec k}\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}\eta^\dagger_{\vec k}|\Phi_A\rangle. \tag{4.3}$$

From Eq. (4.3) we can see that the kinetic energy term may also be neglected. The second term of Eq. (4.3) is small and may be dropped. The first term of Eq. (4.3) is quite small as it stands and also does not contribute to the $\langle Y^{(+)}_{\vec k_1',\vec k_2',B}|H|X^{(+)}_{\vec k',A}\rangle$ matrix elements if the states of the Q_1 space are orthogonalized to those of the P and Q_b spaces. (More generally we may note that the kinetic energy operator does not couple the different Hilbert spaces, $P+Q_b, Q_1, Q_2, \ldots,$ etc., to any significant extent.) This is most clearly seen if use is made of the commutator method, as above.

With these comments in mind, we consider

$$\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}[V,\eta^\dagger_{\vec k}]|\Phi_A\rangle/\rho_B^{1/2}=\tfrac{1}{2}\sum_{\alpha\beta\delta}\langle\Phi_A|\eta_B^\dagger(\eta_{\vec k_2}\eta_{\vec k_1}\eta^\dagger_\alpha\eta^\dagger_\beta)\eta_\delta|\Phi_A\rangle\langle\alpha\beta|v|\chi^{(+)}_{\vec k}\delta\rangle_A/\rho_B^{1/2}. \tag{4.4}$$

We now make use of a relation we shall use several times in this section, viz.

$$(\eta_{\vec k_2}\eta_{\vec k_1}\eta^\dagger_\alpha\eta^\dagger_\beta)=(\delta_{\vec k_2\beta}\delta_{\vec k_1\alpha}-\delta_{\vec k_2\alpha}\delta_{\vec k_1\beta})+(\eta^\dagger_\alpha\eta_{\vec k_2}\delta_{\vec k_1\beta}-\eta^\dagger_\beta\eta_{\vec k_2}\delta_{\vec k_1\alpha}-\eta^\dagger_\alpha\eta_{\vec k_1}\delta_{\vec k_2\beta}+\eta^\dagger_\beta\eta_{\vec k_1}\delta_{\vec k_2\alpha})+\eta^\dagger_\alpha\eta^\dagger_\beta\eta_{\vec k_2}\eta_{\vec k_1}, \tag{4.5}$$

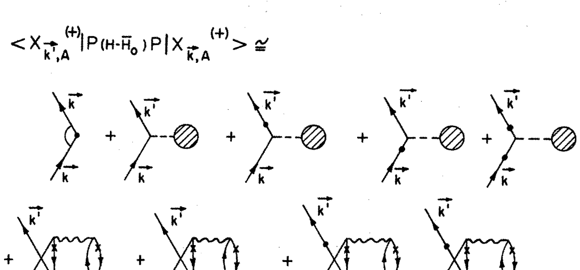

FIG. 2. Diagrammatic representation of Eq. (3.34). Various elements have been defined in Fig. 1, and use has been made of Eqs. (3.39) and (3.40). The crosses in the down-going (hole) lines are a reminder that the holes are associated with their occupation factors, ρ_B.

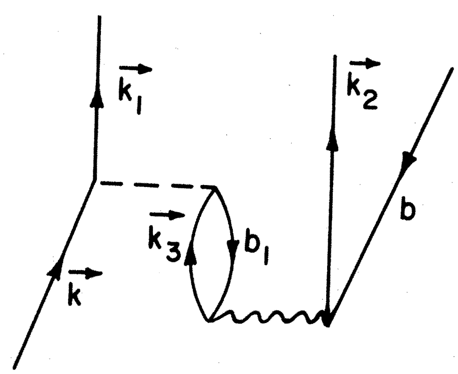

FIG. 3. A diagrammatic representation of a term appearing in the evaluation of Eq. (4.4). The intermediate states $|\chi^{(+)}_{\vec k_3}\rangle$ and $|\phi_{b_1}\rangle$ are summed in this term. The wavy line represents a reaction matrix and the dashed line a potential matrix element.

to reorganize Eq. (4.4) as

$$\langle\Phi_A|\eta_B^\dagger\eta_{\vec k_2}\eta_{\vec k_1}[V,\eta_{\vec k}^\dagger]|\Phi_A\rangle\rho_B^{-1/2}$$

$$=\sum_\delta\langle\Phi_A|\eta_B^\dagger\eta_\delta|\Phi_A\rangle\langle\chi_{\vec k_1}^{(+)}\chi_{\vec k_2}^{(+)}|v|\chi_{\vec k}^{(+)}\delta\rangle_A\rho_B^{-1/2}$$
$$+\tfrac{1}{2}\sum_{\alpha\beta\delta}\langle\Phi_A|\eta_B^\dagger(\eta_\alpha^\dagger\eta_{\vec k_2}\delta_{\vec k_1\beta}-\eta_\beta^\dagger\eta_{\vec k_2}\delta_{\vec k_1\alpha}-\eta_\alpha^\dagger\eta_{\vec k_1}\delta_{\vec k_2\beta}+\eta_\beta^\dagger\eta_{\vec k_1}\delta_{\vec k_2\alpha})\eta_\delta|\Phi_A\rangle\langle\alpha\beta|v|\chi_{\vec k}^{(+)}\delta\rangle\rho_B^{-1/2}$$
$$+\tfrac{1}{2}\sum_{\alpha\beta\delta}\langle\Phi_A|\eta_B^\dagger\eta_\alpha^\dagger\eta_\beta^\dagger\eta_{\vec k_2}\eta_{\vec k_1}\eta_\delta|\Phi_A\rangle\langle\alpha\beta|v|\chi_{\vec k}^{(+)}\delta\rangle\rho_B^{-1/2}. \tag{4.6}$$

The first term on the right-hand side of Eq. (4.6) is

$$\sum_\delta\langle\chi_{\vec k_1}^{(+)}\chi_{\vec k_2}^{(+)}|v|\chi_{\vec k}^{(+)}\delta\rangle_A\langle\delta|\rho|\phi_B\rangle\rho_B^{-1/2}=\sum_{b'}\langle\chi_{\vec k_1}^{(+)}\chi_{\vec k_2}^{(+)}|v|\chi_{\vec k}^{(+)}\phi_{b'}\rangle_A\langle\phi_{b'}|\rho|\phi_B\rangle\rho_B^{-1/2}$$
$$=\langle\chi_{\vec k_1}^{(+)}\chi_{\vec k_2}^{(+)}|v|\chi_{\vec k}^{(+)}\phi_B\rangle_A\rho_B^{-1/2}. \tag{4.7}$$

In obtaining the final equality in Eq. (4.7), we have assumed that the ϕ_b are chosen so as to diagonalize ρ in the space of bound states.

The second term on the right-hand side of Eq. (4.6) gives rise to diagrams of the type shown in Fig. 3. We neglect these two-hole terms for the present. The last term on the right-hand side of Eq. (4.6) gives rise to even more complicated forms which we also neglect in this discussion.

At this point our main concern is to resum the perturbation series, so that the matrix elements of the potential may be replaced by the matrix elements of a reaction matrix. To this end we concentrate on the simplest kinds of diagrams with the *fewest hole lines*.

If we keep only the term given in Eq. (4.7), we then arrive at the approximate result for Eq. (4.1), viz.

$$\langle Y_{\vec k_1,\vec k_2,B}^{(+)}|H|X_{\vec k,A}^{(+)}\rangle\cong\sum_{B'}\int\langle\chi_{\vec k_1}^{(+)}\chi_{\vec k_2}^{(+)}|\sigma_{BB'}|\chi_{\vec k_1'}^{(+)}\chi_{\vec k_2'}^{(+)}\rangle d\vec k_1'd\vec k_2'\langle\chi_{\vec k_1'}^{(+)}\chi_{\vec k_2'}^{(+)}|v|\chi_{\vec k'}^{(+)}\phi_{B'}\rangle_A\rho_{B'}^{1/2}d\vec k'\langle\chi_{\vec k'}^{(+)}|\gamma|\chi_{\vec k}^{(+)}\rangle+\cdots$$
$$\cong\int\langle\chi_{\vec k_1}^{(+)}\chi_{\vec k_2}^{(+)}|\sigma_{BB}|\chi_{\vec k_1'}^{(+)}\chi_{\vec k_2'}^{(+)}\rangle d\vec k_1'd\vec k_2'\langle\chi_{\vec k_1'}^{(+)}\chi_{\vec k_2'}^{(+)}|v|\chi_{\vec k'}^{(+)}\phi_B\rangle_A\rho_B^{1/2}d\vec k'\langle\chi_{\vec k'}^{(+)}|\gamma|\chi_{\vec k}^{(+)}\rangle+\cdots. \tag{4.8}$$

In Eq. (4.8) we have noted that σ is expected to have larger diagonal than off-diagonal elements in the bound-state labels. This "eliminates" the sum over B'. The above matrix element takes one from the P space into the Q_1 space.

Let us now consider the matrix elements of Q_1HQ_1. In the representation in which we are working, we need to study the matrix elements $\langle Y_{\vec k_1'\vec k_2',B'}^{(+)}|H|Y_{\vec k_1\vec k_2,B}^{(+)}\rangle$, which according to the definition of the states $|Y_{\vec k_1'\vec k_2',B}^{(+)}\rangle$ of Eq. (3.9) may, in turn, be obtained from the matrix elements:

$$\langle\hat Y_{\vec k_1',\vec k_2',B'}^{(+)}|H|\hat Y_{\vec k_1,\vec k_2,B}^{(+)}\rangle=\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}H\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger\eta_B|\Phi_A\rangle\rho_B^{-1/2}\rho_{B'}^{-1/2}$$
$$=\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}[H,\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger\eta_B]|\Phi_A\rangle\rho_B^{-1/2}\rho_{B'}^{-1/2}+E_A\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger\eta_B|\Phi_A\rangle\rho_B^{-1/2}\rho_{B'}^{-1/2}. \tag{4.9}$$

In order to evaluate the right-hand side of Eq. (4.9) we must then study the matrix element

$$\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}[H,\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger\eta_B]|\Phi_A\rangle.$$

We first observe that

$$\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}[H,\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger\eta_B]|\Phi_A\rangle=\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}[H,\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger]\eta_B|\Phi_A\rangle+\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger[H,\eta_B]|\Phi_A\rangle. \tag{4.10}$$

With the help of Eq. (4.5) once more, the second term on the right-hand side of Eq. (4.10) is now seen to be

$$\langle\Phi_A|\eta_{B'}^\dagger(\eta_{\vec k_2'}\eta_{\vec k_1'}\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger)[H,\eta_B]|\Phi_A\rangle$$

$$=\langle\Phi_A|\eta_{B'}^\dagger[H,\eta_B]|\Phi_A\rangle\{\delta(\vec k_1-\vec k_1')\delta(\vec k_2-\vec k_2')-\delta(\vec k_1-\vec k_2')\delta(\vec k_2-\vec k_1')\}+\langle\Phi_A|\eta_{B'}^\dagger(\eta_{\vec k_1}^\dagger\eta_{\vec k_2}^\dagger\eta_{\vec k_2'}\eta_{\vec k_1'}+\cdots)[H,\eta_B]|\Phi_A\rangle,$$
$$=-\epsilon_B\,\rho_B\,\delta_{BB'}\{\delta(\vec k_1-\vec k_1')\delta(\vec k_2-\vec k_2')-\delta(\vec k_1-\vec k_2')\delta(\vec k_2-\vec k_1')\}+\cdots. \tag{4.11}$$

The neglected terms on the right-hand side of Eq. (4.11) represent hole interaction terms, which we shall not consider in the present treatment. The identity $\langle\Phi_A|\eta_{B'}^\dagger[H,\eta_B]|\Phi\rangle \equiv -\epsilon_B\,\rho_B\,\delta_{BB'}$ is straightforward and has been established in nuclear structure studies.[9]

We now write the first term on the right-hand side of Eq. (4.10) as

$$\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[H,\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle=\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}\{[\mathcal{K},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]+[V,\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\}\eta_B|\Phi_A\rangle, \tag{4, 12}$$

and proceed to examine the kinetic energy term in this equation.

The kinetic energy operator is

$$\mathcal{K}=\int\epsilon_{\vec{k}}\,\eta_{\vec{k}}^\dagger\eta_{\vec{k}}\,d\vec{k}+\sum_b\int\langle\phi_b|h_0|\chi_{\vec{k}}^{(+)}\rangle\,\eta_b^\dagger\eta_{\vec{k}}\,d\vec{k}+\sum_b\int\langle\chi_{\vec{k}}^{(+)}|h_0|\phi_b\rangle\,\eta_{\vec{k}}^\dagger\eta_b\,d\vec{k}+\sum_{bb'}\langle\phi_b|h_0|\phi_{b'}\rangle\,\eta_b^\dagger\eta_{b'}\,. \tag{4.13}$$

The last three terms on the right-hand side of Eq. (4.13) do not contribute significantly to the matrix element $\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[\mathcal{K},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle$. Thus we evaluate this matrix element as

$$\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[\mathcal{K},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle\cong\int\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[\eta_{\vec{k}}^\dagger\eta_{\vec{k}},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle\,\epsilon_{\vec{k}}\,d\vec{k}\,. \tag{4.14}$$

The commutator in Eq. (4.14) is readily seen to be

$$[\eta_{\vec{k}}^\dagger\eta_{\vec{k}},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]=\eta_{\vec{k}}^\dagger\eta_{\vec{k}_2}^\dagger\,\delta(\vec{k}-\vec{k}_1)-\eta_{\vec{k}}^\dagger\eta_{\vec{k}_1}^\dagger\,\delta(\vec{k}-\vec{k}_2)\,, \tag{4.15}$$

so that we immediately obtain

$$\begin{aligned}\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[\mathcal{K},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle&\cong(\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2})\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger\eta_B|\Phi_A\rangle\\&\cong\rho_B\,\delta_{BB'}(\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2})[\delta(\vec{k}_2-\vec{k}_2')\delta(\vec{k}_1-\vec{k}_1')-\delta(\vec{k}_2-\vec{k}_1')\delta(\vec{k}_1-\vec{k}_2')]+\cdots\,.\end{aligned} \tag{4.16}$$

The last equality in Eq. (4.16) represents still another application of Eq. (4.5), and the terms indicated by the three dots are here ignored for the usual reasons.

We must now evaluate the potential energy matrix element $\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[V,\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle$. To this end we write the potential energy operator V as

$$V=\frac{1}{2}\int\langle\chi_{\vec{k}_3}^{(+)}\chi_{\vec{k}_4}^{(+)}|v|\chi_{\vec{k}_6}^{(+)}\chi_{\vec{k}_5}^{(+)}\rangle\eta_{\vec{k}_3}^\dagger\eta_{\vec{k}_4}^\dagger\eta_{\vec{k}_5}\eta_{\vec{k}_6}\,d\vec{k}_3\,d\vec{k}_4\,d\vec{k}_5\,d\vec{k}_6+\cdots\,. \tag{4.17}$$

The terms which have been indicated by three dots in Eq. (4.17) have been dropped because they represent hole interaction terms in the matrix element under consideration. The potential energy matrix thus becomes

$$\begin{aligned}&\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[V,\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle\\&\quad\cong\frac{1}{2}\int d\vec{k}_3\,d\vec{k}_4\,d\vec{k}_5\,d\vec{k}_6\langle\chi_{\vec{k}_3}^{(+)}\chi_{\vec{k}_4}^{(+)}|v|\chi_{\vec{k}_6}^{(+)}\chi_{\vec{k}_5}^{(+)}\rangle\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[\eta_{\vec{k}_3}^\dagger\eta_{\vec{k}_4}^\dagger\eta_{\vec{k}_5}\eta_{\vec{k}_6},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle+\cdots\,.\end{aligned} \tag{4.18}$$

The commutator on the right-hand side of Eq. (4.18) is easily seen to be

$$\begin{aligned}[\eta_{\vec{k}_3}^\dagger\eta_{\vec{k}_4}^\dagger\eta_{\vec{k}_5}\eta_{\vec{k}_6},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]&=\eta_{\vec{k}_3}^\dagger\eta_{\vec{k}_4}^\dagger\{\delta(\vec{k}_5-\vec{k}_2)\delta(\vec{k}_6-\vec{k}_1)-\delta(\vec{k}_5-\vec{k}_1)\delta(\vec{k}_6-\vec{k}_2)\\&\quad+\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_5}\delta(\vec{k}_2-\vec{k}_6)-\eta_{\vec{k}_2}^\dagger\eta_{\vec{k}_5}\delta(\vec{k}_1-\vec{k}_6)+\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_6}\delta(\vec{k}_2-\vec{k}_5)-\eta_{\vec{k}_2}^\dagger\eta_{\vec{k}_6}\delta(\vec{k}_1-\vec{k}_5)\},\end{aligned} \tag{4.19}$$

and hence

$$\begin{aligned}&\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[\eta_{\vec{k}_3}^\dagger\eta_{\vec{k}_4}^\dagger\eta_{\vec{k}_5}\eta_{\vec{k}_6},\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle\\&\quad=\rho_B\,\delta_{BB'}\{\delta(\vec{k}_2'-\vec{k}_4)\delta(\vec{k}_1'-\vec{k}_3)-\delta(\vec{k}_2'-\vec{k}_3)\delta(\vec{k}_1'-\vec{k}_4)\}\{\delta(\vec{k}_2-\vec{k}_5)\delta(\vec{k}_1-\vec{k}_6)-\delta(\vec{k}_1-\vec{k}_5)\delta(\vec{k}_2-\vec{k}_6)\}+\cdots\,,\end{aligned} \tag{4.20}$$

so that

$$\langle\Phi_A|\eta_{B'}^\dagger\eta_{\vec{k}_2'}\eta_{\vec{k}_1'}[V,\eta_{\vec{k}_1}^\dagger\eta_{\vec{k}_2}^\dagger]\eta_B|\Phi_A\rangle\cong\rho_B\,\delta_{BB'}\langle\chi_{\vec{k}_1'}^{(+)}\chi_{\vec{k}_2'}^{(+)}|v|\chi_{\vec{k}_2}^{(+)}\chi_{\vec{k}_1}^{(+)}\rangle_A+\cdots\,, \tag{4.21}$$

where we remind the reader that

$$|\chi_{\vec{k}_2}^{(+)}\chi_{\vec{k}_1}^{(+)}\rangle_A\equiv|\chi_{\vec{k}_2}^{(+)}\chi_{\vec{k}_1}^{(+)}\rangle-|\chi_{\vec{k}_1}^{(+)}\chi_{\vec{k}_2}^{(+)}\rangle\,. \tag{4.22}$$

We now have all the ingredients required to evaluate the Q_1 matrix elements of the Hamiltonian H to within the approximations indicated above. This result is simply

$$\langle Y^{(+)}_{\vec{k}'_1,\vec{k}'_2,B'}|H|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle \cong (\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2}-\epsilon_B+E_A)\delta_{BB'} \times\{\delta(\vec{k}_1-\vec{k}'_1)\delta(\vec{k}_2-\vec{k}'_2)-\delta(\vec{k}_1-\vec{k}'_2)\delta(\vec{k}_2-\vec{k}'_1)\}+\langle\chi^{(+)}_{\vec{k}'_1}\chi^{(+)}_{\vec{k}'_2}|v|\chi^{(+)}_{\vec{k}_2}\chi^{(+)}_{\vec{k}_1}\rangle_A\delta_{BB'}+\cdots. \quad (4.23)$$

The two results given in Eqs. (4.8) and (4.23) enable us to evaluate $PHQ_1(E-Q_1HQ_1+i\epsilon)^{-1}Q_1HP$, within the indicated approximations.

At this stage in our discussion it proves to be convenient to define propagators $\mathcal{G}_1$ and g_1 as

$$\mathcal{G}_1=\left(\frac{Q_1}{E-Q_1HQ_1+i\epsilon}\right) \quad (4.24)$$

and

$$g_1=\left(\frac{Q_1}{E-Q_1\mathcal{H}_0Q_1+i\epsilon}\right). \quad (4.25)$$

These propagators are connected through the familiar relation

$$\mathcal{G}_1=g_1+g_1Q_1(H-\mathcal{H}_0)Q_1\mathcal{G}_1, \quad (4.26)$$

which may be regarded as a special case of the abstract relation $A^{-1}=B^{-1}+B^{-1}(B-A)A^{-1}$.

Heretofore we have only concerned ourselves with $P\mathcal{H}_0P$ and have left open the complete definition of $\mathcal{H}_0$, except for the fact that we have insisted that $P\mathcal{H}_0Q\equiv 0$. At this point we propose as a useful definition for $Q_1\mathcal{H}_0Q_1$:

$$Q_1\mathcal{H}_0Q_1=\frac{1}{2}\sum_B\int|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle(\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2}-\epsilon_B+E_A)\langle Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}|\,d\vec{k}_1\,d\vec{k}_2. \quad (4.27)$$

The energies $\epsilon_{\vec{k}}$ and ϵ_B in Eq. (4.27) we take to be the kinetic energy of the particle state and the renormalized energy of the Brueckner-Hartree-Fock hole state, respectively.

We note now that

$$\left\langle X^{(+)}_{\vec{k}',A}\left|H\frac{Q_1}{E-Q_1HQ_1+i\epsilon}H\right|X^{(+)}_{\vec{k},A}\right\rangle=\langle X^{(+)}_{\vec{k},A}|H\mathcal{G}_1H|X^{(+)}_{\vec{k},A}\rangle =\langle X^{(+)}_{\vec{k}',A}|H[g_1+g_1(H-\mathcal{H}_0)g_1+\cdots]H|X^{(+)}_{\vec{k},A}\rangle. \quad (4.28)$$

The first term in the perturbation series may be obtained from the knowledge of $\langle Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}|H|X^{(+)}_{\vec{k},A}\rangle$. To obtain the higher term in this series we need the matrix elements, $\langle Y^{(+)}_{\vec{k}'_1,\vec{k}'_2,B'}|(H-\mathcal{H}_0)|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle$. The development of this expression involves various terms, which may be classed as particle particle, particle hole, particle core, etc. Also contained are terms from the kinetic energy operator that give rise to terms involving $\epsilon_{\vec{k}_1}$ and $\epsilon_{\vec{k}_2}$, and terms that depend on the fact that $\sigma\neq 1$. We have dropped the particle-core terms arising from the potential energy (these must be renormalized in any case) and have kept the kinetic energy of the particles and the particle-particle interaction terms. The hole-core terms have been easily taken into account, as in Eq. (4.11); these need not be renormalized. The operator $Q_1\mathcal{H}_0Q_1$ of Eq. (4.27) has been defined such that there are no "self-energy" terms in the evaluation of

$$\langle Y^{(+)}_{\vec{k}'_1,\vec{k}'_2,B'}|(H-\mathcal{H}_0)|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle.$$

For simplicity we have also set the operator σ which orthonormalizes the $|\hat{Y}^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle$ states [as in Eq. (3.9)] equal to unity.

Now, combining Eqs. (4.23) and (4.27), we obtain

$$\langle Y^{(+)}_{\vec{k}'_1,\vec{k}'_2,B'}|(H-\mathcal{H}_0)|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle\cong\langle\chi^{(+)}_{\vec{k}'_1}\chi^{(+)}_{\vec{k}'_2}|v|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle_A\,\delta_{BB'}. \quad (4.29)$$

If we use the approximation in Eqs. (4.8) and (4.29), we then obtain

$$\langle\chi^{(+)}_{\vec{k}',A}|H[g_1+g_1(H-\mathcal{H}_0)g_1+g_1(H-\mathcal{H}_0)g_1(H-\mathcal{H}_0)g_1+\cdots]H|\chi^{(+)}_{\vec{k},A}\rangle$$
$$\cong-\sum_B\left\langle\chi^{(+)}_{\vec{k}'}\phi_B\left|v_{12}\left(\frac{Q_{12}}{e}\right)v_{12}-v_{12}\left(\frac{Q_{12}}{e}\right)v_{12}\left(\frac{Q_{12}}{e}\right)v_{12}+\cdots\right|\chi^{(+)}_{\vec{k}}\phi_B\right\rangle_A\rho_B$$
$$\approx\sum_B\langle\chi^{(+)}_{\vec{k}'}\phi_B|(K_{12}-v_{12})|\chi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B. \quad (4.30)$$

In this equation, Q_{12} is a two-body Pauli operator, K_{12} is a generalized Bethe-Goldstone reaction matrix $[K_{12}=v_{12}-v_{12}(Q_{12}/e)K_{12}]$, $e=h_0(1)+h_0(2)-(\epsilon_B+\epsilon_{\vec{k}})$, and h_0 is the one-body kinetic energy operator.

In this approximation we have

$$Q_{12}=\frac{1}{4}\int|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle_A\, d\vec{k}_1\, d\vec{k}_2\, {}_A\langle\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}|, \tag{4.31}$$

and

$$K_{12}(\omega)=v_{12}-v_{12}\left[\frac{Q_{12}}{Q_{12}[h_0(1)+h_0(2)]Q_{12}-(\omega+i\epsilon)}\right]K_{12}(\omega)$$

$$=v_{12}-v_{12}\int\frac{|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle\, d\vec{k}_1\, d\vec{k}_2\langle\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}|}{\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2}-(\omega+i\epsilon)}\,K_{12}(\omega)\,. \tag{4.32}$$

Equation (4.32) represents the generalization of the Bethe-Goldstone equation (with zero potential for particle states) for positive parametric energy. The modification of these equations to include the effects of the operator σ on the intermediate state propagators is discussed in the Appendix, as the notation in that case becomes somewhat more complicated.

In this approximation ($\sigma=1$), we have

$$\langle X^{(+)}_{\vec{k}',A}|H\mathcal{G}_1H|X^{(+)}_{\vec{k},A}\rangle\cong\sum_B\int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle\, d\vec{k}''\langle\chi^{(+)}_{\vec{k}''}\phi_B|K_{12}-v_{12}|\chi^{(+)}_{\vec{k}'''}\phi_B\rangle_A\,\rho_B\, d\vec{k}'''\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle\,. \tag{4.33}$$

At this point it is useful to rewrite the second term in Eq. (3.34), separating the density matrix into bound and continuum parts. With $\langle\phi_B|\rho|\phi_{B'}\rangle=\delta_{BB'}\rho_B$, we have for that term, designated as I_2,

$$I_2=\sum_B\int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle\, d\vec{k}''\langle\chi^{(+)}_{\vec{k}''}\phi_B|v|\chi^{(+)}_{\vec{k}'''}\phi_B\rangle_A\,\rho_B\, d\vec{k}'''\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle$$

$$+\int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle\, d\vec{k}''\langle\chi^{(+)}_{\vec{k}''}\chi^{(+)}_{\vec{k}_1}|v|\chi^{(+)}_{\vec{k}'''}\chi^{(+)}_{\vec{k}_2}\rangle_A\langle\chi^{(+)}_{\vec{k}_2}|\rho|\chi^{(+)}_{\vec{k}_1}\rangle\, d\vec{k}'''\, d\vec{k}_1\, d\vec{k}_2\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle\,. \tag{4.34}$$

Now we may add Eq. (4.33) to Eq. (4.34). This has the effect of replacing the potential term in the first part of I_2 by the matrix elements of the reaction matrix K_{12}. To accomplish the same replacement of v by K in the second term of Eq. (4.34) we have to consider a selected class of terms arising in the development of $\langle X^{(+)}_{\vec{k}',A}|H\mathcal{G}_2H|X^{(+)}_{\vec{k},A}\rangle$, where

$$\mathcal{G}_2=\left(\frac{Q_2}{E-Q_2HQ_2+i\epsilon}\right). \tag{4.35}$$

To keep this discussion from becoming unreasonably long, we have indicated some of these renormalizations in the diagrammatic representation of Fig. 4.

Once both terms of I_2 have been renormalized, such that K appears instead of v, we may write Eq. (3.22) as

$$\langle X^{(+)}_{\vec{k}',A}|V_{\text{eff}}|X^{(+)}_{\vec{k},A}\rangle\cong(\epsilon_{\vec{k}'}-\epsilon_{\vec{k}})\langle\chi^{(+)}_{\vec{k}'}|(\gamma_p-1)|\chi^{(+)}_{\vec{k}}\rangle$$

$$+\sum_{\alpha\beta}\int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle\, d\vec{k}''\,[\langle\chi^{(+)}_{\vec{k}''}\alpha|K|\chi^{(+)}_{\vec{k}'''}\beta\rangle_A\langle\beta|\rho|\alpha\rangle]\, d\vec{k}'''\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle$$

$$+\frac{1}{2}\sum_{B_1B_2}\int\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}''}\rangle_A\,\langle\phi_{B_1}\phi_{B_2}|K|\chi^{(+)}_{\vec{k}'''}\chi^{(+)}_{\vec{k}_1}\rangle\left(\frac{d\vec{k}_1}{e}\right)$$

$$\times d\vec{k}''\, d\vec{k}'''\langle\chi^{(+)}_{\vec{k}''}\chi^{(+)}_{\vec{k}_1}|K|\phi_{B_1}\phi_{B_2}\rangle_A\,\rho_{B_1}\rho_{B_2}\langle\chi^{(+)}_{\vec{k}'''}|\gamma|\chi^{(+)}_{\vec{k}}\rangle+\cdots\,. \tag{4.36}$$

Some of these terms[10] of V_{eff} are shown diagrammatically in Fig. 5. We note that the quantity appearing in square brackets in Eq. (4.36) should be considered as a single entity despite the factorized notation.[11] A correlation diagram which we have not discussed but which has received a good deal of attention[12] is indicated in Fig. 6. The inclusion of diagrams of this type in our work would involve a detailed study of the role of the Q_2 space.

It is not difficult to see that at high energies Eq. (4.36) will go over to the impulse-approximation result.

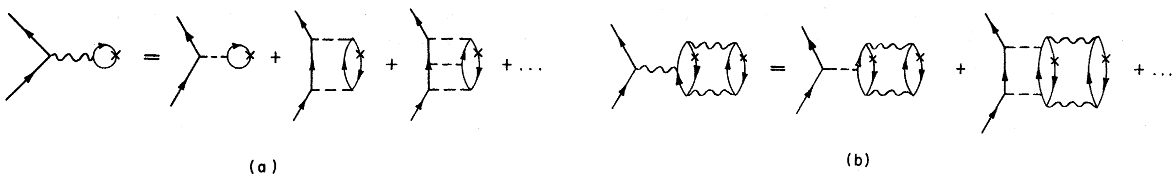

FIG. 4. (a) A diagrammatic representation of the series of interactions which are summed to yield a reaction matrix (wavy line). Only direct terms are shown. The cross on the hole line indicates the inclusion of the occupation probability of this orbit. Except for the leading diagrams, this series arises from the inclusion of the many-body states of the Q_1 space in the theory. (b) A series of diagrams which may be summed to renormalize the potential interaction involving continuum portions of the ground-state density matrix. The wavy line represents a reaction matrix and the dashed line represents the potential matrix elements. Note that the intermediate states in all but the leading diagram of this series arise from the inclusion of the states of the Q_2 space in the theory.

For sufficiently high momenta we can replace γ by unity and $|\chi_{\vec{k}}^{(+)}\rangle$ by $|\vec{k}\rangle$, a plane wave. Also we may write

$$K_{12}=K_{12}^{\text{free}}-K_{12}^{\text{free}}\left[\frac{Q_{12}}{e}-\frac{1}{e}\right]K_{12}, \tag{4.37}$$

where

$$K_{12}^{\text{free}}=v_{12}-v_{12}\frac{1}{e}\,K_{12}^{\text{free}} \tag{4.38}$$

is the free nucleon-nucleon scattering matrix. At high energies $K_{12}\to K_{12}^{\text{free}}$, and if we further neglect the binding energies of the struck particles we may evaluate K_{12}^{free} at the energy of the incident particle, $\epsilon_{\vec{k}}$. These approximations yield the familiar result for the high-energy optical potential:

$$\langle X_{\vec{k}',A}^{(+)}|V_{\text{eff}}|X_{\vec{k},A}^{(+)}\rangle\to\langle\vec{k}',\Phi_A|V_{\text{eff}}|\vec{k},\Phi_A\rangle\cong\sum_{\alpha\beta}\langle\vec{k}',\alpha|K^{\text{free}}(\epsilon_{\vec{k}})|\vec{k},\beta\rangle_A\langle\beta|\rho|\alpha\rangle, \tag{4.39}$$

where $|\vec{k},\Phi_A\rangle\equiv a_{\vec{k}}^{\dagger}|\Phi_A\rangle$. If one further neglects the momentum of the struck particle, one has

$$\langle\vec{k}',\Phi_A|V_{\text{eff}}|\vec{k},\Phi_A\rangle\to K^{\text{free}}(\epsilon_{\vec{k}},\vec{q})\,\rho(\vec{q}), \tag{4.40}$$

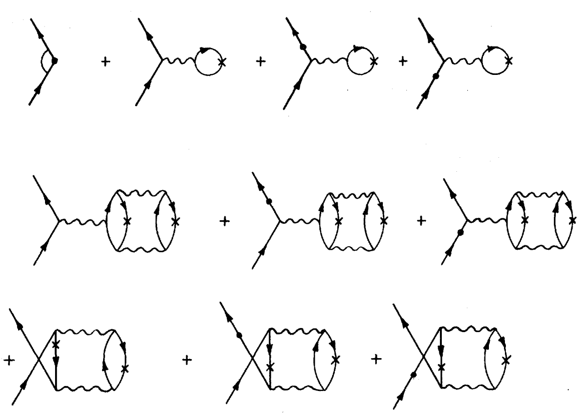

FIG. 5. A diagrammatic representation of some of the (direct) terms of Eq. (4.36). The diagrammatic elements have been defined in Fig. 1. Terms having two black dots are not shown. Again the wavy line represents the reaction matrix, up-going lines refer to orbits $|\chi_{\vec{k}}^{(+)}\rangle$, and down-going lines to orbits $|\phi_b\rangle$. Crosses on the down-going lines indicate that the occupation factors ρ_B should be associated with these lines.

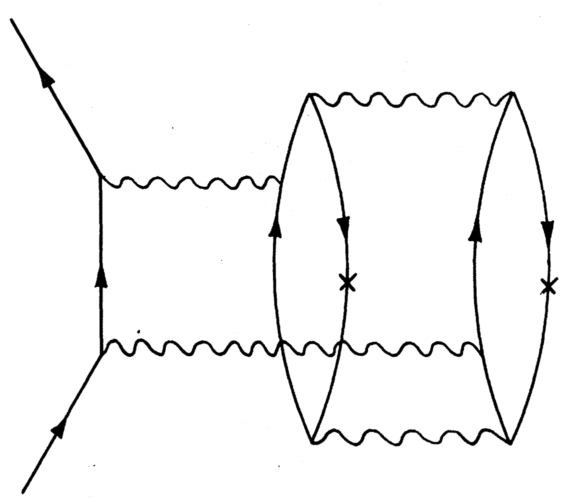

FIG. 6. This diagram has received much attention as a multiple-scattering term which depends on the correlation structure of the target nucleus (Ref. 12). This diagram has not been discussed in this paper because it represents a higher-order term in a systematic hole line expansion. In the language of this paper, this diagram represents a matrix element which involves intermediate states which are in the Q_2 space.

where $\vec{q}$ is the momentum transfer and $\rho(\vec{q})$ is the Fourier transform of the density $\rho(\vec{r})$, normalized such that

$$\int \rho(\vec{r})\, d\vec{r} = A\,, \tag{4.41}$$

with A being the number of target particles.

In Eqs. (4.40) and (4.41) we have neglected the last term in Eq. (4.36) since at very high incident energy the correlated particles of the target are not of sufficiently high momentum so as to undergo exchange with the incident particle.[10]

Recalling Eq. (2.11), we have in the approximation $\gamma_p = 1$ and with the neglect of the last term in Eq. (4.36),

$$\langle \chi_{\vec{k}'}^{(+)} | v_{\text{opt}} | \chi_{\vec{k}}^{(+)} \rangle = \sum_{\alpha\beta} \langle \chi_{\vec{k}'}^{(+)} \alpha | K | \chi_{\vec{k}}^{(+)} \beta \rangle_A \langle \beta | \rho | \alpha \rangle\,. \tag{4.42}$$

The equation for the optical wave function, as discussed in Sec. II is,

$$(\epsilon_{\vec{k}} - p h_0 p - p v_{\text{opt}} p) | \psi_{\vec{k}}^{(+)} \rangle = 0\,, \tag{4.43}$$

or

$$| \psi_{\vec{k}}^{(+)} \rangle = | \chi_{\vec{k}}^{(+)} \rangle + \int \frac{| \chi_{\vec{k}'}^{(+)} \rangle\, d\vec{k}' \langle \chi_{\vec{k}'}^{(+)} | v_{\text{opt}} | \psi_{\vec{k}}^{(+)} \rangle}{\epsilon_{\vec{k}} - \epsilon_{\vec{k}'} + i\epsilon}\,. \tag{4.44}$$

This equation may be compared to the one which obtains if we put $p = 1$ in Eq. (4.43),

$$(\epsilon_{\vec{k}} - h_0 - v_{\text{opt}}) | \tilde{\psi}_{\vec{k}}^{(+)} \rangle = 0\,, \tag{4.45}$$

or

$$| \tilde{\psi}_{\vec{k}}^{(+)} \rangle = | \vec{k} \rangle + \int \frac{| \vec{k}' \rangle\, d\vec{k}' \langle \vec{k}' | v_{\text{opt}} | \tilde{\psi}_{\vec{k}}^{(+)} \rangle}{\epsilon_{\vec{k}} - \epsilon_{\vec{k}'} + i\epsilon}\,. \tag{4.46}$$

The matrix elements of v_{opt} in the plane-wave representation are given by the analog of Eq. (4.42),

$$\langle \vec{k}' | v_{\text{opt}} | \vec{k} \rangle = \sum_{\alpha\beta} \langle \vec{k}' \alpha | K | \vec{k} \beta \rangle_A \langle \beta | \rho | \alpha \rangle\,. \tag{4.47}$$

Now one can ask under what circumstances will $| \tilde{\psi}_{\vec{k}}^{(+)} \rangle$ be a good approximation to $| \psi_{\vec{k}}^{(+)} \rangle$. Clearly, at high energies, where the distinction between $| \chi_{\vec{k}}^{(+)} \rangle$ and $| \vec{k} \rangle$ becomes unimportant, Eq. (4.44) may be replaced by Eq. (4.46). At low energies the validity of this approximation is related to the degree to which v_{opt} is energy dependent through the energy dependence of the reaction matrix K. We recall that the solutions of Eqs. (4.44) and (4.46) are identical if K is energy independent (as in the case of Hartree-Fock theory where we can replace K_{12} by v_{12}). Some numerical investigation is clearly called for to understand the importance of using the more correct Eq. (4.44), rather than the approximate form, Eq. (4.46).

In this section we have seen how the potential terms involving v may be renormalized; however, we have not discussed the role of compound-nucleus formation or virtual excitation of low-lying modes of the target. These (dispersive) effects (which are most important at low energy) will not be discussed at this time.

APPENDIX

In Eq. (4.31) we defined the Pauli operator,

$$Q_{12} \equiv \frac{1}{4} \int | \chi_{\vec{k}_1}^{(+)} \chi_{\vec{k}_2}^{(+)} \rangle_A\, d\vec{k}_1\, d\vec{k}_2\, {}_A\langle \chi_{\vec{k}_1}^{(+)} \chi_{\vec{k}_2}^{(+)} | = \int | \chi_{\vec{k}_1}^{(+)} \chi_{\vec{k}_2}^{(+)} \rangle\, d\vec{k}_1\, d\vec{k}_2 \langle \chi_{\vec{k}_1}^{(+)} \chi_{\vec{k}_2}^{(+)} |\,, \tag{A1}$$

which is appropriate for the Bethe-Goldstone equation for finite systems, if one neglects the potential in the unoccupied states, but maintains the requirement that the particle (unoccupied) states be orthogonal to the hole (occupied) states. The operator given above is the result of approximating the orthogonalization operator σ by unity. We recall that the matrix elements of σ were of the form $\langle \chi_{\vec{k}_1}^{(+)} \chi_{\vec{k}_2}^{(+)} | \sigma_{BB'} | \chi_{\vec{k}_3}^{(+)} \chi_{\vec{k}_4}^{(+)} \rangle$, where the subscripts B, B' indicate that σ is a matrix in the space of bound-state orbitals. It is expected that, in lowest order, σ does not depend on the bound-state labels.

In that approximation, we may generalize Q_{12} to

$$\tilde{Q}_{12} = \int | \chi_{\vec{k}_1'}^{(+)} \chi_{\vec{k}_2'}^{(+)} \rangle\, d\vec{k}_1'\, d\vec{k}_2' \langle \chi_{\vec{k}_1'}^{(+)} \chi_{\vec{k}_2'}^{(+)} | \sigma | \chi_{\vec{k}_1}^{(+)} \chi_{\vec{k}_2}^{(+)} \rangle\, d\vec{k}_1\, d\vec{k}_2 \langle \chi_{\vec{k}_1}^{(+)} \chi_{\vec{k}_2}^{(+)} | \sigma | \chi_{\vec{k}_1''}^{(+)} \chi_{\vec{k}_2''}^{(+)} \rangle\, d\vec{k}_1''\, d\vec{k}_2'' \langle \chi_{\vec{k}_1''}^{(+)} \chi_{\vec{k}_2''}^{(+)} |\,. \tag{A2}$$

To simplify the notation, $\tilde{Q}_{12}$ may be written as

$$\tilde{Q}_{12}=\sigma Q_{12}\sigma\,. \tag{A3}$$

We can introduce an operator $K_{12}=v_{12}-v_{12}(\tilde{Q}_{12}/e)\tilde{K}_{12}$, which satisfies

$$\begin{aligned}\tilde{K}_{12}&=K_{12}-K_{12}\left[\frac{\tilde{Q}_{12}}{e}-\frac{Q_{12}}{e}\right]\tilde{K}_{12}\\ &\approx K_{12}-K_{12}\left[\frac{\tilde{Q}_{12}}{e}-\frac{Q_{12}}{e}\right]K_{12}+\cdots\,.\end{aligned} \tag{A4}$$

While the effects of the operator σ are quite small, it is probably of some interest to indicate the structure of this operator. We may define

$$\langle \hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',B'}|\hat{Y}^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle=\delta_{BB'}[\delta(\vec{k}_1'-\vec{k}_1)\delta(\vec{k}_2-\vec{k}_2')-\delta(\vec{k}_1-\vec{k}_2')\,\delta(\vec{k}_2-\vec{k}_1')]-\langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\Delta_{B'B}|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle\,. \tag{A5}$$

Using the definition of the states $|\hat{Y}^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle$ and keeping the leading contractions, we find

$$\begin{aligned}\langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\Delta_{B'B}|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle_A\simeq\frac{\delta_{BB'}}{\rho_B}\{&\delta(\vec{k}_1-\vec{k}_1')\langle\chi^{(+)}_{\vec{k}_2'}|\rho|\chi^{(+)}_{\vec{k}_2}\rangle-\delta(\vec{k}_1'-\vec{k}_2)\langle\chi^{(+)}_{\vec{k}_2'}|\rho|\chi^{(+)}_{\vec{k}_1}\rangle\\ &+\delta(\vec{k}_2-\vec{k}_2')\langle\chi^{(+)}_{\vec{k}_1'}|\rho|\chi^{(+)}_{\vec{k}_1}\rangle-\delta(\vec{k}_2'-\vec{k}_1)\langle\chi^{(+)}_{\vec{k}_1'}|\rho|\chi^{(+)}_{\vec{k}_2}\rangle\}+\cdots\,.\end{aligned} \tag{A6}$$

Equation (A6) may be written as

$$\Delta\simeq[\rho(1)+\rho(2)]\,, \tag{A7}$$

where we have put $\rho_B\simeq 1$. Equation (A6) may be obtained from Eq. (A7) if one takes the indicated matrix elements.

We recall that the states $|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle$ have been defined such that

$$\begin{aligned}\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B'}|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle&=\delta_{B'B}[\delta(\vec{k}_1'-\vec{k}_1)\delta(\vec{k}_2'-\vec{k}_2)-\delta(\vec{k}_1'-\vec{k}_2)\delta(\vec{k}_2'-\vec{k}_1)]\\ &=\int\sum_{B''B'''}\langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\sigma_{B'B''}|\chi^{(+)}_{\vec{k}_1''}\chi^{(+)}_{\vec{k}_2''}\rangle\,d\vec{k}_1''\,d\vec{k}_2''\langle\hat{Y}^{(+)}_{\vec{k}_1'',\vec{k}_2'',B''}|\hat{Y}^{(+)}_{\vec{k}_1''',\vec{k}_2''',B'''}\rangle\\ &\qquad\times\,d\vec{k}_1'''\,d\vec{k}_2'''\langle\chi^{(+)}_{\vec{k}_1'''}\chi^{(+)}_{\vec{k}_2'''}|\sigma_{B'''B}|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle\,.\end{aligned} \tag{A8}$$

In the approximation given by Eqs. (A5) and (A6) we may then write, in schematic form,

$$\sigma(1-\Delta)\sigma=1\,, \tag{A9}$$

so that

$$\sigma\approx(1+\Delta/2)\simeq 1+\tfrac{1}{2}[\rho(1)+\rho(2)]\,, \tag{A10}$$

or

$$\sigma\approx 1+[\gamma_p(1)-p(1)]+[\gamma_p(2)-p(2)]\,, \tag{A11}$$

where the operator $\gamma_p\cong p+\frac{1}{2}\rho_p$ was introduced previously as a modification of the propagator in the P space.

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*Work supported in part by the National Science Foundation.

[1]R. R. Scheerbaum, C. M. Shakin, and R. M. Thaler, "Scattering from Correlated Systems" to be published in Ann. Phys.

[2]Equation (2.24) is more correctly given as

$$\langle X^{(+)}_{\vec{r}',A}|X^{(+)}_{\vec{r},A}\rangle=\delta(\vec{r}-\vec{r}')-\sum_{b(\eta=1)}\langle\vec{r}'|\phi_b\rangle\langle\phi_b|\vec{r}\rangle\,,$$

where the sum is over those bound states which are *completely* occupied (that is, the eigenvalue of the density matrix, η, is equal to unity for these states).

[3]Our results may also be used at low energies as well; however, in that case our theory does not contain the effects on the optical model of compound-nucleus formation and inelastic scattering. In the present approximation the imaginary part of the optical model arises from particle knockout, for which the threshold would be ~8 MeV.

[4]Note that these states are defined so as to satisfy an orthornormality condition of the form

$$\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B'}|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle = \delta(\vec{k}_1-\vec{k}_1')\delta(\vec{k}_2-\vec{k}_2') - \delta(\vec{k}_1-\vec{k}_2')\delta(\vec{k}_2-\vec{k}_1') .$$

This normalization accounts for the factor of $\frac{1}{2}$ in Eq. (3.10).

[5]It may serve to clarify the nature of the truncation if we examine

$$H^{(2)}_{\rm eff} = H^{(1)}_{\rm eff} + H^{(1)}_{\rm eff}Q_2 \frac{1}{E-Q_2H^{(1)}_{\rm eff}Q_2+i\epsilon} Q_2H^{(1)}_{\rm eff} ,$$

with

$$H^{(1)}_{\rm eff} = H + HQ_1\frac{1}{E-Q_1HQ_1+i\epsilon}Q_1H .$$

The propagator $(E-Q_2H^{(1)}_{\rm eff}Q_2)^{-1}$ may be expanded as

$$\frac{1}{E-Q_2H^{(1)}_{\rm eff}Q_2} = \frac{1}{E-Q_2HQ_2} + \frac{1}{E-Q_2HQ_2}Q_2HQ_1 \times \frac{1}{E-Q_1HQ_1}Q_1HQ_2\frac{1}{E-Q_2HQ_2}Q_2+\cdots .$$

The lowest-order terms of $H^{(2)}_{\rm eff}$ are then

$$H^{(2)}_{\rm eff} = H + H\frac{Q_1}{e_1}H + H\frac{Q_2}{e_2}H + H\frac{Q_2}{e_2}H\frac{Q_1}{e_1}H + H\frac{Q_1}{e_1}H\frac{Q_2}{e_2}H + H\frac{Q_1}{e_1}H\frac{Q_2}{e_2}H\frac{Q_1}{e_1}H + H\frac{Q_2}{e_2}H\frac{Q_1}{e_1}H\frac{Q_2}{e_2}H + \cdots ,$$

where

$$\frac{Q_i}{e_i} = Q_i\frac{1}{E-Q_iHQ_i+i\epsilon}Q_i .$$

The main virtue of this scheme is thus apparent. The propagators never link the orthogonal subspaces.

[6]With the assumption $\langle\chi^{(+)}_{\vec{k}}|\rho|\phi_b\rangle = 0$, discussed above, we have

$$\rho = \sum_{b,b'}|\phi_b\rangle\langle\phi_b|\rho|\phi_{b'}\rangle\langle\phi_{b'}| + \int|\chi^{(+)}_{\vec{k}}\rangle d\vec{k}\,\langle\chi^{(+)}_{\vec{k}}|\rho|\chi^{(+)}_{\vec{k}'}\rangle d\vec{k}'\,\langle\chi^{(+)}_{\vec{k}'}| .$$

This relation may be used to reexpress the matrix element given in Eq. (3.33) in terms of the bound state and continuum parts of the density matrix.

[7]See for example, J. da Providência and C. M. Shakin, Phys. Rev. C 4, 1560 (1971); 5, 53 (1972).

[8]The $|\phi_b\rangle$ may be chosen to accomplish this diagonalization so that

$$\langle\phi_{B_1}|\rho|\phi_{B_2}\rangle = \delta_{B_1B_2}\rho_{B_1} .$$

[9]Here ϵ_B is the Brueckner-Hartree-Fock (renormalized) single-particle energy,

$$\epsilon_B = \langle\phi_B|h_0|\phi_B\rangle + \sum_{B'}\langle\phi_B\,\phi_{B'}|K_{12}|\phi_B\,\phi_{B'}\rangle_A\rho_{B'} .$$

See for example, C. M. Shakin and J. da Providência, Phys. Rev. Letters 27, 1069 (1972). Note that there is an error in sign in the theorem as stated in this reference. A consistent set of definitions leads to

$$\langle\Phi_A|\eta^\dagger_{B'}[H,\eta_B]|\Phi_A\rangle = -\epsilon_B\rho_B\delta_{BB'} .$$

[10]The second term of Eq. (4.36), which appears expanded in the third line of Fig. 5, is usually neglected, following G. Takeda and K. M. Watson, Phys. Rev. 97, 1336 (1955). Estimates of this term have been made by R. R. Scheerbaum and will be reported elsewhere.

[11]The full development of the second term in Eq. (4.36) involves a careful study of the energy dependence of the K matrix elements. If the α and β in this term refer to a bound state, $|\phi_B\rangle$, the energy variable in the K matrix is $\epsilon_{\vec{k}} - |\epsilon_B|$. However, in the case that α and β refer to continuum orbits, the off-shell property is more complicated. See, for example, Fig. 4(b). The bracketed expression must be considered as a shorthand notation. The off-shell character of the K matrix appearing in this equation will be discussed more fully in a future publication.

[12]A. K. Kerman, H. McManus, R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959); E. Kujawski, Phys. Rev. C 1, 1651 (1970); H. Feshbach and J. Hüfner, Ann. Phys. (N.Y.) 56, 268 (1970); H. Feshbach, A. Gal, and J. Hüfner, Ann. Phys. (N.Y.) 66, 20 (1971); E. Lambert and H. Feshbach, Phys. Letters 38B, 487 (1972), and E. Lambert and H. Feshbach, Ann. Phys. (to be published).